Selected Solutions to Underwood Dudley's Elementary Number Theory Second Edition

Greg Kikola

July 12, 2019

Contents

Chapter 1

Integers

1.1 Exercise 1

Which integers divide zero?

Solution. Every integer divides 0. For, if k is any integer, then $0k = 0$ so that $k \mid 0$. $\square$

1.2 Exercise 2

Show that if $a \mid b$ and $b \mid c$ then, $a \mid c$.

Proof. Let $a \mid b$ and $b \mid c$. Then there are integers m and n such that $am = b$ and $bn = c$. But then $a(mn) = (am)n = bn = c$. Since mn is an integer, we have $a \mid c$. $\square$

1.3 Exercise 3

Prove that if $d \mid a$ then $d \mid ca$ for any integer c.

Proof. Again, by definition we can find an integer n such that $dn = a$. But then $cdn = ca$. Since cn is an integer, it follows that $d \mid ca$. $\square$

1.4 Exercise 4

What are $(4, 14)$, $(5, 15)$, and $(6, 16)$?

Solution. By inspection, $(4, 14) = 2$, $(5, 15) = 5$, and $(6, 16) = 2$. $\square$

1.5 Exercise 5

What is $(n, 1)$, where n is any positive integer? What is $(n, 0)$?

Solution. We have $(n, 1) = 1$ since there is no integer greater than 1 which divides 1. We also have $(n, 0) = n$ since no integer larger than n can divide n, and n certainly divides itself and 0. □

1.6 Exercise 6

If d is a positive integer, what is (d, nd)?

Solution. $(d, nd) = d$ since d is a common divisor ($d \mid nd$ by Lemma 2) and there can be no greater divisor of d. □

1.7 Exercise 7

What are q and r if $a = 75$ and $b = 24$? If $a = 75$ and $b = 25$?

Solution. We have

$$75 = 3(24) + 3 \quad \text{and} \quad 75 = 3(25) + 0.$$

So $q = 3$ and $r = 3$ in the first case, and $q = 3$ and $r = 0$ in the second. □