PHYC 3590 - Advanced Classical Mechanics
Assignment 3

Gavin Kerr
B00801584

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Problem 3.5

$$\vec{p_1} + \vec{p_2} = \vec{p_1}' + \vec{p_2}' \tag{1}$$
$$\vec{v_2} = \vec{p_2} = 0 \tag{2}$$
$$m_1\vec{v_1} = m_1\vec{v_1}' + m_2\vec{v_2}' \tag{3}$$

$$\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}m_1v_1'^2 + \tfrac{1}{2}m_2v_2'^2 \tag{4}$$
$$v_2 = 0 \tag{5}$$
$$m_1v_1^2 = m_1v_1'^2 + m_2v_2'^2 \tag{6}$$
$$m_1 = m_2 \tag{7}$$
$$v_1^2 = v_1'^2 + v_2'^2 \tag{8}$$
$$\tag{9}$$

From equation (3) we have

$$m_1\vec{v_1} = m_1\vec{v_1}' + m_2\vec{v_2}' \tag{10}$$
$$m_1^2\vec{v_1}^2 = (m_1\vec{v_1}' + m_2\vec{v_2}')^2 \tag{11}$$
$$m_1^2\vec{v_1}^2 = m_1^2\vec{v_1}'^2 + m_2^2\vec{v_2}'^2 + 2m_1m_2\vec{v_1}' \cdot \vec{v_2}' \tag{12}$$
$$m_1 = m_2 = m \tag{13}$$
$$\vec{v_1}^2 = \vec{v_1}'^2 + \vec{v_2}'^2 + 2\vec{v_1}' \cdot \vec{v_2}' \tag{14}$$

Equating equation (8) and (14) we find

$$v_1'^2 + v_2'^2 = \vec{v_1}'^2 + \vec{v_2}'^2 + 2\vec{v_1}' \cdot \vec{v_2}'$$
$$\vec{v_1}' \cdot \vec{v_2}' = 0$$
$$|v_1'| \cdot |v_2'| \cos\theta = 0$$
$$\theta = \boxed{90^o}$$

Problem 3.11

(a)

Show that the equation of motion is $m\dot{v} = -\dot{m}v_{ex} + F^{ext}$.

$$\begin{aligned} dP &= m\,dv + dm\,v_{ex} \\ \dot{P} &= \frac{dP}{dt} = F^{ext} \\ dP &= F^{ext}dt \\ F^{ext}dt &= m\,dv + dm\,v_{ex} \\ F^{ext} &= m\frac{dv}{dt} + \frac{dm}{dt}v_{ex} \\ m\dot{v} &= -\dot{m}v_{ex} + F^{ext} \end{aligned}$$

QED

(b)

$$\begin{aligned} m\dot{v} &= -\dot{m}v_{ex} - mg \\ m\dot{v} &= kv_{ex} - (m_0 - kt)g \\ m\frac{dv}{dt} &= kv_{ex} - (m_0 - kt)g \\ dv &= \frac{kv_{ex} - m_0 g + ktg}{m}\,dt \end{aligned}$$

(c)

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(d)

...

Problem 3.21

Problem 3.27

Problem 3.32