CS161: FUNDAMENTALS OF ARTIFICIAL INTELLIGENCE

Fall 2011
Assignment 7. Due Wednesday, November 23, 2011, 11:59pm

Please submit your solutions on courseweb. The submitted file should be plain text or a formatted PDF file (no scans or pictures). Text files should have lines no longer than 100 characters and should be well-aligned when viewed with a monospace font.

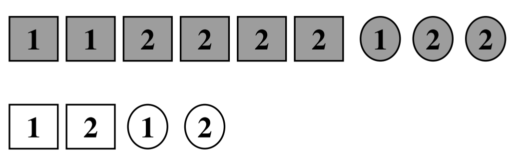
1. Prove:

   Generalized product rule: $\mathsf{Pr}(A, B \mid K) = \mathsf{Pr}(A \mid B, K)\mathsf{Pr}(B \mid K)$.

   Generalized Bayes' rule: $\mathsf{Pr}(A \mid B, K) = \mathsf{Pr}(B \mid A, K)\mathsf{Pr}(A \mid K)/\mathsf{Pr}(B \mid K)$.

2. An oil well may be drilled on Mr. Y's farm in Texas. Based on what has happened to similar farms, we judge the probability of oil being present to be .5, the probability of only natural gas being present to be .2, and the probability of neither being present to be .3. If oil is present, a geological test will give a positive result with probability .9; if only natural gas is present, it will give a positive result with probability .3; and if neither are present, the test will be positive with probability .1. Suppose the test comes back positive. What's the probability that oil is present?

3. (From Bayesian Networks by R. Neapolitan) Consider the set of objects below.



   Mr. Y picked up an object at random from the above set. We want to compute the probabilities of the following events:

   α_1: the object is black;

   α_2: the object is square;

   α_3: if the object is one or black, then it is also square.

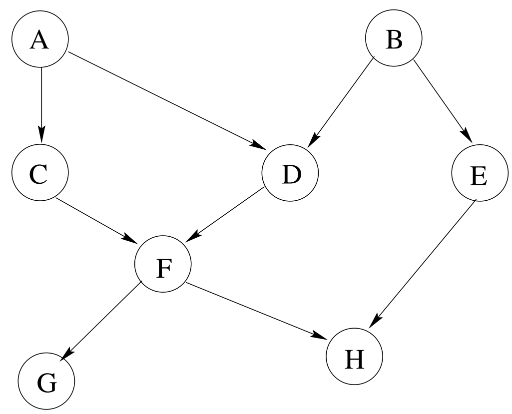


Figure 1: The DAG of a Bayesian network.

Construct the joint probability distribution of this problem. Use it to compute the above probabilities by explicitly identifying the worlds at which each α_i holds. Identify two sets of sentences α, β, γ such that α is independent of β given γ with respect to the constructed distribution.

4. Consider the DAG in Figure 1:
   (a) List the Markovian assumptions asserted by the DAG.
   (b) True or false? Why?
      - $d\_separated(A, BH, E)$
      - $d\_separated(G, D, E)$
      - $d\_separated(AB, F, GH)$
   (c) Express $\mathsf{Pr}(a, b, c, d, e, f, g, h)$ in factored form using the chain rule for Bayesian networks.
   (d) Compute $\mathsf{Pr}(A = 0, B = 0)$ and $\mathsf{Pr}(E = 1 \mid A = 1)$. Justify your answers.

$\mathsf{Pr}(A=0)$	$\mathsf{Pr}(A=1)$
.8	.2

$\mathsf{Pr}(B=0)$	$\mathsf{Pr}(B=1)$
.3	.7

	$\mathsf{Pr}(E=0 \mid B)$	$\mathsf{Pr}(E=1 \mid B)$
$B=0$	.1	.9
$B=1$	.9	.1

	$\mathsf{Pr}(D=0 \mid A,B)$	$\mathsf{Pr}(D=1 \mid A,B)$
$A=0, B=0$	.2	.8
$A=0, B=1$	.9	.1
$A=1, B=0$	.4	.6
$A=1, B=1$	.5	.5