PHYC 3590 - Advanced Classical Mechanics
Assignment 4

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Problem 4.2

(a)

$$
\begin{aligned}
W &= \int_0^P F \cdot dr \\
&= \int_0^Q F \cdot dr + \int_Q^P F \cdot dr \\
&= \int_0^1 F_x dx + \int_Q^P F_y dy \\
&= \int_0^1 x^2 dx + \int_Q^P 2xy\, dy \\
&= \left[\frac{x^3}{3}\right]_0^1 + \int_Q^P 2xy\, dy \\
&= \frac{1}{3} + \int_Q^P 2xy\, dy
\end{aligned}
$$

On the path from Q to P, x is equal to 1.

$$
\begin{aligned}
&= \frac{1}{3} + \int_0^1 2y\, dy \\
&= \frac{1}{3} + [y^2]_0^1 \\
W &= \frac{1}{3} + 1 \\
W &= \boxed{\frac{4}{3}}
\end{aligned}
$$

(b)

$$
\begin{aligned}
W &= \int_0^P F \cdot dr \\
&= \int_0^P F_x dx + \int_0^P F_y dy \\
&= \int_0^P x^2\,dx + \int_0^P 2xy\,dy \\
y =& x^2, \quad dy = 2x\,dx \\
W &= \int_0^P x^2\,dx + \int_0^P 4x^4\,dx \\
&= \int_0^1 x^2\,dx + \int_0^1 4x^4\,dx \\
&= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{4x^5}{5}\right]_0^1 \\
=& \frac{1}{3} + \frac{4}{5} \\
W =& \boxed{\frac{17}{15}}
\end{aligned}
$$

(c)

$$
\begin{aligned}
W &= \int_0^{x=1} x^2\,dx + \int_0^{y=1} 2xy\,dy \\
W &= \int_0^{t=1} t^6\,dx + \int_0^{t=1} 2t^3t^2\,dy \\
dx =& 3t^2\,dt, \quad dy = 2t\,dt \\
W &= \int_0^{t=1} t^6 \cdot 3t^2\,dt + \int_0^{t=1} 2t^5 \cdot 2t\,dt \\
W &= \int_0^1 3t^8\,dt + \int_0^1 4t^6\,dt \\
W &= \left[\frac{2x^9}{9}\right]_0^1 + \left[\frac{4x^7}{7}\right]_0^1 \\
W =& \frac{3}{9} + \frac{4}{7} \\
W =& \boxed{\frac{19}{21}}
\end{aligned}
$$

Problem 4.8

$$\vec{F} = \vec{W} + \vec{N}$$

In the $\hat{x}$ vector we have:

$$F_x = mg\sin\theta$$

In the $\hat{y}$ vector we have:

$$F_y = mg + mg\cos\theta$$

For the total energy of the puck we have

$$\begin{aligned} E &= mgR \\ E &= T + U \\ U &= mgh = mgR\cos\theta \\ T &= \tfrac{1}{2}mv^2 \\ mgR &= \tfrac{1}{2}mv^2 - mgR\cos\theta \end{aligned}$$

Problem 4.22

The goad of this problem is to prove that the Couolomp force is conservative.

$$\begin{aligned}\vec{F} =& \frac{\gamma}{r^2}\hat{r} \\ \Delta \times F =& \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta F_\phi) - \frac{\partial}{\partial\phi}F_\phi\right]\hat{r} + \left[\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}F_r - \frac{1}{r}\frac{\partial}{\partial r}(rF_\phi)\right]\hat{\theta} \\ &+ \frac{1}{r}\left[\frac{\partial}{\partial r}(rF_\theta) - \frac{\partial}{\partial\theta}F_r\right]\hat{\phi}\end{aligned}$$

$\vec{F}$ is equal zero in the $\hat{\theta}$ and $\hat{\phi}$ direction so all terms containing F_ϕ, F_θ, $\frac{\partial}{\partial\theta}$, $\frac{\partial}{\partial\phi}$ go to zero. Every term contains at least of of these listed elements, therefore

$$\Delta \times F = 0$$