Greg Kiar 04/01/2015

SBE II: Homework 7

Experiments 1 → N:

In this homework, we are modeling movement of the arm. The initial position of the arm is shown below. Here and forward, $\theta = \begin{bmatrix} \theta_s \\ \theta_e \end{bmatrix} = \begin{bmatrix} 57 \\ 90 \end{bmatrix}$ in degrees. The parameters used have been taken from the course notes.

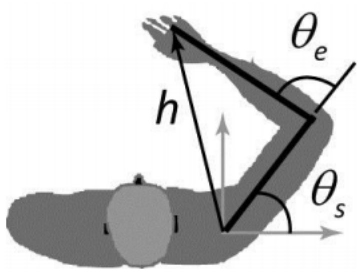


Figure 1: sketch of the arm (copied from course notes)

1. The (x,y) position of the elbow is given by:
$$e = \begin{bmatrix} d_1 \times \cos(\theta_s) \\ d_1 \times \sin(\theta_s) \end{bmatrix} = \begin{bmatrix} 0.1797 \\ 0.2768 \end{bmatrix} m$$

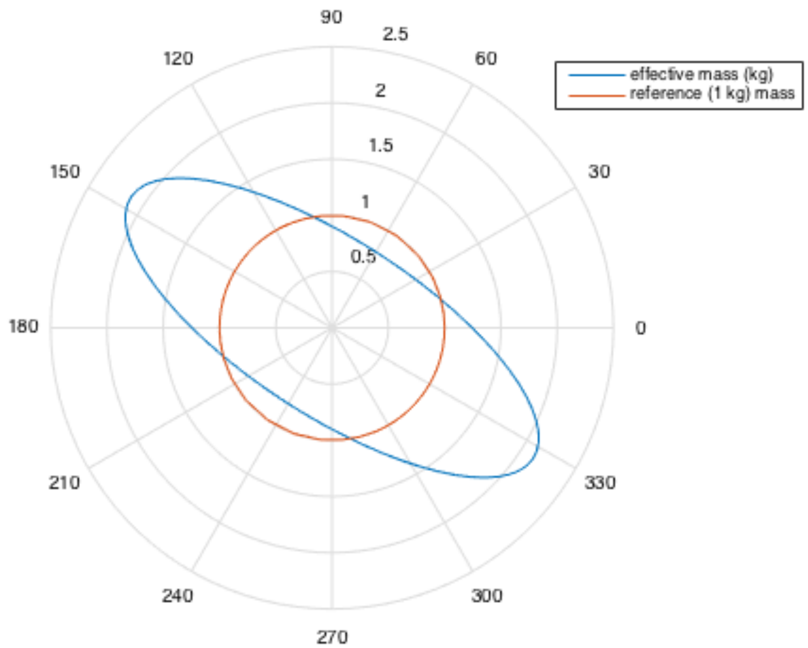
2. The (x,y) position of the hand is given by:
$$h = e + \begin{bmatrix} d_2 \times \cos(\theta_s + \theta_e) \\ d_2 \times \sin(\theta_s + \theta_e) \end{bmatrix} = \begin{bmatrix} 0.1840 \\ 0.5110 \end{bmatrix} m$$

3. The inertia matrix is given by:
$$H = \begin{bmatrix} a_3 + a_1 d_1^2 + a_4 + 2a_2 d_1 \times \cos(\theta_e) & a_2 d_1 \cos(\theta_e) + a_4 \\ a_2 d_1 \times \cos(\theta_s) + a_4 & a_4 \end{bmatrix}$$
$$= \begin{bmatrix} 0.3760 & 0.1439 \\ 0.1439 & 0.1439 \end{bmatrix} kg\ m^2$$

4. The Jacobian of this system is given by:
$$\Lambda = \begin{bmatrix} \frac{dh_x}{d\theta_s} & \frac{dh_x}{d\theta_e} \\ \frac{dh_y}{d\theta_s} & \frac{dh_y}{d\theta_e} \end{bmatrix} = \begin{bmatrix} -0.5110 & -0.2342 \\ -0.1809 & -0.3606 \end{bmatrix} m$$

5. The mass matrix of the system is given by:
$$M = {\Lambda^{-1}}^T H \Lambda^{-1} = \begin{bmatrix} 1.7298 & -0.6179 \\ -0.6179 & 1.1796 \end{bmatrix} kg$$

6. Over the range of $\theta = [0, 2\pi]$, plotted below is the resulting effective mass of the arm for different acceleration directions.



7. The derivative of J, the movement utility, is given as follows:

$$J = \frac{\alpha - mcd^2T}{1+\gamma T}$$

$$\frac{dJ}{dT} = \frac{d}{dT}\left[\frac{\alpha - mcd^2T}{1+\gamma T}\right]$$

$$= \frac{d}{dT}\left(\alpha - \frac{mcd^2}{T}\right)(1+\gamma T)^{-1}$$

$$= \frac{mcd^2/T}{1+\gamma T} - \frac{\alpha - mcd^2/T}{(1+\gamma T)^2}$$

$$\frac{dJ}{dT} = \frac{mcd^2(1+2\gamma T) - \alpha\gamma T^2}{T^2(1+\gamma T)^2}$$

8. The optimal value of T, T^*, is found by setting this derivative to zero and solving for T, as follows:

$$0 = \frac{mcd^2(1+2\gamma T) - \alpha\gamma T^2}{T^2(1+\gamma T)^2}$$

$$0 = mcd^2(1+2\gamma T) - \alpha\gamma T^2$$

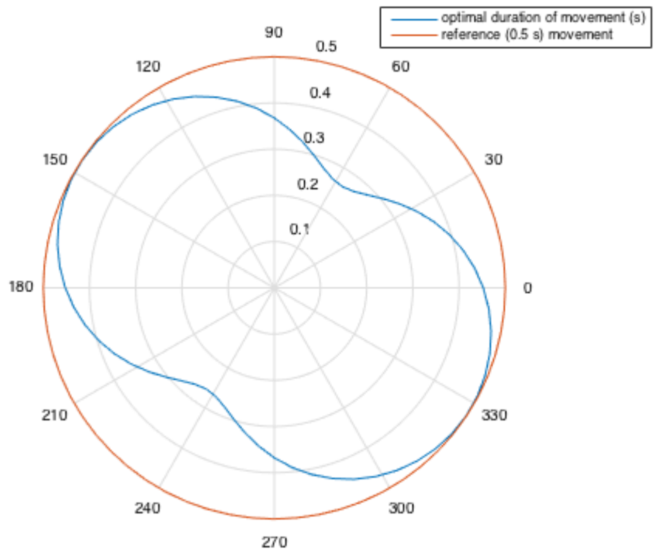
$$0 = mcd^2 + mcd^2 2\gamma T - \alpha\gamma T^2$$

$$0 = T^2 + T\left(-\frac{2mcd^2}{\alpha}\right) - \frac{mcd^2}{\alpha\gamma}$$

$$T = \frac{2mcd^2}{\alpha} + \sqrt{\frac{4m^2c^2d^4}{\alpha^2} + \frac{4mcd^2}{\alpha\gamma}} \; / \; 2$$

$$T^* = \frac{mcd^2 + \sqrt{c^2m^2d^4 + \alpha cmd^2\gamma^{-1}}}{\alpha}$$

9. Over the range of $\theta = [0, 2\pi]$, plotted below is the optimal movement time of an action.



10. Over the range of $\theta = [0, 2\pi]$, plotted below is the optimal utility (i.e. utility corresponding to optimal time) of an action.