ECE M146 | Spring 2019 Quiz
Machine Learning | Wednesday, April 3th, 2019
Instructor: Lara Dolecek | Maximum score: 40 points

Maximum score is 40 points. You have 25 minutes to complete the quiz. Please show your work.

Good luck!

Instructions

- You can attempt the problems in any order as long as it is clear which problem is being attempted and which solution to the problem you want us to grade.
- If you are stuck in any part of a problem do not dwell on it, try to move on and attempt it later.
- Please solve every problem in **the space provided**.
- You may find the following useful.
  - $\int \exp(ax)dx = \frac{\exp(ax)}{a} + C$ where C is an arbitrary constant
  - $||\boldsymbol{x}||_1 = \sum_{i=1}^{n} |x_i|$, $||\boldsymbol{x}||_\infty = \max_i |x_i|$
  - $\frac{d(\ln(x))}{dx} = \frac{1}{x}$

Your Name:

Your ID Number:

Problem	Score	Possible
1		10
2		14
3		16
Total		40

1. Calculus **(10 pts)**

   (a) Let $f(\boldsymbol{x}) = \ln(ax_1x_2 + bx_1 + cx_1^2)$. What is the partial derivative of f with respect to x_1?
   Solution:

$$\begin{aligned}\frac{\partial f(\boldsymbol{x})}{\partial x_1} &= \frac{1}{ax_1x_2 + bx_1 + cx_1^2}\frac{\partial(ax_1x_2 + bx_1 + cx_1^2)}{\partial x_1} \\ &= \frac{ax_2 + b + 2cx_1}{ax_1x_2 + bx_1 + cx_1^2}\end{aligned}$$

   (b) Evaluate $\int_b^\infty a\exp(-a(x-b))dx$ where $a > 0$?
   Solution:
   Substitute $x - b = t$, $dx = dt$. We have

$$\begin{aligned}\int_b^\infty a\exp(-a(x-b))dx &= \int_o^\infty a\exp(-at)dt \\ &= a\left[\frac{\exp(-at)}{-a}\right]_0^\infty \\ &= 1\end{aligned}$$

2. Probability **(14 pts)**
Suppose A and B are two events. Which of these statements is **true/false**? Explain.

(a) A and B are mutually exclusive events then $P(A \cup B) = P(A) + P(B) - P(A)P(B)$
Solution:
False. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B)$.

(b) $P(A \cap B \cap C) = P(A)P(B|A)P(C|B)$
Solution:
False. $P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B)$.

(c) A is a Gaussian random variable with $\mu = 0$ and $\sigma^2 = 1$ then $P(A = 0) = 0.5$
Solution:
False. For continuous random variable $P(A = c) = 0$.

(d) If A and A^c are independent, where A^c denotes the complement of event A then $0 < P(A) < 1$.
Solution:
False. $P(A \cap A^c) = 0 = P(A)P(A^c) = P(A)(1 - P(A))$. Therefore $P(A)$ must be either 0 or 1.

(e) Assume X is a random variable. The variance of X is defined as $Var(X) = E[(X - E[X])^2]$. Prove that $Var(aX + b) = a^2 Var(X)$.
Solution:

$$
\begin{aligned}
Var(aX - b) &= E[(aX + b - E[aX + b])^2] \\
&= E[(a(X - E[X]))^2] \\
&= a^2 E[(X - E[X])^2] \\
&= a^2 Var(X)
\end{aligned}
$$

3. Linear Algebra **(16 pts)**

Consider the vector $\boldsymbol{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(a) Compute $\frac{||\boldsymbol{x}||_2}{||\boldsymbol{x}||_1}$.
Solution:

$$\begin{aligned} ||\boldsymbol{x}||_2 =& \sqrt{x_1^2 + x_2^2} = \sqrt{5} \\ ||\boldsymbol{x}||_1 =& |x_1| + |x_2| = 3 \\ \frac{||\boldsymbol{x}||_2}{||\boldsymbol{x}||_1} =& \frac{\sqrt{5}}{3} \end{aligned}$$

(b) Compute $\boldsymbol{x}\boldsymbol{x}^T$.
Solution:

$$\boldsymbol{x}\boldsymbol{x}^\top = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$

(c) Find $c \in \mathbb{R}$ such that $||c\boldsymbol{x}||_2 < 1$

$$\begin{aligned} ||c\boldsymbol{x}||_2 = |c|||\boldsymbol{x}||_2 < 1 \\ |c| < \frac{1}{\sqrt{5}} \\ c \in (-\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}}) \end{aligned}$$