PHYC 3590 - Advanced Classical Mechanics
Assignment 7

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Problem 7.14

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\begin{aligned}
T =& \tfrac{1}{2}m\dot{x}^2 + \tfrac{1}{2}I\omega^2 \\
T =& \tfrac{1}{2}m\dot{x}^2 + \tfrac{1}{2}(\tfrac{1}{2}mR^2)\omega^2 \\
\dot{x} =& R\omega, \;\; \dot{x}/R = \omega \\
T =& \tfrac{1}{2}m\dot{x}^2 + mR^2(\dot{x}/R)^2 \\
T =& \tfrac{1}{2}m\dot{x}^2 + m\dot{x}^2 = \boxed{\tfrac{3}{4}m\dot{x}^2} \\
U =& mgx \\
L =& \tfrac{3}{4}m\dot{x}^2 - mgx
\end{aligned}
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\begin{aligned}
\frac{dL}{dx} =& \frac{d}{dt}\frac{dL}{d\dot{x}} \\
-mg =& \frac{d}{dt}\frac{3}{2}m\dot{x} \\
-mg =& \frac{3}{2}m\ddot{x} \\
\ddot{x} =& \boxed{-\frac{2}{3}g}
\end{aligned}
$$

QED

Problem 7.27

$$T = \tfrac{1}{2}(4m)\dot{x_1}^2 + \tfrac{1}{2}(3m)\dot{x_2}^2 + \tfrac{1}{2}(m)\dot{x_3}^2$$

where x_1 is the position of mass $4m$, x_2 is the position of mass $3m$, x_3 is the position of mass m, l_1 is the length of rope attached to $4m$, and l_2 is the length of rope attached to $2m$ and $3m$.

$$\begin{aligned} x_2 =& l_1 - x_1 + h, \;\; x_3 = l_1 - x_1 + l_2 - h \\ \dot{x}_2 =& -\dot{x}_1 + \dot{h}, \;\; \dot{x}_3 = -\dot{x}_1 - \dot{h} \end{aligned}$$

$$\begin{aligned} T =& \tfrac{1}{2}(4m)\dot{x}_1^2 + \tfrac{1}{2}(3m)(-\dot{x}_1 + \dot{h})^2 + \tfrac{1}{2}(m)(\dot{x}_1 + \dot{h})^2 \\ T =& \tfrac{1}{2}(4m)\dot{h_1}^2 + \tfrac{1}{2}(3m)(\dot{x}_1^2 - 2\dot{x}_1\dot{h} + \dot{h}^2) + \tfrac{1}{2}(m)(\dot{x}_1^2 + 2\dot{x}_1\dot{h} + \dot{h}^2) \\ T =& \tfrac{1}{2}(4m)\dot{h_1}^2 + \tfrac{1}{2}m[3\dot{x}_1^2 - 6\dot{x}_1\dot{h} + 3\dot{h}^2 + \dot{x}_1^2 + 2\dot{x}_1\dot{h} + \dot{h}_2^2] \\ T =& \tfrac{1}{2}(4m)\dot{x_1}^2 + \tfrac{1}{2}m[4\dot{x}_1^2 - 4\dot{x}_1\dot{h} + 4\dot{h}^2] \\ T =& \tfrac{1}{2}(4m)\dot{x_1}^2 + 2m[\dot{x}_1^2 - \dot{x}_1\dot{h} + \dot{h}^2] \end{aligned}$$

$$\begin{aligned} U =& 4mgx_1 + 3mgx_2 + mgx_3 \\ U =& mg(4x_1 + 3(l_1 - x_1 + h) + l_1 - x_1 + l_2 - h) \\ U =& mg(4l_1 + 2h + l_2) \end{aligned}$$

$$L = \tfrac{1}{2}(4m)\dot{x_1}^2 + 2m[\dot{x}_1^2 - \dot{x}_1\dot{h} + \dot{h}^2] - mg(4l_1 + 2h + l_2)$$

$$\begin{aligned} \frac{dL}{dx_1} &= \frac{d}{dt}\frac{dL}{d\dot{x}_1} \\ \frac{dL}{dx_1} &= 0 \\ \frac{d}{dt}\frac{dL}{d\dot{x}_1} &= 4m\ddot{x}_1 + 4m\ddot{x}_1 - 2m\ddot{h} \\ &\boxed{4m\ddot{x}_1 = m\ddot{h}} \end{aligned}$$

$$\begin{aligned} \frac{dL}{dh} &= \frac{d}{dt}\frac{dL}{d\dot{h}} \\ \frac{dL}{dh} &= -2mg \\ \frac{d}{dt}\frac{dL}{d\dot{h}} &= 4m\ddot{h} - 2m\ddot{x}_1 \\ &\boxed{mg = m\ddot{x}_1 - 2m\ddot{h}} \end{aligned}$$

$$\begin{aligned} mg =& m\ddot{x}_1 - 2m(4\ddot{x}_1) \\ mg =& -7m\ddot{x}_1 \\ &\boxed{\ddot{x}_1 = -g/7} \end{aligned}$$

Problem 7.29

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\begin{aligned}
\vec{P} =& R\omega t = R\cos(\omega t)\hat{x} + R\sin(\omega t)\hat{y} \\
\vec{B'} =& \text{The position of the bob relative to } \vec{P} \text{ is} \\
\vec{B'} =& L(\phi - 3\pi/2) = L\sin\phi\hat{x} - L\cos\phi\hat{y} \\
\vec{B} =& \text{The position of the bob relative to } \vec{O} \text{ is} \\
\vec{B} =& (R\cos(\omega t) + L\sin\phi)\hat{x} + (R\sin(\omega t) - L\cos\phi)\hat{y} \\
\dot{x} =& \frac{d}{dt}(R\cos(\omega t) + L\sin\phi) = -\omega R\sin(\omega t) + \dot{\phi}L\cos\phi \\
\dot{y} =& \frac{d}{dt}(R\sin(\omega t) - L\cos\phi) = \omega R\cos(\omega t) + \dot{\phi}L\sin\phi
\end{aligned}
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\begin{aligned}
T =& \tfrac{1}{2}m(\dot{x}^2 + \dot{y}^2) \\
T =& \tfrac{1}{2}m((-\omega R\sin(\omega t) + \dot{\phi}L\cos\phi)^2 + (\omega R\cos(\omega t) + \dot{\phi}L\sin\phi)^2) \\
T =& \tfrac{1}{2}m(\omega^2R^2\sin^2(\omega t) - 2\omega\dot{\phi}LR\cos\phi\sin\omega t + \dot{\phi}^2L^2\cos^2\phi + \\
& \omega^2R^2\cos^2(\omega t) + 2\omega\dot{\phi}LR\sin\phi\cos\omega t + \dot{\phi}^2L^2\sin^2\phi) \\
T =& \tfrac{1}{2}m[\omega^2R^2(1 - \cos^2\omega t) - 2\omega\dot{\phi}LR\cos\phi\sin\omega t + \dot{\phi}^2L^2\cos^2\phi + \\
& \omega^2R^2\cos^2(\omega t) + 2\omega\dot{\phi}LR\sin\phi\cos\omega t + \dot{\phi}^2L^2(1 - \cos^2\phi)] \\
T =& \tfrac{1}{2}m[\omega^2R^2 - \omega^2R^2\cos^2\omega t - 2\omega\dot{\phi}LR\cos\phi\sin\omega t + \dot{\phi}^2L^2\cos^2\phi + \\
& \omega^2R^2\cos^2(\omega t) + 2\omega\dot{\phi}LR\sin\phi\cos\omega t + \dot{\phi}^2L^2 - \dot{\phi}^2L^2\cos^2\phi] \\
T =& \tfrac{1}{2}m[\omega^2R^2 + 2\omega\dot{\phi}LR(\sin\phi\cos\omega t - \cos\phi\sin\omega t) + \dot{\phi}^2L^2] \\
T =& \tfrac{1}{2}m[\omega^2R^2 + 2\omega\dot{\phi}LR\sin(\phi - \omega t) + \dot{\phi}^2L^2]
\end{aligned}
$$

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\begin{aligned}
U =& mgh \\
h =& B_y = R\sin(\omega t) - L\cos\phi \\
U =& mg[R\sin(\omega t) - L\cos\phi]
\end{aligned}
$$

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\begin{aligned}
L =& \tfrac{1}{2}m[\omega^2R^2 + 2\omega\dot{\phi}LR\sin(\phi - \omega t) + \dot{\phi}^2L^2] - mg[R\sin\omega t - L\cos\phi] \\
\frac{\partial L}{\partial \phi} =& \frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}} \\
\frac{\partial L}{\partial \phi} =& m\omega\dot{\phi}LR\cos(\phi - \omega t) - mgL\sin\phi \\
\frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}} =& \frac{d}{dt}\left(m\omega LR\sin(\phi - \omega t) + m\dot{\phi}L^2\right) = (\dot{\phi} - \omega)m\omega LR\cos(\phi - \omega t) + m\ddot{\phi}L^2
\end{aligned}
$$

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\begin{aligned}
m\omega\dot{\phi}LR\cos(\phi - \omega t) - mgL\sin\phi =& (\dot{\phi} - \omega)m\omega LR\cos(\phi - \omega t) + m\ddot{\phi}L^2 \\
\omega\dot{\phi}LR\cos(\phi - \omega t) - gL\sin\phi =& \dot{\phi}\omega LR\cos(\phi - \omega t) - \omega^2LR\cos(\phi - \omega t) + \ddot{\phi}L^2 \\
-g\sin\phi =& -\omega^2R\cos(\phi - \omega t) + \ddot{\phi}L \\
\ddot{\phi}L =& \boxed{\omega^2R\cos(\phi - \omega t) - g\sin\phi}
\end{aligned}
$$

The equation of a standard pendulum is $L^2\frac{d^2\theta}{dt^2} = -gL\sin\theta$ which agrees with the boxed equation is we set ω to zero.