# PHYC 3590 - Advanced Classical Mechanics
# Assignment 4

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## Problem 4.2

### (a)

$$\begin{aligned}
W &= \int_0^P F \cdot dr \\
&= \int_0^Q F \cdot dr + \int_Q^P F \cdot dr \\
&= \int_0^1 F_x dx + \int_Q^P F_y dy \\
&= \int_0^1 x^2 dx + \int_Q^P 2xy\, dy \\
&= \left[\frac{x^3}{3}\right]_0^1 + \int_Q^P 2xy\, dy \\
&= \frac{1}{3} + \int_Q^P 2xy\, dy
\end{aligned}$$

On the path from Q to P, x is equal to 1.

$$\begin{aligned}
&= \frac{1}{3} + \int_0^1 2y\, dy \\
&= \frac{1}{3} + [y^2]_0^1 \\
W &= \frac{1}{3} + 1 \\
W &= \boxed{\frac{4}{3}}
\end{aligned}$$

**(b)**

$$\begin{aligned}
W &= \int_0^P F \cdot dr \\
&= \int_0^P F_x dx + \int_0^P F_y dy \\
&= \int_0^P x^2\, dx + \int_0^P 2xy\, dy \\
y &= x^2, \;\; dy = 2x\, dx \\
W &= \int_0^P x^2\, dx + \int_0^P 4x^4\, dx \\
&= \int_0^1 x^2\, dx + \int_0^1 4x^4\, dx \\
&= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{4x^5}{5}\right]_0^1 \\
&= \frac{1}{3} + \frac{4}{5} \\
W &= \boxed{\frac{17}{15}}
\end{aligned}$$

**(c)**

$$\begin{aligned}
W &= \int_0^{x=1} x^2\, dx + \int_0^{y=1} 2xy\, dy \\
W &= \int_0^{t=1} t^6\, dx + \int_0^{t=1} 2t^3 t^2\, dy \\
dx &= 3t^2\, dt, \;\; dy = 2t\, dt \\
W &= \int_0^{t=1} t^6 \cdot 3t^2\, dt + \int_0^{t=1} 2t^5 \cdot 2t\, dt \\
W &= \int_0^1 3t^8\, dt + \int_0^1 4t^6\, dt \\
W &= \left[\frac{2x^9}{9}\right]_0^1 + \left[\frac{4x^7}{7}\right]_0^1 \\
W &= \frac{3}{9} + \frac{4}{7} \\
W &= \boxed{\frac{19}{21}}
\end{aligned}$$

## Problem 4.8

## Problem 4.22