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MIDDLE EAST TECHNICAL UNIVERSITY

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

EE 568 Project #3

PM Motor Comparison Analysis

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Introduction

In this project, basic terminologies in machine design is studied such as electrical loading, magnetic loading or material selection. The project gives overall good understanding of the machine design steps. In the first part of the project, magnetic loading is found with the cylindrical stator assumption. Then, in the second part, the electrical loading and machine geometry is defined. Using electrical loading and magnetic loading, the average tangential stress is calculated and torque of the machine is found. In the last part, the machine is optimized to achieve maximum torque density. The trade-offs between machine geometrical parameters and torque is investigated. Also, for the same outer diameter and volume, Ferrite magnets are used and optimization study is presented to increase the torque density of the Ferrite machine. The comparative study showed us that Neodymium design has 44% more volumetric torque density and it is more favourable.

Question I: Magnetic Loading

a) Electrical equivalent circuit

In this part, the stator is assumed to be solid cylinder for 4-pole surface-mount PM machine. In the machine, NdFeB magnets are used with N42 grade, which has remanence flux density of 1.315 T. The machine is shown in Figure 1.

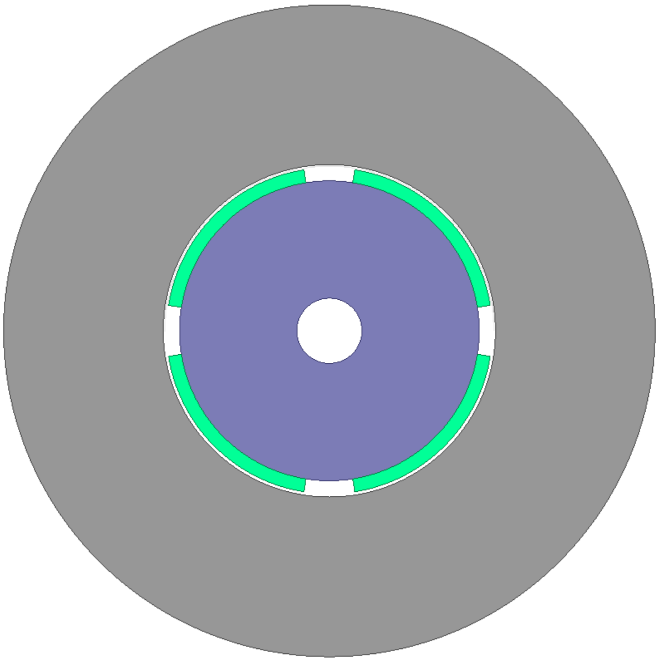

Figure 1: Analysed machine

The magnetic equivalent circuit of a magnet can be shown as in Figure 2.

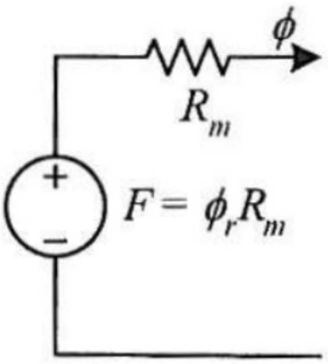

Figure 2: Electrical modelling of a magnet

$$R_m = \frac{l_m}{\mu_0 \mu_r A_m}$$

$$\phi_r = B_r A_m$$

Then, the electrical equivalent of a pole pair can be found easily. Air gap is also modeled as a series added resistance. In this analysis, the cores are assumed to be infinitely permeable, where there is no MMF drop. Then, the electrical circuit model of the machine under one pole can be seen as in Figure 3.

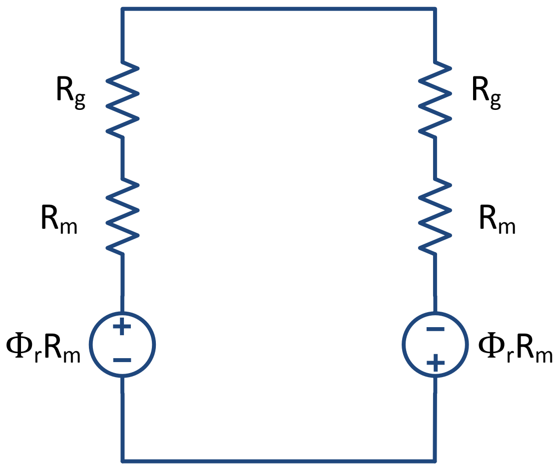

Figure 3: Electrical equivalent of the machine under one pole pair

The analytical calculation of air gap flux density starts with derivation of air gap flux. In the analysis, it is assumed that there is no fringing and leakage flux and air gap flux density has square shape. Then, air gap flux can be derived using electrical equivalent circuit as follows:

$$\phi_m = \phi_g = \frac{2\phi_r R_m}{2(R_m + R_g)} = \frac{\phi_r R_m}{R_m + R_g}$$

$$\phi_r = B_r A_m$$

$$R_m = \frac{l_m}{\mu_0 \mu_r A_m}$$

$$R_g = \frac{l_g}{\phi_0 A_g}$$

$$A_m = A_g$$

$$l_m = 4\ mm \text{ and } l_g = 1\ mm$$

$$\phi_g = A_m * 1.315 * \frac{\frac{4}{1.05}}{\frac{4}{1.05} + 1} = 1.04\ A_m$$

$$B_g = \frac{\phi_g}{A_g} = \frac{\phi_g}{A_m} = 1.04\ T$$

It is found that air gap peak flux density is 1.04 T, analytically. Magnetic field intensity of the magnet can be found as follows

$$B_m = B_r + \mu_0 \mu_r H_m$$

$$H_m = \frac{B_m - B_r}{\mu_0 \mu_r} = -207.2\ kA/m$$

Then, the BH characteristics of the magnet and its operation point can be shown as in Figure 4.

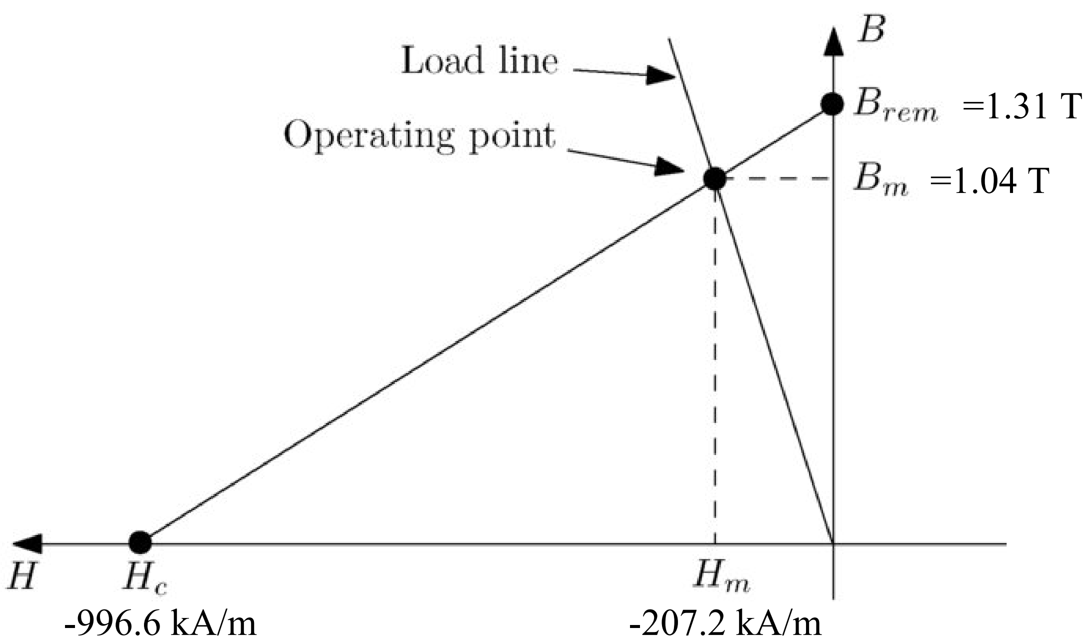

Figure 4: Operation point of the magnet on BH characteristics

b) Magnetic loading

Magnetic loading is defined as average of fundamental component of the air gap flux density. It is assumed in our analysis that air gap flux density has square shaped waveform. Its peak of fundamental component can be found as

$$B_{p1} = B_{square} \frac{4}{\pi} \sin\left(\frac{\theta_m}{2}\right)$$

$$B_{p1} = 1.04 \frac{4}{\pi} \sin(90 * 0.8) = 1.26\ T$$

$$B_{avg} = B_{p1} * \frac{2}{\pi} = 1.26 * \frac{2}{\pi} = 0.8\ T$$

c) FEA results

Using ANSYS Maxwell, the model shown in Figure 1 is solved. The results are shown in Figure 5.

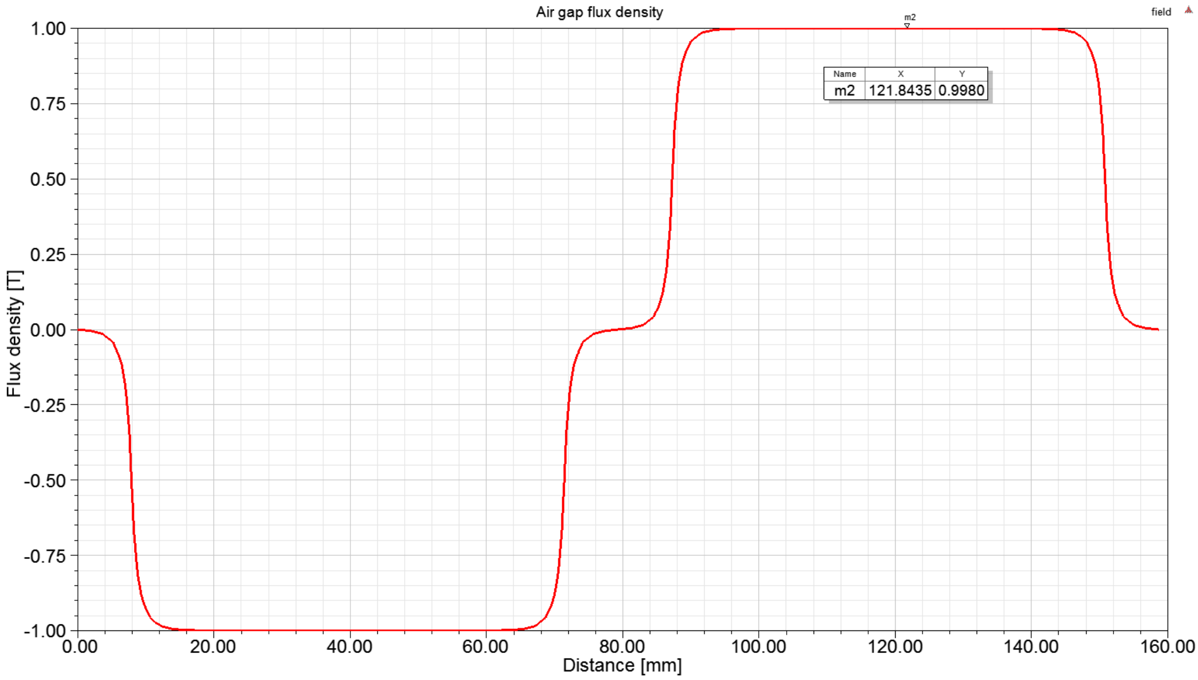

Figure 5: Air gap flux density over one pole-pair

The results obtained using FEA shows that peak flux density is around 1 T. In our analysis, it was 1.04 T. Therefore, there is less than 5% difference, which is mainly caused by the assumption that there is no leakage and fringing effect and there is no saturation in the cores. In overall, the obtained results are in good agreement with analytical calculations.

Question II: Electrical Loading & Machine Sizing

a) Slot number selection

We have four pole machine. Let's assume integral slot winding with number of slots per pole per phase is two. Then, the number of slots is

$$Q_s = q * phase * pole = 2 * 3 * 4$$

$$Q_s = 24\ slots$$

b) Wire selection

Phase current is 2.5 A_{rms}. With maximum current density of 5 A/mm^2, the minimum wire cross sectional area can be found as

$$A_{min} = \frac{I}{J_{max}} = \frac{2.5}{5} = 0.5\ mm^2$$

Using AWG standards, it is found that AWG20 wire has closest value of cross sectional area. Therefore, AWG20 wire is selected with

$$A_{wire} = 0.519\ mm^2$$

c) Slot design back core thickness selection

We can proceed by selecting a slot radio, which is defined as

$$d = \frac{slot\ inner\ diameter}{slot\ outer\ diamer}$$

Using rule of thumbs defined in the lecture, let's pick slot ratio of 0.7. Then, the slot inner diameter can be calculated as

$$0.7 = \frac{102\ mm}{OD}$$

$$OD = 145.7\ mm$$

$$slot\ height = 21.8\ mm$$

In the design, parallel teeth option is preferred. With this selection, the slot area is calculated by drawing the actual stator model as in Figure 6.

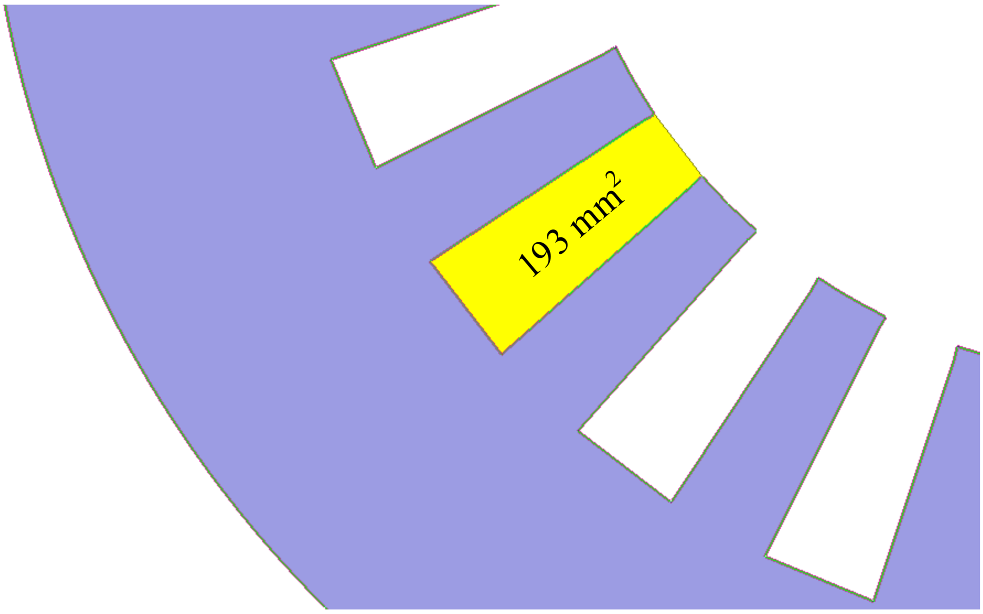

Figure 6: Slot area of the design

It is observed that the slot area is 193 mm^2. Then, assuming that the maximum fill factor is 0.6, the maximum number of turns can be calculated as

$$ff_{max} = \frac{N_{slotmax} * A_{wire}}{A_{slot}}$$

$$N_{slotmax} = \frac{0.6 * 193}{0.516} = 224\ turns$$

Let's choose the number of turns per slot as 200 turns. With this selection, fill factor is

$$ff = \frac{0.516 * 200}{193} = 0.53$$

The back core thickness can be calculated as follows

$$\frac{B_{avg} * A_{pole}}{2} = B_{core} * A_{core}$$

$$h_{core} = \frac{B_{avg} \pi D_{airgap}}{2\ pole\ B_{coremax}} = \frac{0.8 * \pi * 0.101}{2 * 4 * 1} = 32\ mm$$

d) Electrical loading

The electrical loading is calculated as follows

$$A = \frac{N_{slot} I_{wire} Q_s}{\pi D_{airgap}} = \frac{200 * 2.5A * 24}{\pi * 0.101} = 37.8 \frac{kA}{m}$$

For PMSM, as a rule of thumb, the electrical loading should be between 35-65 kA/m. Our calculation is coherent with this values.

e) Average tangential stress and torque

The average tangential stress is calculated as follows

$$\sigma = \frac{A B_{p1} \cos\theta}{\sqrt{2}} = \frac{37.8 * 1.26T * 1}{\sqrt{2}} = 33.7\ kPa$$

The torque of the machine is calculated by tangential stress and surface area as follows

$$T = \sigma \pi D_{airgap} l_{ax} r_{airgap} = 33.7kPa * \pi * 0.101mm * 0.1m * \frac{0.101}{2} = 54\ Nm$$

f) Power calculation

$$P = T * \omega = 54 * 1500 * \frac{\pi}{30} = 8.5\ kW$$

Question III: Comparison & Optimization

a) Slot ratio optimization

First, let's derive the slot ratio at which the torque is maximum. Note that we have parallel teeth design. In that case, the slot area is proportional with slot ratio defined as follows

$$d = \frac{ID_{slot}}{OD_{slot}}$$

$$Slot\ area \rightarrow \ (1 - d^2)$$

$$A \rightarrow \frac{1 - d^2}{d}$$

$$\sigma \rightarrow \frac{1 - d^2}{d}$$

$$Torque \rightarrow (1 - d^2)d$$

Taking the derivative and equating to the zero, the slot ratio that gives maximum torque can be found as follows

$$d = 0.58$$

Now, let's try to find rotor diameter. In our case, the outer diameter is fixed to 160 mm. As we found before, the back core thickness is a function of air gap diameter as follows

$$h_{core} = \frac{B_{avg} \pi D_{airgap}}{2\ pole\ B_{coremax}}$$

$$ID_{slot} = D_{airgap} + 1mm = 0.58 * OD_{slot}$$

$$h_{slot} = \frac{OD_{slot} - ID_{slot}}{2} = 0.21\ OD_{slot}$$

Solving these three equations together yields in

$$OD_{slot} = 117.7\ mm$$

$$ID_{slot} = 68.3\ mm$$

$$D_{airgap} = 67.3\ mm$$

$$h_{slot} = 24.7\ mm$$

$$h_{core} = 21.1\ mm$$

Then, we can find rotor diameter as

$$D_{rotor} = D_{airgap} - 1mm$$

$$D_{rotor} = 66.3\ mm$$

Now, let's calculate the tangential stress and torque values. Let's start with number of conductors in a slot.

$$A_{slot} = 175\ mm^2$$

$$N_{slotmax} = \frac{0.6 * 175}{0.516} = 203\ turns$$

Let's choose the number of conductors in a slot as 200. Then, the electrical loading is

$$A = \frac{N_{slot} I_{wire} Q_s}{\pi D_{airgap}} = \frac{200 * 2.5A * 24}{\pi * 67.3\ mm} = 56.8\ \frac{kA}{m}$$

Then the average tangential stress and the torque values can be calculated as

$$\sigma = \frac{A B_{p1} \cos\theta}{\sqrt{2}} = \frac{56.8 * 1.26T * 1}{\sqrt{2}} = 50.6\ kPa$$

The torque of the machine is calculated by tangential stress and surface area as follows

$$T = \sigma\pi D_{airgap} l_{ax} r_{airgap} = 50.6\ kPa * \pi * 67.3mm * 0.1m * \frac{67.3}{2} = 36\ Nm$$

Now, let's try to verify our results using FEA. In ANSYS Maxwell, the same machine is constructed as shown in Figure 7.

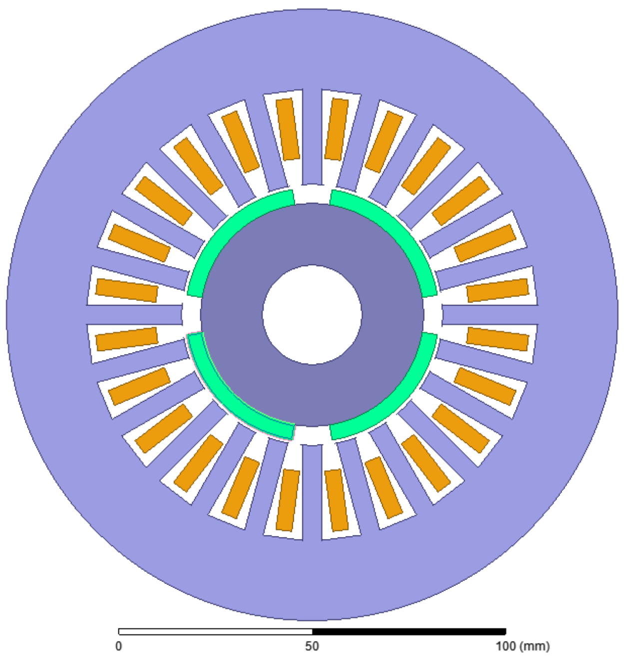

Figure 7: Optimized machine with 0.58 slot ratio

In the FEA analysis, it is found that the machine rated torque is 31 Nm. There is 13% difference between analytical torque and FEA torque. This difference is mainly caused by the ignored harmonics while defining electrical loading. The current harmonics will also contribute to the torque production and it may decrease or increase the torque rating. Additionally, remember that the magnetic loading is calculated with the cylindrical shape rotor approximation. In actual model, we have stator slots and actual magnetic loading should be smaller, which creates the difference.

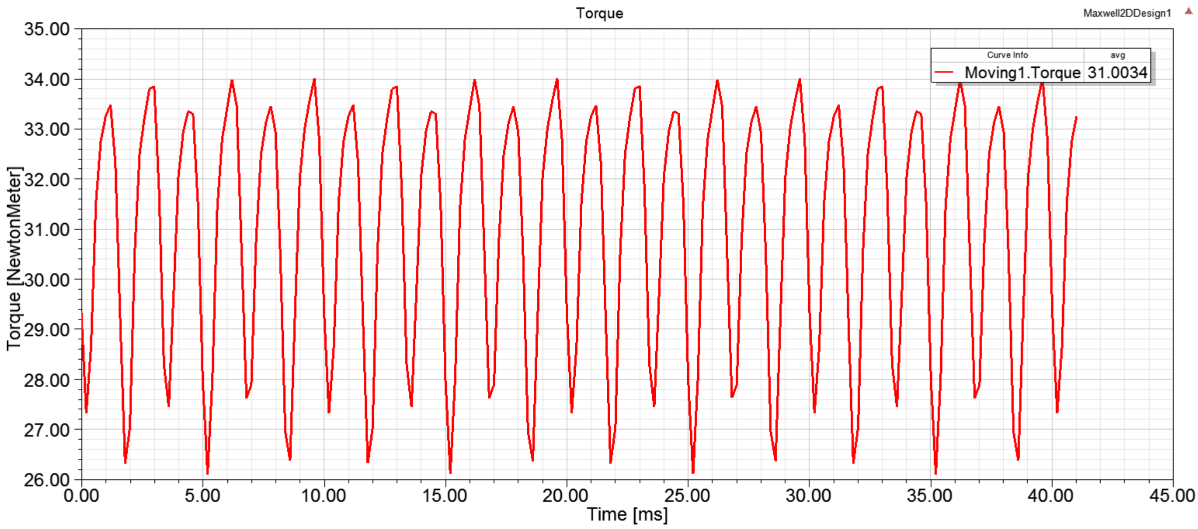

Figure 8: The torque of the machine

The comparison of the machines obtained in this part and in the previous part is tabulated in Table 1. The comparison is carried out on volumetric torque density.

Table 1: Comparison of the machines

	Outer diameter	**Torque**	**Torque density**
The previous machine in QII	210 mm	54 Nm	15 607 Nm/m^3
The optimized machine in QIII	160 mm	36 Nm	17 910 Nm/m^3

It can be seen that optimizing the slot ratio increased the volumetric torque density.

b) Ferrite design

In this part, we will replace NdFeB magnets with ferrite magnets with 0.4 T remanence flux density. First of all, the electrical loading is independent of magnet material and therefore, there is no change in electrical loading. The magnetic loading is directly proportional with magnet remanence flux density. Therefore, the magnetic loading changes as follows

$$B_{avg-ferrite} = B_{avg-NdFeB} * \frac{Br_{ferrite}}{Br_{NdFeB}} = 0.8\,T * \left(\frac{0.4}{1.315}\right) = 0.24\,T$$

The average tangential stress and the torque values also decrease with the same ratio. Therefore,

$$\sigma_{fer} = \sigma * \frac{0.4}{1.315} = 50.6 * \frac{0.4}{1.315} = 15.4\,kPa$$

$$T_{fer} = T * \frac{0.4}{1.315} = 36 * \frac{0.4}{1.315} = 11\,Nm$$

As a result, with the same geometry, the material change to ferrite results in 70% loss in torque value.

c) Ferrite design optimization

I will follow the strategy proposed in the lecture notes as follows

- Double the magnet thickness
- Reduce the teeth width until it saturates
- Reduce back core thickness and increase air gap radius accordingly.

For the magnetic loading calculation, I will again use cylindrical stator approximation in order to simplify my calculations. With new magnet thickness, 8 mm, and 1 mm or air gap clearance, the magnetic loading of the design can be calculated as follows

$$\phi_m = \phi_g = \frac{2\phi_r R_m}{2(R_m + R_g)} = \frac{\phi_r R_m}{R_m + R_g}$$

$$\phi_r = B_r A_m$$

$$R_m = \frac{l_m}{\mu_0 \mu_r A_m}$$

$$R_g = \frac{l_g}{\phi_0 A_g}$$

$$A_m = A_g$$

$$l_m = 8\,mm \text{ and } l_g = 1\,mm$$

$$\phi_g = A_m * 0.4 * \frac{\frac{8}{1.05}}{\frac{8}{1.05} + 1} = 0.35T * A_m$$

$$B_g = \frac{\phi_g}{A_g} = \frac{\phi_g}{A_m} = 0.35\,T$$

Remember that this value is peak of the square shaped air gap flux density characteristics. In our calculations, we use peak or average values fundamental component. Therefore, they can be calculated as

$$B_{p1} = B_{square}\frac{4}{\pi}\sin\left(\frac{\theta_m}{2}\right)$$

$$B_{p1} = 0.35\frac{4}{\pi}\sin(90 * 0.8) = 0.428\,T$$

$$B_{avg} = B_{p1} * \frac{2}{\pi} = 0.428 * \frac{2}{\pi} = 0.273\,T$$

Now, let's redefine stator and rotor geometry. First, let's start with teeth width of the design. In Neodymium design, air gap fundamental peak flux density was 1.26 T. Now, in ferrite core, the air gap peak flux density for fundamental component is 0.428 T. Therefore, teeth flux is reduced with these ratios. Therefore, we can reduce teeth width as follows

$$w_{teeth-ferrite} = \frac{B_{p1-ferrite}}{B_{p1-\text{Neodymium}}} * w_{teeth-\text{Neodymium}}$$

$$w_{teeth-ferrite} = \frac{0.428\,T}{1.26\,T} * 5\,mm = 1.7\,mm$$

We can do the same calculation for back core thickness. However, note that the back core thickness is a function of air gap diameter and air gap flux density. Therefore, we will assume first that the air gap diameter is constant and will modify the back core thickness with the ratios of air gap flux density of two designs. Then, we will obtain an air gap diameter value and modify the back core thickness in this second iteration. The error will decrease with the number of iterations; but, one iteration is enough for this analysis.

$$h_{core-ferrite-init} = h_{core-\text{Neodymium}} \frac{B_{p1-ferrite}}{B_{p1-\text{Neodymium}}}$$

$$h_{core-ferrite-init} = 21.1\ mm * \frac{0.428\ T}{1.26\ T} = 7.2\ mm$$

$$D_{airgap-init} = 67.3\ mm$$

$$D_{airgap-second} = 95.2\ mm$$

$$h_{core-ferrite} = h_{core-\text{Neodymium}} \frac{B_{p1-ferrite}}{B_{p1-\text{Neodymium}}} \frac{D_{airgap-second}}{D_{airgap-init}}$$

$$h_{core-ferrite} = 21.1\ mm * \frac{0.428\ T}{1.26\ T} * \frac{95.2\ mm}{67.3\ mm} = 10\ mm$$

$$D_{airgap-ferrite} = 89.6\ mm$$

Therefore, we conclude that the back core thickness is 10 mm for the Ferrite design. The required geometrical modifications are finished. The Neodymium and Ferrite designs having the same outer diameter can be seen in Figure 9.

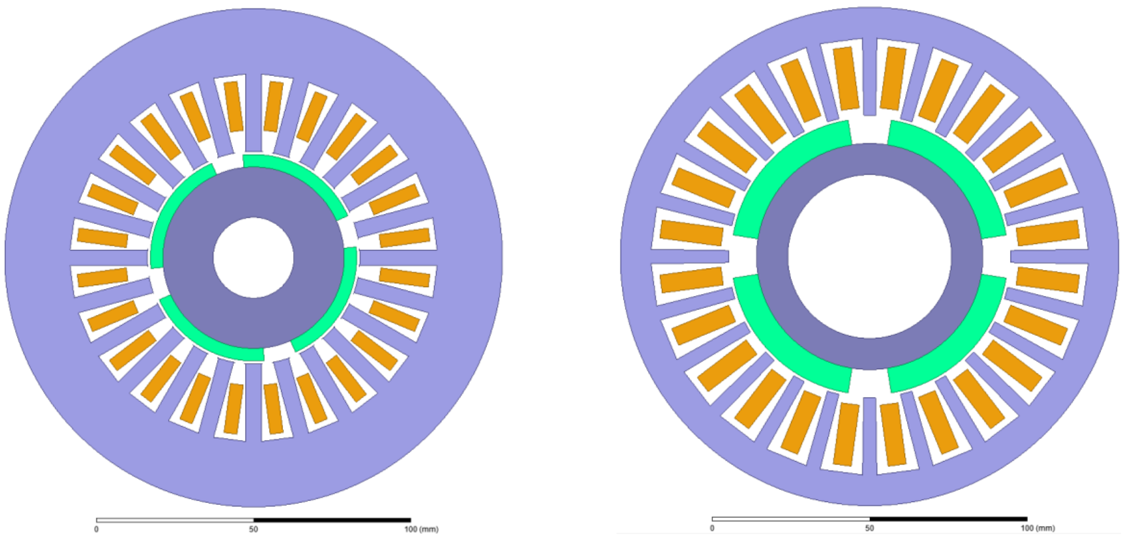

Figure 9: Neodymium (left) and modified Ferrite (right) designs

Now, for the performance comparison, let's calculate the torque for the Ferrite design. Then, we can compare the volumetric torque densities of each design. To achieve this, we need to calculate the electrical loading of the Ferrite design. Let's start with number of conductors in a slot as follows

$$A_{slot-ferrite} = 268\, mm^2$$

$$N_{slotmax} = \frac{0.6 * 268}{0.516} = 311\, turns$$

Let's choose the number of conductors in a slot as 300. Then, the electrical loading is

$$A = \frac{N_{slot} I_{wire} Q_s}{\pi D_{airgap}} = \frac{300 * 2.5A * 24}{\pi * 89.6\, mm} = 63.9\, \frac{kA}{m}$$

Then the average tangential stress and the torque values can be calculated as

$$\sigma = \frac{A B_{p1} \cos\theta}{\sqrt{2}} = \frac{63.9 * 0.428\, T * 1}{\sqrt{2}} = 19.3\, kPa$$

The torque of the machine is calculated by tangential stress and surface area as follows

$$T = \sigma \pi D_{airgap} l_{ax} r_{airgap} = 19.3\, kPa * \pi * 89.6\, mm * 0.1m * \frac{89.6\, mm}{2} = 24.4\, Nm$$

The overall comparison of the Neodymium and Ferrite design are given in Table 2.

Table 2: Comparison of Neodymium and Ferrite designs

	Neodymium design	**Modified Ferrite design**
Outer diameter	160 mm	160 mm
Volume	50 dm^3	50 dm^3
Torque	36 Nm	25 Nm
Torque density	0.72 Nm/dm^3	0.50 Nm/dm^3
Copper cost	32 €	55 €
Magnet cost	28 €	13 €
Core material cost	22 €	14 €
Total cost	82 €	82 €
Copper mass	2.1 kg	3.7 kg
Magnet mass	0.5 kg	0.8 kg
Core material mass	11 kg	7 kg
Total mass	14 kg	12 kg

The results presented in Table 2 show us that with the same outer diameter, torque density of Neodymium design is 44% more compared to Ferrite design. The total costs of the two design are the same, luckily. However, the mass of the Neodymium design is 16 % more compared to Ferrite design. Therefore, it can be concluded that, although the Ferrite design is lighter, the decrease in the torque density makes the Ferrite design non-favorable.

Also, it is seen that increased electrical loading for the Ferrite design comes with a increased cost of copper material. Although the magnet cost is less in Ferrite design, increased copper cost balances the total cost of two designs. In overall, I would prefer NdFeB design due to higher torque density.