Assignment 1

Advanced Classical Mechanics

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Question 1.16

(a)

Show that $|r| = \sqrt{r \cdot r}$. Firstly, let $r = x\hat{x} + y\hat{y} + z\hat{z}$.

$$\begin{aligned} r =& x\hat{x} + y\hat{y} + z\hat{z} \\ r \cdot r =& (x\hat{x} + y\hat{y} + z\hat{z})(x\hat{x} + y\hat{y} + z\hat{z}) \\ r \cdot r =& x^2 + y^2 + z^2 = r^2 \\ r \cdot r =& r^2 \\ \sqrt{r \cdot r} =& \sqrt{r^2} \\ \sqrt{r \cdot r} =& |r| \\ & \text{QED} \end{aligned}$$

(b)

Prove that $r \cdot s$, as defined by (1.7), is the same for any choice of orthogonal axes.

$$|r|$$

Problem 1.26

(a)

For S we have $x = 0, y = v_1 t$, where v_1 is the speed at which the man kicks the puck.

(b)

For S' relative to S we have $x = v_2 t, y = 0$, where v_2 is the speed of the second observer relative to the first observer (S). In the S' frame, the path of the puck would be $x' = x_1 - v_2 t, y' = y_1 + v_1 t$. (x_1, y_1) is the distance between observer 1 & 2.

(c)

For S'' relative to S, we have $x = \frac{1}{2}at^2, y = 0$, where a is the acceleration of the third observer. In the S'' frame, the puck follows the path $x = x_2 - \frac{1}{2}at^2 y = y_2 + v_1 t$. (x_2, y_2) is the distance between the first & third observer.

The S and S' are inertial reference frames. S'' is not an inertial reference frame because it is accelerating.

Problem 1.31

I think this question needs more.

$$\begin{aligned}
P_1 + P_2 =& c \\
\frac{dP_1}{dt} + \frac{dP_2}{dt} =& \frac{dc}{dt} \\
F_1 + F_2 =& 0 \\
F_1 =& -F_2 \\
&\text{QED}
\end{aligned}$$

Problem 1.39

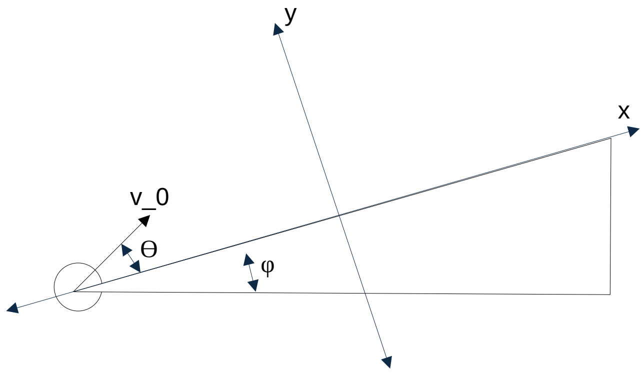

Figure 1: Diagram

$$
\begin{aligned}
g =& -|g|(\sin\phi\hat{x} + \cos\phi\hat{y}) \\
v_0 =& |v_0|(\cos\theta\hat{x} + \sin\theta\hat{y}) \\
x =& |v_0|\cos\theta\, t - \tfrac{1}{2}|g|\sin\phi\, t^2 \\
y =& |v_0|\sin\theta\, t - \tfrac{1}{2}|g|\cos\phi\, t^2
\end{aligned}
$$

$$
\begin{aligned}
0 =& |v_0|\sin\theta\, t - \tfrac{1}{2}|g|\cos\phi\, t^2 \\
0 =& |v_0|\sin\theta - \tfrac{1}{2}|g|\cos\phi\, t \\
\tfrac{1}{2}|g|\cos\phi\, t =& |v_0|\sin\theta \\
t =& \frac{2|v_0|\sin\theta}{|g|\cos\phi}
\end{aligned}
$$

This means the the ball touches the ramp at $t = \dfrac{2|v_0|\sin\theta}{|g|\cos\phi}$. Subbing this into the equation for x to find the final position gives us

$$
\begin{aligned}
x =& |v_0|\cos\theta\left(\frac{2|v_0|\sin\theta}{|g|\cos\phi}\right) - \tfrac{1}{2}|g|\sin\phi\left(\frac{2|v_0|\sin\theta}{|g|\cos\phi}\right)^2 \\
x =& \left(\frac{2v_0^2\sin\theta\cos\theta}{|g|\cos\phi}\right) - \left(\frac{2v_0^2\sin^2\theta\sin\phi}{|g|\cos^2\phi}\right) \\
x =& \frac{2v_0^2}{|g|}\left(\frac{\sin\theta\cos\theta}{\cos\phi} - \frac{\sin^2\theta\sin\phi}{\cos^2\phi}\right) \\
x =& \frac{2v_0^2}{|g|}\left(\frac{\sin\theta\cos\theta\cos\phi}{\cos^2\phi} - \frac{\sin^2\theta\sin\phi}{\cos^2\phi}\right) \\
x =& \frac{2v_0^2}{|g|}\left(\frac{\sin\theta\cos\theta\cos\phi - \sin^2\theta\sin\phi}{\cos^2\phi}\right) \\
x =& \frac{2v_0^2}{|g|}\left(\frac{\sin\theta(\cos\theta\cos\phi - \sin\theta\sin\phi)}{\cos^2\phi}\right) \\
x =& \frac{2v_0^2}{|g|}\left(\frac{\sin\theta\cos(\theta+\phi)}{\cos^2\phi}\right) \\
x =& \boxed{R = \frac{2v_0^2\sin\theta\cos(\theta+\phi)}{|g|\cos^2\phi}} \\
& \text{QED}
\end{aligned}
$$

To solve for R_{max} we need to find a local max for R as a function of θ.

$$
\begin{aligned}
\frac{dR}{d\theta} =& \frac{d}{d\theta}\frac{2v_0^2\sin\theta\cos(\theta+\phi)}{|g|\cos^2\phi} \\
\frac{dR}{d\theta} =& \frac{2v_0^2}{|g|\cos^2\phi}\frac{d}{d\theta}\left(\sin\theta\cos(\theta+\phi)\right) \\
\frac{dR}{d\theta} =& \frac{2v_0^2}{|g|\cos^2\phi}\left(\cos\theta\cos(\theta+\phi) - \sin\theta\sin(\theta+\phi)\right) \\
\frac{dR}{d\theta} =& \frac{2v_0^2}{|g|\cos^2\phi}\sin(2\theta+\phi) \\
0 =& \frac{2v_0^2}{|g|\cos^2\phi}\sin(2\theta+\phi) \\
n\pi =& 2\theta+\phi \\
\theta_{max} =& \frac{n\pi-\phi}{2}
\end{aligned}
$$

I think this solution is wrong. However, next we just sub theta into the equation for $R(\theta_{max})$

Problem 1.46

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