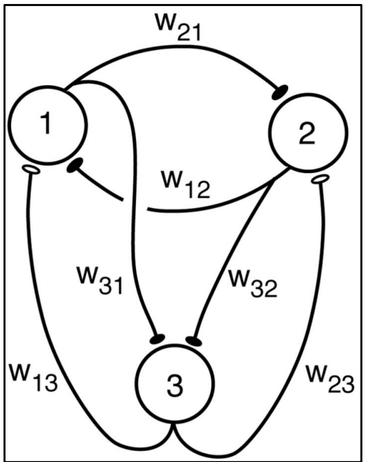
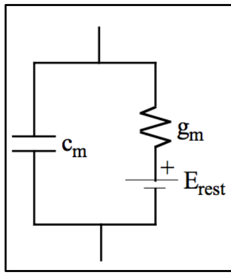


Solid → Inhibitory
Empty → Excitatory



From circuit model shown at left, we require that:

1. Assume that V is the output (no spikes)
2. g_m is linear, $E_m = E_{rest} = 0$
3. $I_{sym} = s\ [\sum w_i V_i]$, where s is a squashing fn, i.e. $s = \frac{1}{1+e^{-x}}$

Perceptron $V = s\ [\vec{w} \cdot \vec{u}\ ]$

$$V(t \to \infty) = \begin{cases} 1, & \vec{w} \cdot \vec{u} \geq \gamma \\ -1, & \vec{w} \cdot \vec{u} < \gamma \end{cases}$$

Where u is input, and w is weight, and γ is a learned constant

3-layer perceptron can solve any continuous mapping function
4-layer perceptron can solve <u>any</u> mapping

Supervised Learning (differentiable $s(x)$)

Patterns: $\vec{u}^1, \dots ,\ \vec{u}^p$
Desired outputs: $\vec{v}^1, \dots, \vec{v}^p$

$$Err[\vec{w}] = \frac{1}{2}\sum_p (v^p - s(\vec{w} \cdot \vec{u}^p))^2$$

$$\frac{\partial Err}{\partial w_k} = -\sum_p [v^p - s(\vec{w} \cdot \vec{u}^p)]\ \frac{\partial s}{\partial h}\Big|\ u_k^p$$

Where $h = \vec{w} \cdot \vec{u}^p$

$$\nabla Err = [\frac{\partial Err}{\partial w_1}, \dots, \frac{\partial Err}{\partial w_k}]$$

<u>Gradient Descent</u>

$$\Delta\,\vec{w} = -\epsilon\vec{\nabla} Err$$

Hebbian (unsupervised) Learning

$$\tau\frac{dV}{dt} = -V + \vec{w} \cdot \vec{u}$$

$$\Delta\vec{w} = \frac{1}{\tau_w} V\vec{u} = \frac{1}{\tau_w}\ \langle V\vec{u}\rangle$$

$$||\Delta\vec{w}||^2 \approx \frac{2}{\tau_w} V^2 > 0$$

Weights can unboundedly grow… so Oja's Rule is req'd:

$$\Delta\vec{w} = \frac{1}{\tau_w}[V\vec{u} - V^2\vec{w}]$$

Hopfield Nets

$$\tau = -S_i + F\left[\sum_i w_{ij} x_j\right]$$

Assume Steady State for analysis → discrete steps from $\vec{S} \to \vec{S}'$
F can be any fn ($sign()$, for instance)

$\vec{S}' = \vec{S}$ is a stable state. Patterns are stable states.

For patterns ξ:

$$\vec{\xi}^j = \{\xi_1^j, \dots, \xi_N^j\}, j = 1 \to P$$

$$w_{ij} = \frac{1}{N}\sum_{k=1} \xi_i^k \xi_j^k$$

Where $w_{ii} = \frac{P}{N}$; $w_{ij} = w_{ji}$

Noisy patterns require more signal than noise for the pattern to be recovered consistently.