GradientDescent

December 3, 2019

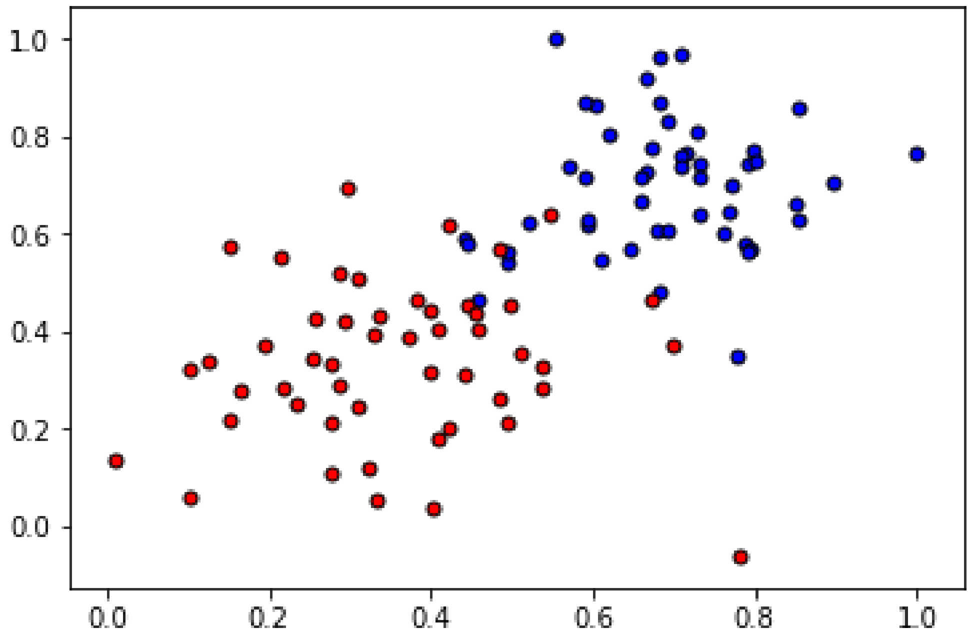
1 Implementing the Gradient Descent Algorithm

In this lab, we'll implement the basic functions of the Gradient Descent algorithm to find the boundary in a small dataset. First, we'll start with some functions that will help us plot and visualize the data.

```
In [8]: import matplotlib.pyplot as plt
        import numpy as np
        import pandas as pd

        #Some helper functions for plotting and drawing lines

        def plot_points(X, y):
            admitted = X[np.argwhere(y==1)]
            rejected = X[np.argwhere(y==0)]
            plt.scatter([s[0][0] for s in rejected], [s[0][1] for s in rejected], s = 25, color
            plt.scatter([s[0][0] for s in admitted], [s[0][1] for s in admitted], s = 25, color

        def display(m, b, color='g--'):
            plt.xlim(-0.05,1.05)
            plt.ylim(-0.05,1.05)
            x = np.arange(-10, 10, 0.1)
            plt.plot(x, m*x+b, color)
```

1.1 Reading and plotting the data

```
In [9]: data = pd.read_csv('data.csv', header=None)
        X = np.array(data[[0,1]])
        y = np.array(data[2])
        plot_points(X,y)
        plt.show()
```



1.2 TODO: Implementing the basic functions

Here is your turn to shine. Implement the following formulas, as explained in the text. - Sigmoid activation function

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

- Output (prediction) formula

$$\hat{y} = \sigma(w_1 x_1 + w_2 x_2 + b)$$

- Error function

$$Error(y, \hat{y}) = -y \log(\hat{y}) - (1 - y) \log(1 - \hat{y})$$

- The function that updates the weights

$$w_i \longrightarrow w_i + \alpha(y - \hat{y}) x_i$$

$$b \longrightarrow b + \alpha(y - \hat{y})$$

```
In [13]: # Implement the following functions

         # Activation (sigmoid) function
         def sigmoid(x):
             return 1/(1+np.exp(-x))

         # Output (prediction) formula
         def output_formula(features, weights, bias):
             return sigmoid(np.dot(features , weights) + bias)

         # Error (log-loss) formula
         def error_formula(y, output):
             return ((-y * np.log(output))-(1-y)*(np.log(1-output)))

         # Gradient descent step
         def update_weights(x, y, weights, bias, learnrate):
             weights[0] = weights[0] + learnrate*(y-output_formula(x,weights,bias))*x[0]
             weights[1] = weights[1] + learnrate*(y-output_formula(x,weights,bias))*x[1]
             bias  = bias + learnrate*(y-output_formula(x,weights,bias))

             return weights, bias
```

1.3 Training function

This function will help us iterate the gradient descent algorithm through all the data, for a number of epochs. It will also plot the data, and some of the boundary lines obtained as we run the algorithm.

```
In [18]: np.random.seed(44)

         epochs = 100
         learnrate = 0.01

         def train(features, targets, epochs, learnrate, graph_lines=False):

             errors = []
             n_records, n_features = features.shape
             last_loss = None
             weights = np.random.normal(scale=1 / n_features**.5, size=n_features)
             bias = 0
             for e in range(epochs):
                 del_w = np.zeros(weights.shape)
                 for x, y in zip(features, targets):
                     output = output_formula(x, weights, bias)
                     error = error_formula(y, output)
                     weights, bias = update_weights(x, y, weights, bias, learnrate)

                 # Printing out the log-loss error on the training set
```

```
        out = output_formula(features, weights, bias)
        loss = np.mean(error_formula(targets, out))
        errors.append(loss)
        if e % (epochs / 10) == 0:
            print("\n========== Epoch", e,"==========")
            if last_loss and last_loss < loss:
                print("Train loss: ", loss, "  WARNING - Loss Increasing")
            else:
                print("Train loss: ", loss)
            last_loss = loss
            predictions = out > 0.5
            accuracy = np.mean(predictions == targets)
            print("Accuracy: ", accuracy)
        if graph_lines and e % (epochs / 100) == 0:
            display(-weights[0]/weights[1], -bias/weights[1])


    # Plotting the solution boundary
    plt.title("Solution boundary")
    display(-weights[0]/weights[1], -bias/weights[1], 'black')

    # Plotting the data
    plot_points(features, targets)
    plt.show()

    # Plotting the error
    plt.title("Error Plot")
    plt.xlabel('Number of epochs')
    plt.ylabel('Error')
    plt.plot(errors)
    plt.show()
```

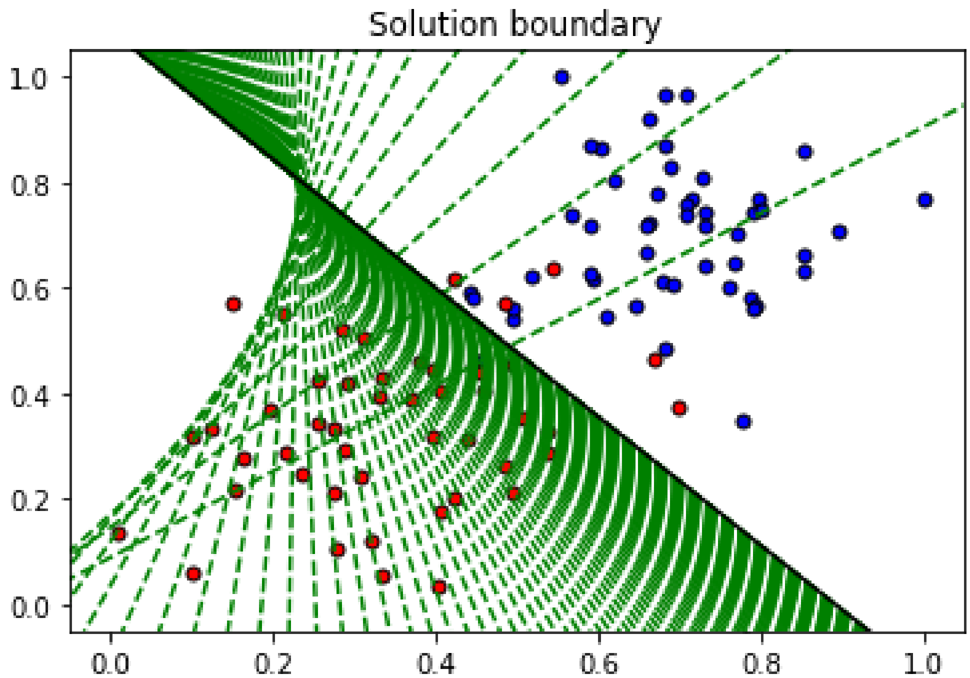
1.4 Time to train the algorithm!

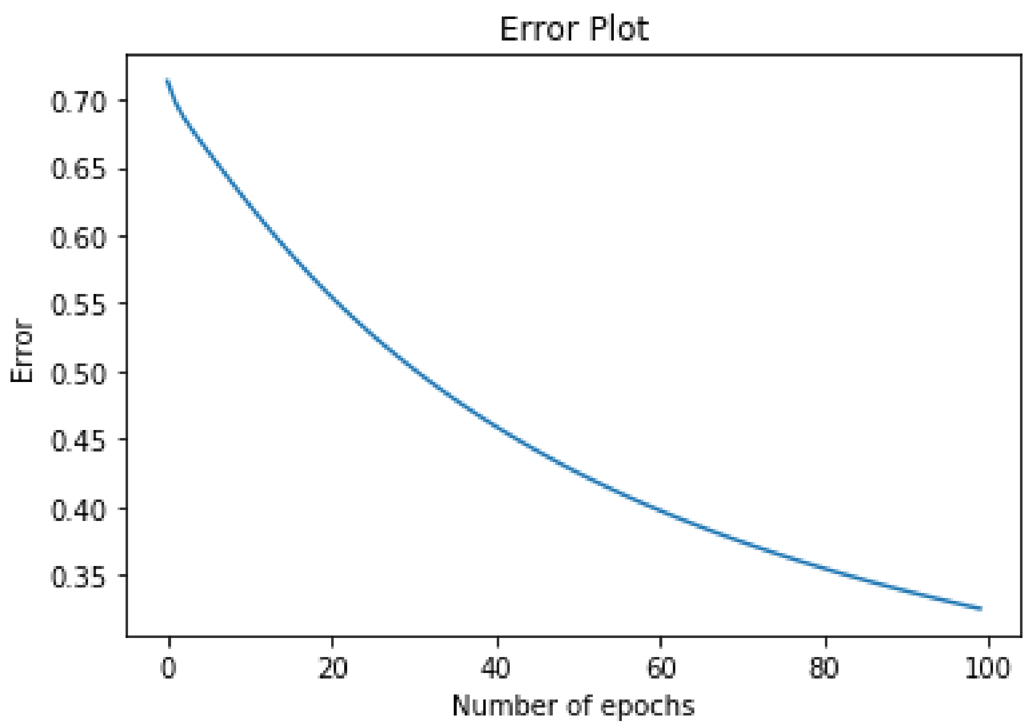
When we run the function, we'll obtain the following: - 10 updates with the current training loss and accuracy - A plot of the data and some of the boundary lines obtained. The final one is in black. Notice how the lines get closer and closer to the best fit, as we go through more epochs. - A plot of the error function. Notice how it decreases as we go through more epochs.

```
In [19]: train(X, y, epochs, learnrate, True)
```

```
========== Epoch 0 ==========
Train loss:  0.713602073584
Accuracy:  0.4

========== Epoch 10 ==========
Train loss:  0.622420886019
```

```
Accuracy:  0.59

========== Epoch 20 ==========
Train loss:  0.554686046141
Accuracy:  0.74

========== Epoch 30 ==========
Train loss:  0.501413981765
Accuracy:  0.84

========== Epoch 40 ==========
Train loss:  0.459147439113
Accuracy:  0.86

========== Epoch 50 ==========
Train loss:  0.425079866313
Accuracy:  0.93

========== Epoch 60 ==========
Train loss:  0.397182481214
Accuracy:  0.93

========== Epoch 70 ==========
Train loss:  0.373995215262
Accuracy:  0.93

========== Epoch 80 ==========
Train loss:  0.354459264302
Accuracy:  0.94

========== Epoch 90 ==========
Train loss:  0.337797309481
Accuracy:  0.94
```


Solution boundary
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


Error Plot
Error
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0
20
40
60
80
100
Number of epochs

```
In [ ]:
```

```
In [ ]:
```