1. **Equilibrium**

Free energy (equal at equil $\mu_i = \mu_i^o + RT \ln C_i + z_i \mathcal{F} V$

Equilibrium is defined as $\Delta G = 0$ or $\mu_s^1 = \mu_s^2$

Nernst Equation $$E = \frac{RT}{z\mathcal{F}} \ln \frac{C_1}{C_2}$$

2. **Ion Flux**

Nernst-Planck Equation
$$J = -uC \left[RT \frac{d \ln C}{dx} + z\mathcal{F} \frac{dV}{dx} \right]$$
$$I = -z\mathcal{F}uC \left[RT \frac{d \ln C}{dx} + z\mathcal{F} \frac{dV}{dx} \right]$$

GHK Equation
$$I_i = \frac{\left(z\mathcal{F}\right)^2 u_i}{d} \Delta V \frac{C_i(d) e^{\frac{z_o \mathcal{F} \Delta V}{RT}} - C_i(0)}{e^{\frac{z_o \mathcal{F} \Delta V}{RT}} - 1}$$
$$\Delta V_{rest} = \frac{RT}{\mathcal{F}} \ln \left(\frac{u_K K_o + u_{Na} Na_o + u_{Cl} Cl_i}{u_K K_i + u_{Na} Na_i + u_{Cl} Cl_o} \right)$$

Rate Constants $$k_i = (const) e^{\frac{-(G + \lambda z \mathcal{F} \Delta V)}{RT}}$$

Flux Across Barrier
$$J_{AB} = k_i A, J_{BA} = k_{-i} B,$$
$$J = J_{AB} - J_{BA}$$
$$J = (const) e^{\frac{-(G + \lambda z \mathcal{F} \Delta V)}{RT}} \left(A - B e^{\frac{z \mathcal{F} \Delta V}{RT}} \right)$$

$$\frac{RT}{\mathcal{F}} = 26mV$$

3. **Phase Planes**

Corelated Variables
$$m_\infty = 0.5 \left(1 + \tanh \left(\frac{V - V_1}{V_2} \right) \right)$$
$$\omega_\infty = 0.5 \left(1 + \tanh \left(\frac{V - V_3}{V_4} \right) \right)$$
$$\tau_\infty = \frac{1}{\cosh \left(\frac{V - V_3}{2V_4} \right)}$$

State Vector $$\dot{\vec{X}} = \begin{bmatrix} \dot{V} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} F_1(\vec{x}) \\ F_2(\vec{x}) \end{bmatrix} = \vec{F}(\vec{X})$$

Nullclines for MLE case
$$\dot{V} = 0 \rightarrow \omega = \frac{I_{ext} - \vec{G_{Ca}} m_\infty (V - E_{Ca}) - \vec{G_L}(V - E_L)}{\vec{G_K}(V - E_K)}$$
$$\dot{\omega} = 0 \rightarrow \omega = \omega_\infty(V)$$

Jacobian $$\vec{x}(t) = \sum_{i=1}^{N} a_i \vec{e_i} e^{\lambda_i t}$$

$$\text{Eigen values}\lambda_i : \begin{aligned} \mathbb{R}^- &\rightarrow \text{stable eq} \\ \mathbb{R}^+ &\rightarrow \text{unstable eq} \\ \mathbb{C}, \lambda_1 = \lambda_2^* &\rightarrow \text{possible Hopf bifurcation nearby} \\ \mathbb{R}, sign(\lambda_1) \neq sign(\lambda_2) &\rightarrow \text{saddle eq} \end{aligned}$$

4. **Limit Cycles**

$$V = \frac{RT}{\mathcal{F}} \ln \left(\frac{\sum g_i [C_i]_o}{\sum g_i [C_i]_i} \right)$$