PHYC 3590xx - Advanced Classical Mechanics
Assignment 3

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Problem 3.5

$$\begin{aligned}
\vec{p_1} + \vec{p_2} =& \vec{p_1}' + \vec{p_2}' && (1)\\
\vec{v_2} = \vec{p_2} =& 0 && (2)\\
m_1\vec{v_1} =& m_1\vec{v_1}' + m_2\vec{v_2}' && (3)
\end{aligned}$$

$$\begin{aligned}
\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 =& \tfrac{1}{2}m_1v_1'^2 + \tfrac{1}{2}m_2v_2'^2 && (4)\\
v_2 =& 0 && (5)\\
m_1v_1^2 =& m_1v_1'^2 + m_2v_2'^2 && (6)\\
m_1 =& m_2 && (7)\\
v_1^2 =& v_1'^2 + v_2'^2 && (8)\\
& && (9)
\end{aligned}$$

From equation (3) we have

$$\begin{aligned}
m_1\vec{v_1} =& m_1\vec{v_1}' + m_2\vec{v_2}' && (10)\\
m_1^2\vec{v_1}^2 =& (m_1\vec{v_1}' + m_2\vec{v_2}')^2 && (11)\\
m_1^2\vec{v_1}^2 =& m_1^2\vec{v_1}'^2 + m_2^2\vec{v_2}'^2 + 2m_1m_2\vec{v_1}'\cdot\vec{v_2}' && (12)\\
m_1 =& m_2 = m && (13)\\
\vec{v_1}^2 =& \vec{v_1}'^2 + \vec{v_2}'^2 + 2\vec{v_1}'\cdot\vec{v_2}' && (14)
\end{aligned}$$

Equating equation (8) and (14) we find

$$\begin{aligned}
v_1'^2 + v_2'^2 =& \vec{v_1}'^2 + \vec{v_2}'^2 + 2\vec{v_1}'\cdot\vec{v_2}'\\
\vec{v_1}'\cdot\vec{v_2}' =& 0\\
|v_1'|\cdot|v_2'|\cos\theta =& 0\\
\theta =& \boxed{90^o}
\end{aligned}$$

Problem 3.11

(a)

Show that the equation of motion is $m\dot{v} = -\dot{m}v_{ex} + F^{ext}$.

$$\begin{aligned}
dP =& m\,dv + dm\,v_{ex} \\
\dot{P} =& \frac{dP}{dt} = F^{ext} \\
dP =& F^{ext} dt \\
F^{ext} dt =& m\,dv + dm\,v_{ex} \\
F^{ext} =& m\frac{dv}{dt} + \frac{dm}{dt} v_{ex} \\
m\dot{v} =& -\dot{m}v_{ex} + F^{ext} \\
& \text{QED}
\end{aligned}$$

(b)

$$\begin{aligned}
m\dot{v} =& -\dot{m}v_{ex} - mg \\
m\dot{v} =& kv_{ex} - (m_0 - kt)g \\
m\frac{dv}{dt} =& kv_{ex} - (m_0 - kt)g \\
dv =& \frac{kv_{ex} - m_0 g + ktg}{m}\,dt
\end{aligned}$$

I am not sure how to finishe this problem.

(c)

...

(d)

...

Problem 3.21

$$
\begin{aligned}
CM &= \int r\sigma dA \\
\sigma &= \frac{m}{A} = \frac{m}{\pi R^2} \\
dA &= r\,dr\,d\theta \\
CM &= \int\int \frac{m}{\pi R^2} r^2\,dr\,d\theta \\
CM &= \frac{m}{\pi R^2}\int_0^R r^2 dr \int_0^{2\pi} d\theta \\
CM &= \frac{m}{\pi R^2}(R^3/3)(2\pi) \\
CM &= \boxed{\frac{2mR}{3}}
\end{aligned}
$$

Problem 3.27

(a)

$$
\begin{aligned}
l =& \vec{r} \times \vec{p} \\
\vec{p} =& m\vec{v} \\
l =& \vec{r} \times m\vec{v} \\
\vec{v} =& \frac{d\vec{r}}{dt} = \dot{r}\hat{r} + r\dot{\phi}\hat{\phi} \\
l =& \vec{r} \times m(\dot{r}\hat{r} + r\dot{\phi}\hat{\phi}) \\
l =& r\hat{r} \times m(\dot{r}\hat{r} + r\dot{\phi}\hat{\phi}) \\
l =& r\hat{r} \times m(\dot{r}\hat{r} + r\dot{\phi}\hat{\phi}) \\
l =& r\hat{r} \times mr\dot{\phi}\hat{\phi} \\
\omega =& \dot{\phi} \\
l =& r\hat{r} \times mr\omega\hat{\phi} \\
l =& \boxed{mr^2\omega(\hat{r} \times \hat{\phi})}
\end{aligned}
$$

(b)

Show that $\frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{l}{2m}$. Ler r be the distance between the planet and the sun, let x be the distance the planet is traveling perpendicular to r.

$$
\begin{aligned}
A =& \frac{1}{2}rx \\
dA =& \frac{1}{2}r\,dx \\
x =& r\sin\phi \\
dx =& r\,d\phi \\
dA =& \frac{1}{2}r\,r\,d\phi \\
\frac{dA}{dt} =& \frac{1}{2}r^2\frac{d\phi}{dt} \\
\frac{dA}{dt} =& \boxed{\frac{1}{2}r^2\omega}
\end{aligned}
$$

From part (a) we have

$$
\begin{aligned}
l =& mr^2\omega && (15) \\
\frac{l}{2m} =& \frac{1}{2}r^2\omega && (16) \\
\frac{dA}{dt} =& \frac{1}{2}r^2\omega && (17) \\
\frac{dA}{dt} =& \boxed{\frac{l}{2m}} && (18)
\end{aligned}
$$

Problem 3.32

$$
\begin{aligned}
I =& \sum m_\alpha \rho_\alpha^2 \\
I =& \int_v Density \cdot \rho^2 \, dV \\
I =& \int_v \frac{m}{V} \rho^2 \, r^2 dr \sin\theta d\theta d\phi \\
I =& \int_v \frac{3m}{4\pi R^3} \rho^2 \, r^2 dr \sin\theta d\theta d\phi \\
\rho^2 =& (r_r \hat{r} + r_\theta \hat{\theta} + r_\phi \hat{\phi})^2 = r_r^2 + r_\theta^2 + r_\phi^2 = r^2 \\
I =& \int_v \frac{3m}{4\pi R^3} r^2 \, r^2 dr \sin\theta d\theta d\phi \\
I =& \frac{3m}{4\pi R^3} \int_0^R r^4 dr \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi \\
I =& \frac{3m}{4\pi R^3} (R^5/5)(-\cos\pi)(2\pi) \\
I =& \tfrac{3}{10} m R^2
\end{aligned}
$$