PHYC 3590 - Advanced Classical Mechanics
Assignment 4

Gavin Kerr
B00801584

2023-02-07

Question 6.3

$$
\begin{aligned}
\tau =& s/c \\
P_1 =& (0, y_1, 0),\ \ Q = (x, 0, z),\ \ P_2 = (x_2, y_2, 0). \\
|P_1 \to Q| =& |Q - P_1| = \sqrt{(x-0)^2 + (0-y_1)^2 + (z-0)^2} \\
|Q \to P_2| =& |P_2 - Q| = \sqrt{(x_2-x)^2 + (y_2-0)^2 + (0-z)^2} \\
s =& |P_1 \to P_2| = \sqrt{x^2 + y_1^2 + z^2} + \sqrt{(x_2-x)^2 + y_2^2 + z^2} \\
\tau =& \boxed{\frac{\sqrt{x^2 + y_1^2 + z^2} + \sqrt{(x_2-x)^2 + y_2^2 + z^2}}{c}}
\end{aligned}
$$

We know that τ is a minimums with respect to the variable z when $\frac{dz}{dt} = 0$.

$$
\begin{aligned}
\frac{\partial \tau}{\partial z} =& \frac{z}{c\sqrt{x^2 + y_1^2 + z^2}} + \frac{z}{c\sqrt{(x_2-x)^2 + y_2^2 + z^2}} = 0 \\
& \boxed{z = 0}
\end{aligned}
$$

Therefore τ is minimized at $z = 0$.

$$
\begin{aligned}
\frac{\partial \tau}{\partial z} =& \frac{z}{c\sqrt{x^2 + y_1^2 + z^2}} + \frac{x_2 - x}{c\sqrt{(x_2-x)^2 + y_2^2 + z^2}} = 0 \\
\frac{\partial \tau}{\partial z} =& \frac{\sin\theta_1}{c\sqrt{x^2 + y_1^2 + z^2}} + \frac{\sin\theta_2}{c\sqrt{(x_2-x)^2 + y_2^2 + z^2}} = 0 \\
& \boxed{\theta_1 = \theta_2}
\end{aligned}
$$

Question 6.4

$$\begin{aligned}
L &= \int_{P_1}^{P_2} ds = \int_{P_1}^{Q} ds + \int_{Q}^{P_2} ds \\
\tau &= \frac{n_1\sqrt{x^2+y_1^2+z^2} + n_2\sqrt{(x_2-x)^2+y_2^2+z^2}}{c} \\
\frac{\partial \tau}{\partial z} &= \frac{n_1 z}{c\sqrt{x^2+y_1^2+z^2}} + \frac{n_2 z}{c\sqrt{(x_2-x)^2+y_2^2+z^2}} = 0 \\
&\boxed{z = 0}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \tau}{\partial x} &= \frac{n_1 x}{c\sqrt{x^2+y_1^2+z^2}} + \frac{n_2(x_2-x)}{c\sqrt{(x_2-x)^2+y_2^2+z^2}} \\
0 &= \frac{n_1 \sin\theta_1}{c\sqrt{x^2+y_1^2+z^2}} + \frac{n_2 \sin\theta_2}{c\sqrt{(x_2-x)^2+y_2^2+z^2}} \\
&\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}
\end{aligned}$$

Question 6.5

$$\begin{aligned}
APB &= 2R\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right) + 2R\sin\left(\frac{\pi}{4} + \frac{\theta}{2}\right) \\
\tau &= \frac{APB}{c} \\
\tau &= \frac{2R\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right) + 2R\sin\left(\frac{\pi}{4} + \frac{\theta}{2}\right)}{c}
\end{aligned}$$

τ is minimized for $\theta = 0$ where $P = P_0$.