JHU BME 580.422 Systems Bioengineering II

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Introduction to voluntary motor control:

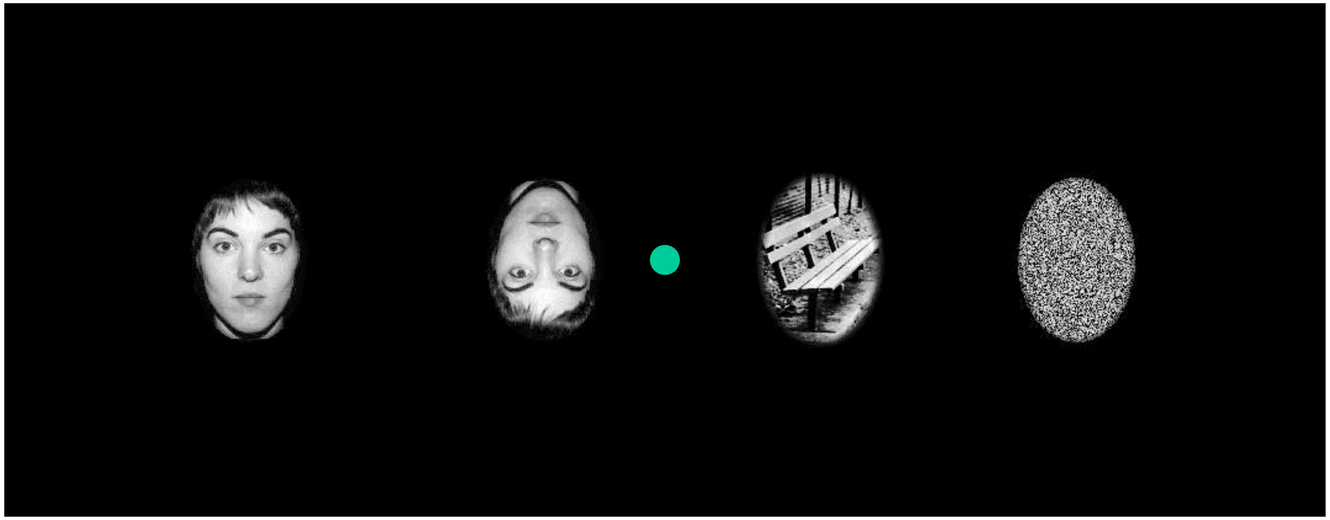
Movement as an interplay between effort and reward

What separates plants and animals is that animals can move voluntarily. To control movement, multi-cell organisms developed a nervous system.

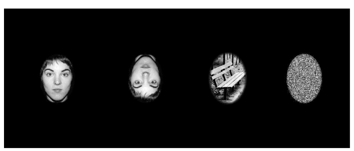
Development of the nervous system began when multi-cell organisms began to move.

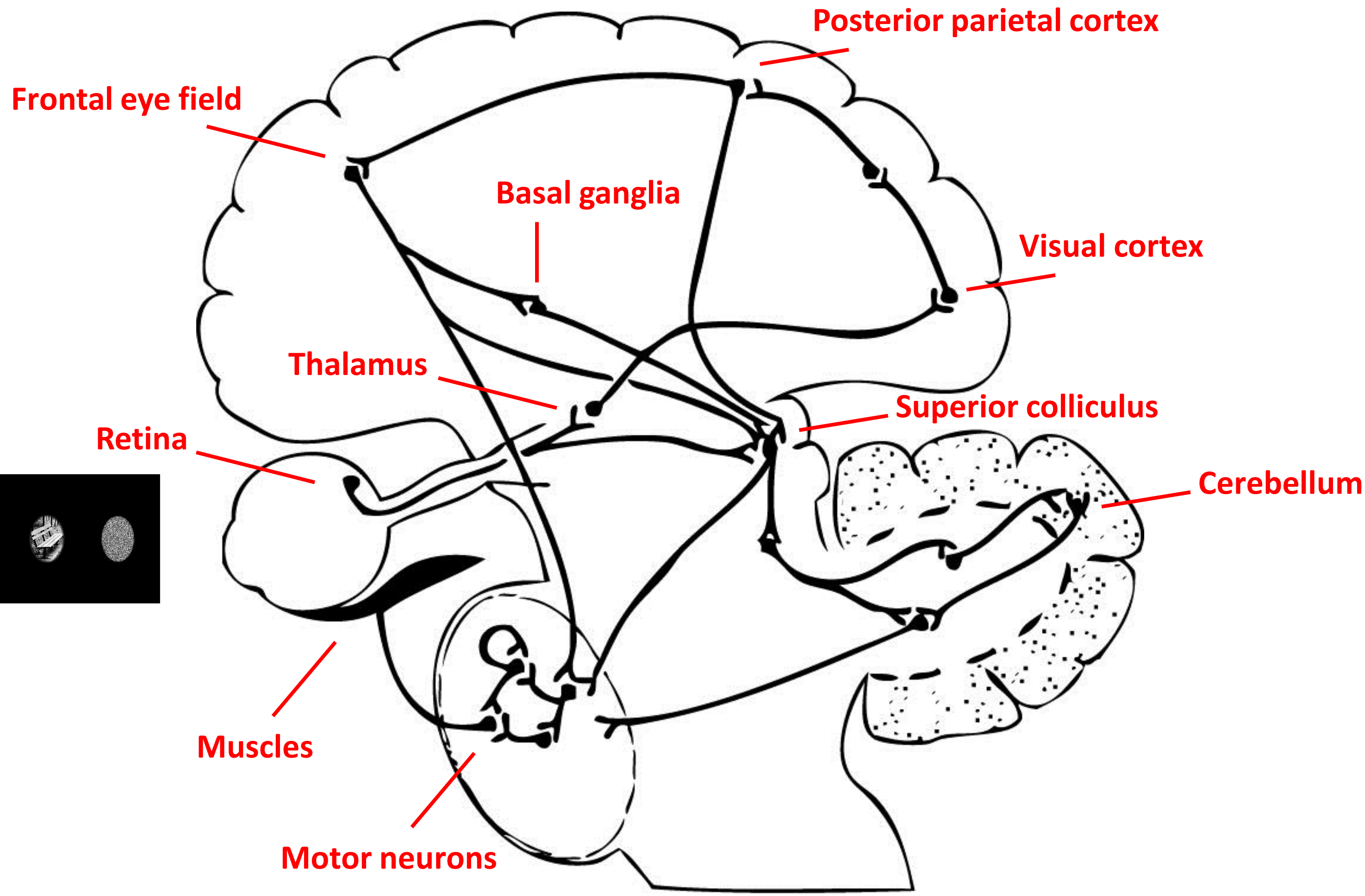
The sea squirt: In larval form, is briefly free swimming and is equipped with a brain-like structure of 300 cells. Upon finding a suitable substrate, it buries its head and starts absorbing most of its own brain.

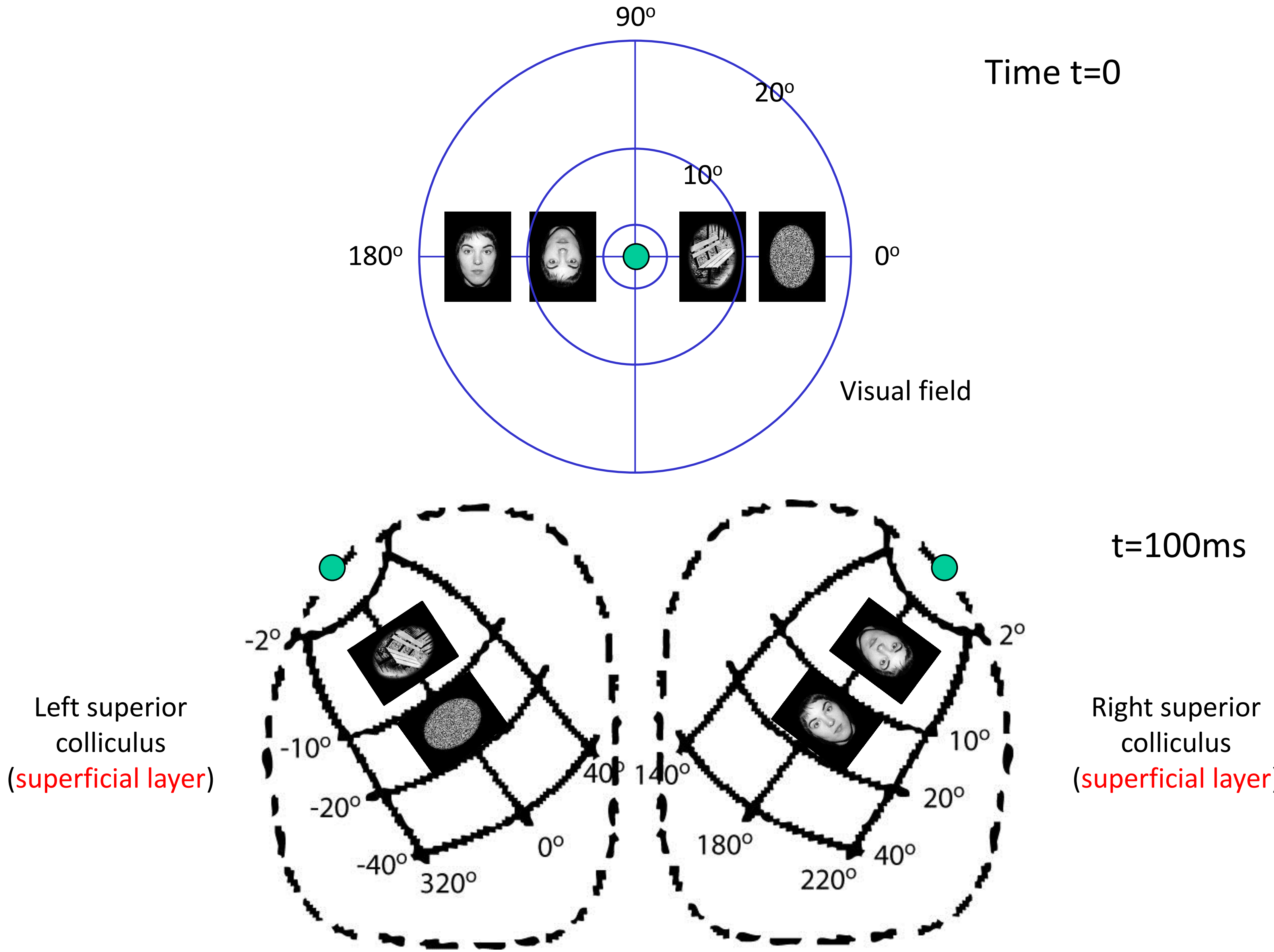
The brain assigns value to stimuli that we sense.
We tend to move toward the most valuable stimulus.



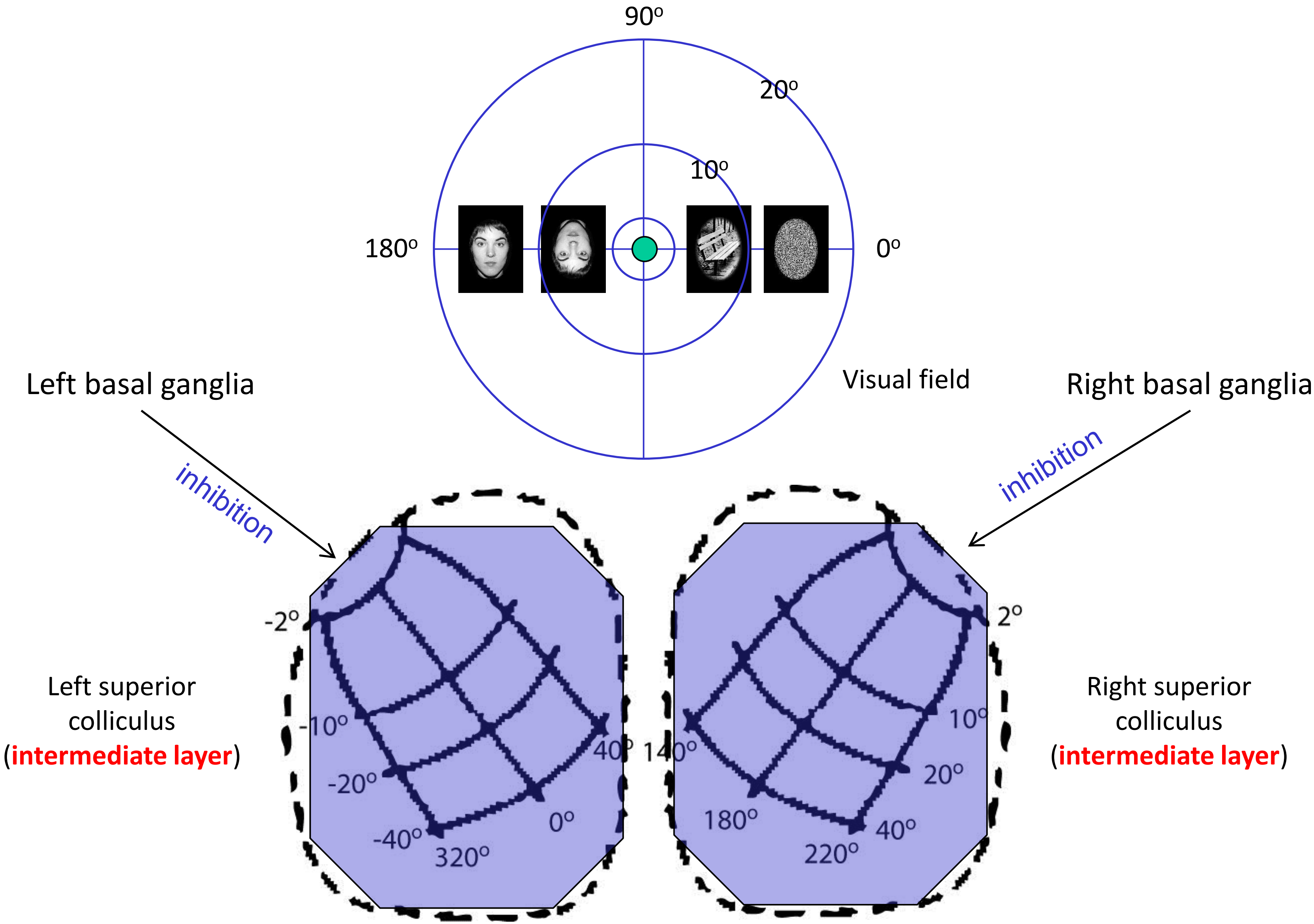
Xu-Wilson, Zee, and Shadmehr (2009)



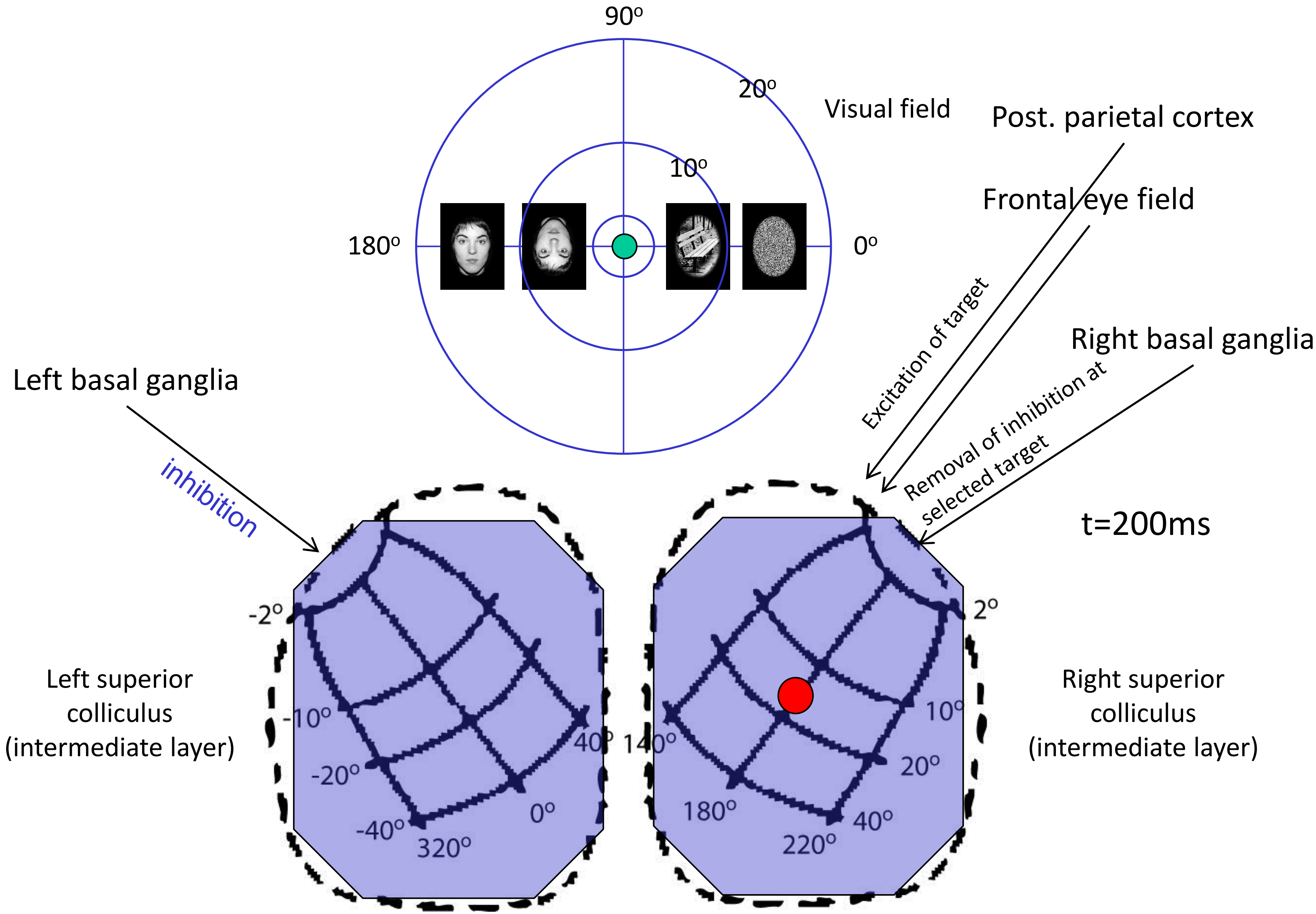

Posterior parietal cortex
Frontal eye field
Basal ganglia
Visual cortex
Thalamus
Superior colliculus
Retina
Cerebellum
Muscles
Motor neurons



Superficial layers of superior colliculus (SC) have neurons that respond to visual information on the retina. Right SC has a map of the left visual field. Left SC has a map of the right visual field.



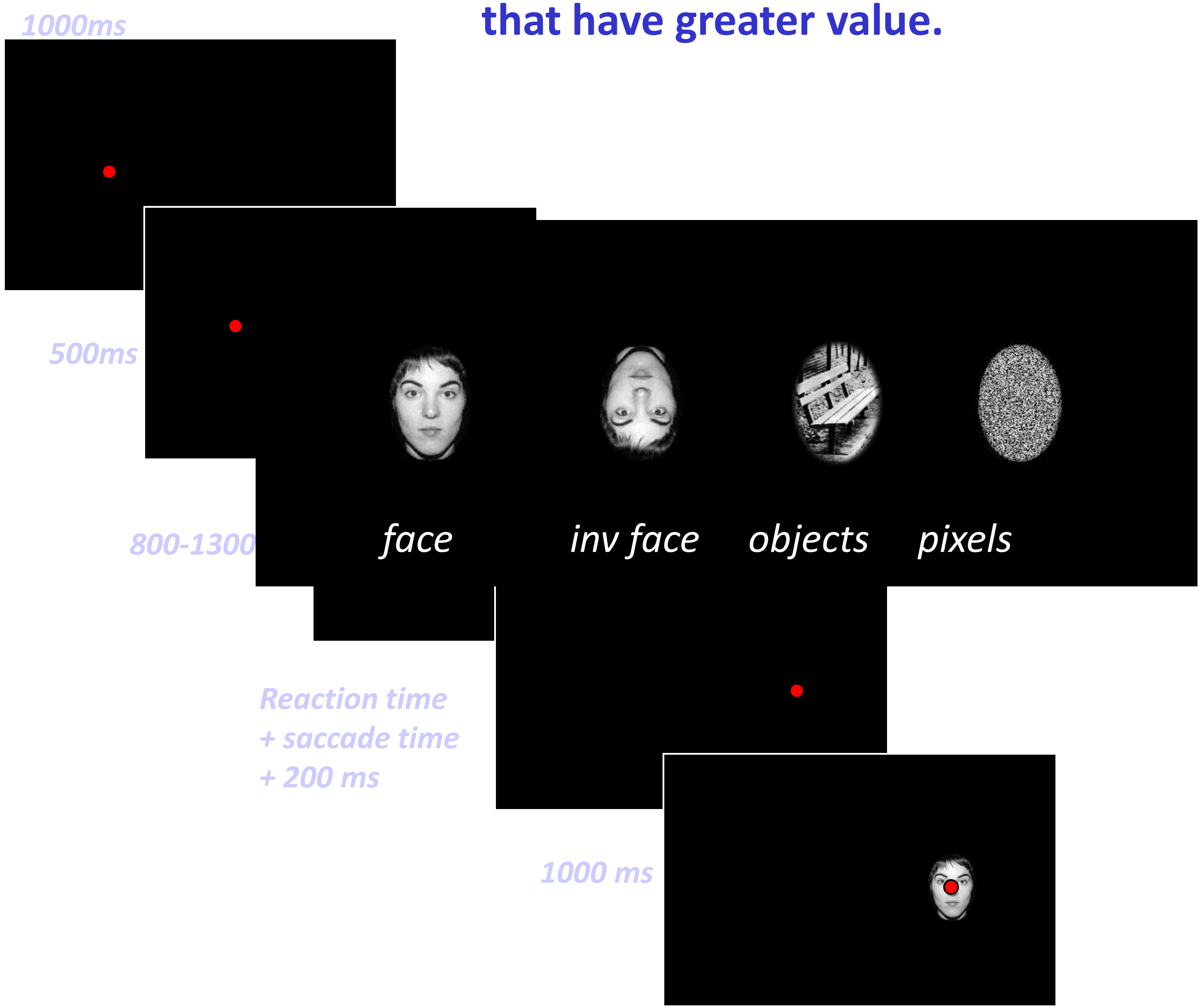
For the eyes to move, neurons in the intermediate layers of the superior colliculus must fire. However, at this point the basal ganglia is inhibiting the colliculus, and therefore the eyes stay fixated at the center.



A saccadic eye movement occurs where there is a burst of activity on the map of visual space on the intermediate layers of superior colliculus.

- Soon after visual information falls on the retina, superficial layers of superior colliculus (SC) respond. Right SC has a map of the left visual field. Left SC has a map of the right visual field.
- For the eyes to move, neurons in the intermediate layers of the superior colliculus must fire. However, at this point the basal ganglia is inhibiting the colliculus, and therefore the eyes stay fixated at the center.
- The neurons in the cortex, along with the basal ganglia, evaluate the visual information, and assign a value to each image. The image with the highest value is the up-right face. The basal ganglia reflects this value assignment by removing the inhibition for the neurons on the intermediate layer of SC corresponding to the location of the upright face.
- Posterior parietal cortex and frontal eye field excite this same region of the colliculus. As a result, the neurons in the spatial region corresponding to the upright face in the intermediate superior colliculus fire.
- A saccade is made to the face.

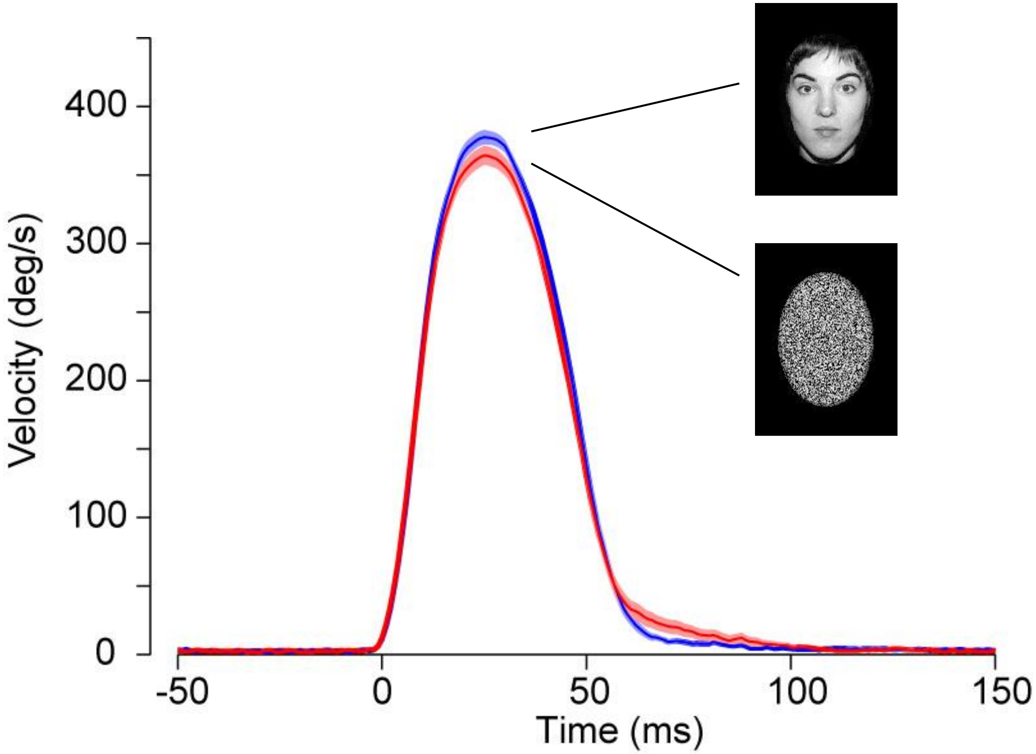
The brain assigns value to stimuli around us. We tend to move toward the most valuable stimulus. Movements are faster to stimuli that have greater value.



Xu-Wilson et al. (2009)

Saccades to faces are faster

Eye velocity



Xu-Wilson et al. (2009)

Reaching for a candy bar

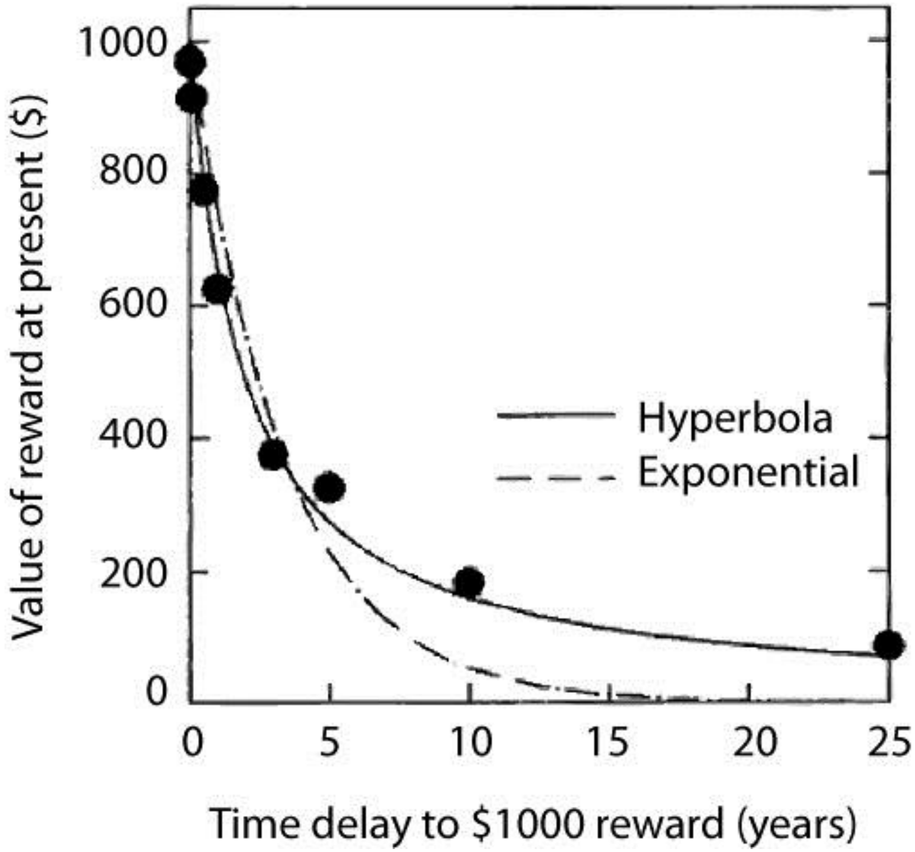
Sackaloo et al. (2015)

- 39 subjects were tested by asking them to reach to a single candy bar from a constant start position.
- Various candies were considered in random order.
- After the reach trials the subjects completed a survey describing their preferences for the candies.
- Reach duration was significantly shorter for most preferred as compared to least preferred.
- Therefore, people reach faster toward stimuli that they value more as compared to stimuli that they value less.

Summary

- The basal ganglia, along with the posterior parietal cortex and the frontal lobe, assign value to the various available actions (different movements that can be done).
- We tend to produce a movement toward the stimulus that has the highest subjective value.
- The relative value of the stimulus affects movement speed: we move faster toward stimuli that our brain assigns a greater value.

Subjective value of a stimulus is not constant. Value drops as a function of time.



Myerson and Green (1995)

Subjective value of reward at time *T*

Subjective value of reward now

$$SV(\alpha, t = T) = \frac{SV(\alpha, t = 0)}{1 + \gamma T}$$

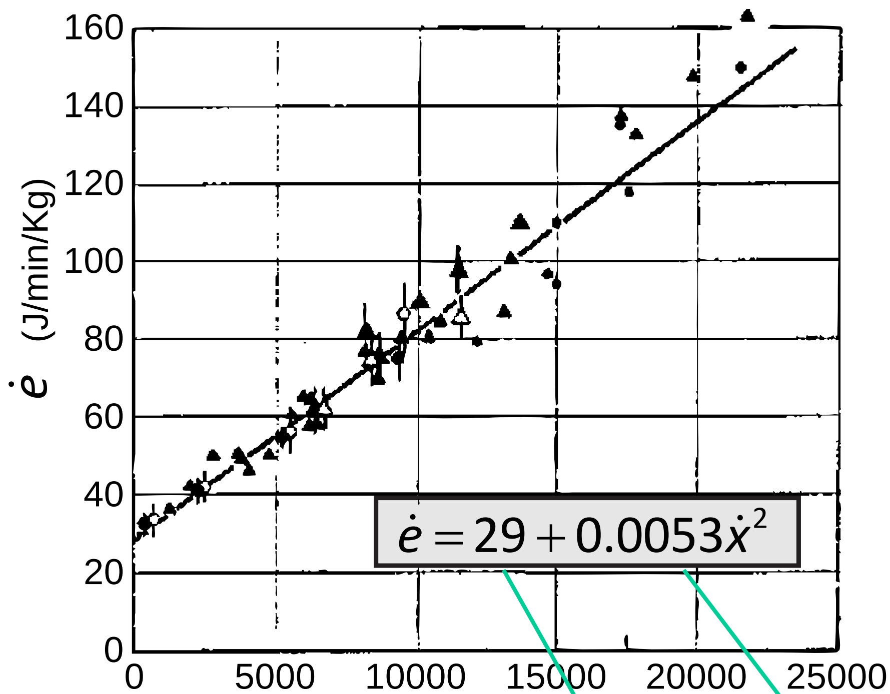
Objective value of reward

Because time discounts the value of the stimulus, the faster we acquire the rewarding stimulus, the better.
However, fast movement require greater energy.

The faster the movement, the greater the metabolic cost.

Energy consumption pre minute of walking increases with the squared speed of walking.

Energy consumption
per min per kg



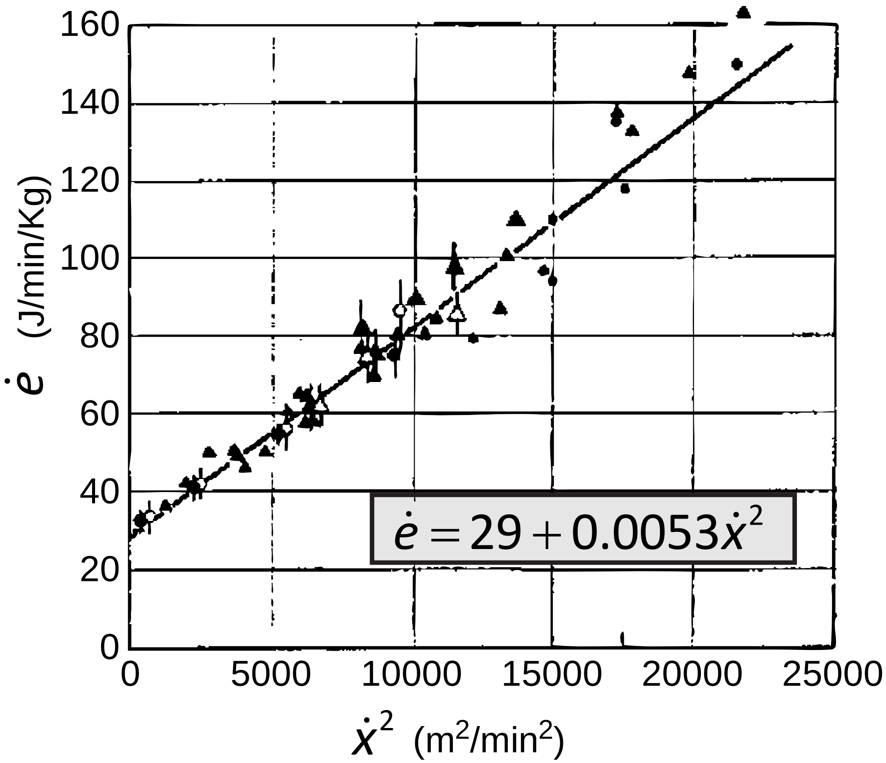
Average speed of walking (squared). $\dot{x}^2$ (m^2/min^2)

Metabolic cost (per min) of walking

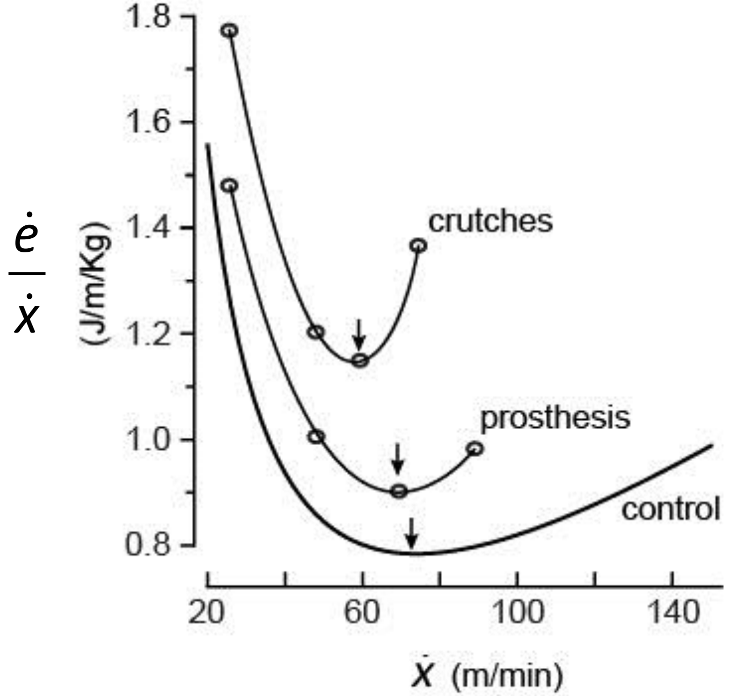
Metabolic cost (per min) of being alive

Ralston (1958)

Animals move at approximately the speed that minimizes energetic cost.

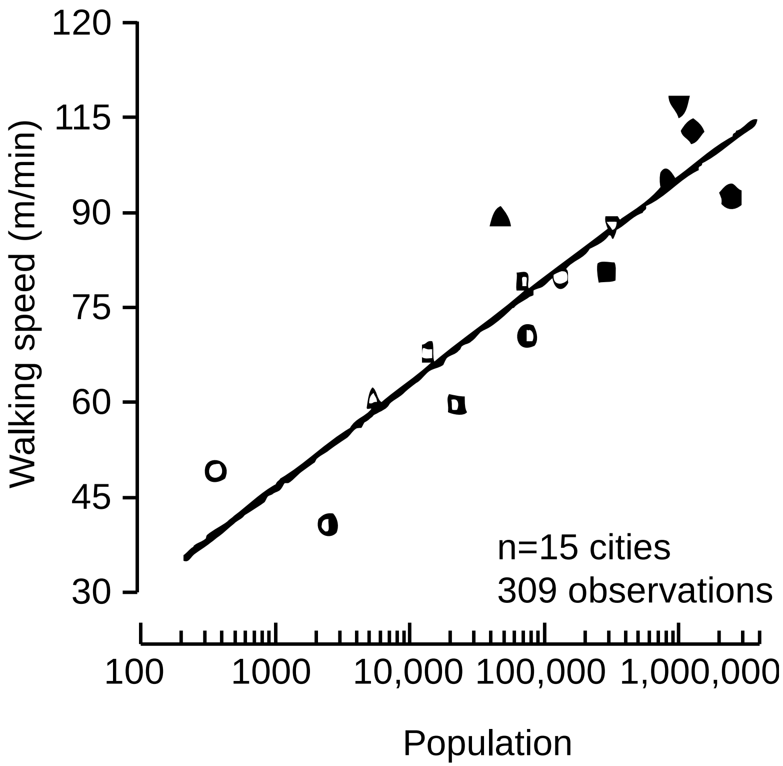


Energy per unit distance $\frac{\dot{e}}{\dot{x}}$



Ralston (1958)

Energetic cost is only one concern for the brain: people walk twice as fast in some cities as compared to small towns



Bornstein and Bornstein (1976)

Economics of motor control

We move to acquire a rewarding state. This is the **gain**.

However, we must pay for that action by consuming energy. This is the **loss**.

The utility of a movement is the sum of gains and losses of that movement, discounted by time.

We should move in a way that maximizes the utility.

Subjective value of reward

Energetic cost of the movement

Utility of the movement

$$J = \frac{\alpha - e(T)}{1 + \gamma T}$$

Temporal discounting factor

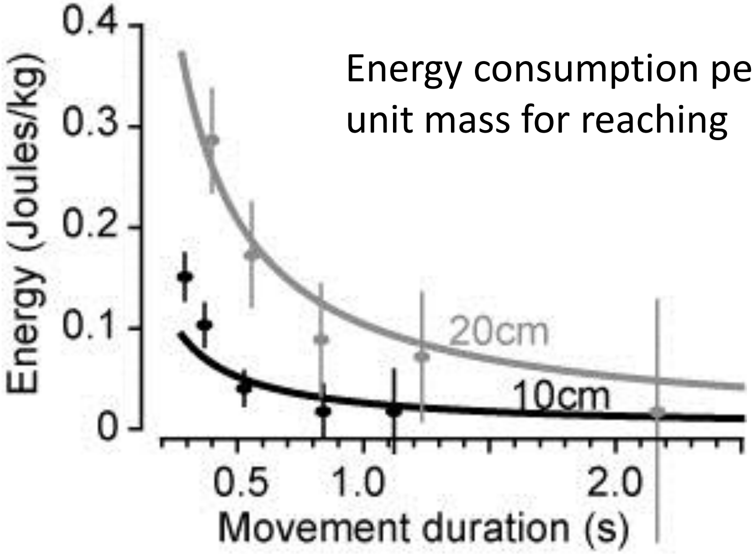
Duration of the movement

Movement involves expenditure of energy.
How does energy vary with duration and distance of a movement?

$\dot{e} = a + cE\left[\dot{x}\right]^2$ Total rate of energy consumption per unit mass during walking

$\dot{e}_m = cE\left[\dot{x}\right]^2$ Rate of energy consumption per unit mass for the act of walking

$e_m = c\dfrac{d^2}{T}$ Total energy consumption per unit mass as a function of distance and duration



Energy consumption per unit mass for reaching

$c = 2.6 \text{ J.sec / Kg / m}^2$

$e = mc\dfrac{d^2}{T}$ Total energy consumption as a function of mass, distance, and duration of the movement.

Movement utility as sum of reward and effort

Utility of the movement

$$J = \frac{\overbrace{\alpha}^{\text{reward}} - \overbrace{mcd^2 / T}^{\text{effort}}}{1 + \gamma T}$$

We want to find the *T* (movement duration) that maximizes this utility.

$$\frac{dJ}{dT} = \frac{cmd^2\left(1 + 2\gamma T\right) - \alpha\gamma T^2}{T^2(1 + \gamma T)^2}$$

Duration of the movement that maximizes the utility

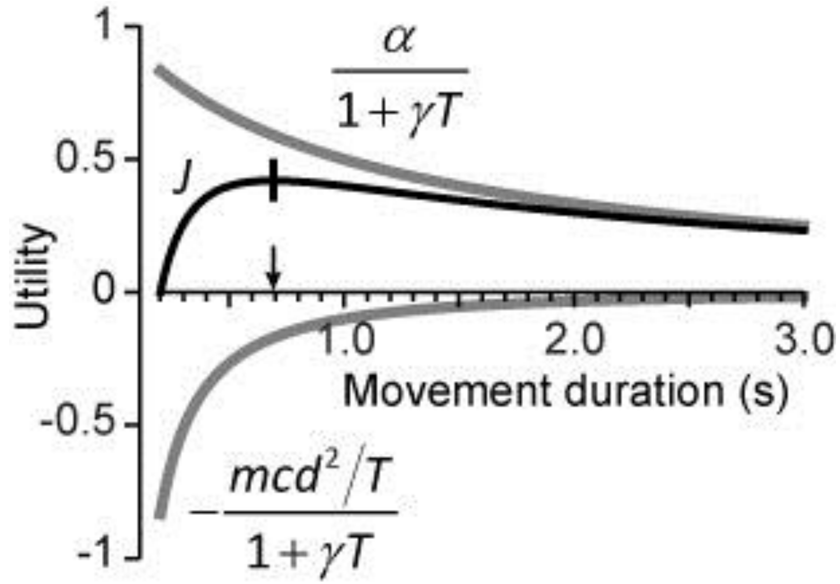
$$T^* = \frac{cd^2m + \sqrt{c^2d^4m^2 + \alpha cd^2m\gamma^{-1}}}{\alpha}$$

- As mass increases, utility decreases (option is less preferred), and duration of the movement increases (movement is slower).
- As reward increases, utility increases (option is more preferred), and duration of the movement decreases (movement is faster).

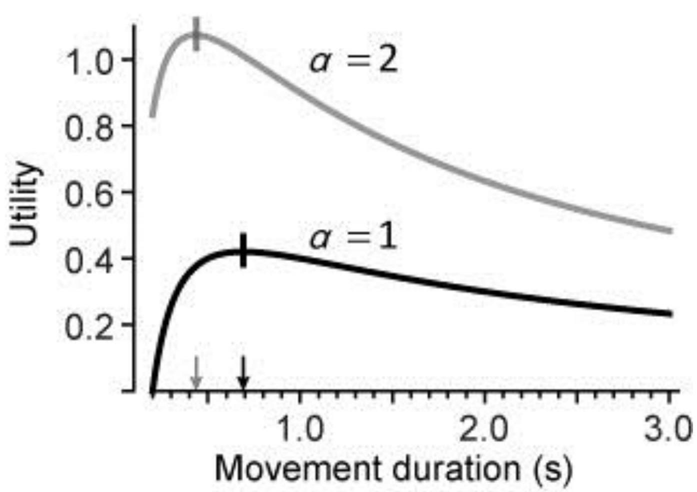
Movement utility as sum of reward and effort

reward effort

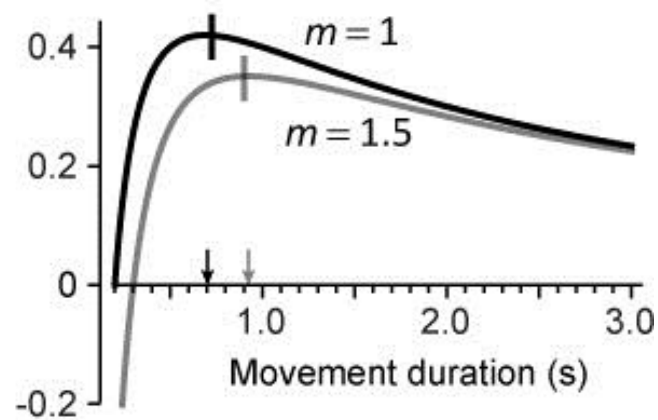
$$J = \frac{\alpha - mcd^2/T}{1+\gamma T}$$



Increased reward

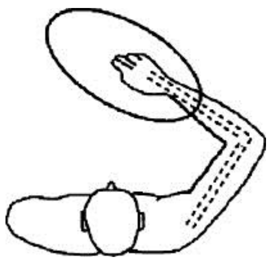


Increased effort

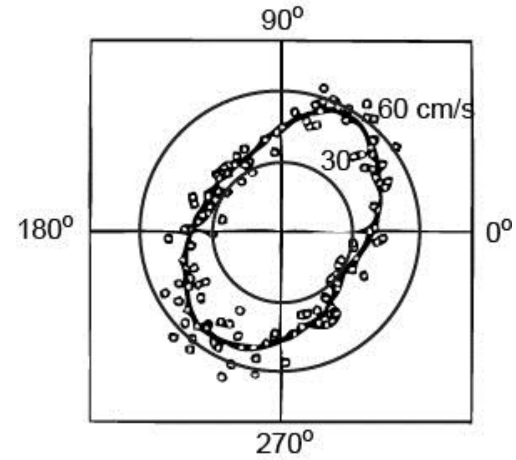


Reaching speed is slower for directions that have greater mass

Gordon et al. (1994)



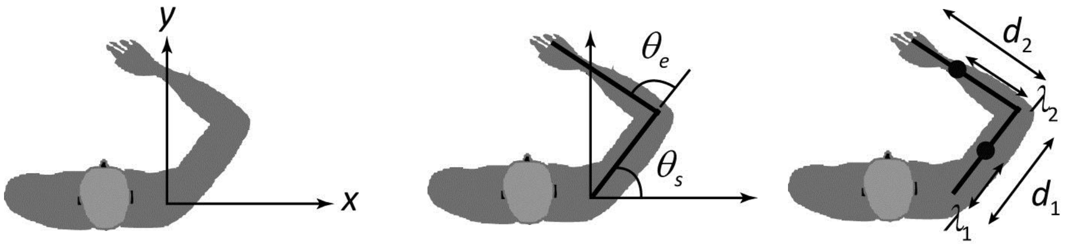
Peak velocity



Authors asked their subjects to make a single, uncorrected movement to a target at 10cm, with no time constraints on the movement.

Estimating the mass of the arm for movements in various directions

1. Define the inertia of the arm



$$d_1 = 0.33 \quad d_2 = 0.43 \quad \text{meters}$$
$$m_1 = 1.93 \quad m_2 = 1.52 \quad \text{kg}$$
$$\lambda_1 = \frac{d_1}{2} \quad \lambda_2 = \frac{2d_2}{3} \quad \text{meters}$$
$$I_1 = 0.014 \quad I_1 = 0.019 \quad \text{kg m}^2$$

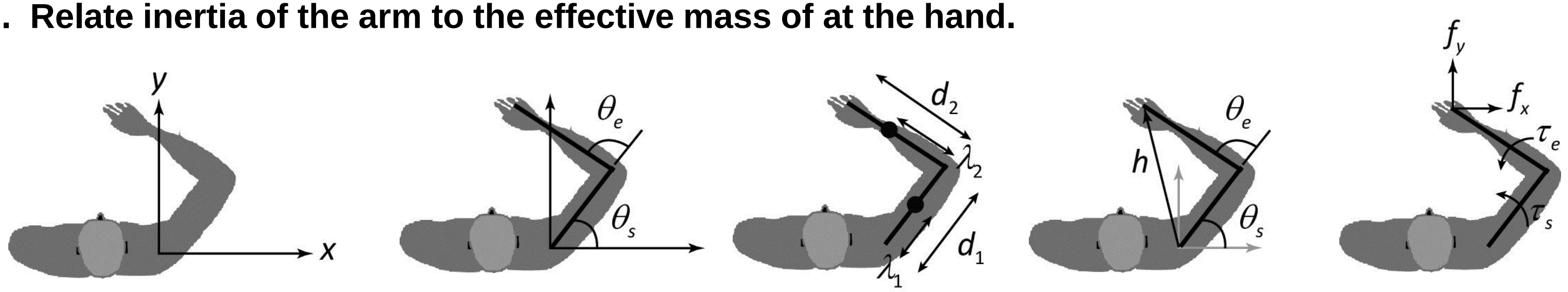
$$H = \begin{bmatrix} a_3 + a_1 d_1^2 + a_4 + 2a_2 d_1 \cos(\theta_e) & a_2 d_1 \cos(\theta_e) + a_4 \\ a_2 d_1 \cos(\theta_e) + a_4 & a_4 \end{bmatrix}$$

When the arm is in the horizontal plane,
the inertia of the arm is a 2x2 matrix

$$a_1 = m_2 \qquad a_3 = m_1 \lambda_1^2 + I_1$$
$$a_2 = m_2 \lambda_2 \qquad a_4 = m_2 \lambda_2^2 + I_2$$

Estimating the mass of the arm for movements in various directions

2. Relate inertia of the arm to the effective mass of at the hand.



$$\tau = \begin{bmatrix} \tau_s \\ \tau_e \end{bmatrix} \quad \theta = \begin{bmatrix} \theta_s \\ \theta_e \end{bmatrix}$$

$$\tau = H\ddot{\theta}$$

At rest, inertia describes the relationship between forces and torques in joint coordinates

$$h = \begin{bmatrix} d_1\cos(\theta_s) + d_2\cos(\theta_s+\theta_e) \\ d_1\sin(\theta_s) + d_2\sin(\theta_s+\theta_e) \end{bmatrix}$$

Location of the hand in Cartesian coordinates written in terms of joint angles

$$f = \begin{bmatrix} f_x \\ f_y \end{bmatrix} \quad f = M\ddot{h}$$

At rest, the mass matrix *M* describes the relationship between forces and acceleration in Cartesian coordinates

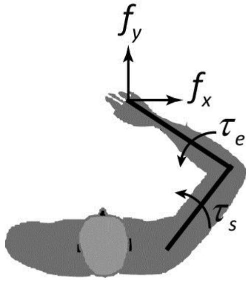
$$\Lambda = \frac{dh}{d\theta} = \begin{bmatrix} \dfrac{dh_x}{d\theta_s} & \dfrac{dh_x}{d\theta_e} \\ \dfrac{dh_y}{d\theta_s} & \dfrac{dh_y}{d\theta_e} \end{bmatrix}$$

Jacobian matrix

$$= \begin{bmatrix} -d_1\sin(\theta_s) - d_2\sin(\theta_s+\theta_e) & -d_2\sin(\theta_s+\theta_e) \\ d_1\cos(\theta_s) + d_2\cos(\theta_s+\theta_e) & d_2\cos(\theta_s+\theta_e) \end{bmatrix}$$

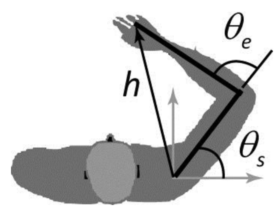
Estimating the mass of the arm for movements in various directions

2. Relate inertia of the arm to the effective mass of at the hand.



$$\tau = \Lambda^T f$$

Force and torques are related by the Jacobian matrix



$$\dot{h} = \Lambda \dot{\theta}$$

$$\ddot{h} = \frac{d\Lambda}{d\theta}\dot{\theta}\dot{\theta} + \Lambda \ddot{\theta}$$

Motion at the hand is related to motion at the joints by the Jacobian

$$\left.\begin{aligned} \tau &= H\ddot{\theta} \\ \tau &= \Lambda^T f \end{aligned}\right\} H\ddot{\theta} = \Lambda^T f$$

$$f = \left(\Lambda^{-1}\right)^T H\ddot{\theta}$$

$$f \approx \underbrace{\left(\Lambda^{-1}\right)^T H\Lambda^{-1}}_{M} \ddot{h}$$

$$f \approx M\ddot{h}$$

$$M = \left(\Lambda^{-1}\right)^T H\Lambda^{-1}$$

The above equation relates inertia *H* to mass *M*.

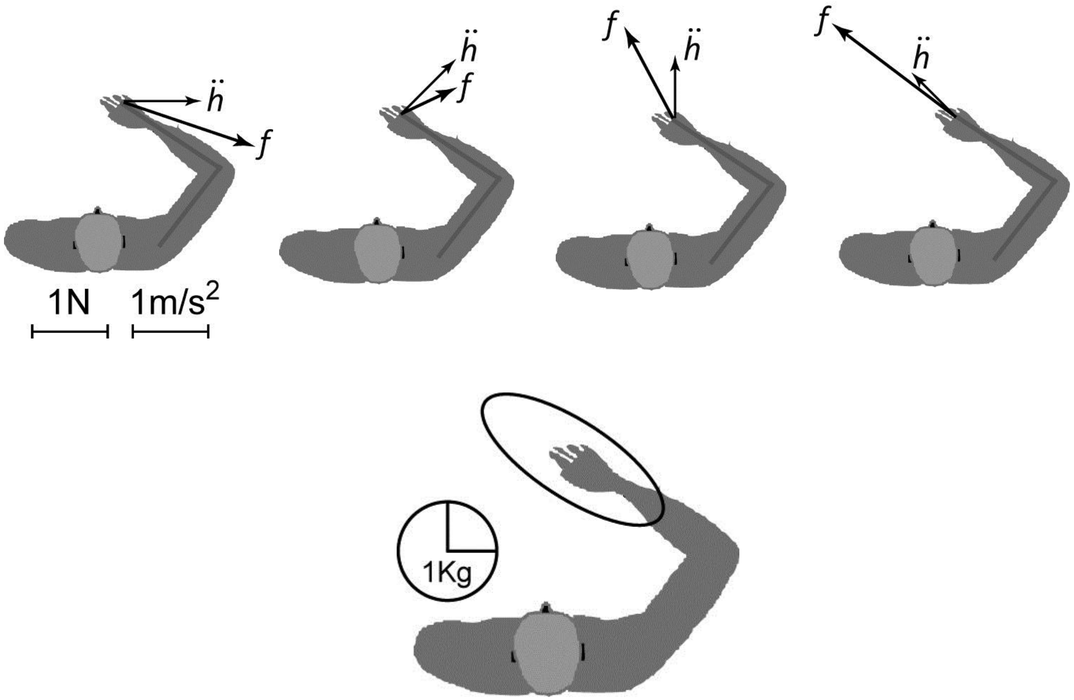
The mass at the hand *M* is a 2x2 matrix, relating acceleration of the hand to forces at the hand.

Estimating the mass of the arm for movements in various directions

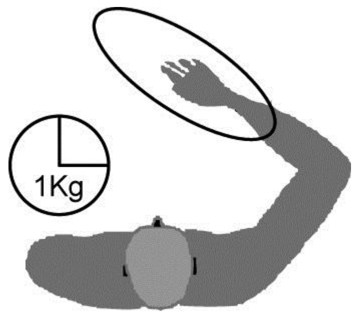
$$f \approx M\ddot{h}$$

$$M = \left(\Lambda^{-1}\right)^{T} H \Lambda^{-1}$$

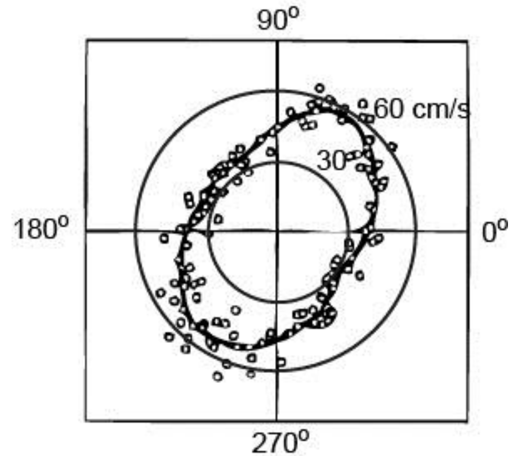
Pick a unit length acceleration of the hand in direction *p*. Using the mass matrix we can compute the required force. The magnitude of the resulting force vector is the effective mass *m* for a movement in direction *p*.



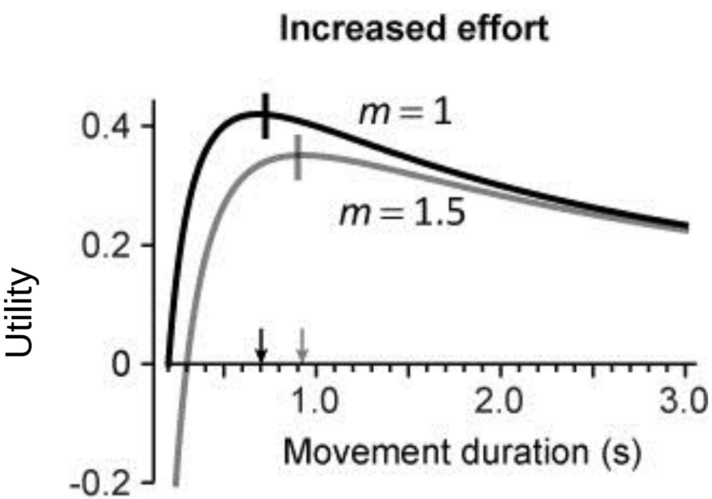
Reaching speed is slower for directions that have greater mass

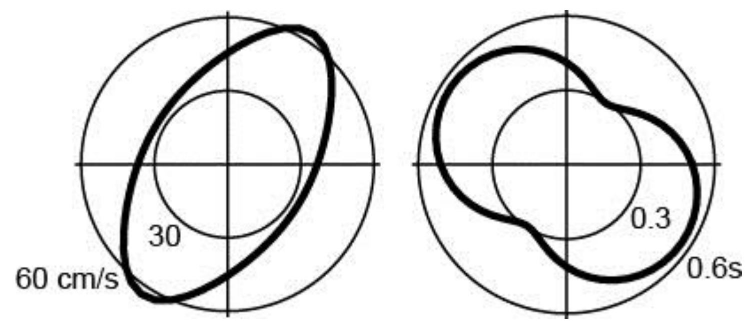


Peak velocity



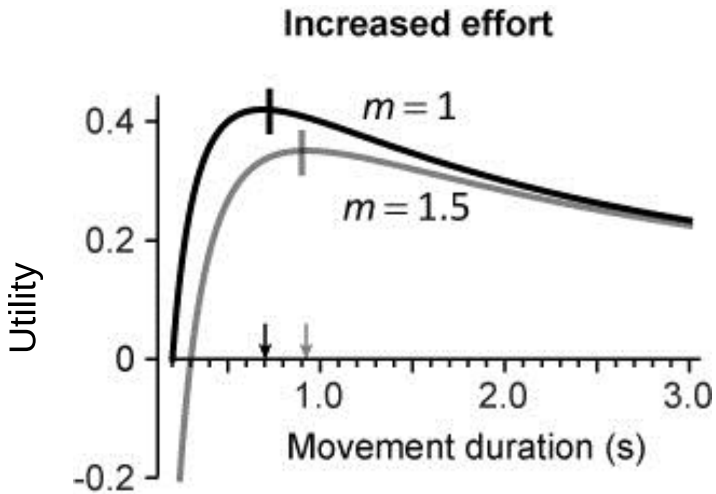
$$J = \frac{\alpha - mcd^2/T}{1+\gamma T} \qquad T^* = \frac{cd^2m + \sqrt{c^2d^4m^2 + \alpha cd^2 m\gamma^{-1}}}{\alpha}$$





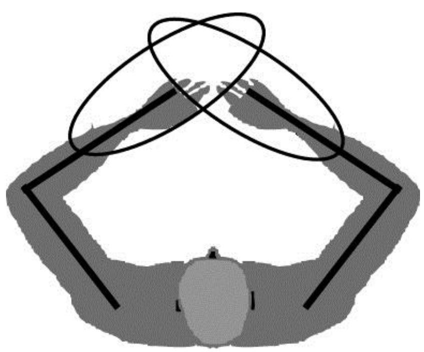
Peak velocity Duration

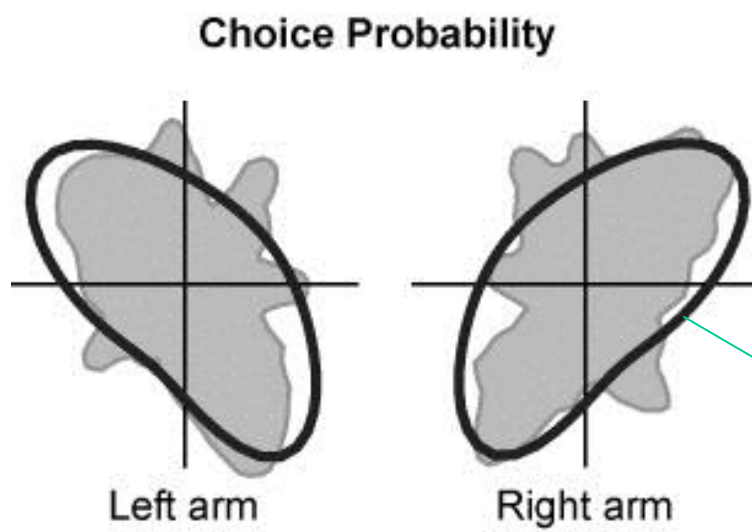
Probability of choosing a reaching direction is accounted for by the utility of the movement



Increased mass makes the movement less valuable (utility of that movement is lower).

Subjects were asked to reach in any direction that they chose (out-and-back movements of constant amplitude). The investigators measure probability of the reach direction.





$$J = \frac{\alpha - mcd^2/T}{1+\gamma T}$$

Goble et al. (2007)
Wang and Dounskaia (2012)

Summary

- The basal ganglia, along with the posterior parietal cortex and the frontal lobe, assign value to the various available actions (different movements that can be done).
- We tend to produce a movement toward the stimulus that has the highest subjective value.
- The relative value of the stimulus affects movement speed: we move faster toward stimuli that our brain assigns a greater value.
- Because time discounts the value of the stimulus, the faster we acquire the rewarding stimulus, the better. However, fast movement require greater energy.
- The utility of a movement is the sum of gains and losses of that movement, discounted by time. Reward we attain is the gain, the energy we spend is the loss.

Subjective value of reward

Energetic cost of the movement

Utility of the movement

$$J = \frac{\alpha - e(T)}{1 + \gamma T}$$

Temporal discounting factor

Duration of the movement