1)Introduction

Our problem here is to simulate a population that has a hospital and people get sick with a rate of 1/300. People get sick and they decide whether to go to hospital or not. We use exponential random values while deciding if a person gets sick, if a person decides to go to hospital while he is sick. And also we are given the rate of healing, with random exponential variables we determine the time it takes to heal. Our simulation is a multiple server process based simulation. It works as intended.

2) Abstract

Objective of the work is modeling a pandemic in the most basic design to asses business of an hospital, average healing time etc. It is modeled as a discrete event simulation and implemented with simpy, python 3.8.

3) Problem and Model

3.1) Problem Description

We have a problem such that there are people with a determined population, like an isolated village in Japan, in which people are continuously getting sick and healing back, and a person can get sick as soon as she feels good. There is a hospital with fixed number of beds. People got sick chooses whether they want to go to the hospital or not. The probability of a person's choosing to go to the hospital is 0.2. Probability of choosing to heal in home is 1-0.2 = 0.8.
If all the beds in the hospital are in use, the sick person is sent to his home and he is expected to heal in his home. These people get healed a little slower than the ones in the hospital. People who choose to heal in home are also get healed a little slower than the others.

3.2) Model

We try and create discrete event simulation with exponential arrival and departure rates. There are three different departure rates for the people who get sick. Our system(hospital) has fixed number of servers(beds and homes), which have three different service rate.

They are: ceiling(1453/24) = 61 beds and 1453 homes.

beds service rate is 1/6 days^{-1}, chosen home service rate is 1/10 days^{-1}, and obligatory home service rate is 1/6*r days^{-1}.
r is a real number uniformly distributed between [1,2] (U[1,2]).

Our arrival rate (people getting sick) is 1453/300 patients/day.

In other words, λ = N/300[patients/day], $(\mu_1)^{-1}$ = 6[days], $(\mu_1)^{-1}$ = 10[days], and $(\mu_3)^{-1} = (\mu_1)^{-1}$*r.

Our simulation model is subject to the fix arrival rate model, that is, it does not represent a simulation of a realistic pandemic, in which our arrival rate would be variable with respect to current sick and healthy people amount.
Our model takes advantage of memoryless property of exponentially distributed arrival rate, as it creates a person and makes him sick immediately, then determines his decision of place to heal with probability using a random number generator. Then, it determines his healing time(service time) according to his decision and the service rate of his decision, using a random.expovariate with the related service rate. Thus, his healing time is determined as soon as he created, in other words, he gets sick. After this the simulation determines an interarrival time according to our arrival rate, using a random.expovariate. Thus, the next person's arrival time(getting sick time) is determined.
-This way of thinking lacks the requirements of the pandemic simulation and differential equations to calculate them.-
After getting sick, a person get healed and a departure event occurs. Our model do not record a person's past data related to his sickness time. But it records every single arrival and departure events time as a timestamp to be used in calculating model responses. They are:
Our model also records interarrival times (as tuples of arrival and departure times) of current number of full beds in a list so that it will be used in calculating model responses. (bed_list)

Our model also records every sick persons service time in a list (service_times)
Our model also records number of sick people in the system at the moment of an event occurs (num_of_sick)
Our model also records number of used beds in the system at the moment of an event occurs. It also determines index of <bed_list> list variable. (num_of_used_beds)

Part 4) Numerical Analysis

The below 3 table represents 3 runs of the simulation with empty hospital, half full hospital, full hospital, with the same random number generator seed, and it shows the first 50 event occurred. Beds_Full represents number of beds full when that event occurs.

Empty beds at start:

Event_No	Sick_No	Simulation_Time	Num_of_Sick	Beds_Full	Treatment_Way	Event_Type
1	P1	0.321766	1	0	2	A
2	P2	0.49003	2	1	1	A
3	P3	0.627744	3	1	2	A
4	P4	0.77467	4	2	1	A
5	P5	1.24933	5	2	2	A
6	P6	1.32991	6	2	2	A
7	P7	1.64325	7	2	2	A
8	P8	1.91233	8	2	2	A
9	P2	1.98468	7	1	1	D
10	P9	2.14992	8	1	2	A
11	P10	2.1704	9	1	2	A
12	P11	2.35443	10	1	2	A
13	P12	2.62299	11	1	2	A
14	P13	2.76844	12	1	2	A
15	P14	2.79634	13	1	2	A
16	P15	2.9724	14	1	2	A
17	P16	3.06722	15	1	2	A
18	P17	3.11166	16	1	2	A
19	P18	3.29814	17	1	2	A
20	P19	3.39188	18	1	2	A
21	P20	3.71386	19	1	2	A
22	P6	3.75079	18	1	2	D
23	P11	3.92109	17	1	2	D
24	P21	4.08976	18	1	2	A
25	P22	4.17272	19	1	2	A
26	P23	4.18468	20	1	2	A
27	P5	4.27202	19	1	2	D
28	P24	4.44442	20	1	2	A
29	P25	4.72829	21	1	2	A
30	P21	4.74129	20	1	2	D
31	P26	5.30165	21	1	2	A
32	P27	5.31075	22	1	2	A
33	P28	5.51794	23	1	2	A
34	P29	5.55627	24	1	2	A
35	P30	5.57275	25	1	2	A
36	P31	5.63227	26	1	2	A
37	P32	5.67749	27	1	2	A

38	P33	5.67869	28	1	2	A
39	P32	5.74014	27	1	2	D
40	P14	5.78889	26	1	2	D
41	P12	5.91983	25	1	2	D
42	P34	5.93585	26	1	2	A
43	P35	5.98566	27	1	2	A
44	P36	6.44219	28	1	2	A
45	P37	6.89141	29	1	2	A
46	P4	7.3759	28	0	1	D
47	P38	7.46509	29	0	2	A
48	P39	7.5957	30	0	2	A
49	P1	7.73544	29	0	2	D
50	P40	8.04827	30	0	2	A

Half full beds at start:

Note that first 31 of them are arrivals at time 0 in order to simulate half full beds at the beginning and there are the next 50 events afterwards. You can see that their treatment way is 1 so they are in the hospital. As this is Markovian, memoryless property of exponential distribution allows us to do this.

Event_No	Sick_No	Simulation_Time	Num_of_Sick	Beds_Full	Treatment_Way	Event_Type
1	P1	0	1	1	1	A
2	P2	0	2	2	1	A
3	P3	0	3	3	1	A
4	P4	0	4	4	1	A
5	P5	0	5	5	1	A
6	P6	0	6	6	1	A
7	P7	0	7	7	1	A
8	P8	0	8	8	1	A
9	P9	0	9	9	1	A
10	P10	0	10	10	1	A
11	P11	0	11	11	1	A
12	P12	0	12	12	1	A
13	P13	0	13	13	1	A
14	P14	0	14	14	1	A
15	P15	0	15	15	1	A
16	P16	0	16	16	1	A
17	P17	0	17	17	1	A
18	P18	0	18	18	1	A
19	P19	0	19	19	1	A
20	P20	0	20	20	1	A
21	P21	0	21	21	1	A
22	P22	0	22	22	1	A
23	P23	0	23	23	1	A
24	P24	0	24	24	1	A
25	P25	0	25	25	1	A
26	P26	0	26	26	1	A
27	P27	0	27	27	1	A
28	P28	0	28	28	1	A
29	P29	0	29	29	1	A

30	P30	0	30	30	1	A
31	P31	0	31	31	1	A
32	P28	0.0158383	30	30	1	D
33	P30	0.296412	29	29	1	D
34	P23	0.472603	28	28	1	D
35	P21	0.550606	27	27	1	D
36	P11	0.560035	26	26	1	D
37	P4	0.94051	25	25	1	D
38	P32	1.02674	26	25	2	A
39	P33	1.19161	27	26	1	A
40	P25	1.19595	26	25	1	D
41	P24	1.21534	25	24	1	D
42	P9	1.24251	24	23	1	D
43	P22	1.25914	23	22	1	D
44	P34	1.37542	24	22	2	A
45	P35	1.55883	25	22	2	A
46	P36	1.76072	26	22	2	A
47	P33	1.9673	25	21	1	D
48	P37	1.99674	26	21	2	A
49	P38	2.05636	27	21	2	A
50	P12	2.06165	26	20	1	D
51	P39	2.17102	27	21	1	A
52	P34	2.43059	26	21	2	D
53	P40	2.57582	27	21	2	A
54	P32	2.66775	26	21	2	D
55	P17	3.13884	25	20	1	D
56	P41	3.14992	26	20	2	A
57	P42	3.15699	27	20	2	A
58	P43	3.26417	28	20	2	A
59	P26	3.38481	27	19	1	D
60	P44	3.39208	28	19	2	A
61	P18	3.48845	27	18	1	D
62	P45	3.53232	28	18	2	A
63	P5	3.74638	27	17	1	D
64	P8	4.01551	26	16	1	D
65	P1	4.0185	25	15	1	D
66	P45	4.16977	24	15	2	D
67	P46	4.23554	25	15	2	A
68	P36	4.28421	24	15	2	D
69	P47	4.4452	25	15	2	A
70	P48	4.54478	26	15	2	A
71	P40	4.69244	25	15	2	D
72	P49	4.78013	26	15	2	A
73	P50	5.03974	27	15	2	A
74	P51	5.0954	28	16	1	A
75	P52	5.1818	29	16	2	A
76	P43	5.18985	28	16	2	D
77	P53	5.35332	29	17	1	A
78	P27	5.35424	28	16	1	D
79	P54	5.84752	29	16	2	A
80	P55	6.02236	30	16	2	A

81	P54	6.18853	29	16	2	D

Beds are full at start:

Note that first 61 of them are arrivals at time 0 in order to simulate full beds at the beginning and there are the next 50 events afterwards. You can see that their treatment way is 1 so they are in the hospital. As this is Markovian, memoryless property of exponential distribution allows us to do this.

Event_No	Sick_No	Simulation_Time	Num_of_Sick	Beds_Full	Treatment_Way	Event_Type
1	P1	0	1	1	1	A
2	P2	0	2	2	1	A
3	P3	0	3	3	1	A
4	P4	0	4	4	1	A
5	P5	0	5	5	1	A
6	P6	0	6	6	1	A
7	P7	0	7	7	1	A
8	P8	0	8	8	1	A
9	P9	0	9	9	1	A
10	P10	0	10	10	1	A
11	P11	0	11	11	1	A
12	P12	0	12	12	1	A
13	P13	0	13	13	1	A
14	P14	0	14	14	1	A
15	P15	0	15	15	1	A
16	P16	0	16	16	1	A
17	P17	0	17	17	1	A
18	P18	0	18	18	1	A
19	P19	0	19	19	1	A
20	P20	0	20	20	1	A
21	P21	0	21	21	1	A
22	P22	0	22	22	1	A
23	P23	0	23	23	1	A
24	P24	0	24	24	1	A
25	P25	0	25	25	1	A
26	P26	0	26	26	1	A
27	P27	0	27	27	1	A
28	P28	0	28	28	1	A
29	P29	0	29	29	1	A
30	P30	0	30	30	1	A
31	P31	0	31	31	1	A
32	P32	0	32	32	1	A
33	P33	0	33	33	1	A

34	P34	0	34	34	1	A
35	P35	0	35	35	1	A
36	P36	0	36	36	1	A
37	P37	0	37	37	1	A
38	P38	0	38	38	1	A
39	P39	0	39	39	1	A
40	P40	0	40	40	1	A
41	P41	0	41	41	1	A
42	P42	0	42	42	1	A
43	P43	0	43	43	1	A
44	P44	0	44	44	1	A
45	P45	0	45	45	1	A
46	P46	0	46	46	1	A
47	P47	0	47	47	1	A
48	P48	0	48	48	1	A
49	P49	0	49	49	1	A
50	P50	0	50	50	1	A
51	P51	0	51	51	1	A
52	P52	0	52	52	1	A
53	P53	0	53	53	1	A
54	P54	0	54	54	1	A
55	P55	0	55	55	1	A
56	P56	0	56	56	1	A
57	P57	0	57	57	1	A
58	P58	0	58	58	1	A
59	P59	0	59	59	1	A
60	P60	0	60	60	1	A
61	P61	0	61	61	1	A
62	P28	0.0150295	60	60	1	D
63	P22	0.0453485	59	59	1	D
64	P62	0.135696	60	59	2	A
65	P63	0.165238	61	59	2	A
66	P64	0.196993	62	59	2	A
67	P19	0.231765	61	58	1	D
68	P65	0.246564	62	58	2	A
69	P66	0.513221	63	58	2	A
70	P4	0.599672	62	57	1	D
71	P35	0.819331	61	56	1	D
72	P11	0.870772	60	55	1	D
73	P38	0.915473	59	54	1	D
74	P67	1.0041	60	54	2	A
75	P34	1.10329	59	53	1	D
76	P52	1.1307	58	52	1	D
77	P68	1.27787	59	52	2	A
78	P69	1.28476	60	52	2	A
79	P54	1.41855	59	51	1	D
80	P9	1.50897	58	50	1	D
81	P16	1.58508	57	49	1	D
82	P50	1.7072	56	48	1	D
83	P27	1.76624	55	47	1	D
84	P24	1.82978	54	46	1	D

85	P18	1.89288	53	45	1	D
86	P1	1.90524	52	44	1	D
87	P14	2.0732	51	43	1	D
88	P48	2.26002	50	42	1	D
89	P49	2.31691	49	41	1	D
90	P40	2.37181	48	40	1	D
91	P42	2.56996	47	39	1	D
92	P10	2.64148	46	38	1	D
93	P70	2.70935	47	38	2	A
94	P71	2.82378	48	38	2	A
95	P72	2.87145	49	38	2	A
96	P31	2.89941	48	37	1	D
97	P51	2.90378	47	36	1	D
98	P58	2.95571	46	35	1	D
99	P73	3.03012	47	35	2	A
100	P74	3.03537	48	36	1	A
101	P72	3.17407	47	36	2	D
102	P39	3.17643	46	35	1	D
103	P75	3.17977	47	35	2	A
104	P76	3.34078	48	35	2	A
105	P45	3.38796	47	34	1	D
106	P77	3.42137	48	34	2	A
107	P67	3.65086	47	34	2	D
108	P78	3.76942	48	35	1	A
109	P41	3.78344	47	34	1	D
110	P79	3.87995	48	34	2	A
111	P80	4.1999	49	34	2	A

In this part we conduct numerical analysis to compare results of different simulation runs with each other and with the theoretical values.

Theoretical values:

Probability of the hospital being empty: {[$\sum_{n=0}^{60}(61\rho)^n/n!$] + [$(61\rho)^{61} * \left(\frac{1}{61!}\right) * \frac{1}{1-\rho}$]}$^{-1}$

Average number of occupied beds in the hospital: $\hat{L}_1$ (t)= λ*W, where W = $\frac{0.8}{10} + 0.2 * \frac{1-q}{6} + q * 0{,}2 * \frac{1{,}5}{6}$,

where q = P_k, which is probability of hospital being full.

Average proportion of sick people: L/N = λ*W/N = W/300, where W = (above)

Average sickness time: $\hat{L}$ =(1/T)*$\int L(t)dt$

Seed1=123

Time	Bed start	Probability of being empty	Sample mean of occupied beds	Sample variance of occupied beds	Average proportion of sick people	Sample mean of sickness times	Sample variance of sickness times
1000	Empty	0.0031943845650746496	5.85297989477414	6.063881484836947	0.030895619555830672	9.180232150229442	88.97537548990672

1000	Half full	0.002336995351904022	5.891563592551417	7.995821843732945	0.03137501944364821	9.315059920641776	94.49521058289425
1000	Full	0.005490851106226614	5.99213499468393	18.689179834120296	0.03061837431380165	9.107164355773552	88.20404927076879
10000	Empty	0.003712792510544932	5.71769477770153	5.700146321448181	0.030492456037007953	9.166157444093958	90.13663006554061
10000	Half full	0.003170733492554504	5.851010934756371	6.012463833545147	0.03126107519820915	9.25060939737646	90.67791656561654
10000	Full	0.00527002093216521	5.6678807223614385	6.5495416675801374	0.03044752500926478	9.2259454951747	90.53100971728107
100000	Empty	0.0027288954028976105	5.838789696690722	5.731188151485721	0.030703581711942648	9.198678771645506	89.64665032441356
100000	Half full	0.00357434742890985	5.766492553370028	5.883626077391689	0.03071513396006736	9.203657955145509	89.52467743660296
100000	Full	0.0032107711517664786	5.803447294396889	5.785701922313949	0.030675034152113145	9.206793654106193	89.81160517227711

Seed2 = 246

Time	Bed start	Probability of being empty	Sample mean of occupied beds	Sample variance of occupied beds	Average proportion of sick people	Sample mean of sickness times	Sample variance of sickness times
1000	Empty	0.004410002510374786	6.029567283705626	5.85449066348369	0.030722223526913325	9.111939331415607	87.61641087196365
1000	Half full	0.0067077192979913885	5.740515990219585	7.457975866360358	0.031234175178362952	9.22986710070396	87.26428289137128
1000	Full	0.00382943849628154	6.173126070551145	17.554873370288117	0.03103355798285887	9.131583586288768	89.83744541505159
10000	Empty	0.002955223578584041	5.766186854892481	5.929724713363326	0.03083092446707773	9.22401129404603	89.62299396107076
10000	Half full	0.003362026621624998	5.755111605080259	6.04895094271447	0.030572154341827515	9.150172051553211	87.8367493569232
10000	Full	0.0036824628695981574	5.831993338766509	6.358915340455512	0.030393738575015594	9.08685229413532	86.63868345672634

100000	Empty	0.002933373417142122	5.787234169128349	5.781352963484322	0.030784083573281135	9.222568635742311	90.0130062816964
100000	Half full	0.0027968992318267156	5.829035456617678	5.84068338070469	0.030750456723370435	9.216180893527536	90.06715900013755
100000	Full	0.003388821709059788	5.800519213315356	5.947531500340241	0.03057663656176024	9.185875217456616	89.65041942092365

Comparison:

-Probability of being empty is expected to be lower when the hospital is full. But in our case, our simulation converges to an optimal value. Because of this speed we cannot see a difference depending on bed fullness.

-We expected sample mean of occupied beds to differ as bed start condition changes. However, this change gets insignificant as the time interval increases.

-Sample variance of occupied beds are very similar with each other. We expected time independency because of the fact that our simulation converges fast. Also, because of the same reason we don't see any fluctuations in different starting conditions of beds.

-Average proportion of sick people has converged to 0.03 even at the least time interval. We can say that it is independent of time because it is affected by healing rate.

-Healing rate determines sample mean of sickness times. So, it is irrelevant to total time or other factors. And we can see it directly from our table, time or fullness of hospital didn't change the result.

-Sample variance of sickness times occurs due to healing rate differences which are given as mu1, mu2 and mu3. And this difference doesn't change with time or fullness of the hospital as we can observe at our table.

-Seed2's numerical values are parallel with Seed1. Since the simulation depends on lambda and mu values, random numbers do not change the behaviour of simulation.

5) Conclusion

Our model works as intended, and gives correct results. Comparisons between theoretical values are lacked, due to time limitations.