

### Objective #3

Learn how a pure plane wave code functions

### How?

Implement a (toy) plane wave code to calculate the band structure of Si

- To plot the band structure of Si (as predicted by Slater exchange)
  - 1). Initial setup (reciprocal-space grid (basis), kinetic energy cutoff, etc.)
  - 2). Converge density
  - 3). Determine eigenvalues at “special” KPs (irreducible BZ)

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### How?

Implement a (toy) plane wave code to calculate the band structure of Si

- 1). Initial setup
  - Choice of k-point mesh: Monkhorst-Pack (216 k-points)
  - Kinetic energy cutoff: 150 Ry
  - Si lattice parameter: 10.26 Bohr
  - FCC (real and reciprocal) lattice vectors and atomic positions
  - Pseudopotential (local and/or non-local): Appelbaum-Hamann

$$v(G) = \exp(-G^2/4\alpha) \left( \underbrace{-Z_{ion} \frac{4\pi}{G^2}}_{\text{Long-Range}} + \underbrace{\left(\frac{\pi}{\alpha}\right)^{3/2} \left( v1 + \frac{v2}{\alpha} \left( \frac{3}{2} - \frac{G^2}{4\alpha} \right) \right)}_{\text{Short-Range}} \right)$$

- Similar to the local component of the GTH pseudopotentials used in PySCF

$$V_{\text{loc}}(g) = -\frac{4\pi Z_{\text{ion}}}{\Omega g^2} e^{-(gr_{\text{loc}})^2/2} + \sqrt{8\pi^3} \frac{r_{\text{loc}}^3}{\Omega} e^{-(gr_{\text{loc}})^2/2} \times \{ C_1 + C_2(3 - g^2 r_{\text{loc}}^2) + C_3[15 - 10(gr_{\text{loc}})^2 + (gr_{\text{loc}})^4] + C_4[105 - 105(gr_{\text{loc}})^2 + 21(gr_{\text{loc}})^4 - (gr_{\text{loc}})^6] \}$$

$$\alpha = \frac{1}{2r_{\text{loc}}^2}$$

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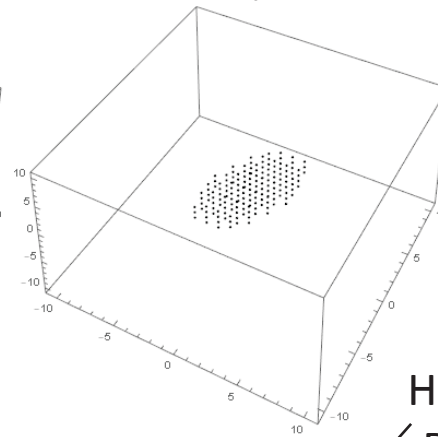
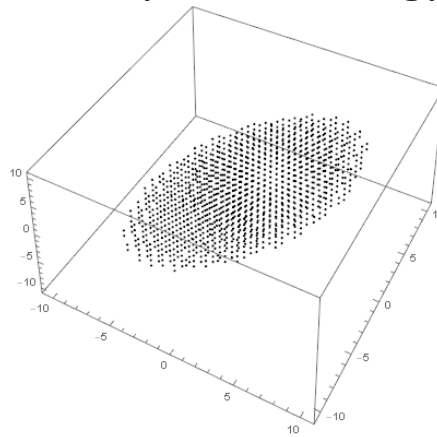
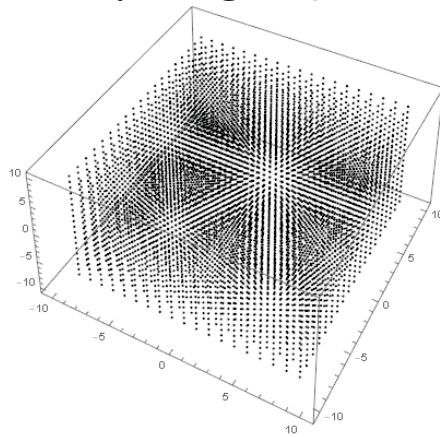
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### How?

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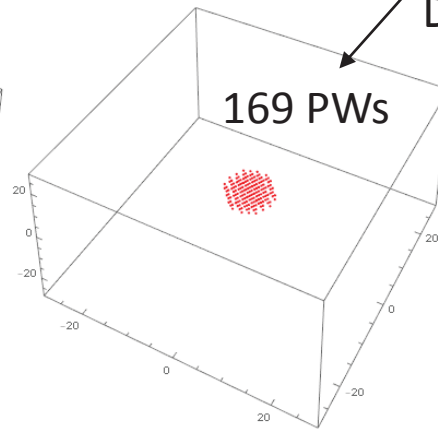
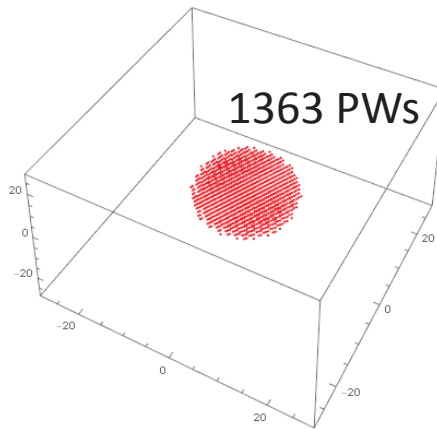
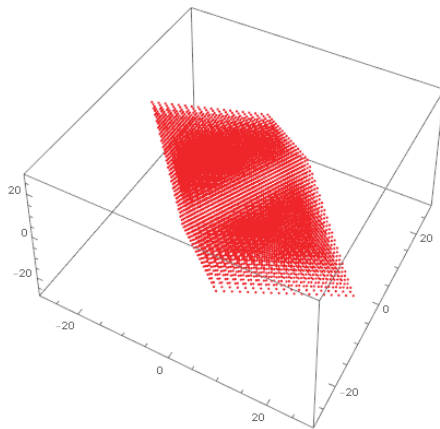
- 1). Initial setup
  - Reciprocal-space grid (determined by kinetic energy cutoff, ECUT)

Miller Indices



Hamiltonian Dimension

Reciprocal Space



Initial Space

$\rho$  Grid:  $G^2 < 4(\text{ECUT})$

$\phi$  Grid:  $G^2 < \text{ECUT}$

- # of plane waves for density  $\approx 8 \times$  (# of plane waves for orbitals)

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- 2). Converge density
  - Form Hamiltonian
    - Diagonal elements get kinetic energy contribution,  $\propto (\mathbf{k} + \mathbf{G})^2$
    - All elements get potential (pseudo x structure factor + Coulomb +  $V_{xc}$ )
  - Diagonalize Hamiltonian
    - This gives eigenvalues (for plotting band structure) as well as eigenvectors for evaluating the density
  - Accumulate density (over all k-points) by summing over occupied bands
- Check convergence of density
- Evaluate  $V_{xc}$  (on real space grid, then FFT to reciprocal space)
- Evaluate Coulomb (in reciprocal space, after IFFT of density to reciprocal space)
  - $4\pi\rho(\mathbf{G})/\mathbf{G}^2$
- Iterate until convergence

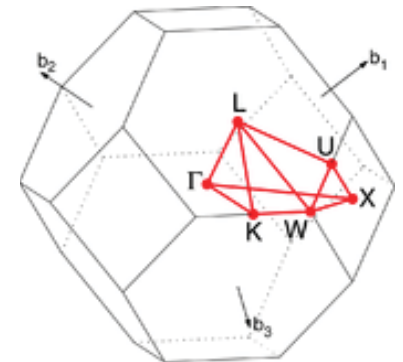
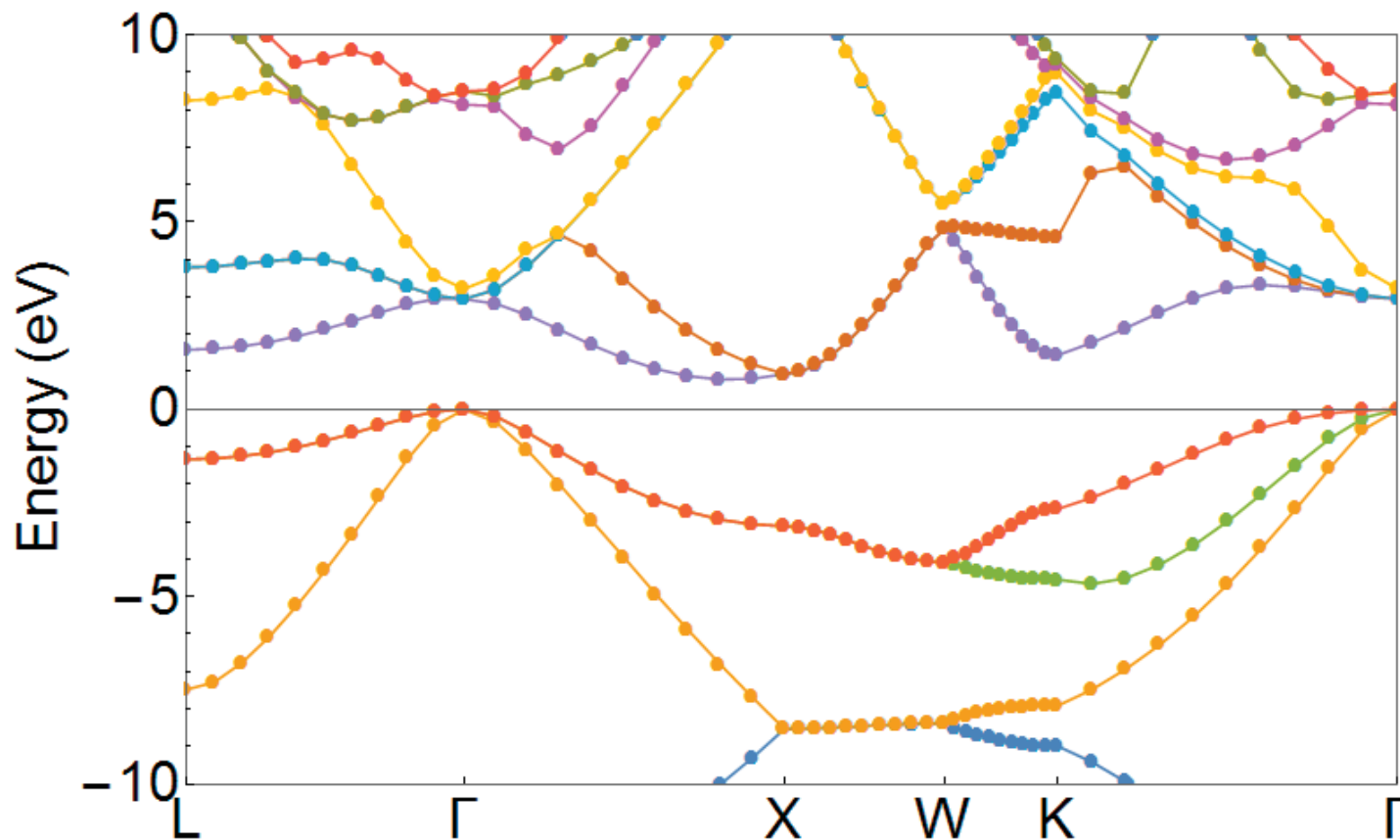
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### How?

Implement a (toy) plane wave code to calculate the band structure of Si

- 3). Determine eigenvalues at “special” KPs (non-self-consistently)
  - Set up a new k-point mesh (over N special points)
  - Form Hamiltonian (using converged density) and diagonalize N times



The indirect band gap is 0.795 eV

The direct band gap is 2.94 eV