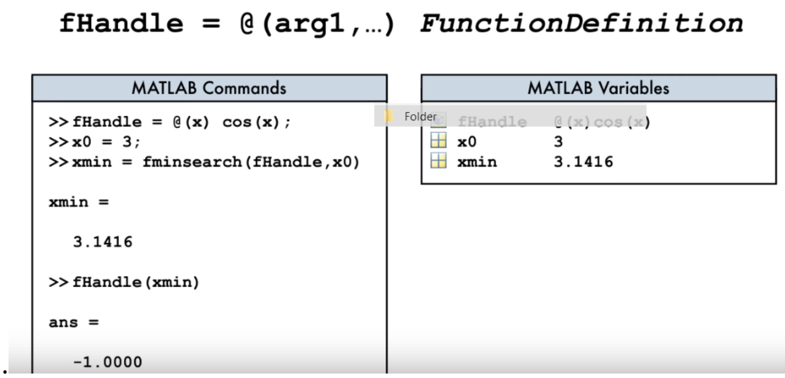
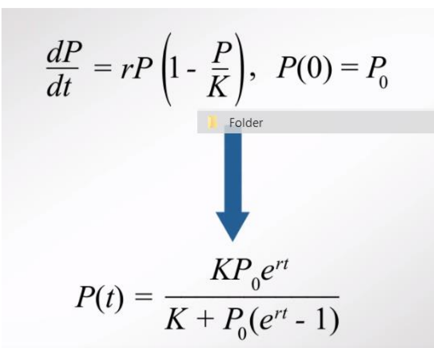
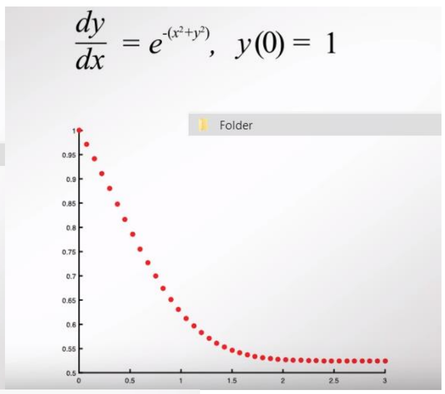
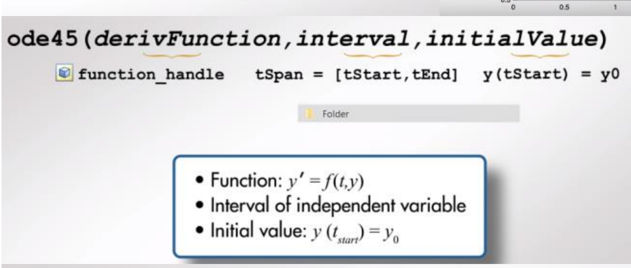
1. 

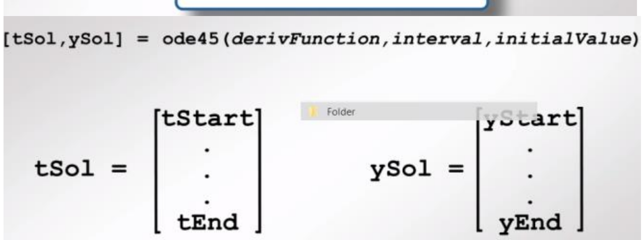

2.Solving ordinary diff equation is very diff.so we go with numerical approx solving

We use numerical based approximation for ODE euqtions



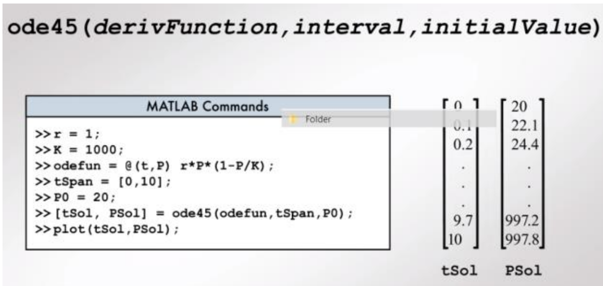



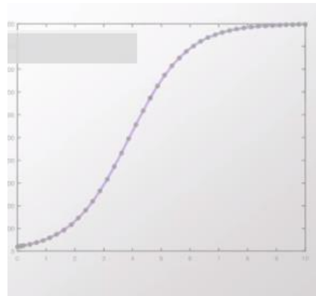




both tSol and ySol are vectors

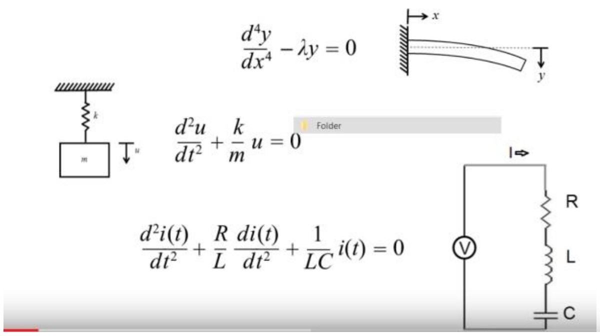
3.eg

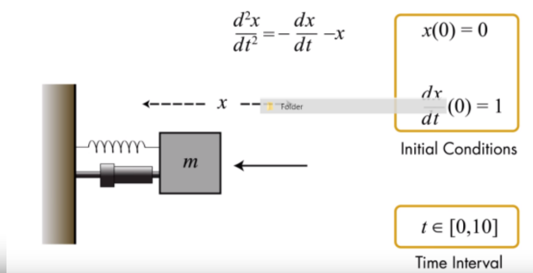


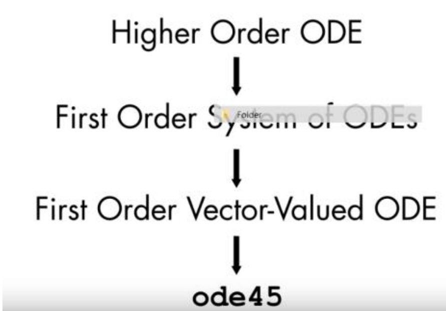
For corresponding tsol we have respective psol values

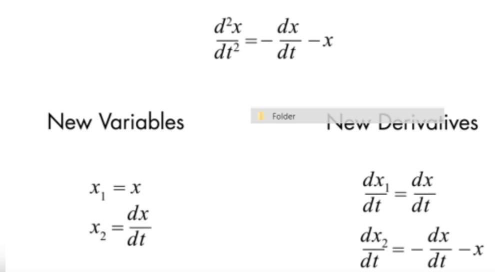
5.

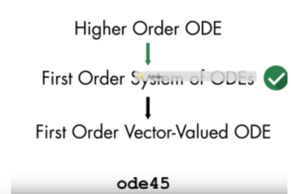


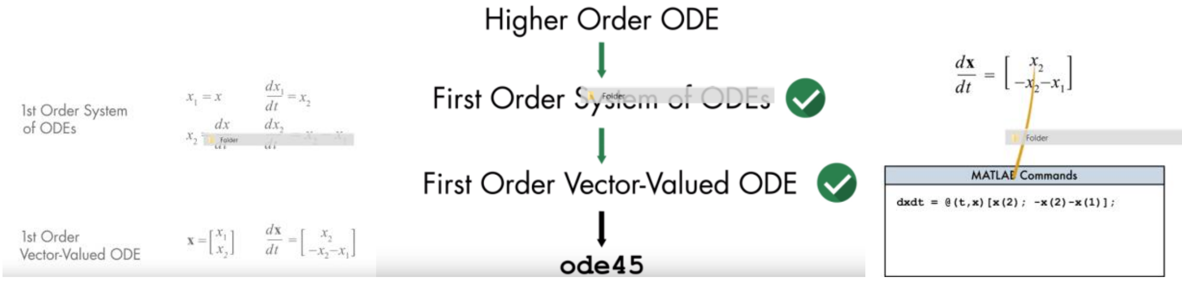

Note: Since Matlab ODE45 tool box only can solve 1$^{st}$ order ODE ,any higher order diff euqtions need to be converted to 1$^{st}$ order first and then given ti MATLAB api

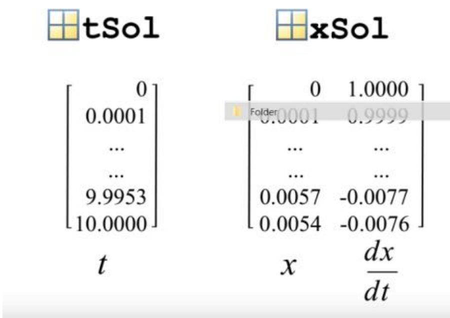


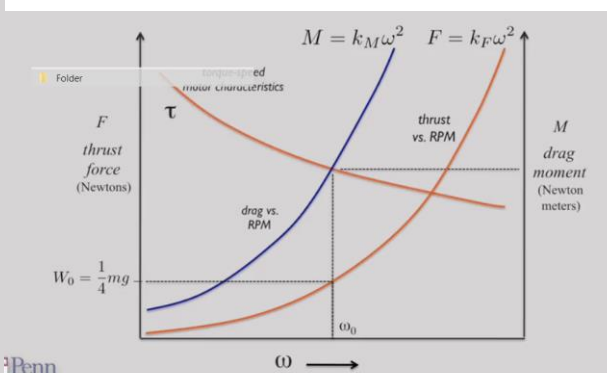

Now multiple variables need to be repsented in to single variabe




//////////////////////////////////////////////////////////////////////////////////////////////////////////////

### Basic Mechanics:

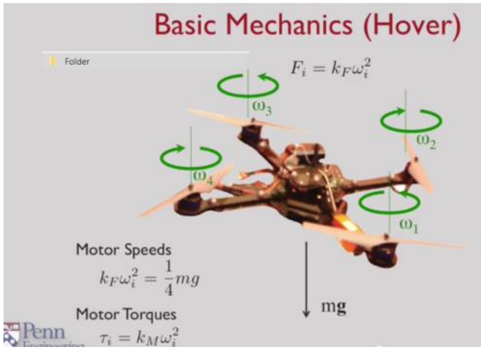




w=rotor speed

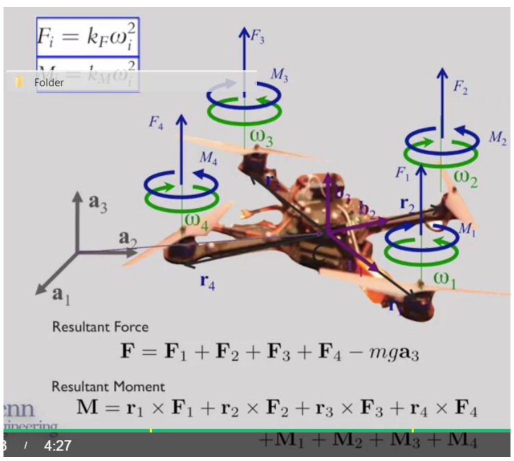
a.when rotor spins it has to overcome drag

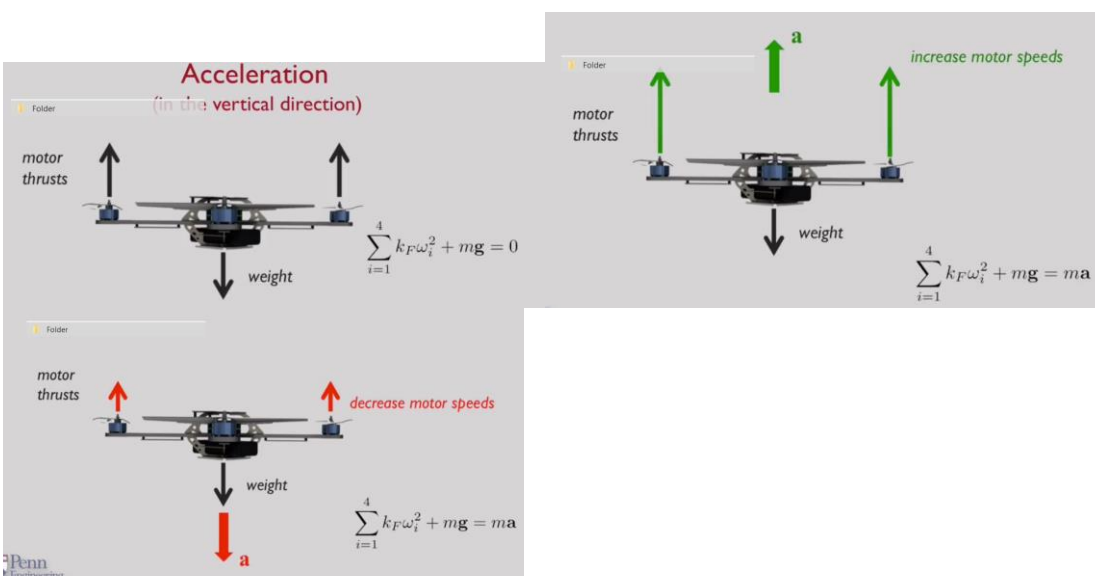
b.since 4 rotors..each rotor has to support w0=1/4wg to be in equilibrium

c. A quadrotor is at hover and the speeds of all its motors double, it will **Accelerate Up**




kf=force w=rpm Mi=Drag moment Km=drag force



# Dynamical Systems

Systems where the effects of actions do not occur immediately

State: a collection of variables that completely characterizes the motion of a system

Eg:A Thermostat in room.Temp in room will increase slowly but not instantly

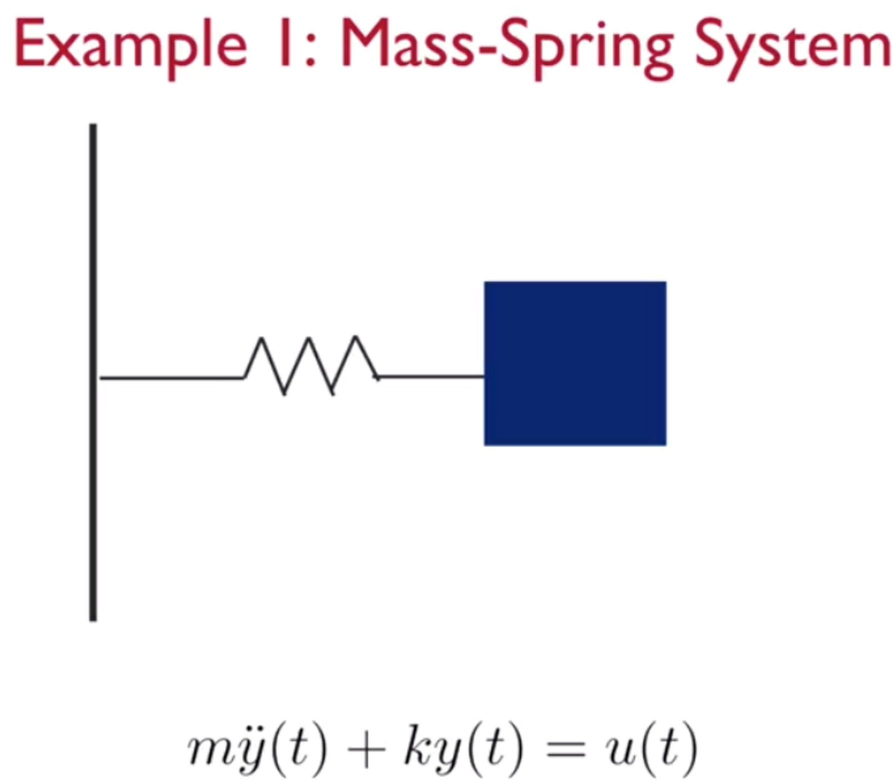
# Dynamical Systems

Evolution of these states over time is often given by a set of governing ordinary differential equations

- Order: highest derivative that appears in the equations

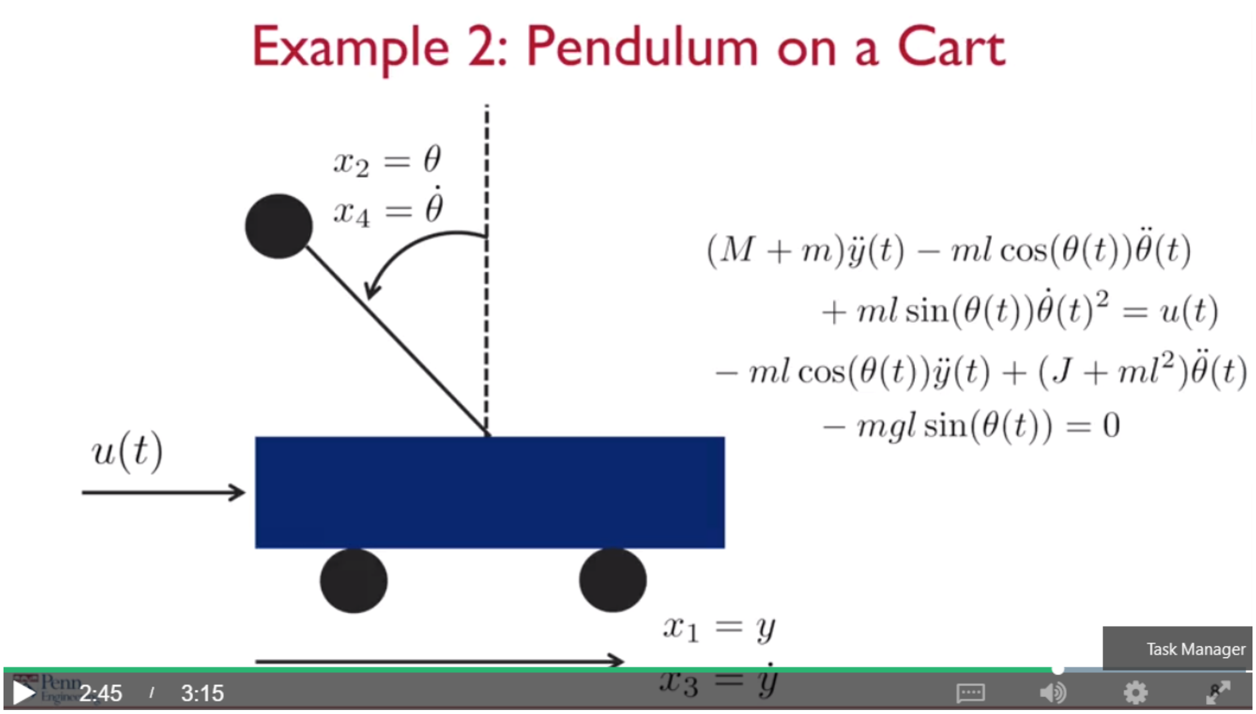
$$\boxed{\ddot{x}(t)} = u(t)$$ Second-order system

SInce



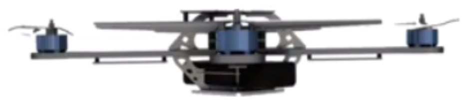

Since highest deruvative is 2nd order.the example 1 is 2nd order dynamic system

State of system = position and velocity of MASS



Here above system has 4 states = vel of cart ,vel of pendulum ,pos of cart,pos of pendulum

## Example 3: Quadrotor



$$x_1 = x \qquad x_7 = \dot{x}$$
$$x_2 = y \qquad x_8 = \dot{y}$$
$$x_3 = z \qquad x_9 = \dot{z}$$
$$x_4 = \phi \qquad x_{10} = p$$
$$x_5 = \theta \qquad x_{11} = q$$
$$x_6 = \psi \qquad x_{12} = r$$

States -12

3 Lin vel ,3 ang vel ,3 pos,3 angular orientation

## Rates of Convergence

How fast do we want this error to go to 0?

- The error *exponentially converges* to 0 if there exists constants $\alpha$ and $\beta$ and time $t_0$, such that for all $t \geq t_0$:

$$\|e(t)\| \leq \alpha e^{-\beta t}$$

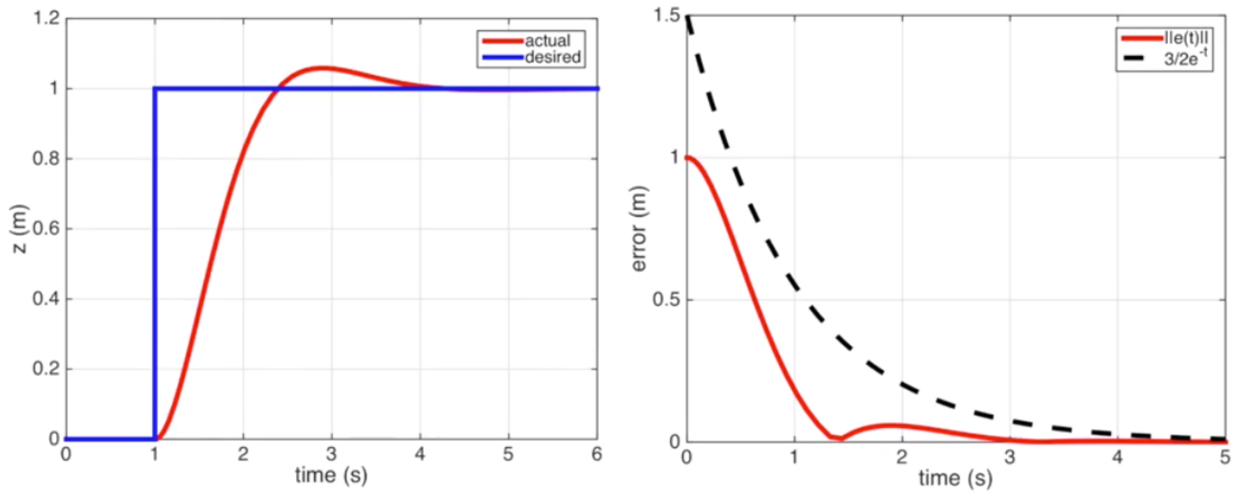
## Feedback Control

*Recall the control problem*

Determine the appropriate input that will cause the error between the desired state and the actual state of a dynamical system to eventually reach 0.

$$e(t) = x^{des}(t) - x(t) \rightarrow 0 \text{ as } t \rightarrow \infty$$

## Example 1: PD Controller




## Feedback Control

Here we will accomplish this using a PD (or PID) controller.

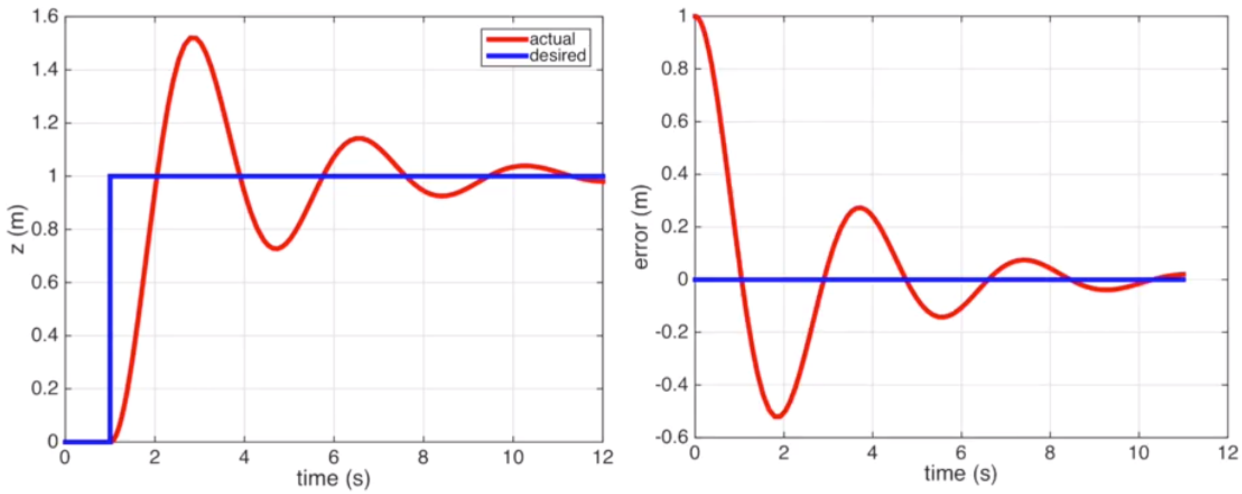
$$u(t) = \ddot{x}^{\text{des}}(t) + K_v \dot{e}(t) + K_p e(t)$$

Consider the controllers we used before to control the height of a quadrotor.

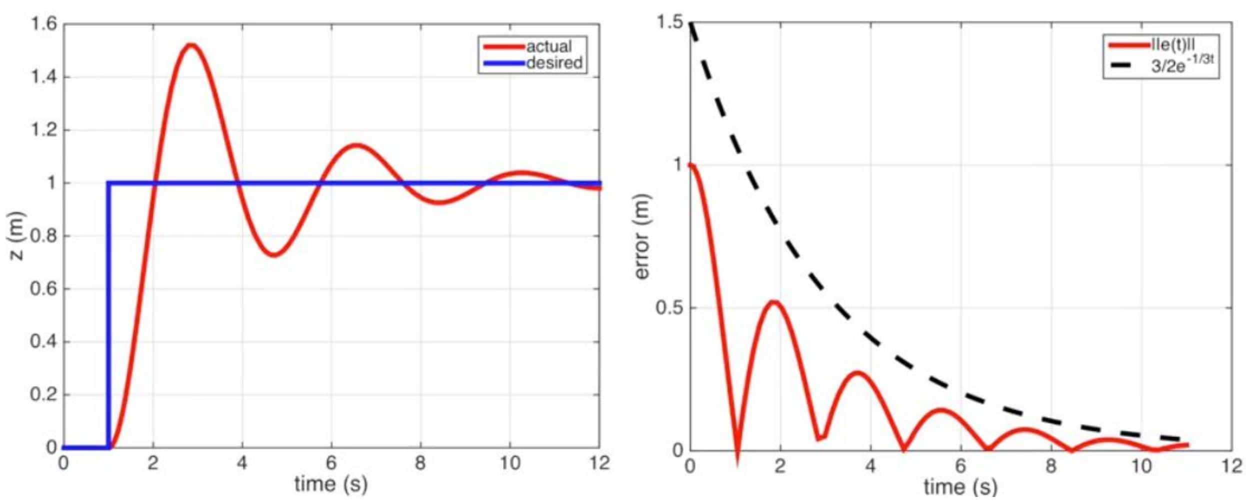
Left Fig : Reaching desired state

Right Fig: Error of feedback control getting converged to zero(Red=absolute error ,black=exp error)

# Example 2: High $K_p$




# Example 2: High $K_p$




Right fig: Absolute error.

SO even though we see oscillation but convergence is happening exponentillay.

Therefore in PD controller whatever is error function either oscillatory or other,Error fucntion converges exponentiall