Assignment - 3

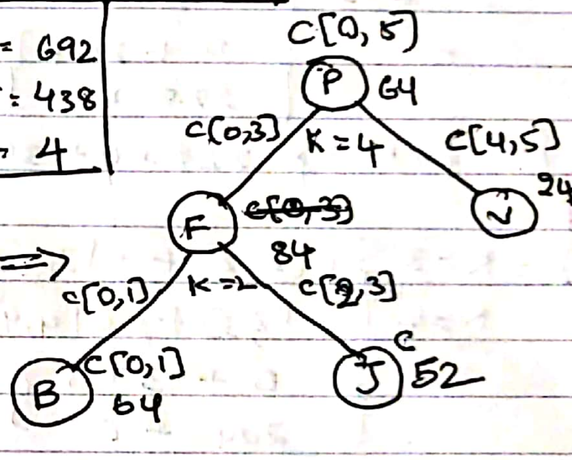
(1)

		p_1	p_2	p_3	p_4	p_5
p		64	84	52	64	24
q	0	68	76	116	190	42
	q_0	q_1	q_2	q_3	q_4	q_5

Sol:-

$j-i=0$	$C_{00}=0$ $W_{00}=0$ $r_{00}=0$	$C_{11}=0$ $W_{11}=68$ $r_{11}=0$	$C_{22}=0$ $W_{22}=76$ $r_{22}=0$	$C_{33}=0$ $W_{33}=116$ $r_{33}=0$	$C_{44}=0$ $W_{44}=190$ $r_{44}=0$	$C_{55}=0$ $W_{55}=42$ $r_{55}=0$
$j-i=1$	$C_{01}=132$ $W_{01}=132$ $r_{01}=1$	$C_{12}=228$ $W_{12}=228$ $r_{12}=2$	$C_{23}=244$ $W_{23}=244$ $r_{23}=3$	$C_{34}=370$ $W_{34}=370$ $r_{34}=4$	$C_{45}=256$ $W_{45}=256$ $r_{45}=5$	
$j-i=2$	$C_{02}=424$ $W_{02}=292$ $r_{02}=2$	$C_{13}=624$ $W_{13}=396$ $r_{13}=3$	$C_{24}=742$ $W_{24}=498$ $r_{24}=4$	$C_{35}=692$ $W_{35}=438$ $r_{35}=4$		
$j-i=3$	$C_{03}=836$ $W_{03}=460$ $r_{03}=2$	$C_{14}=1248$ $W_{14}=650$ $r_{14}=3$	$C_{25}=1064$ $W_{25}=564$ $r_{25}=4$			
$j-i=4$	$C_{04}=1508$ $W_{04}=714$ $r_{04}=3$	$C_{15}=1596$ $W_{15}=716$ $r_{15}=4$				
$j-i=5$	$C_{05}=1872$ $W_{05}=780$ $r_{05}=4$					

$\Rightarrow$



llowing.

$$C[i,j] = \min_{i<k\le j} \{ C[i, k-1] + C[k, j] \} + W[i,j]$$

$$W[i,j] = W[i, j-1] + P_j + q_j$$

$$W[i,i] = q_i$$

$c[0,1] = k=1 \{ c[0,0] + c[1,1] \} + w[0,1]$

$0 + 0 + 132 = 132$

Similarly

$c[1,2] = 0 + 0 + w[1,2] = 228.$

$c[3,4] = 0 + 0 + w[3,4] = 370$

$c[4,5] = 0 + 0 + w[4,5] = 256.$

$c[0,2] = \min \begin{cases} k=1 & c[0,0] + c[1,2] \\ k=2 & c[0,1] + c[2,2] \end{cases} + w[0,2]$

$\therefore \min \begin{cases} 0 + 228 \\ 132 + 0 \end{cases} + 292$

Min for $k=2$, $132 + 292 = 424.$

$c[1,3] = \min \begin{cases} k=2 & c[1,1] + c[2,3] \\ k=3 & c[1,2] + c[3,3] \end{cases} + w[1,3]$

$= \begin{cases} 0 + 244 \\ 228 + 0 \end{cases} + 396$

Min for $k=3 \{ 228 + 0 + 396 \} = 624.$

$c[2,4] = \min \begin{cases} k=3 & c[2,2] + c[3,4] \\ k=4 & c[3,3] + c[4,4] \end{cases} + w[2,4]$

$\begin{cases} 0 + 370 \\ 244 + 0 \end{cases} + 498$

Min for $k=4 \{ 244 + 0 + 498 \} = 742.$

$c[3,5] = \begin{cases} k=4 & c[3,3] + c[4,5] \\ k=5 & c[3,4] + c[5,5] \end{cases} + w[3,5]$

Min for $k=4$ $\begin{cases} 0 + 256 \\ 370 + 0 \end{cases} + 436$

Min for $k=4$ = $370 + 0 + 436 = 692$

$c[0,3] = \begin{cases} k=1 & c[0,0] + c[1,3] \\ k=2 & c[0,1] + c[2,3] \\ k=3 & c[0,2] + c[3,3] \end{cases} + w[0,3]$

Min for $k=2 \{ 132 + 244 + 460 \}$ 836//

$c[2,5] = \begin{cases} k=3 & c[2,2] + c(3,5) \\ k=4 & c[2,3] + c(4,5) \\ k=5 & c[2,4] + c[5,5] \end{cases} + w[2,5].$

$\therefore \begin{cases} 0 + 692 \\ 244 + 256 \\ 742 + 0 \end{cases} + 564.$

Min for $k=4 \{ 244 + 256 + 564 \}$

$= 1064.$

$c[1,4] = \begin{cases} k=2 & c[1,1] + c(2,4) \\ k=3 & c[1,2] + c(3,4) \\ k=4 & c[1,3] + c(4,4) \end{cases} + w(1,4)$

$= \min \begin{cases} 0 + 742 \\ 228 + 370 \\ 624 + 0 \end{cases} + 650$

Min for $k=3 = \{ 228 + 370 + 650 \}$

$= 1248.$

$c[0,4] = \begin{cases} k=1 & c[0,0] + c[1,4] \\ k=2 & c[0,1] + c[2,4] \\ k=3 & c[0,2] + c[3,4] \\ k=4 & c[0,3] + c[4,4] \end{cases} + w[0,4]$

$\min \begin{cases} 0 + 1248 \\ 132 + 742 \\ 424 + 370 \\ 836 + 0 \end{cases} + 714$

Min for $k=3 \{ 424 + 370 + 714 \}$

$= 1,508$//

$$C[1,5] = \min_{\substack{K=2\\K=3\\K=4\\K=5}} \left\{ \begin{array}{l} C[1,1] + C[2,5] \\ C[1,2] + C[3,5] \\ C[1,3] + C[4,5] \\ C[1,4], + C[5,5] \end{array} \right\} + \omega(1,5)$$

$$\text{Min} \left\{ \begin{array}{l} 0 + 1064 \\ 228 + 692 \\ 624 + 256 \\ 1248 \quad + 0 \end{array} \right\} + 716.$$

$$\text{Min for } K=4 \ \{624 + 256 + 716\} = 1596$$

$$C[0,5] = \min_{\substack{K=1\\K=2\\K=3\\K=4\\K=5}} \left\{ \begin{array}{l} C[0,0] + C[1,5] \\ C[0,1] + C[2,5] \\ C[0,2] + C[3,5] \\ C[0,3] + C[4,5] \\ C[0,4] + C[5,5] \end{array} \right\} + \omega(0,5)$$

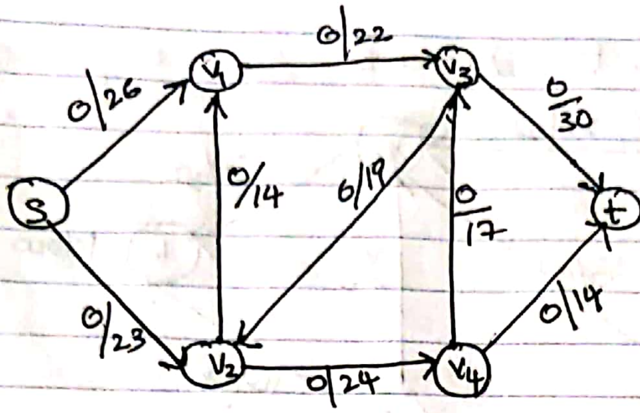
$$\text{Min} \left\{ \begin{array}{l} 0 + 1596 \\ 132 + 1064 \\ 434 + 692 \\ 836 + 256 \\ 1508 \quad + 0 \end{array} \right\} + 780$$

Min for $K=4$

$$K \ \{836 + 256 + 780\} = 1872$$

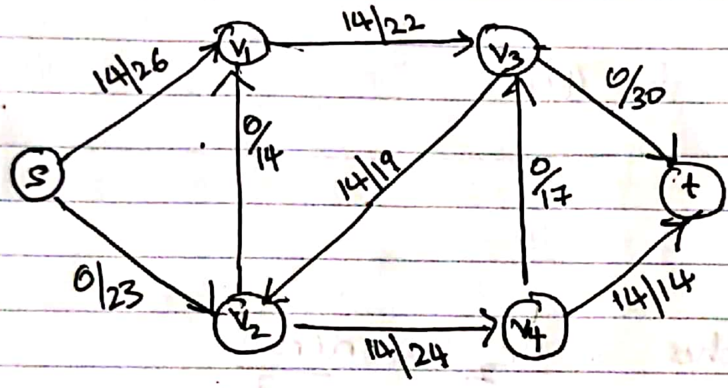
Scanned with CamScanner

(2) solⁿ



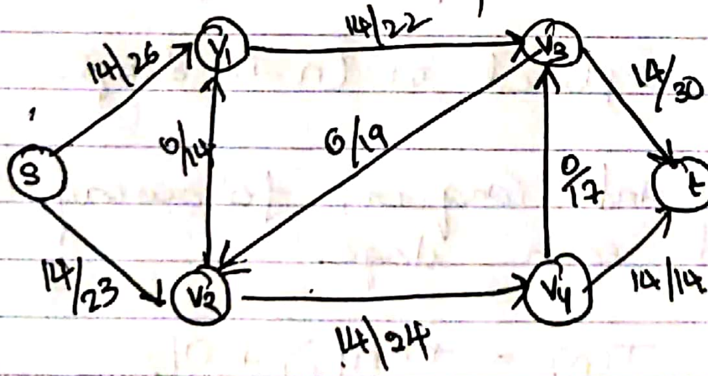
Flow = 0

1- Path. $S - V_1 - V_3 - V_2 - V_4 - t$ Min. $\Delta(e) = 14$



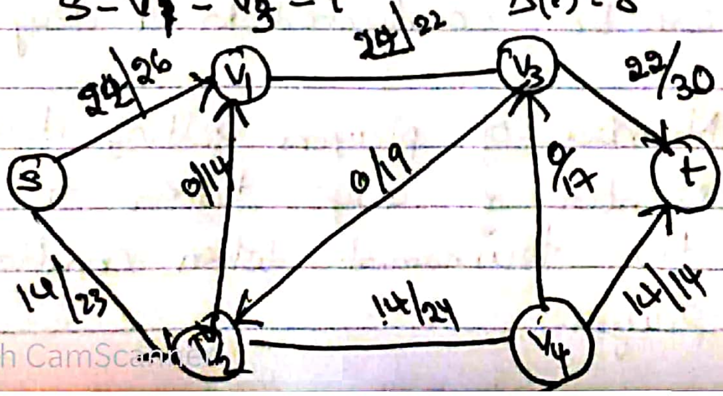
Flow = 14

2-Path $S - V_2 - V_3 - t$ $\Delta(e) = 14$



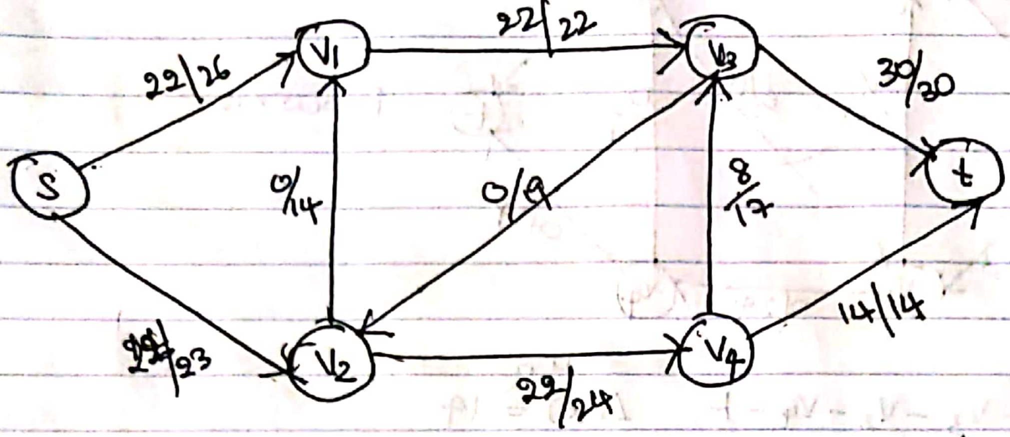
Flow = 28

3rd-path $S - V_1 - V_3 - t$ $\Delta(e) = 8$



Flow = 36

(4) Path $S - V_2 - V_4 - V_3 - t$ $\Delta(e) = 8$



Flow = 44

The Total Max Flow is 44.

---

(3) If n is 2^K.

Sol:

Total no. of matches $= {}^nC_2 = \frac{n(n-1)}{2}$.

No. of matches that can be played in single day is $n/2$

$\frac{n(n-1)}{2}$ matches can be played in $(n-1)$ days.

We can use divide and Conquer, following round robin algorithm at each stage.

For Example Consider $T(n) = T(n/2) + n/2$

Where $T(2) = 1 \Rightarrow$ which is the base condition for 2 players can be conducted in 1 day.

and $T(n/2)$ is the Number of players getting divided

$n/2$ is the Number of days taken to schedule the tournament when combining all the players.

For Example:- Consider. [1 2 3 4 5 6 7 8.]

③ — [1 2 3 4] | [5 6 7 8]

② — => [1 2] [3 4] | [5 6] [7 8]

① — [1] [2] [3] [4] | [5] [6] [7] [8].

At stage ① (1,2) (3,4), (5,6) (7,8) → plays in one day

	①	②	①	②	
stage ②	(1,2)	(1,2)	(5,6)	(5,6)	→ plays in two days.
	(3,4)	(4,3)	(7,8)	(8,7)	

	①	②	③	④
Stage ③	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4
	5 6 7 8	6 7 8 5	7 8 5 6	8 5 6 7

↳ They play in 4four days.

The total No. of days for 8 players is 7 days.
for n players is (n-1) days.

∴ Hence the least number of days required for $n = 2^k$ players is $(n-1)$ days.

(4) For $n = odd$, The least number of days required
soln to schedule a ~~algos~~ tournament is ~~(n) days~~.
n days for 'n' players

~~To~~ It uses the divide & conquer strategy and follows round robin algorithm at each stage.

For Example Consider. $N = 7$. 'X' = no player.
↳ requires 4 days.

(1)

1	2	3	4
5	6	7	X

1	2	3	4
X	5	6	7

1	2	3	4
7	X	5	6

4 day ←

1	2	3	4
5	7	X	6

We can see that each day, a free player is left for odd number of players - 7

(2)

①	②
(1, 2)	(1, 2)
(3, 4)	(4, 3)

①	②
(5, 6)	(5, 6)
(7, X)	(X, 7)

→ we requires 2 days

(3) (1) (2) (3) (4) (1, 2) (3, 4) (5, 6) (7, X)
(5) (6) ..
↳ requires 1 day.

Hence, the total no. of days required for 7 players is 7 days.

When $n = odd$

$$T(n) = T\left(\frac{n+1}{2}\right) + \left(\frac{n+1}{2}\right)$$

and $T(2) = 1$.

for n players, it is n days. When $n = odd$