Greg Kiar 02/25/2015

SBE II: Homework 4

Experiment-2:

Given that we know the following:

$$\lambda^2 \frac{\partial^2 V}{\partial x^2} = \tau \frac{\partial V}{\partial t} + V$$
$$V(x, t = 0) = 0$$
$$I_i(x = 0, t) = I_0(t)$$
$$I_i(L) = 0$$

With the boundary conditions:

$$\left.\frac{\partial V}{\partial x}\right|_{x=0} = -r_i I_0(t)$$
$$\left.\frac{\partial V}{\partial x}\right|_{x=L} = -r_i I_i(L) = 0$$

We are asked to transform the differential equation into the Laplace domain. Recall that $\mathcal{L}$: f(t) $\rightarrow$ g(s). Thus, $\mathcal{L}\{V(x,t)\} = \tilde{V}(x,s)$.
In order to simplify the computation moving forward both relative length and time, χ, T, will be used. Recall, from their definition, that $\chi = \frac{x}{\lambda}$ and $T = \frac{t}{\tau}$.

$$\lambda^2 \frac{\partial^2 V}{\partial x^2} = \tau \frac{\partial V}{\partial t} + V$$
$$\frac{\partial^2 V}{\partial \chi^2} = \frac{\partial V}{\partial T} + V$$
$$\mathcal{L}\left\{\frac{\partial^2 V}{\partial \chi^2}\right\} = \mathcal{L}\left\{\frac{\partial V}{\partial T} + V\right\}$$
$$\frac{\partial^2 \tilde{V}}{\partial \chi^2} = s\tilde{V} - V(\chi, T = 0) + \tilde{V}$$
$$\frac{\partial^2 \tilde{V}}{\partial \chi^2} = (s+1)\tilde{V}(\chi, s)$$

We are also able to transform our initial conditions from before so that they now exist in out Laplace domain.

$$\left.\frac{\partial V}{\partial x}\right|_{x=0} = -\frac{I_0(s)}{sG_\infty}$$
$$\left.\frac{\partial V}{\partial x}\right|_{x=L} = -r_i I_i(L): \tilde{V}(\chi, T) < \infty \,\forall \chi, T$$

Combining the above two results, we can obtain the closed form solution (solved for constants in next part):

$$\tilde{V}(\chi, T) = A(s)e^{\sqrt{s+1}\chi} + B(s)e^{-\sqrt{s+1}\chi}$$