580.422 System Bioengineering II: Neurosciences (Spring 2015)

Homework #2: *Neural selectivity and hyperacuity*

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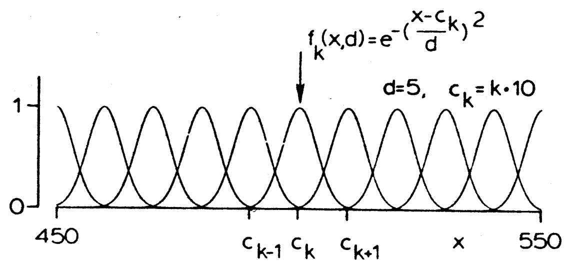
Due on Friday, February 13, 2015 at 1:00pm

Problem:

Sensory systems can achieve needed stimulus resolution (along a particular stimulus dimension) by relying on individual neurons whose receptive fields (RFs) are sufficiently small. In this case, the overall resolution of the system is limited by the size of the RFs of individual neurons. However, in many situations, the behavioral performance of humans and animals commonly reveal a degree of stimulus resolution that is greater than what can be explained by the size of the RFs of individual neurons, a phenomena often referred to as "hyperacuity". It has been suggested that the *hyperacuity* is achieved by combining information from a population of neurons. In this problem, you are asked to examine one of these such models as described below.

Suppose we have an array of 100 periphery (PNS) neurons that have bell-shaped sensitivity profiles, $f_k(x,d)$ of width parameter d and center $c_k=10*k$, k=1,...,100. This array of neurons is defined over the range of the stimulus variable x from 0 to 1000, with the range from 450 to 550 displayed at the following figure for d=5.



Assume that a CNS neuron G sums the outputs of the above PNS neurons using the following equation:

$$G(x,d) = \sum_{k=1}^{100} k * f_k(x,d) \quad \text{for } 450<x<550 \quad (Eq.1)$$

Question 1:

Plot $G(x,d)$ as a function of x for d = 2, 4, 8, 16, 32, respectively. Discuss the properties of $G(.)$ as a function of increasing d.

Question 2:

If we define the relative resolution of the CNS neuron G as the ratio of its response slope $dG(x,d)/dx$ over its response magnitude $G(x,d)$ at a given x value,

$$S(d) = [\, dG(x,d)/dx \,] / G(x,d) \quad \text{for } x=500 \qquad (Eq.2)$$

Calculate and plot $S(d)$ for $5 \le d \le 100$ (on a logarithmic scale). What is the range of d values where $S(d)$ reaches saturation?

If a subject relies on the CNS neuron G for making perceptual decisions regarding <u>changes</u> in stimulus parameter x, discuss the implications from results obtained in **Question 2**. Specifically, how are these results related to the *hyperacuity* phenomena?

Question 3:

Suppose that a CNS neuron G sums the outputs of the above PNS neurons using the following equation instead of *Eq.1*:

$$G(x,d)=\sum_{k=1}^{100} k^2 * f_k(x,d) \quad \text{for } 450<x<550 \qquad (Eq.3)$$

Calculate and plot $S(d)$ as defined in *Eq.2* above. Compare the results of **Question 2** and **3**. Which CNS neuron has better relative resolution (for large d values) in detecting changes in stimulus parameter x? The difference between the two results demonstrates that CNS neurons can achieve different resolution by employing different computational algorithms.

Question 4:

Prove mathematically that given a sufficiently large d, $G(x,d)$ is approximately a linear function of x. You are free to use any mathematical tools in your proofs, but it must be clear in each step.