Reasoning about GADT Pattern Matching in Haskell

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Partners

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Joint work with

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Background

Algebraic Data Types

```
data List a
  = Nil
  | Cons a (List a)
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l :: List Int
l = Cons 1 (Cons 2 (Cons 3 Nil))
```

Example

```
foo :: List a -> List b -> Int
foo (Cons x xs) (Cons y ys) = 1
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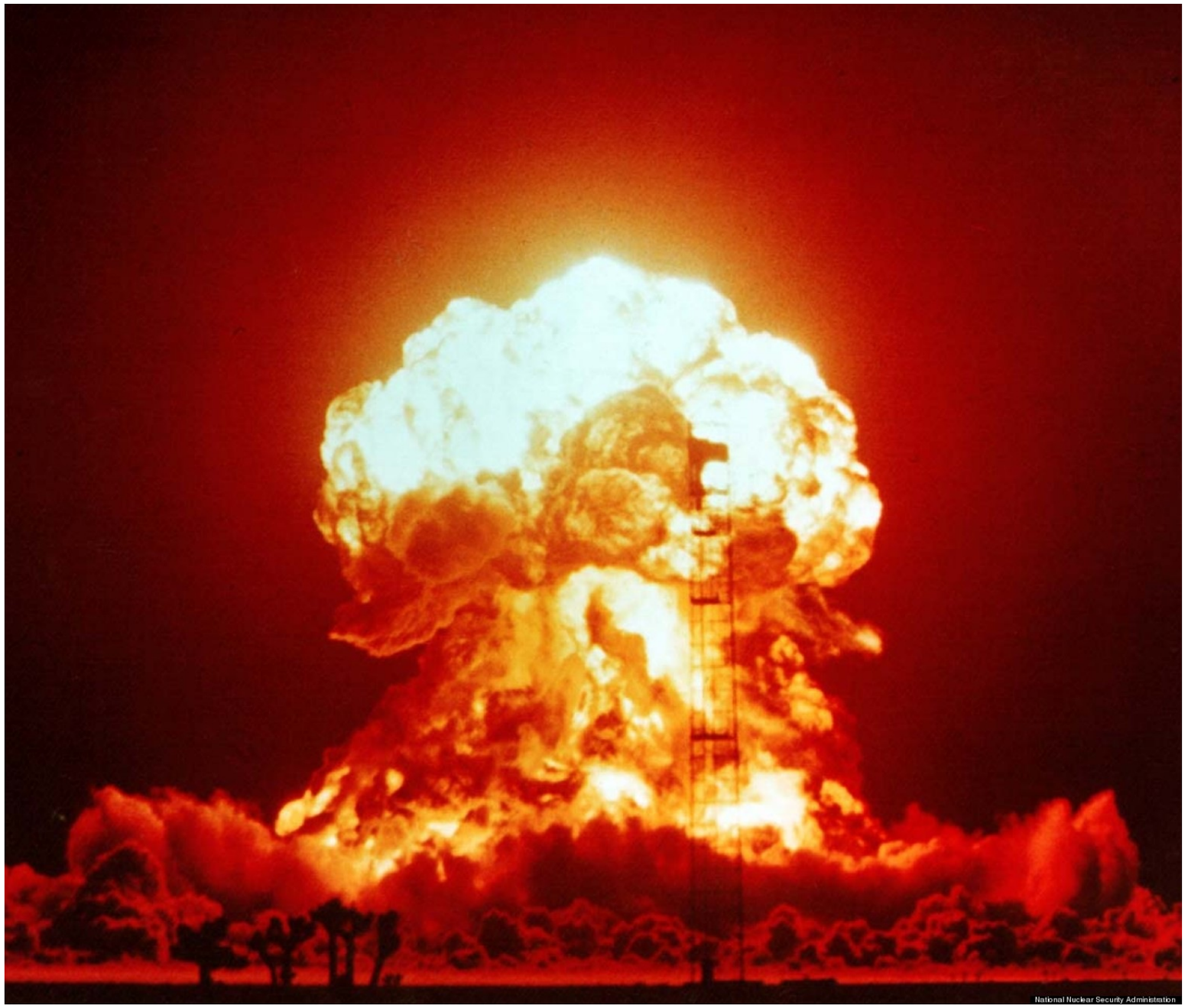
Non-Exhaustiveness



Exhaustiveness Checking

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Patterns not matched:
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```
Pattern matches are overlapped:
    foo Nil Nil = 3
```

Summary

- Everything is rosy thanks to GHC's exhaustiveness/overlapping checker
- Based on Maranget's first technique [Mara94] for compiling lazy pattern matching

[Mara94] Luc Maranget, *"Two Techniques for Compiling Lazy Pattern Matching"*, Rapport de recherche RR-2385, INRIA, 1994. Projet PARA.

Generalized Algebraic Data Types (since 2006)

Not so rosy
anymore

GADTs

```
data Vec :: Nat -> * -> * where
  Nil  :: Vec Zero a
  Cons :: a -> Vec n a -> Vec (Succ n) a

kind Nat = Zero | Succ Nat
```

Example

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zip :: Vec n a -> Vec n b -> Vec n (a,b)
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Attempted Fix

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zip :: Vec n a -> Vec n b -> Vec n (a,b)
zip Nil          Nil          = Nil
zip (Cons x xs) (Cons y ys) = Cons (x,y) (zip xs ys)
zip Nil          (Cons y ys) = error “Nil (Cons _ _)”
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Error:
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[Vyti11] Dimitrios Vytiniotis, Simon Peyton Jones, Tom Schrijvers and Martin Sulzmann, *“OutsideIn(X) modular type inference with local assumptions”*, J. Funct. Program., vol. 21, no. 4-5, pp. 333–412, September 2011.

Silencing the Compiler

```
zip :: Vec n a -> Vec n b -> Vec n (a,b)
zip Nil          Nil          = Nil
zip (Cons x xs) (Cons y ys) = Cons (x,y) (zip xs ys)
zip _            _            = error "type error”
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The result?

- No trust in the emitted warnings
- Bogus overlapped patterns to suppress them

My Way

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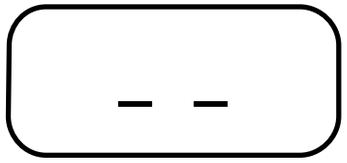

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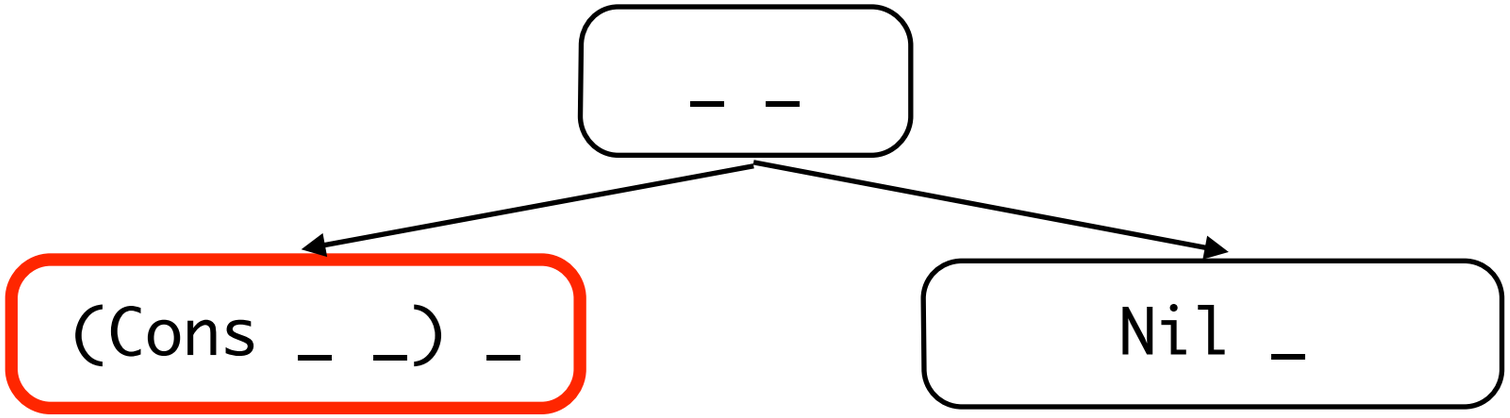

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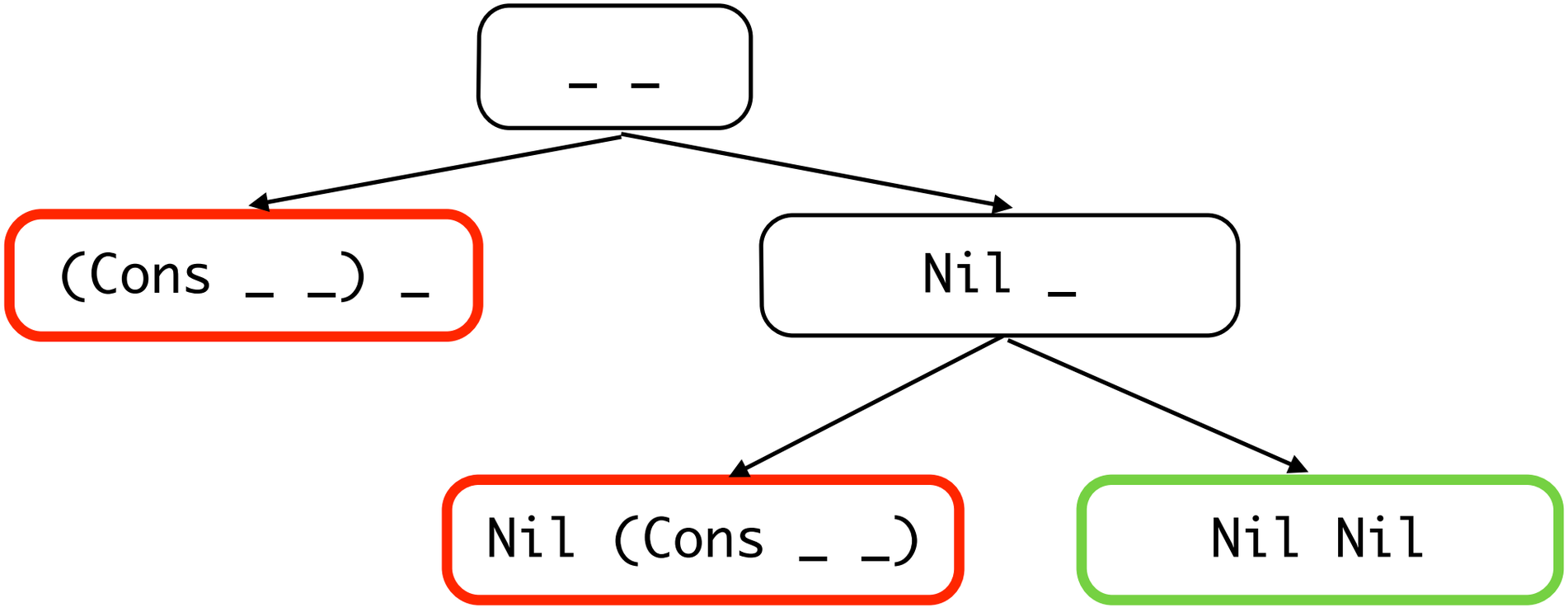

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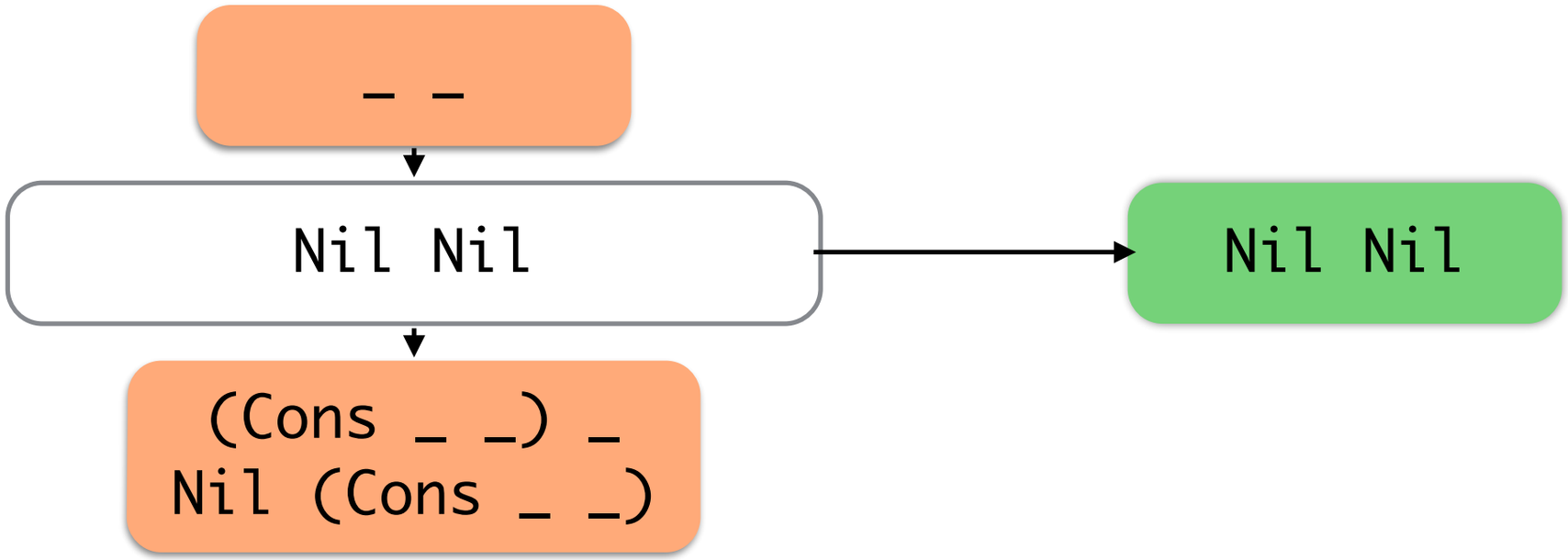

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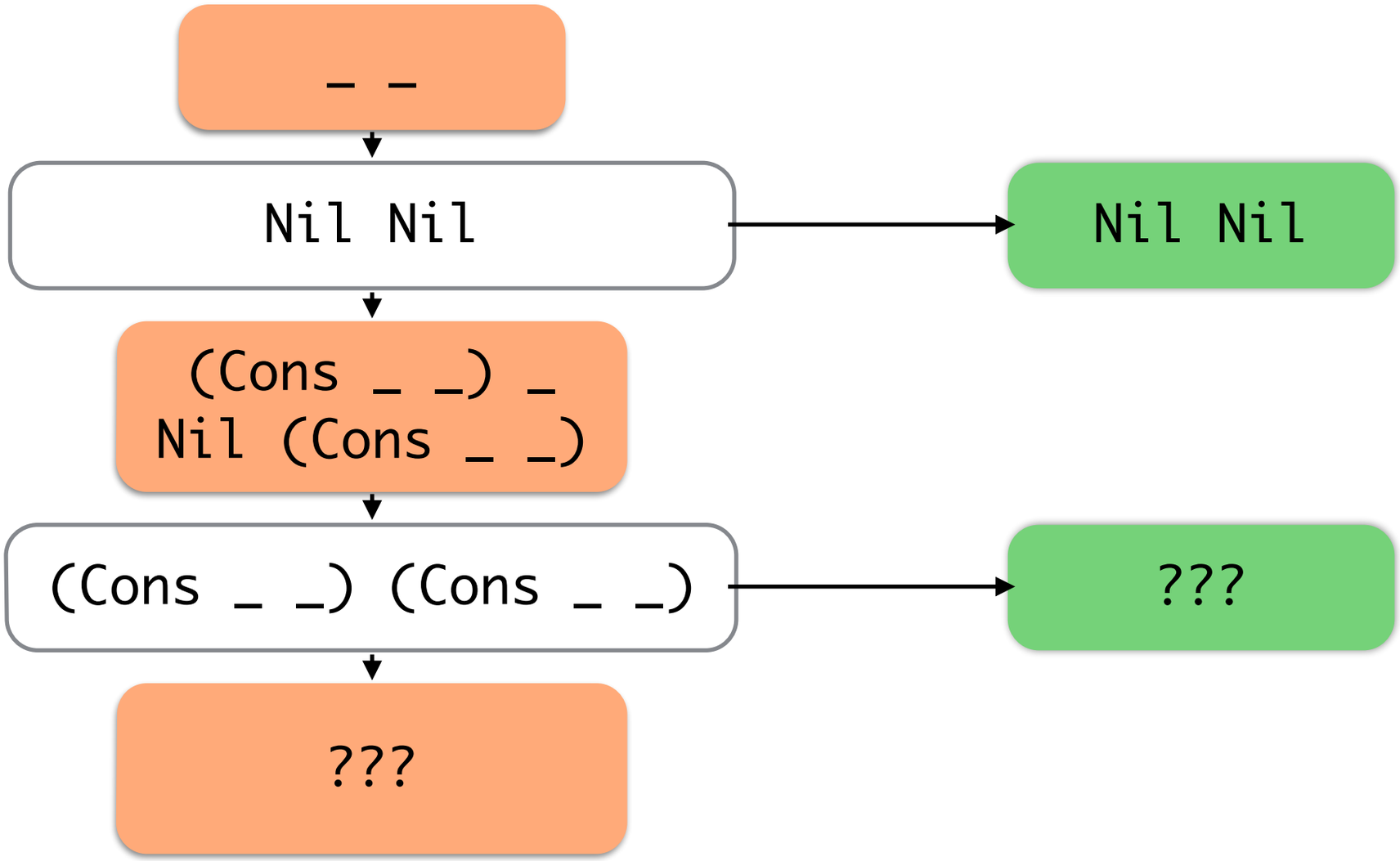

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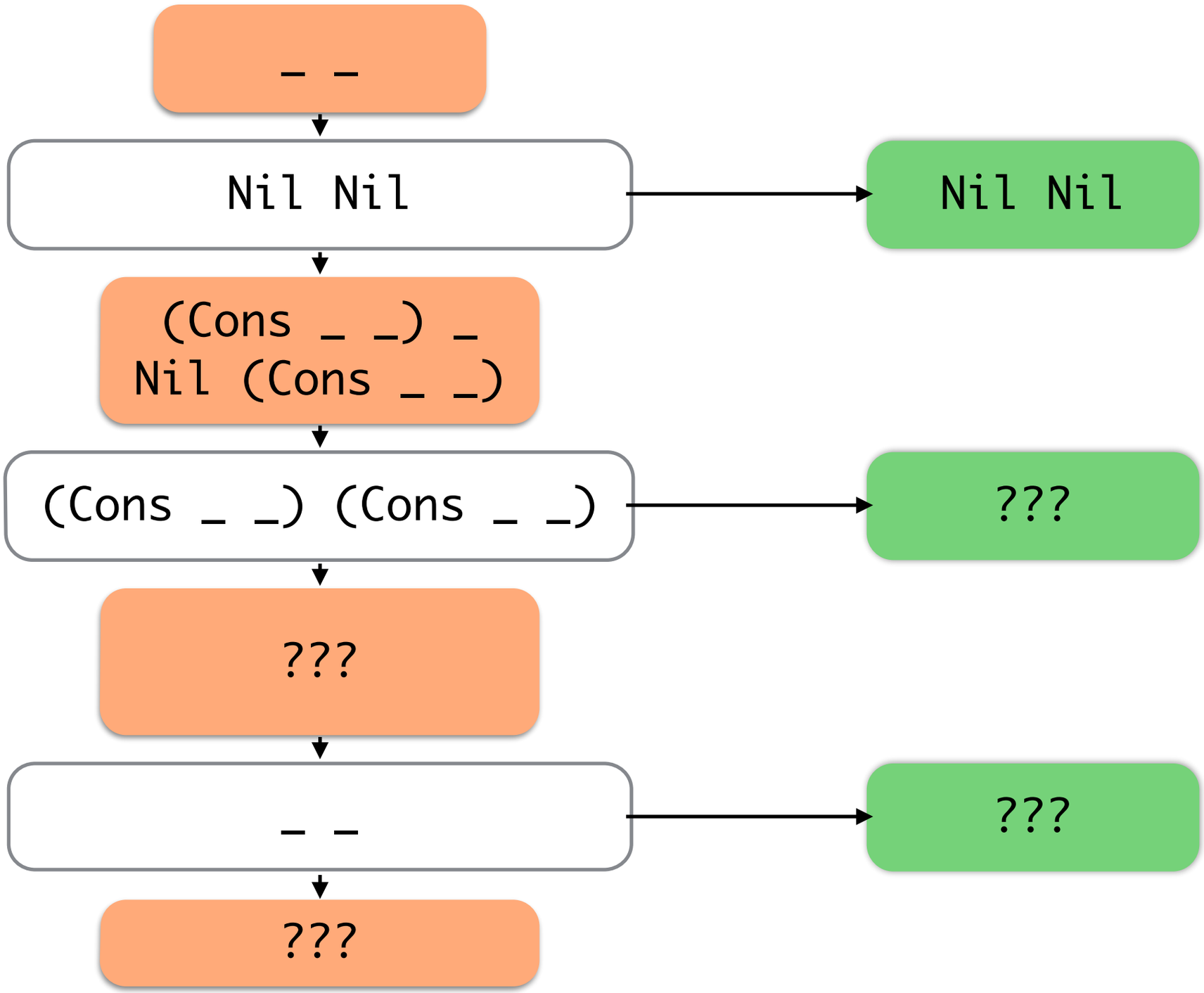

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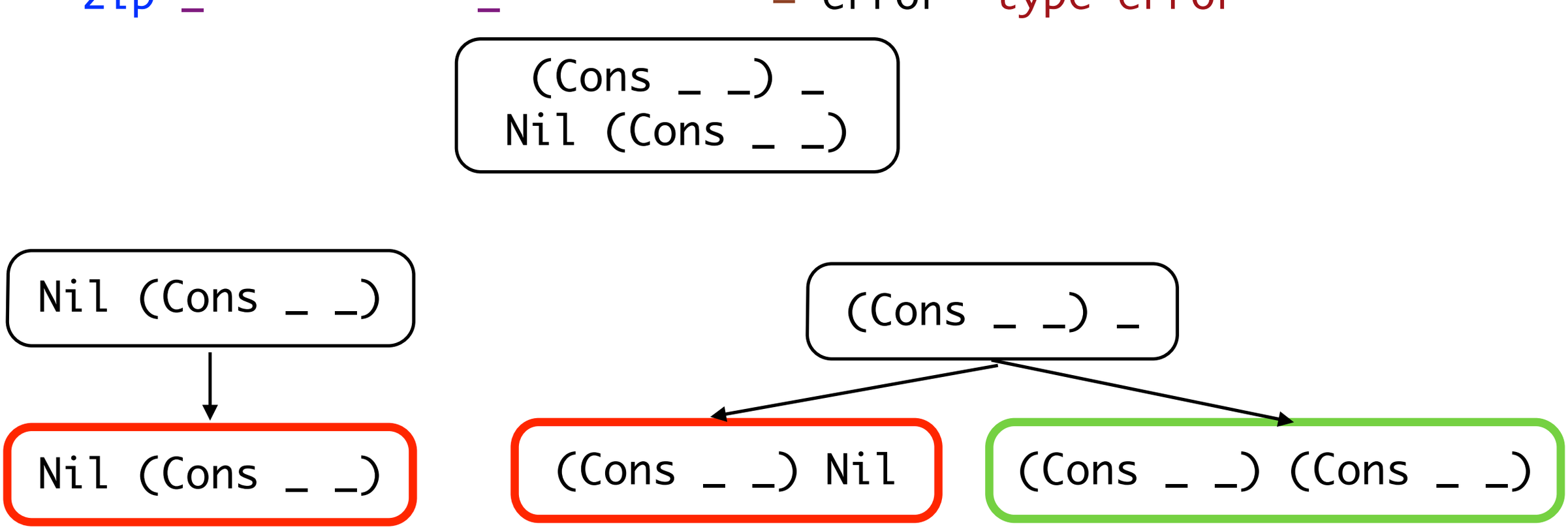

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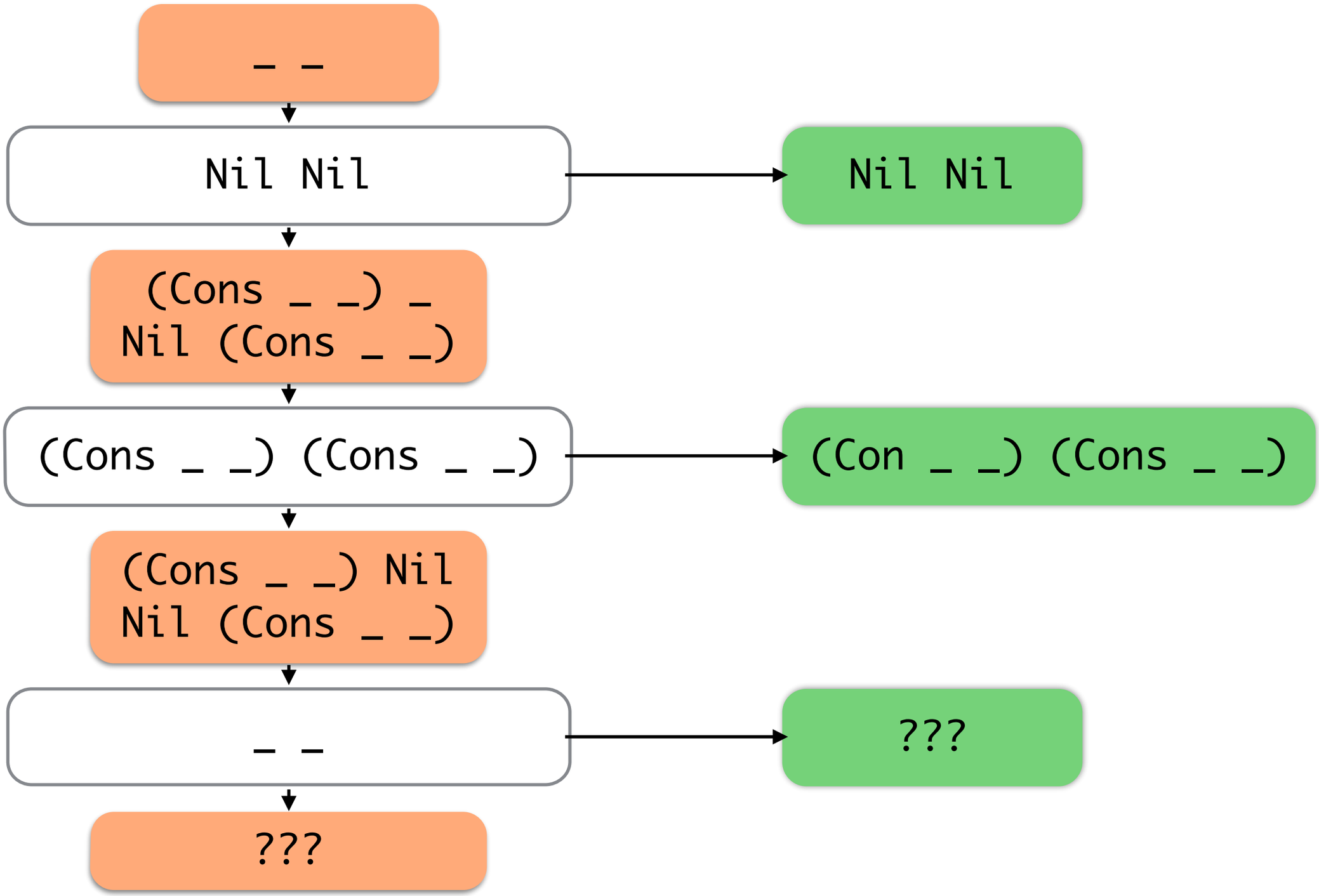
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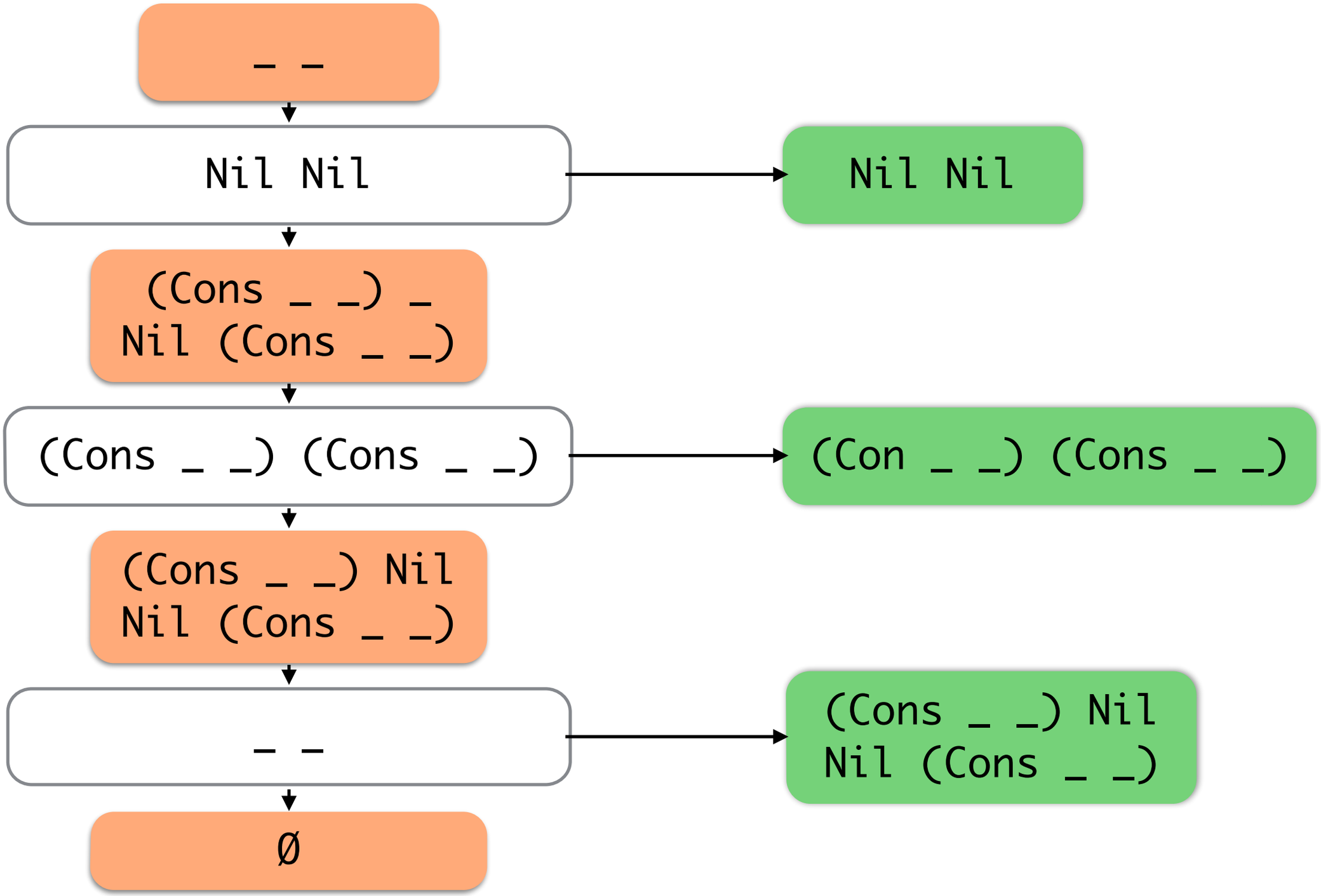
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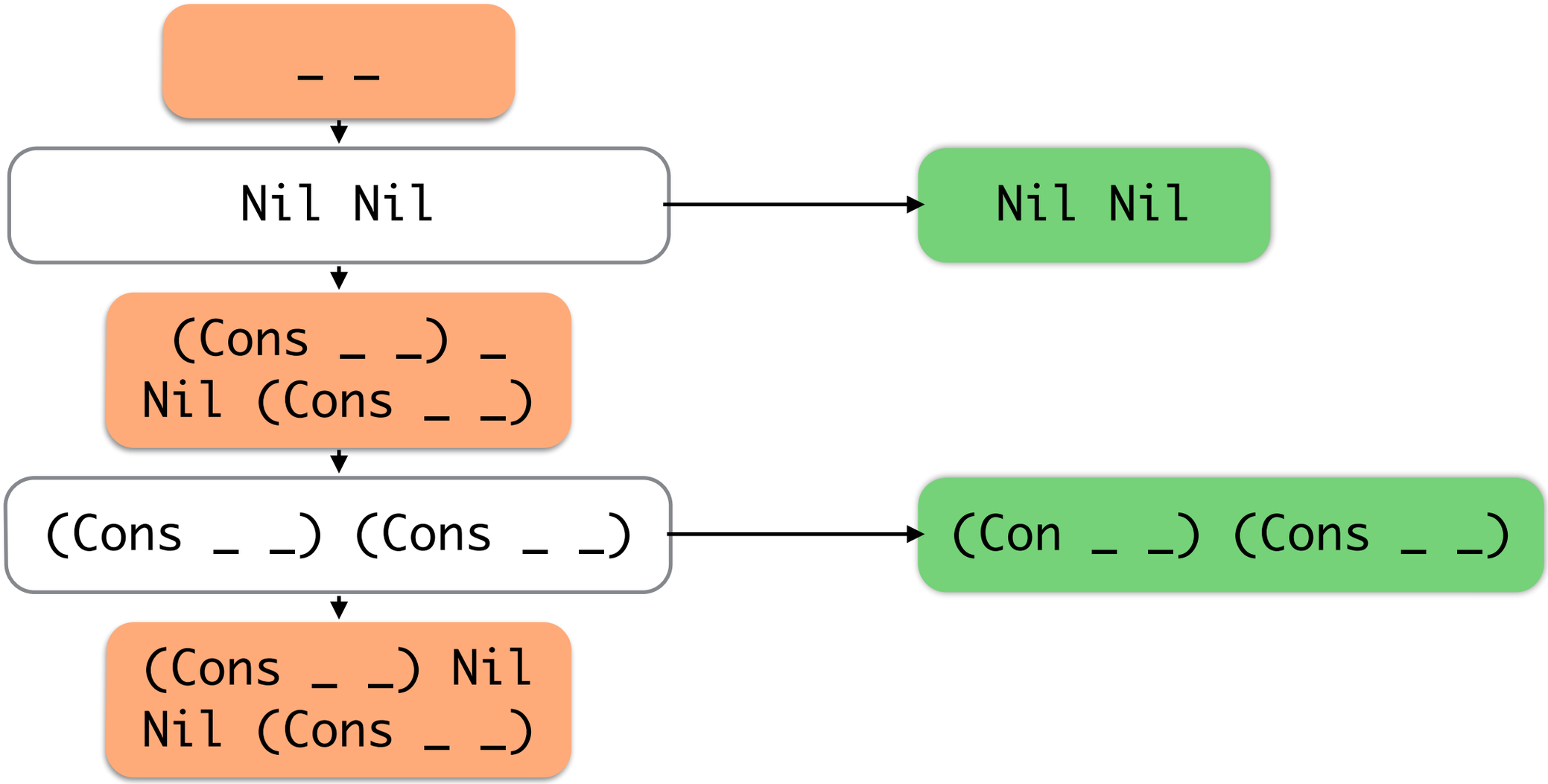
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Totality & Usefulness

Exhaustiveness Check

- A match is total if the final set of ***well-typed*** uncovered cases is empty.

Redundancy Check

- A clause is useful if the set of ***well-typed*** cases it covers is non-empty.

Type-checking

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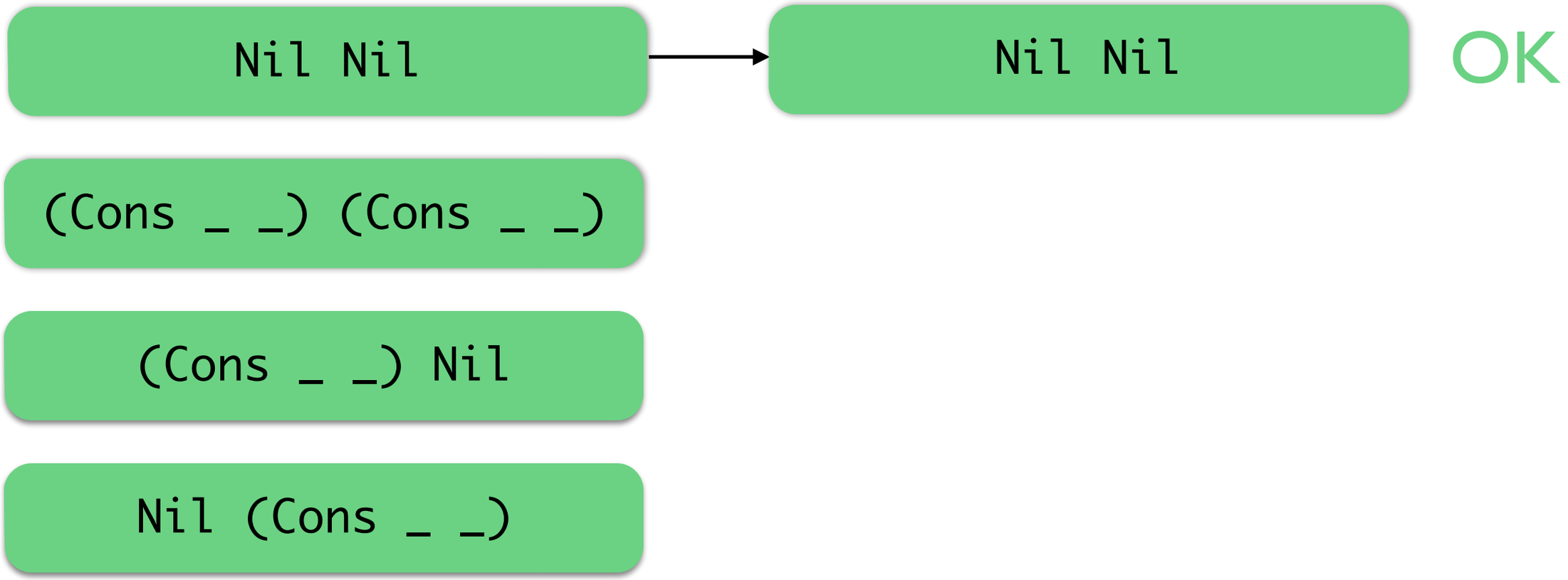

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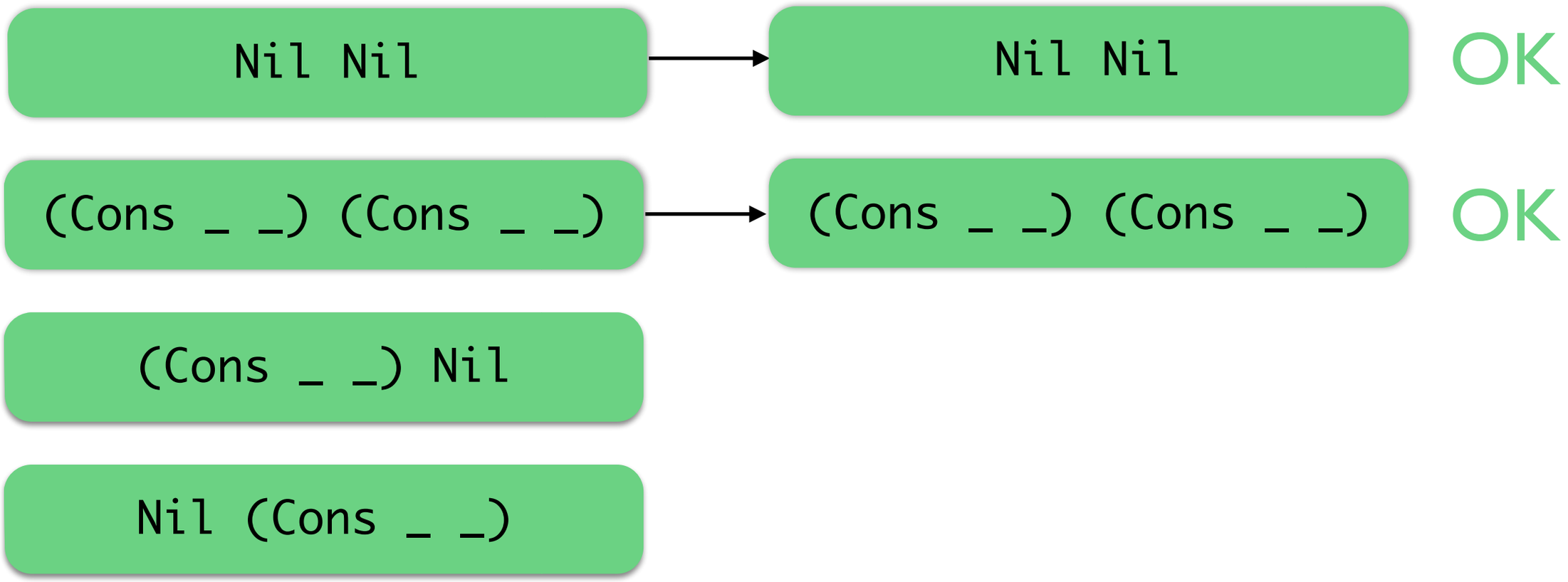

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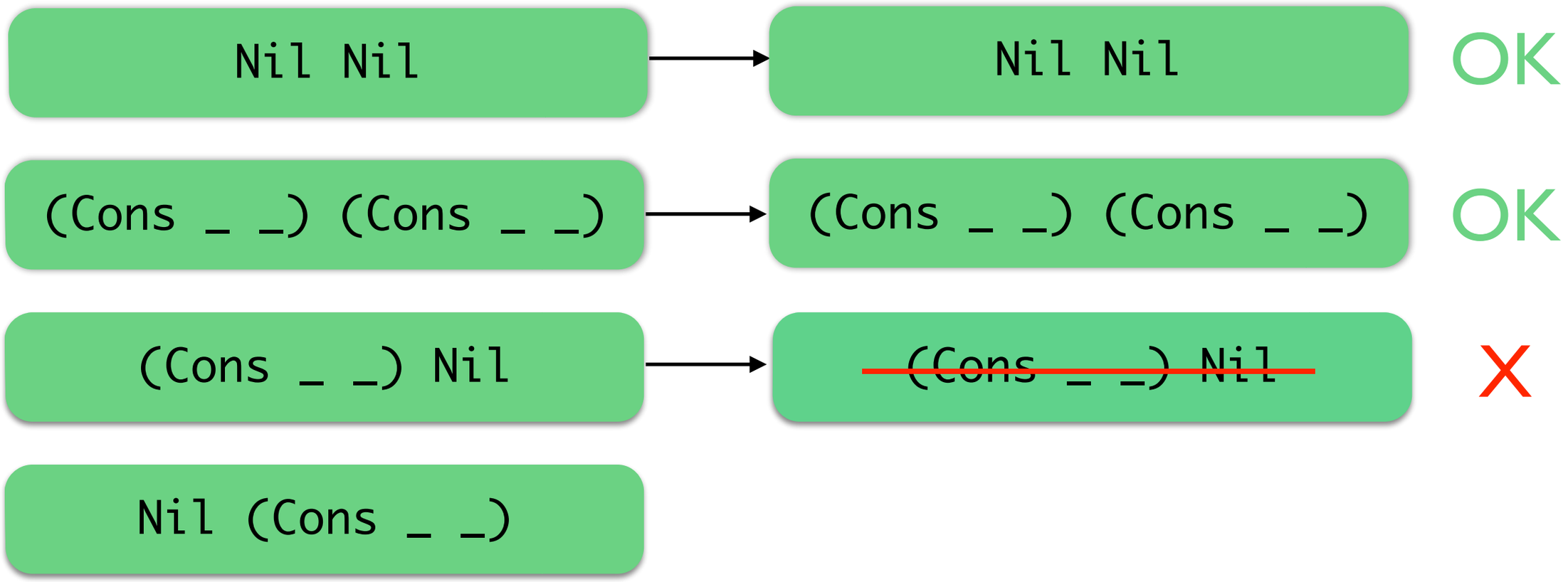

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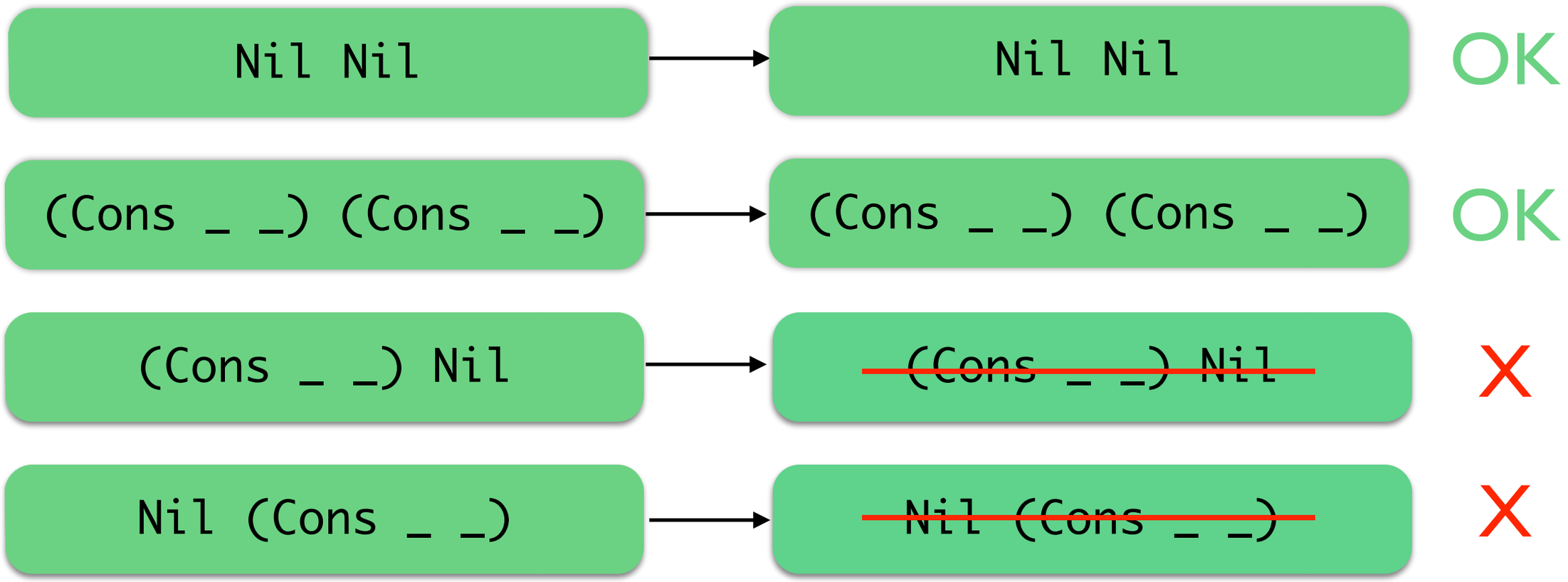

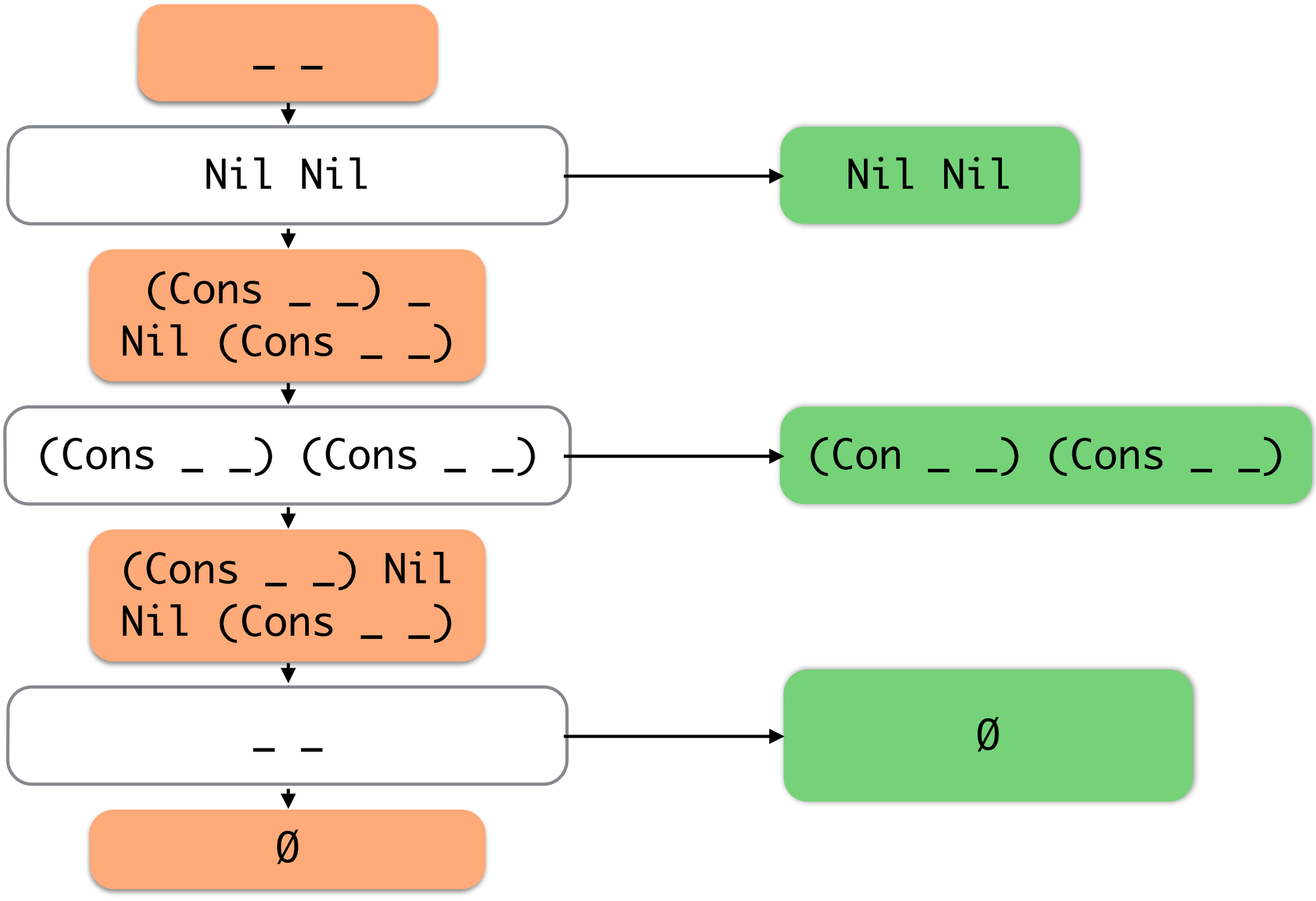
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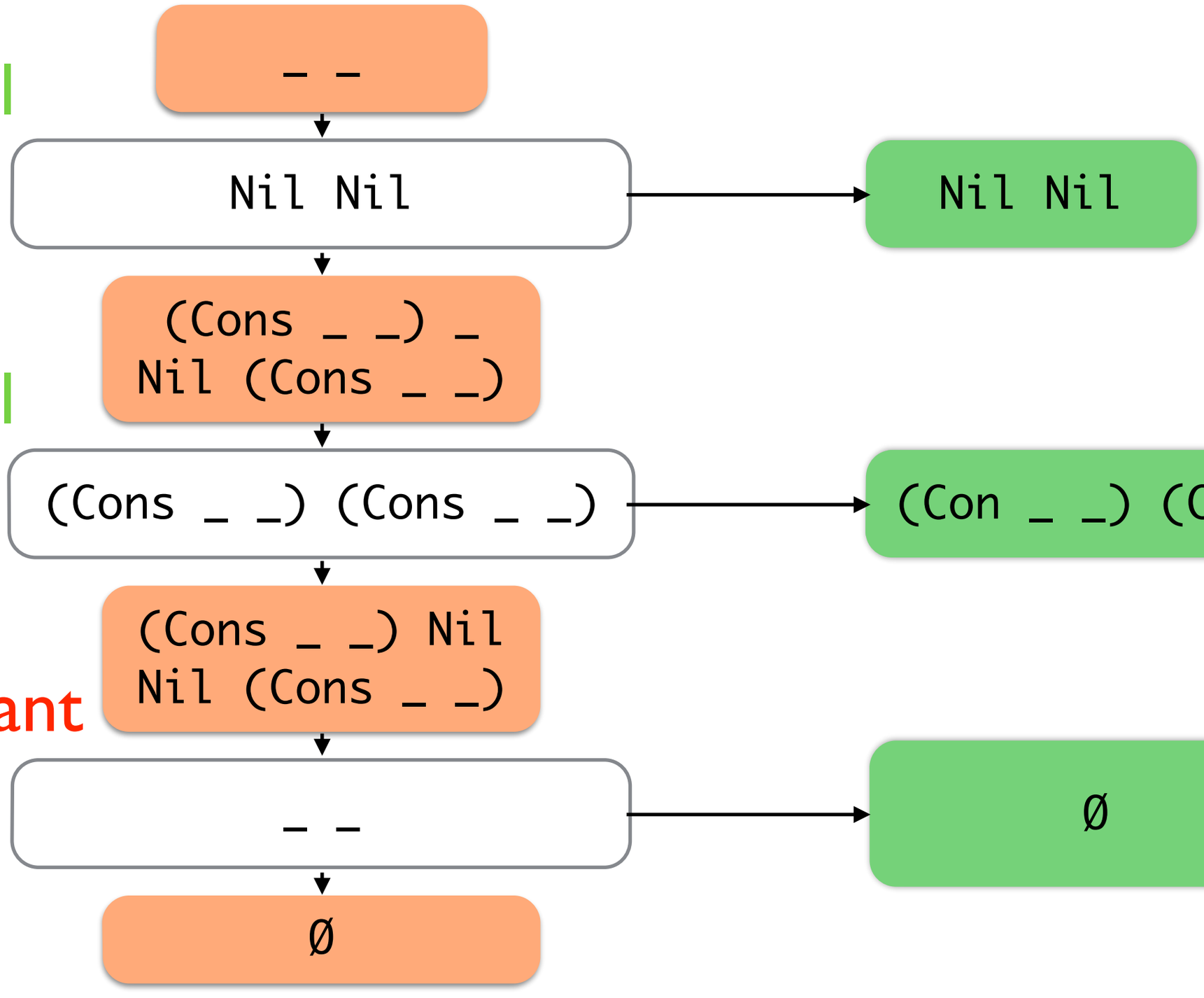

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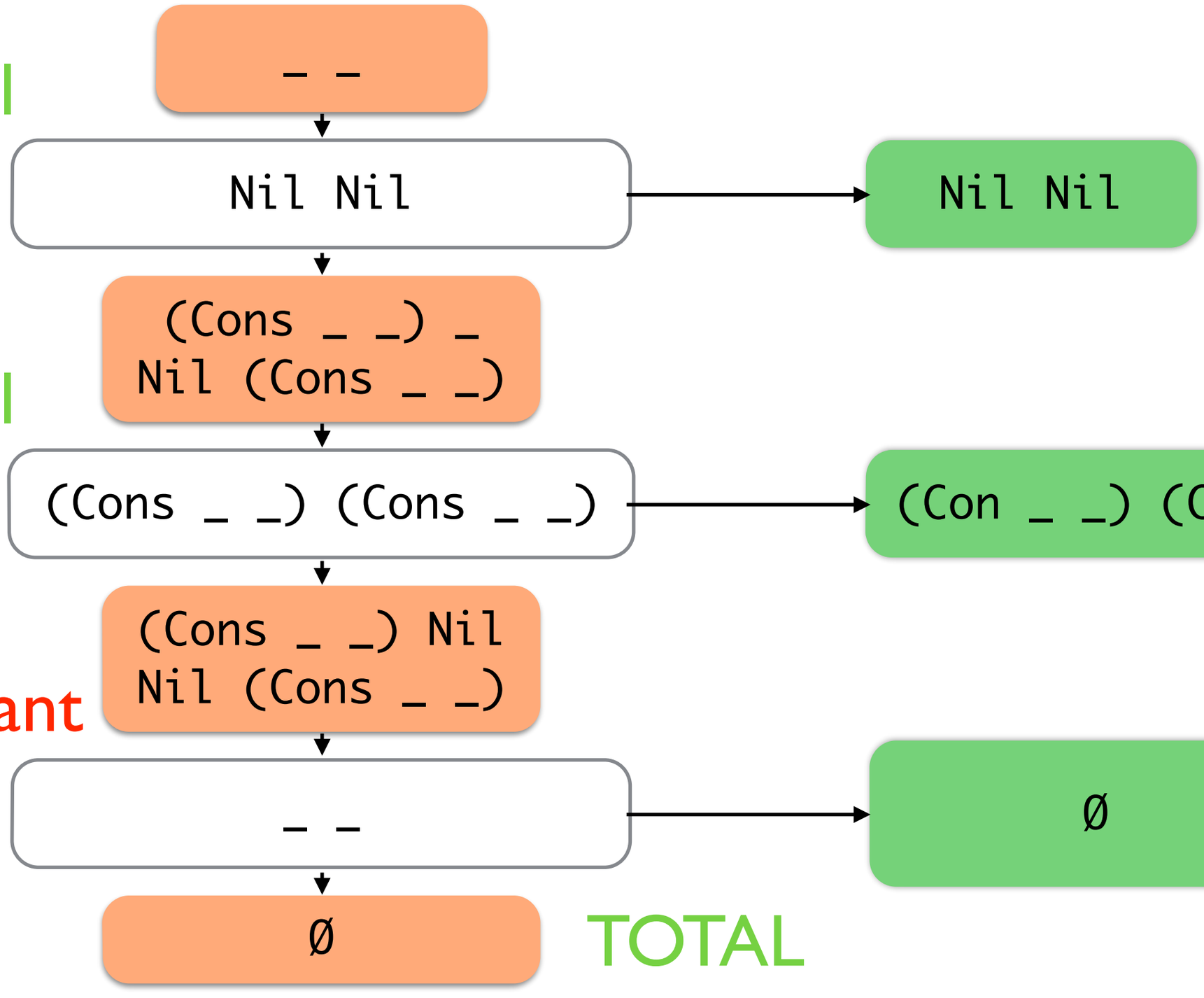

Useful

Redundant

TOTAL

Summary

Discussed

- Pattern Matching in Haskell
- Basic static checks for pattern matching (totality & redundancy)
- Our incremental approach for detecting these problems
- Easy integration with type checking

As of Today

- Formalisation of the algorithm
- Prototype implementation in GHC
  - Support for guards & literals
  - Reasoning about laziness
  - Tested (GHC bootstrapping & testsuite)

Future Work

- Optimise implementation (space & time)
- Test it on Hackage libraries
- GHC Extensions
- Better support for guards
  - ESC Haskell (Dana N. Xu, 2006)
  - Catch (Neil Mitchell et al, 2008)
  - Liquid Types (Niki Vazou et al, 2014)

Questions?