ECE M146 Discussion 4
Introduction to Machine Learning Friday, April 26, 2019
Instructor: Lara Dolecek
TA: Zehui (Alex) Chen, Ruiyi (John) Wu

1. Introduction on optimization problem
   (a) Convex sets.
   (b) Convex functions.
   (c) Optimization problem in standard form.
      - Convex optimization.
   (d) Global and local optimal.
   (e) Duality.
      - Lagrange dual problem.
      - Geometric interpretation.
      - KKT conditions.

2. Find the dual problem of the following Quadratic program

$$\begin{array}{ll} \text{minimize}_x & x^T P x \\ \text{subject to} & Ax \leq b \end{array}$$

Assume $P \in \mathcal{S}^n_{++}$.

3. Quadratic program example Consider the objective function

$$J(x_1, x_2) = 5x_1^2 + 4x_1x_2 + 2x_2^2 + 2x_1 - 4x_2.$$

Find the optimal x that minimize $J(x)$ under the following constrains:

(a) No constrain.

(b) $x_1 + x_2 + 2 = 0$.

(c) $x_1 + x_2 + 2 \leq 0$.

(d) $x_1 + x_2 + 2 \geq 0$.

4. Relation soft SVM loss and logistic regression

   We learned that the loss function for the primal problem of SVM is of the form

$$\frac{1}{2}\|w\|^2 + C\sum_{n=1}^{m}\xi_n.$$

   We have seen that for data points that are on the correct side of the margin boundary, and which therefore satisfy $y_n h(x_n) \geq 1$, we have $\xi_n = 0$. For the remaining points we have $\xi_n = 1 - y_n h(x_n)$. Thus the objective function can be written in the form

$$\sum_{n=1}^{N} E_{SV}(y_n h(x_n)) + \lambda\|w\|^2.$$

   where $\lambda = (2C)^{-1}$, find out what is the error function $E_{SV}(.)$ in terms of $y_n h(x_n)$.