PHYC 3590 - Advanced Classical Mechanics
Assignment 6

Gavin Kerr
B00801584

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1 6.16

From problem 6.1 we have $L = R\int_{\theta_1}^{\theta_2} \sqrt{1+\sin^2\theta\phi'(\theta)^2}d\theta$.

$$\begin{aligned}
\frac{\partial f}{\partial \phi'} =& c \\
\frac{\partial f}{\partial \phi'} =& \sqrt{1+\sin^2\theta\phi'(\theta)^2} \\
\frac{\partial f}{\partial \phi'} =& \frac{\sin^2\theta\phi'}{\sqrt{1+\sin^2\theta\phi'(\theta)^2}} \\
\frac{\sin^2\theta\phi'}{\sqrt{1+\sin^2\theta\phi'(\theta)^2}} =& c
\end{aligned}$$

2 6.19

The area of the surface of revolution is defined by the equation.

$$
\begin{aligned}
A =& 2\pi \int_{x_1}^{x_2} y\sqrt{1+x'(y)^2}dy \\
f =& y\sqrt{1+x'(y)^2} \\
0 =& \frac{\partial f}{\partial x} - \frac{d}{dy}\frac{\partial f}{\partial x'} \\
0 =& \frac{d}{dy}\frac{\partial f}{\partial x'} \\
y_0 =& \frac{\partial f}{\partial x'} \\
y_0 =& \frac{\partial}{\partial x'} y\sqrt{1+x'^2} \\
y_0 =& y\frac{x'}{\sqrt{1+x'^2}} \\
y_0^2(1+x'^2) =& y^2x'^2 \\
y_0^2 =& y^2x'^2 - y_0^2x'^2 \\
y_0^2 =& (y^2 - y_0^2)x'^2 \\
\frac{y_0^2}{y^2 - y_0^2} =& x'^2 \\
x' = & \frac{y_0}{\sqrt{y^2 - y_0^2}}
\end{aligned}
$$

I am not sure how to simplify this too the desired equation

3 6.27

$$
\begin{aligned}
L &= \int ds \\
ds &= \sqrt{dx^2 + dy^2 + dz^2} \\
ds &= \sqrt{x'^2 + y'^2 + z'^2} du \\
L &= \int \sqrt{x'^2 + y'^2 + z'^2} du \\
f &= \sqrt{x'^2 + y'^2 + z'^2} \\
\frac{\partial f}{\partial x'} &= \frac{x'}{\sqrt{x'^2 + y'^2 + z'^2}} \\
\frac{\partial f}{\partial y'} &= \frac{y'}{\sqrt{x'^2 + y'^2 + z'^2}} \\
\frac{\partial f}{\partial z'} &= \frac{z'}{\sqrt{x'^2 + y'^2 + z'^2}}
\end{aligned}
$$

The first order derivative of f with respect to x, y, and z are all equal to zero therefore

$$
\begin{aligned}
\frac{d}{dx}\frac{\partial f}{\partial x'} &= 0 \\
\frac{\partial f}{\partial x'} &= c_1 \\
\frac{d}{dx}\frac{\partial f}{\partial y'} &= 0 \\
\frac{\partial f}{\partial y'} &= c_2 \\
\frac{d}{dx}\frac{\partial f}{\partial z'} &= 0 \\
\frac{\partial f}{\partial z'} &= c_3
\end{aligned}
$$

These derivatives are all equal to a constant, therefore x', y' and z' are all equal to constants. This can only be the case if the path is a straight line.

4 7.1

5 7.4

6 7.8