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SBE II: Homework 4

Experiment-3:

Given that we know the following:

$$\tilde{V}(\chi, T) = A(s)e^{\sqrt{s+1}\chi} + B(s)e^{-\sqrt{s+1}\chi}$$

With the following boundary conditions:

$$\left.\frac{\partial V}{\partial x}\right|_{x=0} = -\frac{I_0(s)}{sG_\infty}$$

$$\left.\frac{\partial V}{\partial x}\right|_{x=L} = -r_i I_i(L): \tilde{V}(\chi, T) < \infty \, \forall \chi, T$$

We are asked to solve for the constants $A(s), B(s)$, and represent this solution as a $cosh$ function. Applying boundary conditions to the above gives us the following:

$$\left.\frac{\partial V}{\partial x}\right|_{x=0} = \left.\left[\sqrt{s+1}A(s)e^{\sqrt{s+1}\chi} - \sqrt{s+1}B(s)e^{-\sqrt{s+1}\chi}\right]\right|_{\chi=0}$$

$$= \sqrt{s+1}\big(A(s) - B(s)\big) = -\frac{I_0}{sG_\infty}$$

For the case at the limit of the cylinder, $\tilde{V}(\chi, T) < \infty \, \forall \chi, T$ requires that $A(s) = 0 \because$ $\lim_{\chi \to \infty} A(s) = \infty$.

This leaves us with:

$$\tilde{V}(\chi, s) = \frac{I_0 e^{-\sqrt{s+1}\chi}}{s\sqrt{s+1}G_\infty}$$

And, to convert to the prescribed form by the question is then trivial:

$$\tilde{V}(\chi, s) = \frac{2I_0}{s\sqrt{s+1}G_\infty}\cosh(\sqrt{s+1}\chi)$$

Or, in spatial form,

$$\tilde{V}(x, s) = \frac{2I_0\lambda r_i}{s\sqrt{s+1}}\cosh(\sqrt{s+1}\frac{x}{\lambda})$$

Where,

$$A(s) = \frac{2I_0\lambda r_i}{s\sqrt{s+1}}$$

$$g(s) = \frac{\sqrt{s+1}}{\lambda}$$