CSE-551 Homework-2

ASU ID: 1218506822

(1) Given :

Soln

$F(n) = F(n-1) + F(n-2) + F(n-3) + F(n-4)$ — (1)

$F(0) = 0, \quad F(1) = 1, \quad F(2) = 1, \quad F(3) = 1$ — (2)

for $F(n) = F(n-1) + F(n-2) \Rightarrow [\ F(n-1) \quad F(n)\ ]$

Similarly for $F(n) = F(n-1) + F(n-2) + F(n-3) + F(n-4)$

We have $[\ F(n-3) \quad F(n-2) \quad F(n-1) \quad F(n)\ ]$

Replace $F(n)$ with equation (1)

$[\ F(n-3) \quad F(n-2) \quad F(n-1) \quad F(n-1) + F(n-2) + F(n-3) + F(n-4)\ ]$

$[\ F(n-4) \quad F(n-3) \quad F(n-2) \quad F(n-1)\ ] \times A.$

Where

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$\Rightarrow [\ F(n-5) \quad F(n-4) \quad F(n-3) \quad f(n-2)\ ] \times A \times A.$

$[\ F(n-6) \quad F(n-5) \quad F(n-4) \quad F(n-3)\ ] \times A^3$

$[\ F(0) \quad F(1) \quad F(2) \quad F(3)\ ] \times A^{n-3}$ —

$\Rightarrow [\ 0 \quad 1 \quad 1 \quad 1\ ] \times A^{n-3}$ — from (2)

The matrix A is constant, just as A^{n-1}, the matrix Multiplication A^{n-3} can be computed in $O(\log n)$ complexity

(2) Given $T(n) = 7T(n/2) + n^2$

Using Master's Theorm

$T(n) = aT(n/b) + f(n)$

Where $f(n) = \Theta(n^k \log^p n)$.

Case (1) $\log_b a > k \Rightarrow \log_2 7 > 2 \Leftrightarrow 2.80 > 2$

then $\therefore O(n^{\log_2 7})$

Where $T'(n) = aT'(n/4) + n^2$ should be faster than $T(n)$.

$$O(n^{\log_4 a}) < O(n^{\log_2 7})$$

$$\log_4 a < \log_2 7$$

$$\frac{\log a}{\log 4} < \frac{2 \times \log 7}{2 \times \log 2}$$

$$\frac{\log a}{\log 4} < \frac{\log 49}{\log 4}$$

$$\log_4 a < \log_4 49$$

$$a < 49$$

$\therefore$ The largest value of a is 48. such that A' is asymptotically faster than A.

There won't be a stable matching, where a good man is not married to good woman.

3) let's prove it by Example:-

Consider $n=2$, i.e; 2 men & 2 women.
let $k=1$, $n-k = 2-1 = 1$, i.e; 1 good man, 1 good women.
(gm) (gw).

Each man, woman has ranking from good to bad.

gm	gw	bw
bm	gw	bw

Men's Preference list

gw	gm	bm
bw	gm	bm

Women's prefrence list.

When gm = goodman, gw = good woman
bm = badman, bw = badwoman.

We have stable Matching {(gm, gw), (bm, bw)}

If gm is assigned to bw then gw is paired with bm

but The first preference of gm is gw & gw is gm
Hence they pair will elope, and there won't be any stability.

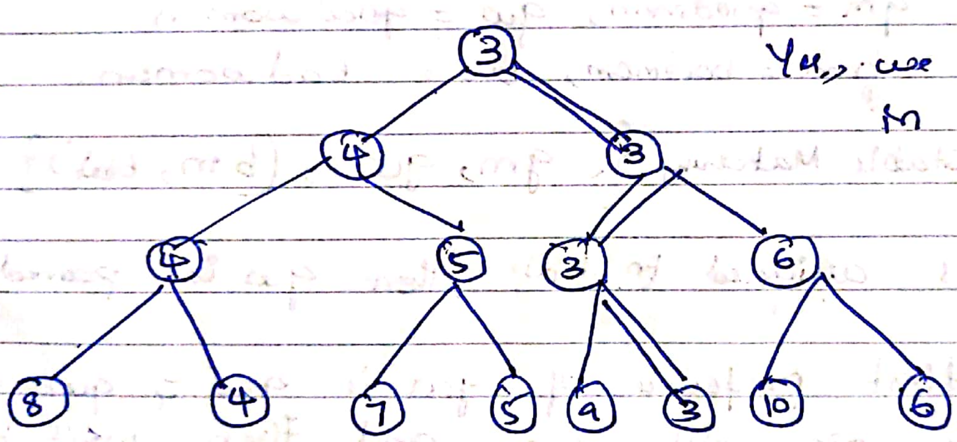
In general, let's consider a stablematching where one of goodman (namely gm) is already married to bad woman (bw) and K-1 are the remaining good men. Even If all of them are married to a good women, there would be still some good women who is married to badman (bm).

Let 'gm' be such good woman. Who is married to bad man (bm). Now we can find the instability in this kind of matching. If we consider the pairs (gw, bm), (gm, bw) where each of them is good & married to bad partner. Thus, each of gm, gw prefers to other to their current partner. Hence (gw, bm) (gm, bw) is an instability.

Hence, There cannot be a stable matching where a good man is not married to a good woman.

(4) For solving this problem let's consider a Tournament tree.

Sol:



Yes, we can solve it in $n + \log_2 \lceil n \rceil - 2$ comparison.

The first smallest of all the numbers can be found it $(n-1)$ comparisons. $= (8-1) = 7$ comparison.

The second smallest can be found by analysing the comparisons with first smallest which is $\lceil \log_2 n \rceil - 1$ comparisons, where $\log n$ is the height of the tree.

Hence the Total Comparisons is $(n-1) + (\lceil \log n \rceil - 1) = n + \lceil \log_2 n \rceil - 2$ Comparisons

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(5) Given $F_0 = 0$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2} \; \forall n \geq 2$

$F_n = F_{n-1} + F_{n-2} \Rightarrow$ difference Equation.

$F_n = r^n$, $F_{n-1} = r^{n-1}$, $F_{n-2} = r^{n-2}$

$$r^n = r^{n-1} + r^{n-2}$$

dividing by r^n:

$$1 = \frac{1}{r} + \frac{1}{r^2}$$

$$r^2 - r - 1 = 0 \quad \text{— (5)}$$

When $r = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$

$$= \frac{+1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$\phi_1 = \dfrac{1+\sqrt{5}}{2}$ $\phi'_2 = \dfrac{1-\sqrt{5}}{2}$ let ϕ, ϕ' be roots of eq (5)

$F_n = c_1 \phi_1^n + c_2 \phi_2^n$ ($\because$ linear Combination of solution)

$F_0 = 0$ for $n = 0$, $0 = c_1 \phi^0 + c_2 \phi'^0$ — (4)

$$\therefore \boxed{c_1 = -c_2}$$

$F_1 = 1$, for $n = 1$ $c_1\left(\dfrac{1+\sqrt{5}}{2}\right) + c_2\left(\dfrac{1-\sqrt{5}}{2}\right) = 1$

$\therefore c$ $-c_2\left(\dfrac{1+\sqrt{5}}{2}\right) + c_2\left(\dfrac{1-\sqrt{5}}{2}\right) = 1$ — (1)

$\boxed{\therefore \cancel{As}\; c_1 = -c_2}$ substituting in eq (1)

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$$c_2\left(\frac{\cancel{1}-\sqrt{5}\,\cancel{-1}-\sqrt{5}}{2}\right) = 1$$

$$c_2\left(-\frac{\cancel{2}\sqrt{5}}{\cancel{2}}\right) = 1$$

$$\boxed{c_2 = -\frac{1}{\sqrt{5}}} \quad \text{—— (2)}$$

$$c_1 = -c_2 = \frac{1}{\sqrt{5}} \quad \therefore \boxed{c_1 = \frac{1}{\sqrt{5}}} \quad \text{—— (3)}$$

$$F_n = c_1\phi_1^n + c_2\hat{\phi}_2^n$$

On substituting $F_n = \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$

$$\therefore F_n = \frac{1}{\sqrt{5}}(\phi^n) - \frac{1}{\sqrt{5}}(\hat{\phi})^n$$

$$F_n = \frac{(\phi^n - \hat{\phi}^n)}{\sqrt{5}} \quad \text{(Hence proved)}$$