Systems Bioengineering 3
Homework 11

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1. We are given that:

$$
\begin{aligned}
G_T &= G + GX_n + GY_n \\
G + nX &\rightleftharpoons GX_n \\
G + nY &\rightleftharpoons GY_n
\end{aligned}
$$

(a)

$$
\begin{aligned}
GX^n &= GX_n \\
\frac{GX_n}{G} &= X^n
\end{aligned}
$$

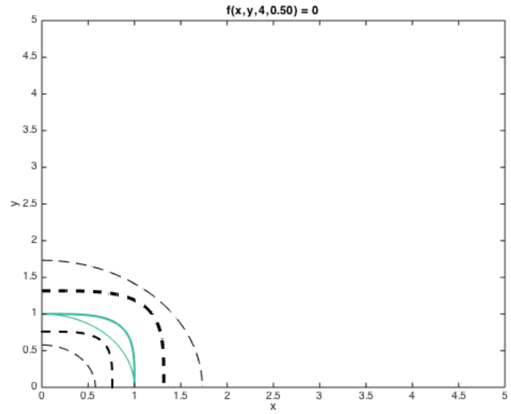
(b)

$$
\begin{aligned}
GY^n &= GY_n \\
\frac{GY_n}{G} &= Y^n
\end{aligned}
$$

(c)

$$
\begin{aligned}
f &= \frac{GX_n + GY_n}{G_T} \\
&= \frac{GX_n + GY_n}{G + GX_n + GY_n} \\
&= \frac{X^nG + Y^nG}{G + X^nG + Y^nG} \\
f &= \frac{X^n + Y^n}{1 + X^n + Y^n}
\end{aligned}
$$

(d) The figure plotted for the previous calculations is shown below.



(e) I would describe the behaviour of this function in logic terms as an OR gate; the logic function is $f = a + b$. Upon inspection of the graph, you can see that when either the X or Y term is large, they contribute to activate the function. However, when both X and Y are small, the function goes to zero. This, of course, is equivalent to a $NAND$ gate as well.

2. We are given that:

$$G + nX + nY \rightleftharpoons GX_nY_n$$

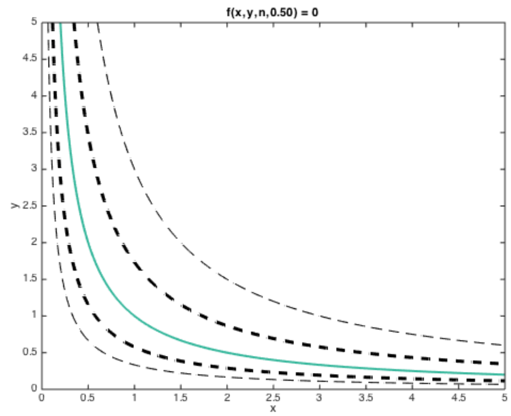
(a)

$$\begin{aligned} GX^nY^n &= GX_nY_n \\ \frac{GX_nY_n}{G} &= X^nY^n \end{aligned}$$

(b)

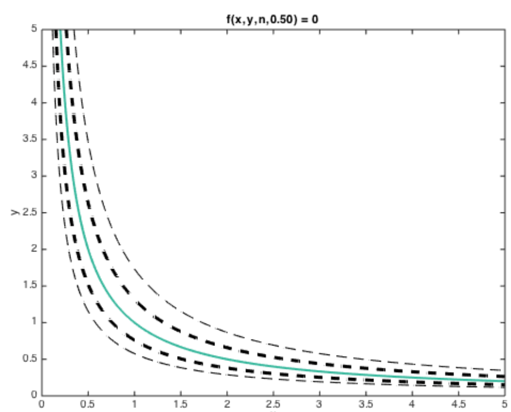
$$\begin{aligned} f &= \frac{GX_nY_n}{G_T} \\ &= \frac{GX_nY_n}{G + GX_nY_n} \\ &= \frac{GX^nY^n}{G + GX^nY^n} \\ f &= \frac{X^nY^n}{1 + X^nY^n} \end{aligned}$$

(c) The figure plotted for the previous calculations is shown below.



(d) I would describe the behaviour of this function in logic terms as an AND gate; the logic function is $f = ab$. Upon inspection of the graph, you can see that only when the X and Y terms are large, do they contribute make the function nonzero. However, when either X or Y are small, the function goes to zero. This, of course, is equivalent to a NOR gate as well.

(e) As can be seen in the figure below, when n increases, the sharpness of this relationship increases. This means that as the value of n increases, the function more closely matches the AND logic function. Also the $f = 0.75$ and $f = 0.25$ lines approach the $f = 0.5$ line more closely.



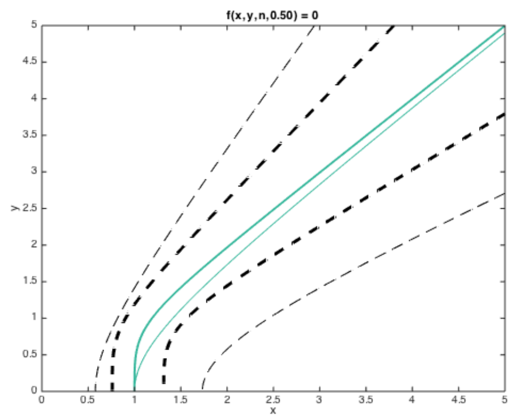
3. We are given that:

$$G + nX \rightleftharpoons GX_n$$
$$G + nY \rightleftharpoons GY_n$$

(a)

$$
\begin{aligned}
f &= \frac{GX_n}{G_T} \\
&= \frac{GX_n}{G + GX_n + GY_n} \\
&= \frac{GX^n}{G + GX^n + GY^n} \\
f &= \frac{X^n}{1 + X^n + Y^n}
\end{aligned}
$$

(b) The figure plotted for the previous calculations is shown below.



(c) I would describe the behaviour of this function in logic terms as an AND gate with an inverted a input; the logic function is $f = \bar{a}\bar{b}$ where $\bar{a}$ is the inverted value of a. Upon inspection of the graph, you can see that only when the Y term is large and the X is small, does the function grow. However, when X is large or Y is small, the function goes to zero.

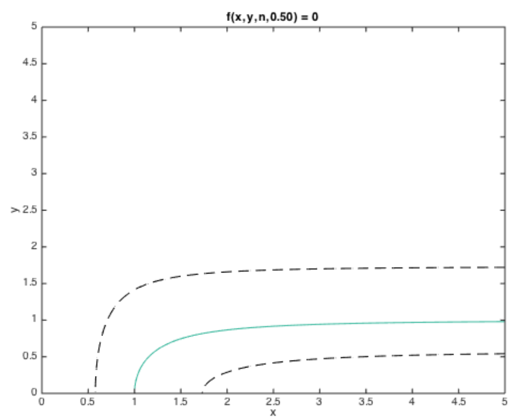
4. We are given that:

$$
\begin{aligned}
G + nX &\rightleftharpoons GX_n \\
GX_n + nY &\rightleftharpoons GX_nY_n
\end{aligned}
$$

(a)

$$
\begin{aligned}
f &= \frac{GX_n}{G_T} \\
&= \frac{GX_n}{G + GX_n + GX_nY_n} \\
&= \frac{GX^n}{G + GX^n + GX^nY^n} \\
f &= \frac{X^n}{1 + X^n + X^nY^n}
\end{aligned}
$$

(b) The figure plotted for the previous calculations is shown below.



(c) I would describe the behaviour of this function in logic terms as an OR gate with an inverted a input; the logic function is $f = \bar{a} + b$ where $\bar{a}$ is the inverted value of a. Upon inspection of the graph, you can see that only when the X term is small or the Y is large, does the function grow. However, when X is large and Y is small, the function goes to zero.

(d) Comparing this trend to that obtained in the section 3 above, we can see a sharper response in this model. When a repressor is used instread of a competetive activator, we see a much more consistently controlled response.

5. We are given that:

$$
f = \frac{X^2 + Y^2 + X^2Y^2}{1 + X^2 + Y^2 + X^2Y^2 + Z^2 + X^2Y^2Z^2}
$$

(a) Based on the equation f given, we can derive that the following are the transciptional complexes which can bind to this promoter:

$$
\begin{array}{lll}
GX_2 & GY_2 & GX_2Y_2 \\
GZ_2 & GX_2Y_2Z_2 &
\end{array}
$$

(b) Assuming that trascriptionally active refers to having active involvement in the process of transcription, then we can isolate the transcriptional complexes that appear within the numerator of f as being our transcriptionally active complexes. They are:

$$GX_2 \qquad\qquad GY_2 \qquad\qquad GX_2Y_2$$

(c) Given the revised formula depicting the forward rate of transcription, the coefficients $\beta_{1,2,3}$ would represent rates of the binding of each type of transcriptional complex. For instance, this could be shown as follows:

$$G + 2X \overset{\beta_1}{\rightleftharpoons} GX_2$$
$$G + 2Y \overset{\beta_2}{\rightleftharpoons} GY_2$$
$$G + 2X + 2Y \overset{\beta_3}{\rightleftharpoons} GX_2Y_2$$