Πεδία Α | 2024Κ

Θέμα 1

(Α) $E(\omega)=0 \Leftrightarrow \underbrace{\omega^4-\omega_1^2\omega^2+\omega_2^4=0}$

Δύο λύσεις: $\Delta>0 \Leftrightarrow \omega_1^4-4\omega_2^4 \Rightarrow \boxed{\omega_1^2>2\omega_2^2}$

(Β) $\Delta=0 \Rightarrow \boxed{\omega_1^2=2\omega_2^2} \to \underbrace{E(\omega)=\frac{(\omega^2-\omega_2^2)^2}{(\omega^2-\omega_3^2)\omega_3^2}}$

Μετάδοση χωρίς παραμόρφωση $\Rightarrow E(\omega)H(\omega)=$ σταθερό.

$$\Rightarrow \frac{(\omega^2-\omega_2^2)^2}{\omega_3^2(\omega^2-\omega_3^2)}\cdot\frac{3\omega_4^2}{\omega^2-2\omega_4^2}=\frac{3\omega_4^2}{\omega_3^2}\cdot\frac{\omega^2-\omega_2^2}{\omega^2-\omega_3^2}\cdot\frac{\omega^2-\omega_2^2}{\omega^2-2\omega_4^2}=\text{σταθ}$$

$$\Rightarrow \begin{cases}\dfrac{\omega^2-\omega_2^2}{\omega^2-\omega_3^2}=A \\[2ex] \dfrac{\omega^2-\omega_2^2}{\omega^2-2\omega_4^2}=B\end{cases} \Rightarrow \begin{cases}\omega^2-\omega_2^2=A\omega^2-A\omega_3^2 \\ \omega^2-\omega_2^2=B\omega^2-B2\omega_4^2\end{cases} \Leftrightarrow \begin{cases}A=1,\ \omega_2^2=\omega_3^2 \\ B=1,\ \omega_2^2=2\omega_4^2\end{cases}$$

Άρα $\{\omega_1,\omega_2,\omega_3,\omega_4\}=\{\sqrt{2}\omega_2,\omega_2,\omega_2,\frac{\omega_2}{\sqrt{2}}\}$

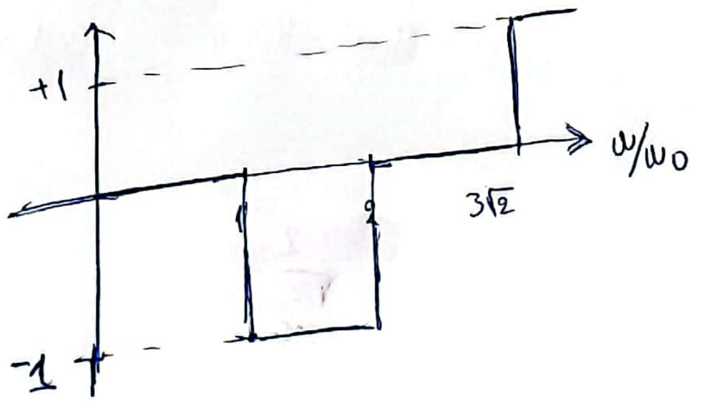
$$E(\omega)H(\omega)=\frac{3\omega_4^2}{\omega_2^2}=\frac{3\frac{\omega_2^2}{2}}{\cancel{\omega_2^2}} \Rightarrow \boxed{E(\omega)H(\omega)=3/2}$$

(Γ)

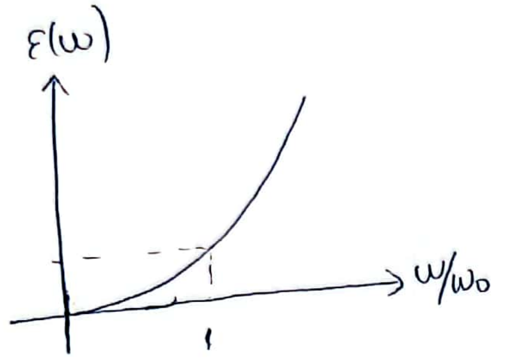
$$\cancel{\varepsilon(\omega) = \frac{\omega^2(\omega^2-\omega_0^2)}{(\omega^2-4\omega_0^2)\,\omega_0^2} = \frac{\omega^2}{\omega_0^2(\omega^2-4\omega_0^2)}}$$

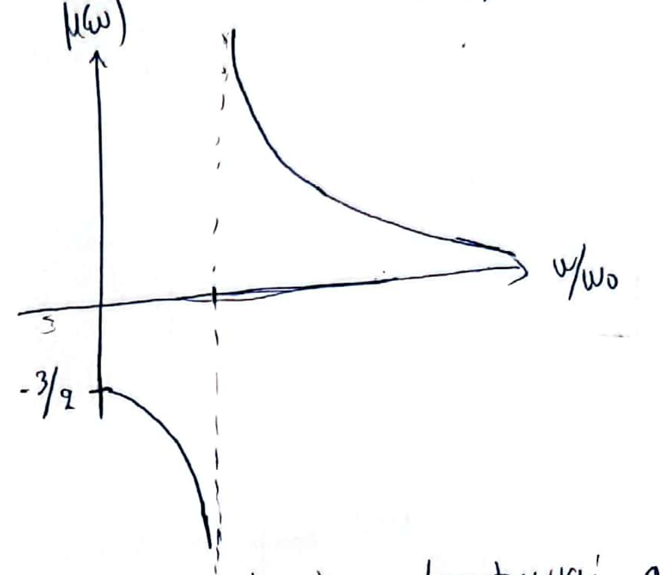
$$\varepsilon(\omega) = \frac{\omega^4 - \omega_0^2\omega^2}{(\omega^2-4\omega_0^2)\,9\omega_0^2} = \frac{\omega^2(\omega^2-\omega_0^2)}{9\omega_0^2(\omega^2-4\omega_0^2)}, \quad \mu(\omega) = \frac{27\omega_4^2}{\omega^2-18\omega_0^2}$$

	0		ω_0		$2\omega_0$		$3\sqrt{2}\,\omega_0$		$+\infty$
$\omega^2-\omega_0^2$		−	0	+		+		+	
$\omega^2-4\omega_0^2$		−		−	0	+		+	
$\omega^2-18\omega_0^2$		−		−		−	0	+	
$\varepsilon(\omega)$		+		−		+		+	
$\mu(\omega)$		−		−		−		+	
$\varepsilon(\omega)\mu(\omega)$		0		−1		0		+1	



(Δ) $\varepsilon(\omega) = \left(\frac{\omega}{\omega_0}\right)^2 \quad , \mu(\omega) = \frac{3\omega_0^2}{\omega^2 - 2\omega_0^2} = \frac{3}{\left(\frac{\omega}{\omega_0}\right)^2 - 2}$





η μετάδοση μεγιστοποιείται όταν έχουμε μηδενική ανάκλαση: (διατήρηση ενέργειας)

$$R = \frac{\sqrt{\frac{\varepsilon_0 \varepsilon}{\mu_0 \mu}} - \sqrt{\frac{\varepsilon_0}{\mu_0}}}{\sqrt{\frac{\varepsilon_0 \varepsilon}{\mu_0 \mu}} \quad \sqrt{\frac{\varepsilon_0}{\mu_0}}}$$

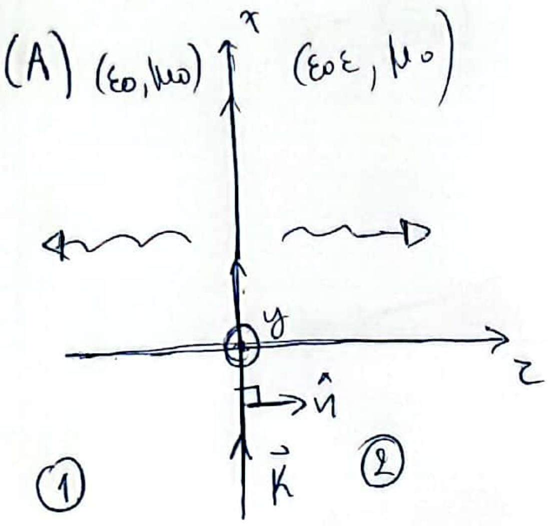
$$R = \frac{\sqrt{\frac{\mu_0 \mu}{\varepsilon_0 \varepsilon}} - \sqrt{\frac{\mu_0}{\varepsilon_0}}}{\sqrt{\frac{\mu_0 \mu}{\varepsilon_0 \varepsilon}} - \sqrt{\frac{\mu_0}{\varepsilon_0}}} = \frac{\sqrt{\mu} - \sqrt{\varepsilon}}{\sqrt{\mu} + \sqrt{\varepsilon}} \overset{set}{=} 0 \Leftrightarrow \varepsilon(\omega) = \mu(\omega)$$

$$\rightarrow \frac{\omega^2}{\omega_0^2} = \frac{3\omega_0^2}{\omega^2 - \omega_0^2} \Rightarrow \omega^4 - \omega^2\omega_0^2 - 3\omega_0^4 = 0$$

$$\omega^2 = \frac{\omega_0^2 \pm \sqrt{\omega_0^4 + 12\omega_0^4}}{2} \xrightarrow[\omega_m^2 > 0]{+} \boxed{\omega_m^2 = \frac{(\sqrt{13}+1)\omega_0^2}{2}}$$

Θέμα 2ο

(A)



Σ.Σ. $\hat{n}\times(\vec{E}_2-\vec{E}_1)=\vec{0}$

$$\underbrace{\hat{n}}_{\hat{z}}\times\underbrace{(\vec{H}_2-\vec{H}_1)}_{\hat{y}}=\underbrace{\vec{K}}_{\hat{x}}$$

$\vec{H}$: "$\hat{y}$" $\Rightarrow$ $\vec{E}$: "$\hat{x}$"

$$\vec{E}_1=\hat{x}A_1(\omega)e^{+ik_0z},\quad \vec{E}_2=\hat{x}A_2e^{-ik_0\sqrt{\varepsilon}z}$$

$$\hat{n}\times(\vec{E}_2-\vec{E}_1)\big|_{z=0}=\vec{0}\Rightarrow\boxed{A_2(\omega)=A_1(\omega)=A(\omega)}$$

$$\vec{H}=\frac{i}{k_0\eta_0}\nabla\times\vec{E}=\frac{i}{k_0\eta_0}\begin{vmatrix}\hat{x}&\hat{y}&\hat{z}\\ \partial/\partial x&\partial/\partial y&\partial/\partial z\\ E_x&0&0\end{vmatrix}.$$

$$\Rightarrow \underline{H_y=\frac{i}{k_0\eta_0}\frac{\partial E_x}{\partial z}}\quad\begin{cases}\vec{H}_{y,1}=\hat{y}\dfrac{i}{\cancel{k_0\eta_0}}A(\omega)(\cancel{i}k_0)A(\omega)e^{ik_0z}\\ \vec{H}_{y,2}=\hat{y}\dfrac{i}{\cancel{k_0\eta_0}}A(\omega)(-\cancel{i}k_0\sqrt{\varepsilon})A(\omega)e^{-ik_0\sqrt{\varepsilon}z}\end{cases}$$

$$\hat{n}\times(\vec{H}_2-\vec{H}_1)\big|_{z=0}=\vec{K}\Rightarrow\frac{A(\omega)}{\eta_0}\sqrt{\varepsilon}-\left(\frac{-A(\omega)}{\eta_0}\right)=-K(\omega)$$

$$\Rightarrow\frac{A(\omega)(\sqrt{\varepsilon}+1)}{\eta_0}=-K(\omega)\Rightarrow\boxed{A(\omega)=\frac{-\eta_0K(\omega)}{1+\sqrt{\varepsilon}}}$$

<u>Άρα:</u>

$$\vec{E}_1=\frac{-\hat{x}K(\omega)\eta_0}{\sqrt{\varepsilon}+1}e^{ik_0z},\quad \vec{E}_2=\frac{-\hat{x}(K(\omega))\eta_0}{\sqrt{\varepsilon}+1}e^{-ik_0\sqrt{\varepsilon}z}$$

(Β)

Fourier: $\vec{K}(\omega) = \hat{x}\left[K_1 \frac{\delta(\omega-\omega_1)+\delta(\omega+\omega_1)}{2} + K_2 \frac{\delta(\omega-\omega_2)-\delta(\omega+\omega_2)}{2i}\right]$

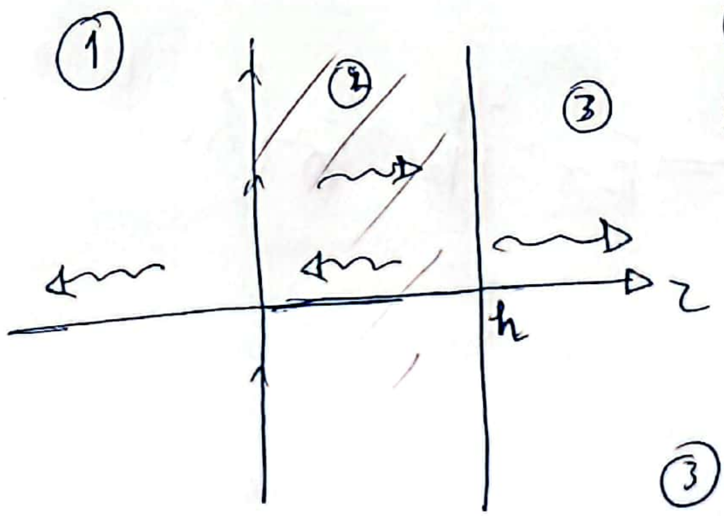
άρα κρατάμε ~~τους~~ συγκεκριμένους τόνους. Δηλαδή έχουμε:

$$\vec{E}_1 = \hat{x}\,\mathrm{Re}\left\{\frac{-n_0K_1}{\sqrt{\varepsilon(\omega_1)}+1} e^{ik_0z+i\omega_1t}\right\}$$
$$+\hat{x}\,\mathrm{Im}\left\{\frac{-n_0K_2}{\sqrt{\varepsilon(\omega_2)}+1} e^{+ik_0z+i\omega t}\right\}$$

$$\Rightarrow \vec{E}_1(z,t) = -\hat{x}\left[\frac{n_0K_1}{1+\sqrt{\varepsilon(\omega_1)}}\cos(k_0z+\omega_1t) + \frac{n_0K_2}{1+\sqrt{\varepsilon(\omega_2)}}\sin(k_0z+\omega_2t)\right]$$

όμοια: $$\vec{E}_2(z,t) = -\hat{x}\left[\frac{n_0K_1}{1+\sqrt{\varepsilon(\omega_1)}}\cos(-k_0\sqrt{\varepsilon}z+\omega_1t) + \frac{n_0K_2}{1+\sqrt{\varepsilon(\omega_2)}}\sin(k_0\sqrt{\varepsilon}z+\omega_2t)\right]$$

(Γ)



1: κύματα διαδίδονται προς το "$-\infty$"

$E_1 \sim \cos(\omega_1t+k_0z+\varphi) + \sin(\omega_2t+k_0z+\varphi)$

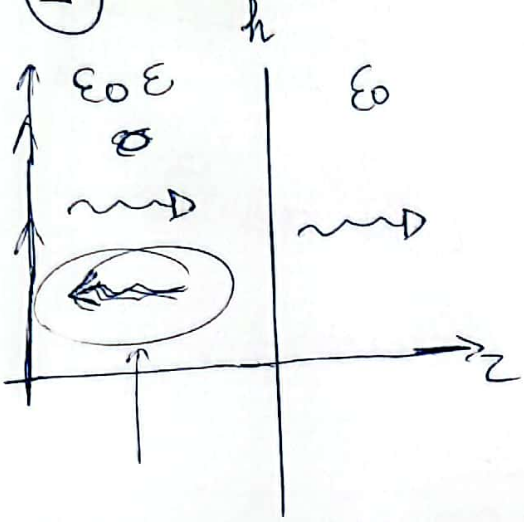
2:

$E_2 \sim \cos(\omega_1t \pm k_0\sqrt{\varepsilon}z+\varphi')$
$+\sin(\omega_2t \pm k_0\sqrt{\varepsilon}z+\varphi')$

"και προς τις δυο κατευθύνσεις"

3: $E_3 \sim \cos(\omega_1t - k_0z+\varphi'')$
$+\sin(\omega_2t - k_0z+\varphi'')$

Δ



Δεν έχω ανάκλαση μόνο όταν το υλικό $[0,h]$ είναι κενό. Όμως μας ενδιαφέρουν μόνο οι συχνότητες εκπομπής. Δηλαδή πρέπει

$$\varepsilon(\omega_1)=\varepsilon(\omega_2)=1.$$

Επιπλέον θέλω $\varepsilon(0)=2$, $\varepsilon(\infty)=3$

$$\boxed{\varepsilon(\omega)=1+2\cdot\frac{(\omega^2-\omega_1^2)(\omega^2-\omega_2^2)}{\omega^4+\omega_0^4}}$$

$$\varepsilon(0)=2\to 1+2\frac{\omega_1^2\omega_2^2}{\omega_0^4}=2\Rightarrow \boxed{\omega_0^4=2\omega_1^2\omega_2^2}$$

Θέμα 3

$$H_{y,inc}=A\,e^{-ik_0\sin\theta x+ik_0\cos\theta z}$$

$$E=\frac{\eta_0}{ik_0}\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \partial_x & \partial_y & \partial_z\\ 0 & H_y & 0\end{vmatrix}\Rightarrow E_x=\frac{-\eta_0}{ik_0}\frac{\partial}{\partial z}\left(A e^{-ik_0\sin\theta x+ik_0\cos\theta z}\right)$$

$$=\frac{-\eta_0}{ik_0}(ik_0\cos\theta)A e^{-ik_0\sin\theta x+ik_0\cos\theta z}$$

$$H_{y,ref}=A\cdot R\,e^{-ik_0\sin\theta x-ik_0\cos\theta z},\quad E_{x,ref}=AR\eta_0\cos\theta\, e^{-ik_0\sin\theta x-ik_0\cos\theta z}$$

(B) Σ.Σ.: $n\left[\cancel{A e^{-ik_o \sin\theta x}} + \cancel{AR e^{-ik_o\sin\theta x}}\right] + A\left[-n_o\cos\theta\, \cancel{e^{-ik_o\sin\theta x}} + R n_o \cos\theta\, \cancel{e^{-ik_o\sin\theta x}}\right] = 0$

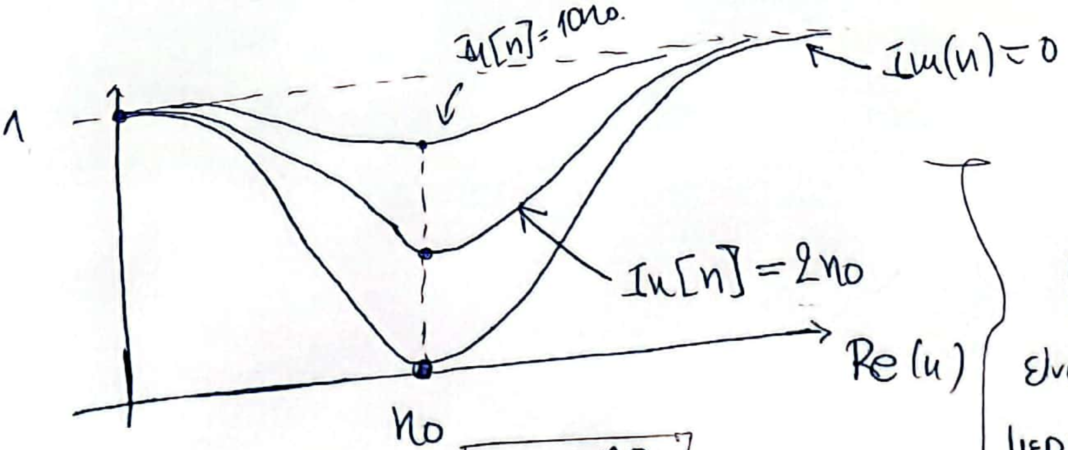
(A) $n(1+R) - n_o\cos\theta(1-R) = 0$

$\Rightarrow R(n + n_o\cos\theta) = n_o\cos\theta - n \Leftrightarrow$

$$\boxed{R = \frac{n_o\cos\theta - n}{n_o\cos\theta + n}}$$

(Γ) ποσοστό ισχύος $= |R|^2 = \dfrac{(n_o\cos\theta - \mathrm{Re}\,n)^2 + \mathrm{Im}^2[n]}{(n_o\cos\theta + \mathrm{Re}\,n)^2 + \mathrm{Im}^2 n}$

(Δ) $|R|^2 = \dfrac{(n_o - \mathrm{Re}\,n)^2 + \mathrm{Im}^2(n)}{(n_o + \mathrm{Re}\,n)^2 + \mathrm{Im}^2(n)}$



Η λογική είναι ότι βάζουμε μερικά χαρακτ. σημεία και κοιτάμε τι γίνεται στο +∞.

(Ε) $R(\theta = 60°) = 0 \Rightarrow \boxed{n = \frac{n_o}{2}}$