Newton's Laws of Motion

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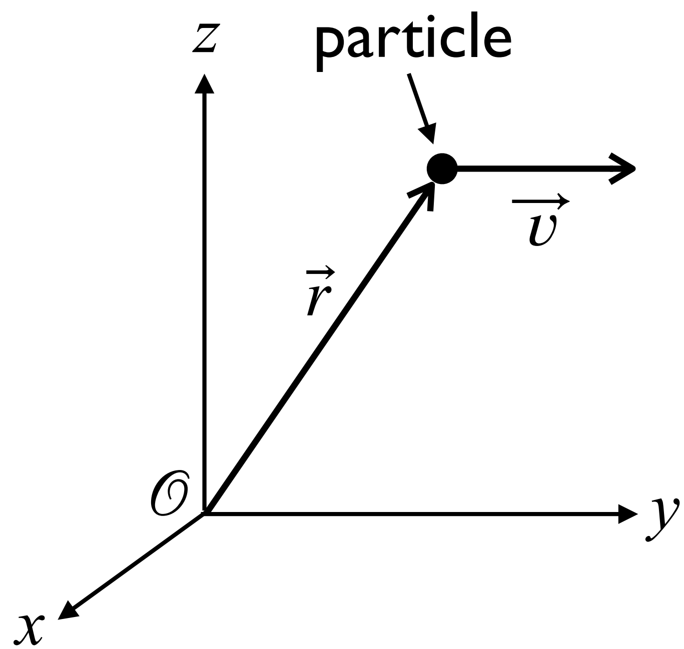
Newton's First Law

(the law of inertia)

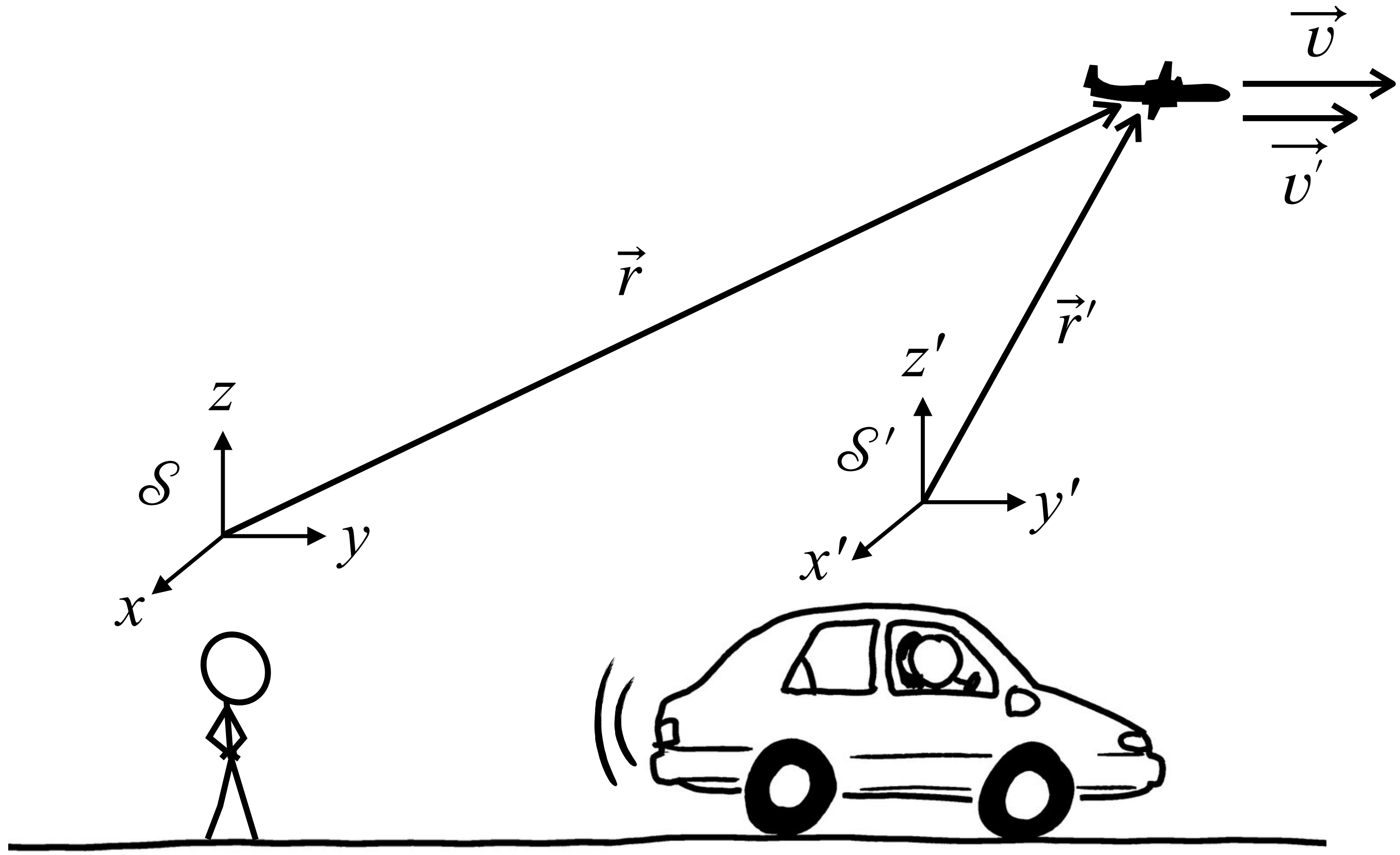
"In the absence of a net force, a particle moves with constant velocity $\vec{v}$."



By definition,

$$\vec{v} = \frac{d\vec{r}}{dt} \equiv \dot{\vec{r}}$$

A *frame of reference* is a system of coordinate axes relative to which positions, velocities, and accelerations can be measured.



An *inertial frame* is a frame of reference in which Newton's first law holds.

Newton's laws only apply in inertial frames.

Image source: xkcd

Newton’s Second Law

“For any particle of inertial mass m, the net force $\overrightarrow{F}$ on the particle is related to particle’s acceleration $\overrightarrow{a}$ by

$$\boxed{\overrightarrow{F} = m\overrightarrow{a}}$$ ”

Here, $$\overrightarrow{a} = \frac{d\overrightarrow{v}}{dt} \equiv \dot{\overrightarrow{v}} = \frac{d^2\vec{r}}{dt^2} \equiv \ddot{\vec{r}} \; .$$

Defining linear momentum as $\overrightarrow{p} = m\vec{v}$ allows us to rewrite Newton’s second law as

$$\boxed{\overrightarrow{F} = \dot{\overrightarrow{p}}}$$

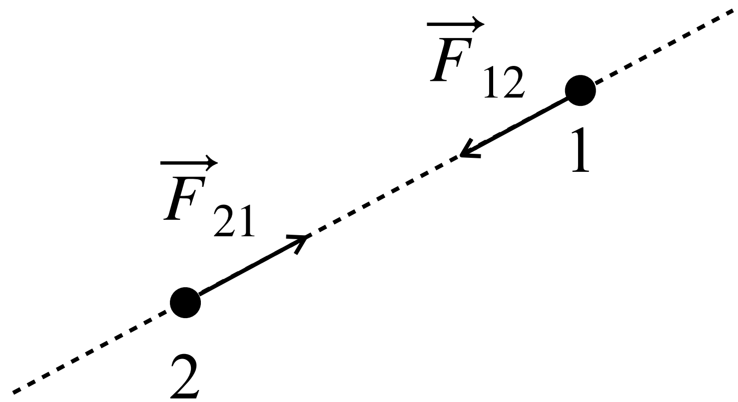
(assuming m is conserved).

Newton's Third Law

"If object 1 exerts a force $\overrightarrow{F}_{21}$ on object 2, then object 2 exerts a reaction force $\overrightarrow{F}_{12}$ on object 1 given by

$$\boxed{\overrightarrow{F}_{12} = -\overrightarrow{F}_{21}}$$"

e.g. electrostatic or gravitational attraction



Forces that act along the line through the body centres are called *central forces*.

Newton's Law of Gravitation

"Every two particles attract each other with a force proportional to the product of their gravitational masses divided by the squared distance between them."

The magnitude of the force is

$$\boxed{F_{\mathrm{g}} = \frac{GmM}{r^2}}$$

where $G = 6.6743 \times 10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}$ is the universal gravitational constant.

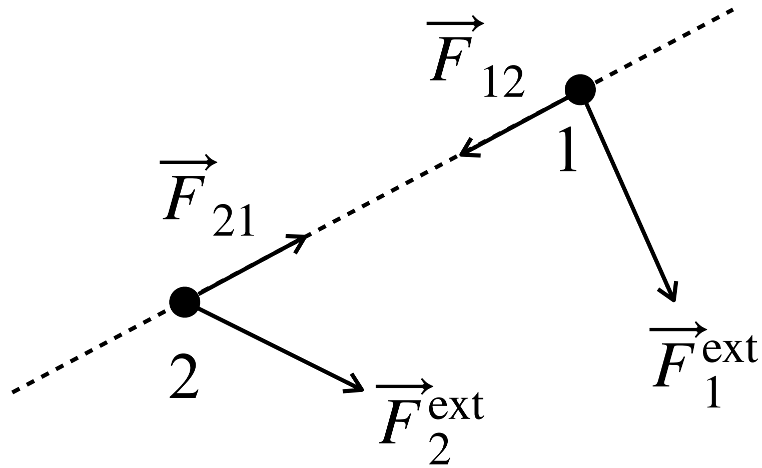
Gravity is a central force.

It is an experimental fact that inertial mass and gravitational mass are equal.

Conservation of Linear Momentum

Two Particles

Consider two particles that exert central forces on each other and are also subjected to external forces.



The net forces on each particle are

$$\overrightarrow{F}_1 = \overrightarrow{F}_{12} + \overrightarrow{F}_1^{\text{ext}} \quad \text{and} \quad \overrightarrow{F}_2 = \overrightarrow{F}_{21} + \overrightarrow{F}_2^{\text{ext}}$$

Substituting Newton's second law gives

$$\dot{\vec{p}}_1 = \vec{F}_{12} + \vec{F}_1^{\text{ext}} \quad \text{and} \quad \dot{\vec{p}}_2 = \vec{F}_{21} + \vec{F}_2^{\text{ext}}$$

The total momentum is

$$\vec{P} = \vec{p}_1 + \vec{p}_2$$

$$\begin{aligned} \rightarrow \dot{\vec{P}} &= \dot{\vec{p}}_1 + \dot{\vec{p}}_2 \\ &= \vec{F}_{12} + \vec{F}_1^{\text{ext}} + \vec{F}_{21} + \vec{F}_2^{\text{ext}} \end{aligned}$$

But $\vec{F}_{12} = -\vec{F}_{21}$ (Newton's third law), whence

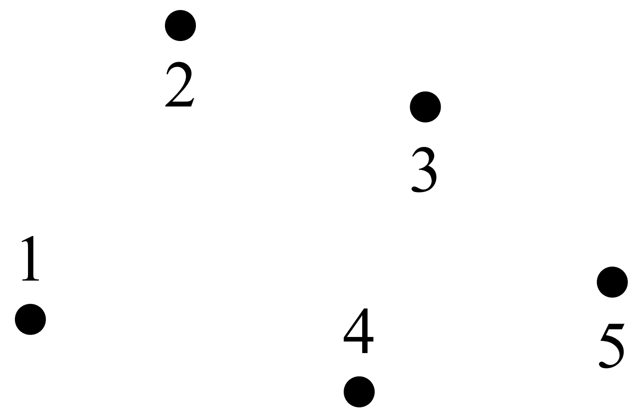
$$\dot{\vec{P}} = \vec{F}_1^{\text{ext}} + \vec{F}_2^{\text{ext}}$$

$$\rightarrow \boxed{\dot{\vec{P}} = \vec{F}^{\text{ext}}}$$

$\Rightarrow$ In the absence of a net external force ($\vec{F}^{\text{ext}} = 0$), total linear momentum is conserved ($\vec{P} = \text{const}$).

Many Particles

Consider N particles.



Let $\vec{F}_\alpha$ be the net force on particle α.

Using Newton's second law,

$$\dot{\vec{p}}_\alpha = \vec{F}_\alpha = \sum_{\beta \neq \alpha} \vec{F}_{\alpha\beta} + \vec{F}_\alpha^{\text{ext}}$$

The total momentum for all N particles is

$$\vec{P} = \sum_\alpha \vec{p}_\alpha$$

$$\rightarrow \dot{\vec{P}} = \sum_\alpha \dot{\vec{p}}_\alpha = \sum_\alpha \sum_{\beta \neq \alpha} \vec{F}_{\alpha\beta} + \sum_\alpha \vec{F}_\alpha^{\text{ext}}$$

But

$$\sum_\alpha \sum_{\beta \neq \alpha} \vec{F}_{\alpha\beta} = \sum_\alpha \sum_{\beta > \alpha} (\vec{F}_{\alpha\beta} + \vec{F}_{\beta\alpha})$$

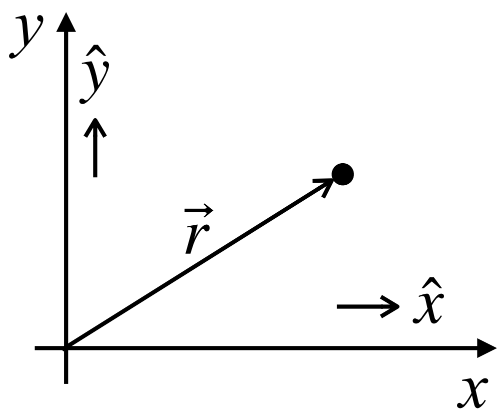
which is zero since $\vec{F}_{\alpha\beta} = -\vec{F}_{\beta\alpha}$ (Newton's third law).

$$\rightarrow \dot{\vec{P}} = \sum_\alpha \vec{F}_\alpha^{\text{ext}} \qquad \rightarrow \boxed{\dot{\vec{P}} = \vec{F}^{\text{ext}}}$$

$\Rightarrow$ In the absence of a net external force ($\vec{F}^{\text{ext}} = 0$), total linear momentum is conserved ($\vec{P} = \text{const}$).

Newton's Second Law in Cartesian Coordinates

(also called *rectangular* coordinates)



$\hat{x}$ and $\hat{y}$ are invariant with position

The position vector is written

$$\vec{r} = x\hat{x} + y\hat{y}$$

The unit vectors $\hat{x}$ and $\hat{y}$ are constant in time, so

$$\ddot{\vec{r}} = \ddot{x}\hat{x} + \ddot{y}\hat{y}$$

The force vector can similarly be written

$$\overrightarrow{F} = F_x \hat{x} + F_y \hat{y}$$

Substituting into Newton's second law, $\overrightarrow{F} = m\ddot{\vec{r}}$, gives

$$F_x \hat{x} + F_y \hat{y} = m\ddot{x}\hat{x} + m\ddot{y}\hat{y}$$

Comparing like components yields

$$\boxed{\begin{array}{c} F_x = m\ddot{x} \\ F_y = m\ddot{y} \\ \text{and} \\ F_z = m\ddot{z} \end{array}}$$

where a third coordinate, z, has been added.

$\Rightarrow$ In Cartesian coordinates, a scalar form of Newton's second law applies independently to each dimension.