CSE 551 Homework 2

March 1, 2020

Submission Instructions: Deadline is 11:59pm on 03/13. Late submissions will be penalized, therefore please ensure that you submit (file upload is **completed**) before the deadline. Additionally, you can download the submitted file to verify if the file was uploaded correctly. Submit your answers electronically, in a **single PDF**, via canvas. You can type up the answers or scan (or take pictures) your handwritten answers.

Furthermore, please note that the graders will grade 3 out of the **5 questions** randomly. Therefore, if the grader decides to check questions 1, 2 and 3 and you haven't answered question 3, you'll lose points for question 3. Hence, please answer all the questions.

1. The Fibonacci series can be computed as follows,

$$F(n) = F(n-1) + F(n-2) \tag{1}$$

   In class, we showed how this can be done in $\mathcal{O}(log\ n)$ computation time. Now suppose that the definition is changed in the following way,

$$F'(n) = F'(n-1) + F'(n-2) + F'(n-3) + F'(n-4) \tag{2}$$

   Can $F'(n)$ be computed in $\mathcal{O}(log\ n)$? If yes, please show how it can be done. If no, show a counterexample where this fails. Please provide your rationale for both. Assume that $F'(0) = 0, F'(1) = 1, F'(2) = 1, F'(3) = 1$.

2. The recurrence relation $T(n) = 7T(n/2) + n^2$ describes the running time for an algorithm A. A competing algorithm A' has a running time of $T'(n) = aT'(n/4) + n^2$. What is the largest integer value for a, such that A' is asymptotically faster than A?

3. Consider a town with n men and n women seeking to get married to one another. Each man has a preference list that ranks all the women, and each woman has a preference list that ranks all the men.

   The set of all $2n$ people is divided into two categories: *good* people and *bad* people. Suppose that for some number k, $1 \leq k \leq n-1$, there are k

good men and k good women, thus there are $n-k$ bad men and $n-k$ bad women.

Given the preference list of men and women, is there a stable matching where a good man is not married to a good woman?

4. Can the second smallest of n elements be found with $n + \lceil log_2\ n \rceil - 2$ number of comparisons? If your answer is yes, show how it can be done. If your answer is no, explain why it cannot be done.

5. The Fibonacci numbers are defined by the following recurrence: $F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}, \forall n \geq 2$. Fibonacci numbers are related to the golden ratio Φ and to its conjugate, and are given by the following formulae:

   $\Phi = (1+\sqrt{5})/2$

   $\hat{\Phi} = (1-\sqrt{5})/2$

   Show that, $F_n = (\Phi^n - \hat{\Phi}^n)/\sqrt{5}$.