Computer Vision — Homework #1 — Greg Kiar — 09/10/2014

(p1) GK

[citation]: Quandt, Richard E. Princeton University. "Some basic matrix theorems"

1\. a) $S v = \lambda v$ (1)

$\rightarrow$ Assume $\lambda \in \mathbb{C}$, $\therefore$

$S \bar{v} = \bar{\lambda} \bar{v}$ (2)

$\bar{v} S v = \bar{v} \lambda v$ (1)' $\quad v S \bar{v} = v \bar{\lambda} \bar{v}$ (2)'

(1)' $-$ (2)'

$\bar{v} S v - v S \bar{v} = \bar{v} \lambda v - v \bar{\lambda} \bar{v}$

$\because$ S is symmetric, $\bar{v} S v - v S \bar{v} = 0$, $\therefore$

$\bar{v} v (\lambda - \bar{\lambda}) = 0 \ldots$

$\because \bar{v} \cdot v \neq 0$, $\lambda - \bar{\lambda} = 0$, $\therefore \lambda$ must be real.

b) $S v = \lambda v$

$\rightarrow S v_i = \lambda_i v_i$ (1)

$S v_j = \lambda_j v_j$ (2)

$v_j S v_i = v_j \lambda_i v_i$ (1)' $\quad v_i S v_j = v_i \lambda_j v_j$ (2)'

$v_j S v_i - v_i S v_j = v_j \lambda_i v_i - v_i \lambda_j v_j$

$\because$ S is symmetric, $v_2 S v_1 - v_1 S v_2 = 0$, $\therefore$

$v_j v_i (\lambda_j - \lambda_i) = 0 \ldots$

$\because \lambda_j - \lambda_i \neq 0$, $v_i \perp v_j$

c) $\because$ S is symmetric, and all $\lambda|_i^n \in \mathbb{R}$

$$\left.\begin{aligned} S v_1 &= \lambda_1 v_1 \\ S v_2 &= \lambda_2 v_2 \\ &\vdots \\ S v_n &= \lambda_n v_n \end{aligned}\right\}$$

$\therefore v_i \perp v_j$, and λ_i can be scaled such that $|v_i| = 1$, $\forall$ ~~satisfies being orthonormal~~ vectors in V which are orthonormal

Also, $\because S$ is $n \times n$, Eigen Value Decomposition Applies:

$$S = V \Lambda V^{-1}$$

$\because$ V is always invertible, V must have n vectors.

$\therefore$ V is orthonormal basis of n elements.