

PHYC 3590xx - Advanced Classical Mechanics

Assignment 3

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Problem 3.5

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2' \quad (1)$$

$$\vec{v}_2 = \vec{p}_2 = 0 \quad (2)$$

$$m_1 \vec{v}_1 = m_1 \vec{v}_1' + m_2 \vec{v}_2' \quad (3)$$

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2 \quad (4)$$

$$v_2 = 0 \quad (5)$$

$$m_1 v_1^2 = m_1 v_1'^2 + m_2 v_2'^2 \quad (6)$$

$$m_1 = m_2 \quad (7)$$

$$v_1^2 = v_1'^2 + v_2'^2 \quad (8)$$

$$(9)$$

From equation (3) we have

$$m_1 \vec{v}_1 = m_1 \vec{v}_1' + m_2 \vec{v}_2' \quad (10)$$

$$m_1^2 \vec{v}_1^2 = (m_1 \vec{v}_1' + m_2 \vec{v}_2')^2 \quad (11)$$

$$m_1^2 \vec{v}_1^2 = m_1^2 \vec{v}_1'^2 + m_2^2 \vec{v}_2'^2 + 2m_1 m_2 \vec{v}_1' \cdot \vec{v}_2' \quad (12)$$

$$m_1 = m_2 = m \quad (13)$$

$$\vec{v}_1^2 = \vec{v}_1'^2 + \vec{v}_2'^2 + 2\vec{v}_1' \cdot \vec{v}_2' \quad (14)$$

Equating equation (8) and (14) we find

$$v_1'^2 + v_2'^2 = \vec{v}_1'^2 + \vec{v}_2'^2 + 2\vec{v}_1' \cdot \vec{v}_2'$$

$$\vec{v}_1' \cdot \vec{v}_2' = 0$$

$$|v_1'| \cdot |v_2'| \cos \theta = 0$$

$$\theta = \boxed{90^\circ}$$

Problem 3.11

(a)

Show that the equation of motion is $m\dot{v} = -\dot{m}v_{ex} + F^{ext}$.

$$\begin{aligned}
 dP &= m dv + dm v_{ex} \\
 \dot{P} &= \frac{dP}{dt} = F^{ext} \\
 dP &= F^{ext} dt \\
 F^{ext} dt &= m dv + dm v_{ex} \\
 F^{ext} &= m \frac{dv}{dt} + \frac{dm}{dt} v_{ex} \\
 m\dot{v} &= -\dot{m}v_{ex} + F^{ext} \\
 &\text{QED}
 \end{aligned}$$

(b)

$$\begin{aligned}
 m\dot{v} &= -\dot{m}v_{ex} - mg \\
 m\dot{v} &= kv_{ex} - (m_0 - kt)g \\
 m \frac{dv}{dt} &= kv_{ex} - (m_0 - kt)g \\
 dv &= \frac{kv_{ex} - m_0g + ktg}{m} dt
 \end{aligned}$$

I am not sure how to finish this problem.

(c)

...

(d)

...

Problem 3.21

$$CM = \int r \sigma dA$$

$$\sigma = \frac{m}{A} = \frac{m}{\pi R^2}$$

$$dA = r dr d\theta$$

$$CM = \int \int \frac{m}{\pi R^2} r^2 dr d\theta$$

$$CM = \frac{m}{\pi R^2} \int_0^R r^2 dr \int_0^{2\pi} d\theta$$

$$CM = \frac{m}{\pi R^2} (R^3/3)(2\pi)$$

$$CM = \boxed{\frac{2mR}{3}}$$

Problem 3.27

(a)

$$l = \vec{r} \times \vec{p}$$

$$\vec{p} = m\vec{v}$$

$$l = \vec{r} \times m\vec{v}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{r}\hat{r} + r\dot{\phi}\hat{\phi}$$

$$l = \vec{r} \times m(\dot{r}\hat{r} + r\dot{\phi}\hat{\phi})$$

$$l = r\hat{r} \times m(\dot{r}\hat{r} + r\dot{\phi}\hat{\phi})$$

$$l = r\hat{r} \times m(\dot{r}\hat{r} + r\dot{\phi}\hat{\phi})$$

$$l = r\hat{r} \times mr\dot{\phi}\hat{\phi}$$

$$\omega = \dot{\phi}$$

$$l = r\hat{r} \times mr\omega\hat{\phi}$$

$$l = \boxed{mr^2\omega(\hat{r} \times \hat{\phi})}$$

(b)

...

Problem 3.32

$$L_z = I\omega$$

$$I = \sum m_{\alpha} \rho_{\alpha}^2$$

$$I = \int_v \text{Density} \cdot \rho_{\alpha}^2 dV$$

$$I = \int_v \frac{m}{V} r^2 dr \sin \theta d\theta d\phi$$

$$I = \int_v \frac{3m}{4\pi R^3} r^2 dr \sin \theta d\theta d\phi$$

$$I = \frac{3m}{4\pi R^3} \int_0^R r^2 dr \int_0^{\pi} \sin \theta d\theta d\phi$$