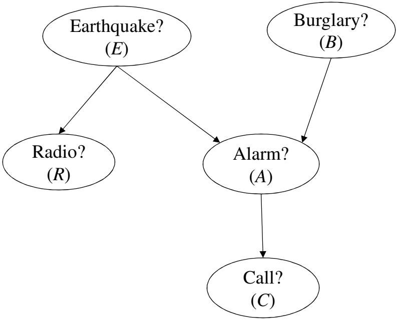
Chapter 4: Bayesian Networks

Adnan Darwiche[1]

October 6, 2009

[1]Lecture slides for *Modeling and Reasoning with Bayesian Networks*, Adnan Darwiche, Cambridge University Press, 2009.

Capturing Independence Graphically

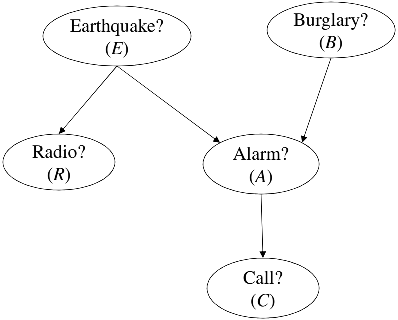


Assume that edges in this graph represent direct causal influences among these variables.

Example

The alarm triggering (A) is a direct cause of receiving a call from a neighbor (C).

Capturing Independence Graphically



We expect our belief in C to be influenced by evidence on R

If we get a radio report that an earthquake took place in our neighborhood, our belief in the alarm triggering would probably increase, which would also increase our belief in receiving a call from our neighbor.

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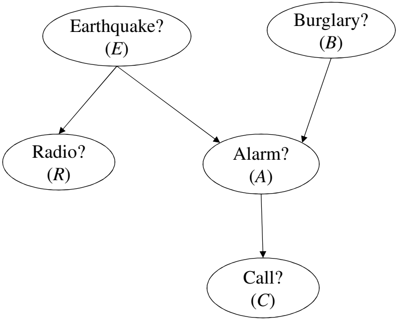

We would not change this belief, however, if we knew for sure that the alarm did not trigger.

C independent of R given $\neg A$

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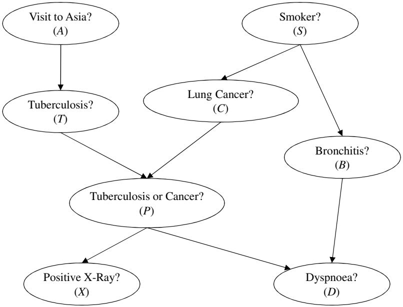

We would clearly find a visit to Asia relevant to our belief in the X-Ray test coming out positive, but we would find the visit irrelevant if we know for sure that the patient does not have Tuberculosis.

X is dependent on A, but is independent of A given $\neg T$

Capturing Independence Graphically

These examples of independence are all implied by a formal interpretation of each DAG as a set of conditional independence statements.

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variables N with an edge from N to V

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Descendants(V)

variables N with a directed path from V to N.
V is said to be an ancestor of N

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variables N with an edge from N to V

Descendants(V)

variables N with a directed path from V to N.
V is said to be an ancestor of N

Non_Descendants(V)

variables other than V, Parents(V) and Descendants(V)

Capturing Independence Graphically

Markovian assumptions of a DAG

We will formally interpret each DAG G as a compact representation of the following independence statements, denoted $\mathrm{Markov}(G)$:

$$I(V, \mathrm{Parents}(V), \mathrm{Non\_Descendants}(V)),$$

for all variables V in DAG G

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DAG as a causal structure

$\mathrm{Parents}(V)$ denote the direct causes of V and $\mathrm{Descendants}(V)$ denote the effects of V

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DAG as a causal structure

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Markovian assumptions restated

Given the direct causes of a variable, our beliefs in that variable become independent of its non–effects.

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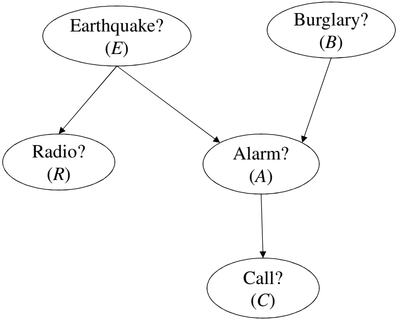

Markovian assumptions, Markov(G):

$I(C,$

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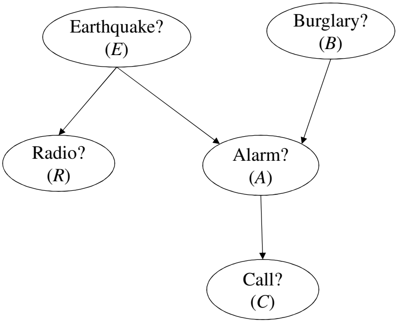

Markovian assumptions, $\text{Markov}(G)$:

$$I(C, A, \{B, E, R\})$$

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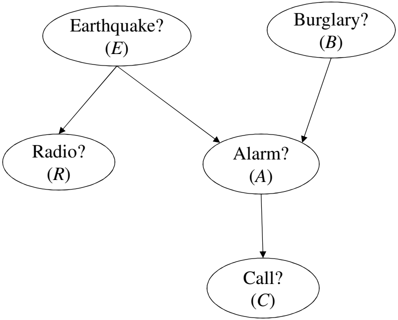

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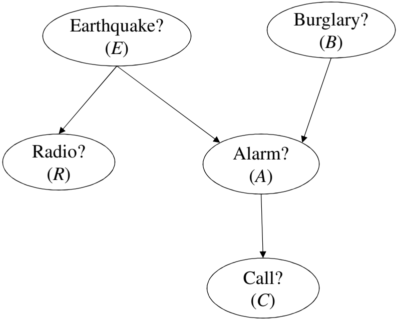

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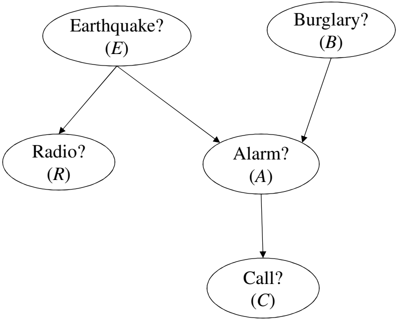

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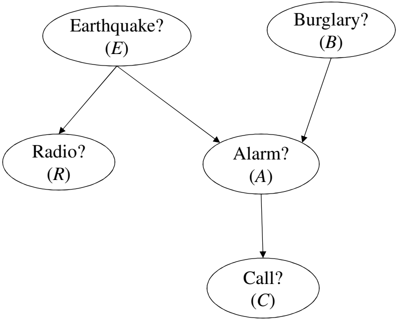

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$$I(A, \{B, E\}, R)$$

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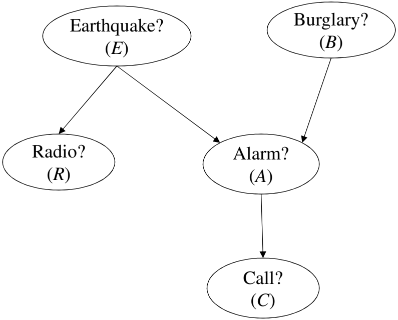

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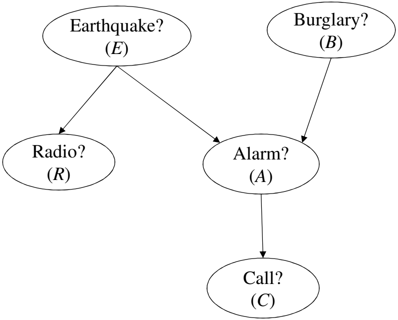

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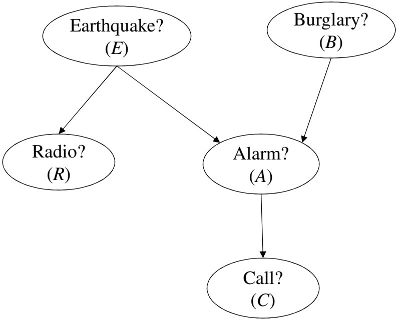

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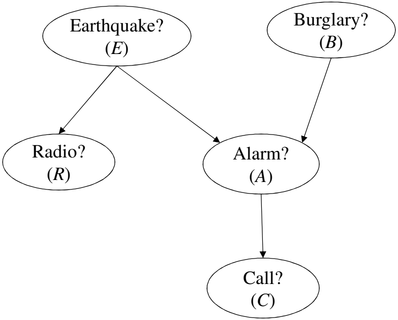

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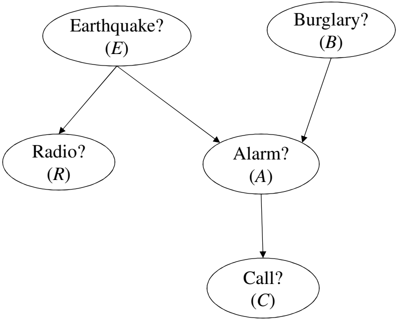

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$$I(A, \{B, E\}, R)$$
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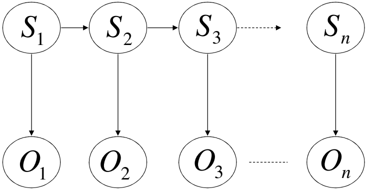
Variables B and E have no parents, hence, they are marginally independent of their non-descendants.

Capturing Independence Graphically

Hidden Markov Model



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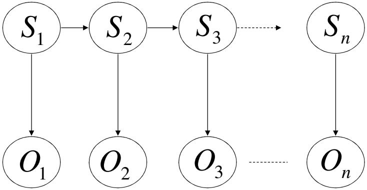

$S_1, S_2, \ldots, S_n$

The state of a dynamic system at time points $1, 2, \ldots, n$

Capturing Independence Graphically

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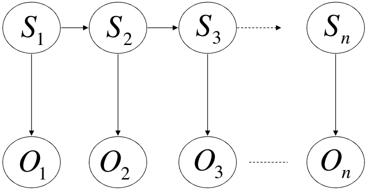

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$O_1, O_2, \ldots, O_n$

Sensors that measure the system state at the corresponding time points.

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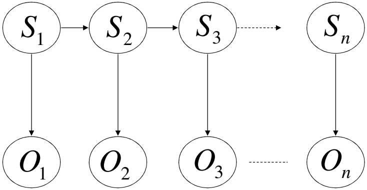

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The state of a dynamic system at time points $1, 2, \ldots, n$

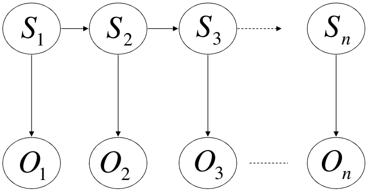
$O_1, O_2, \ldots, O_n$

Sensors that measure the system state at the corresponding time points.

Usually, one has some information about the sensor readings and is interested in computing beliefs in the system states.

Capturing Independence Graphically

Hidden Markov Model



The Markovian assumptions imply that

once we know the state of the system at the previous time point, $t-1$, our belief in the present system state, at t, is no longer influenced by any other information about the past.

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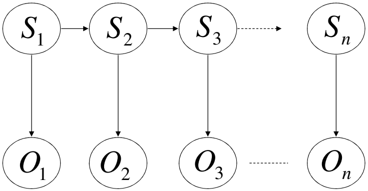

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Characteristic property of HMMs

$$I(S_t, \{S_{t-1}\}, \{S_1, \ldots, S_{t-2}, O_1, \ldots, O_{t-1}\})$$

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Interpretation of DAGs in terms of conditional independence makes no reference to causality

even though we have used causality to motivate this interpretation.

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If one constructs the DAG based on causal perceptions

one tends to agree with the independencies declared by the DAG.

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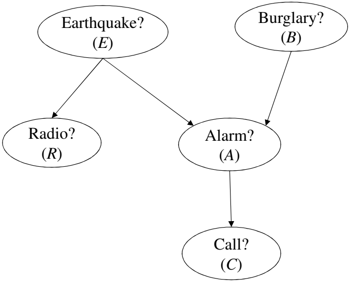
Possible to have a DAG that does not match our causal perceptions

yet we agree with the independencies declared by the DAG.

Capturing Independence Graphically

DAG is causal



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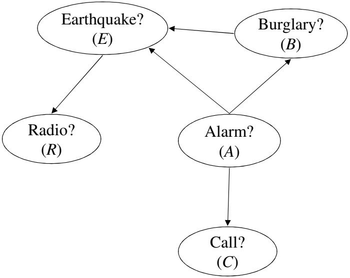

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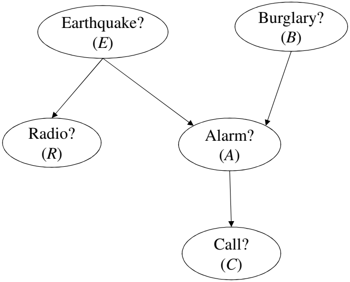

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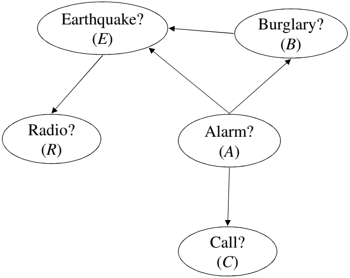
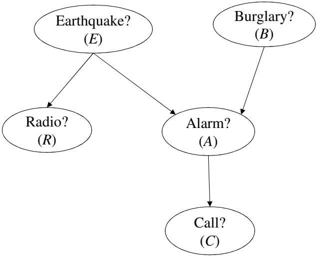


Every independence which is declared (or implied) by the second DAG is also declared (or implied) by the first one. Hence, if we accept the first DAG, then we must also accept the second.

Parameterizing the Independence Structure



DAG G is a partial specification of our state of belief Pr

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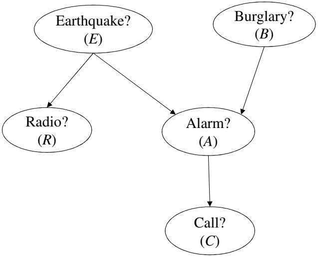

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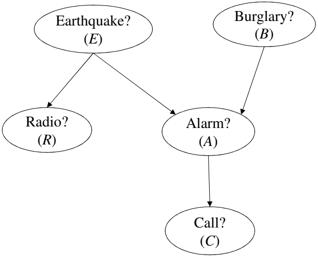

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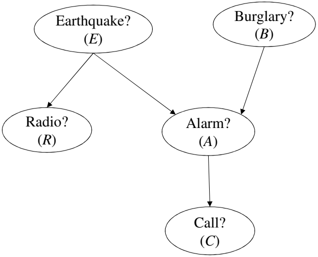

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We can augment the DAG G by a set of conditional probabilities that together with $\mathrm{Markov}(G)$ define the distribution Pr uniquely.

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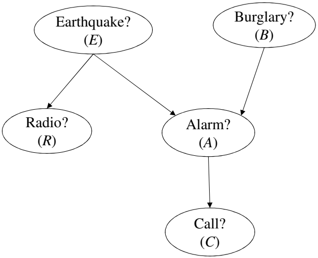

For every variable X and its parents $\mathbf{U}$

Need probability $\Pr(x|\mathbf{u})$ for every value x and every instantiation $\mathbf{u}$

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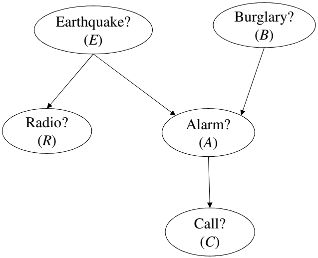

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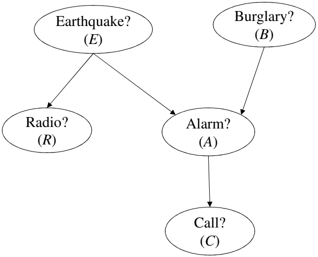
We need to provide the following conditional probabilities

$\Pr(c|a)$, $\Pr(r|e)$, $\Pr(a|b,e)$, $\Pr(e)$, $\Pr(b)$,
where a, b, c, e and r are values of variables A, B, C, E and R

Parameterizing the Independence Structure

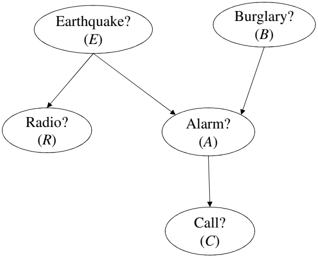


Conditional probabilities for variable C

A	C	$\Pr(c\|a)$
true	true	.80
true	false	.20
false	true	.001
false	false	.999

Conditional Probability Table (CPT)

Parameterizing the Independence Structure



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Conditional Probability Table (CPT)

$$\Pr(c|a) + \Pr(\bar{c}|a) = 1 \text{ and } \Pr(c|\bar{a}) + \Pr(\bar{c}|\bar{a}) = 1$$

Two of the probabilities in the above CPT are redundant and can be inferred from the other two. We only need 10 independent probabilities to completely specify the CPTs for this DAG.

Definition

A Bayesian network for variables $\mathbf{Z}$ is a pair (G, Θ), where

- G is a directed acyclic graph over variables $\mathbf{Z}$, called the network structure.
- Θ is a set of conditional probability tables (CPTs), one for each variable in $\mathbf{Z}$, called the network parametrization.

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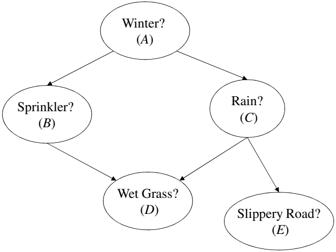
- $\Theta_{X|\mathbf{U}}$: CPT for variable X and its parents $\mathbf{U}$
- $X\mathbf{U}$: called a network family
- $\theta_{x|\mathbf{u}} = \Pr(x|\mathbf{u})$: called a network parameter

We must have $\sum_x \theta_{x|\mathbf{u}} = 1$ for every parent instantiation $\mathbf{u}$

An Example Bayesian Network



A	B	$\Theta_{B\|A}$
true	true	.2
true	false	.8
false	true	.75
false	false	.25

A	C	$\Theta_{C\|A}$
true	true	.8
true	false	.2
false	true	.1
false	false	.9

A	Θ_A
true	.6
false	.4

B	C	D	$\Theta_{D\|B,C}$
true	true	true	.95
true	true	false	.05
true	false	true	.9
true	false	false	.1
false	true	true	.8
false	true	false	.2
false	false	true	0
false	false	false	1

C	E	$\Theta_{E\|C}$
true	true	.7
true	false	.3
false	true	0
false	false	1

Notation

A network instantiation

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Example

θ_a, $\theta_{b|a}$, $\theta_{\bar{c}|a}$, $\theta_{d|b,\bar{c}}$, and $\theta_{\bar{e}|\bar{c}}$ are all the network parameters compatible with network instantiation $a, b, \bar{c}, d, \bar{e}$

The Distribution of a Bayesian Network

A Bayesian network induces distribution

$$\Pr(\mathbf{z}) \stackrel{\mathit{def}}{=} \prod_{\theta_{x|\mathbf{u}} \sim \mathbf{z}} \theta_{x|\mathbf{u}}$$

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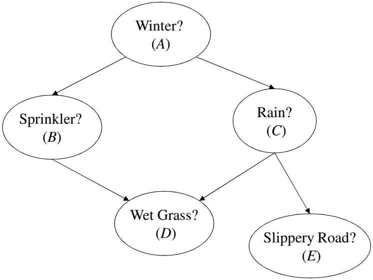
This is called the chain rule of Bayesian networks.

The Distribution of a Bayesian Network



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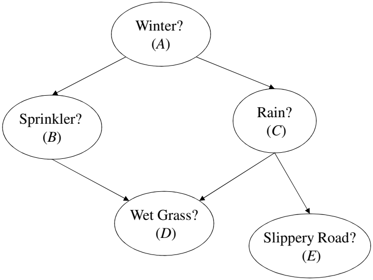
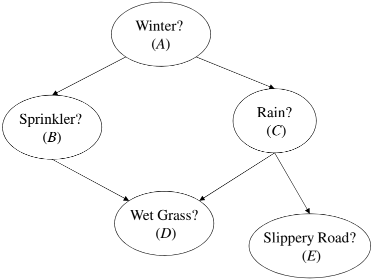


$\Pr(a, b, \bar{c}, d, \bar{e})$

$$
\begin{aligned}
&= \theta_a \; \theta_{b|a} \; \theta_{\bar{c}|a} \; \theta_{d|b,\bar{c}} \; \theta_{\bar{e}|\bar{c}} \\
&= (.6)(.2)(.2)(.9)(1) \\
&= .0216
\end{aligned}
$$

The Distribution of a Bayesian Network



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&= (.6)(.2)(.2)(.9)(1) \\
&= .0216
\end{aligned}
$$

$\Pr(\bar{a}, \bar{b}, \bar{c}, \bar{d}, \bar{e})$

$$
\begin{aligned}
&= \theta_{\bar{a}} \; \theta_{\bar{b}|\bar{a}} \; \theta_{\bar{c}|\bar{a}} \; \theta_{\bar{d}|\bar{b},\bar{c}} \; \theta_{\bar{e}|\bar{c}} \\
&= (.4)(.25)(.9)(1)(1) \\
&= .09
\end{aligned}
$$

The Size of a Bayesian Network

The CPT $\Theta_{X|\mathbf{U}}$ is exponential in the number of parents $\mathbf{U}$

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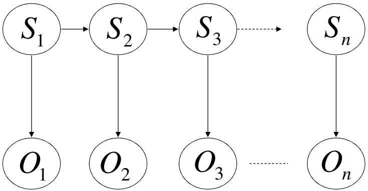
This number is quite reasonable

as long as the number of parents per variable is relatively small.

Variable S_i has m values and similarly for variables O_i

Hidden Markov Model



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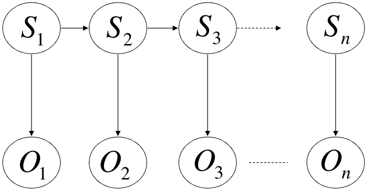

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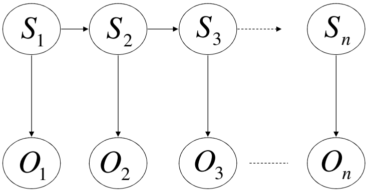

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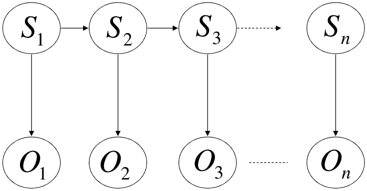

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The CPT for S_1 has m parameters.

A Graphical Test of Independence

The inferential power of the graphoid axioms can be captured using a graphical test, known as d-separation, which allows one to mechanically derive the independencies implied by these axioms.

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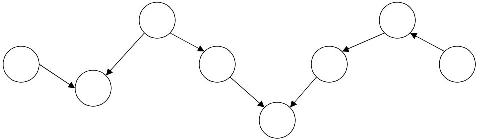
$\text{dsep}_G(\mathbf{X}, \mathbf{Z}, \mathbf{Y})$ implies $I_{\Pr}(\mathbf{X}, \mathbf{Z}, \mathbf{Y})$

for every probability distribution $\Pr$ induced by G

d-separation



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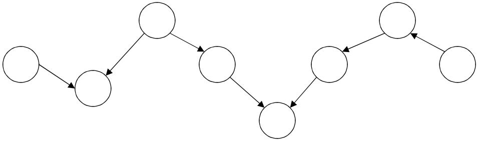
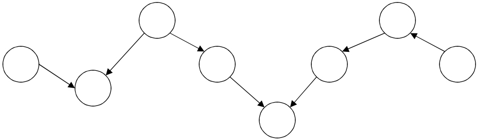


View the path as a pipe

and view each variable W on the path as a valve.

d-separation



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A valve W is either open or closed

depending on some conditions that we state later.

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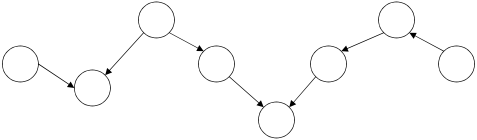

View the path as a pipe

and view each variable W on the path as a valve.

A valve W is either open or closed

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If at least one of the valves on the path is closed

the whole path is blocked. Otherwise, the path is not blocked.

d-separation

The type of a valve

is determined by its relationship to its neighbors on the path.

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sequential $\longrightarrow W \longrightarrow$

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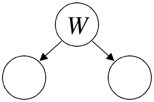

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sequential $\rightarrow W \rightarrow$

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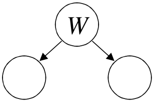
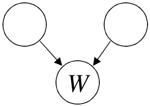
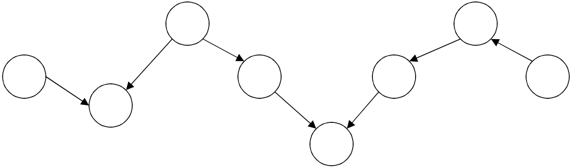




d-separation

A path with 6 valves



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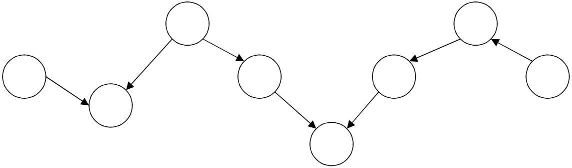

From left to right

d-separation

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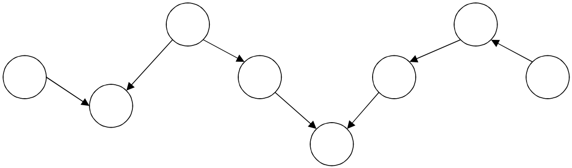

From left to right

convergent,

d-separation

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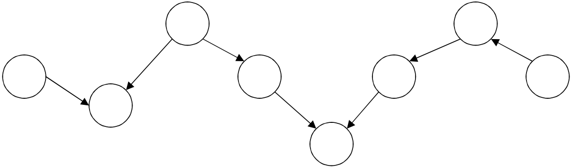

From left to right

convergent, divergent,

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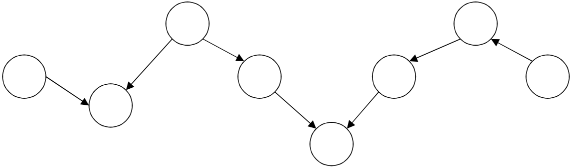

From left to right

convergent, divergent, sequential,

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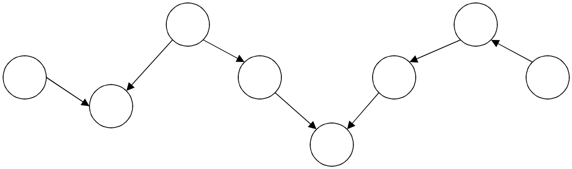

From left to right

convergent, divergent, sequential, convergent,

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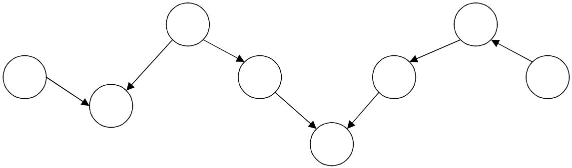

From left to right

convergent, divergent, sequential, convergent, sequential,

d-separation

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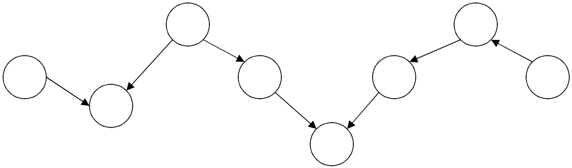


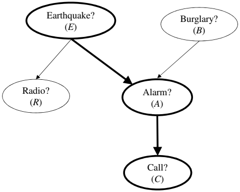
From left to right

convergent, divergent, sequential, convergent, sequential, and sequential.

d-separation

d-separation

sequential valve

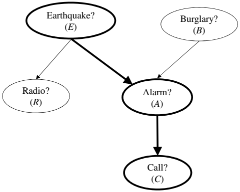


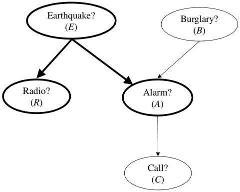
A is intermediary between cause E and effect C

d-separation

sequential valve

divergent valve

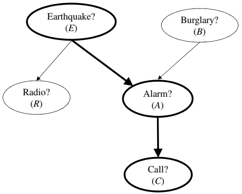
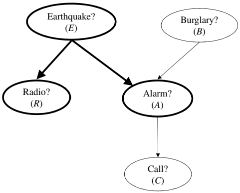
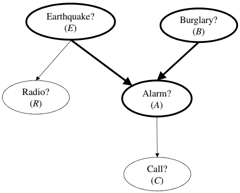




A is intermediary between cause E and effect C

E is common cause of effects R and A

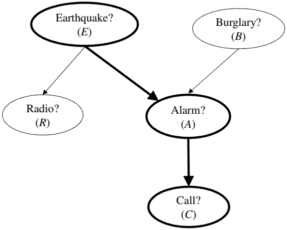
d-separation

sequential valve	divergent valve	convergent valve
		
A is intermediary between cause E and effect C	E is common cause of effects R and A	A is common effect of causes E and B

d-separation

Given that we know Z

when is a sequential valve closed?



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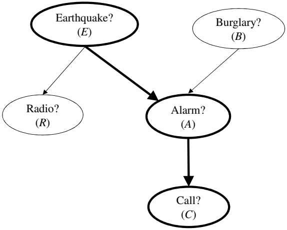


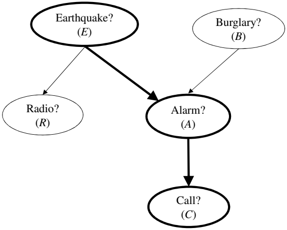
Valve $E \rightarrow A \rightarrow C$ is closed iff

we know the value of variable A, otherwise an earthquake E may change our belief in getting a call C.

d-separation

Given that we know $\mathbf{Z}$

when is a sequential valve closed?



Valve $E \rightarrow A \rightarrow C$ is closed iff

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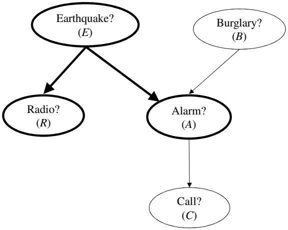
A sequential valve $\rightarrow W \rightarrow$ is closed iff variable W appears in $\mathbf{Z}$

d-separation

Given that we know Z

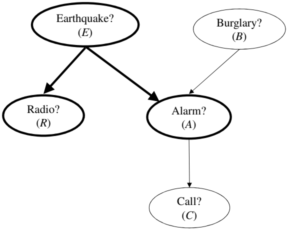
when is a divergent valve closed?



d-separation

Given that we know $\mathbf{Z}$

when is a divergent valve closed?



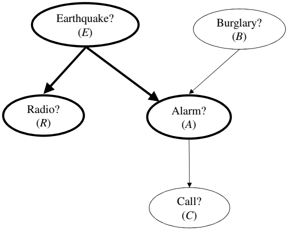
Valve $R \leftarrow E \rightarrow A$ is closed iff

we know the value of variable E, otherwise a radio report on an earthquake may change our belief in the alarm triggering.

d-separation

Given that we know $\mathbf{Z}$

when is a divergent valve closed?



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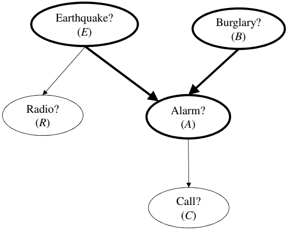
A divergent valve $\leftarrow W \rightarrow$ is closed iff variable W appears in $\mathbf{Z}$

d-separation

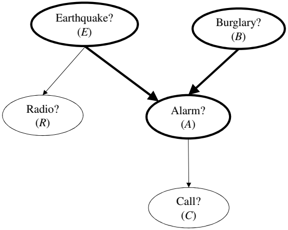
Given that we know **Z**

when is a convergent valve closed?



Given that we know $\mathbf{Z}$

when is a convergent valve closed?



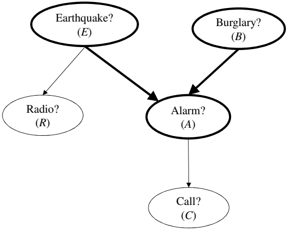
Valve $E \rightarrow A \leftarrow B$ is closed iff

neither the value of variable A nor the value of C are known, otherwise, a burglary may change our belief in an earthquake.

d-separation

Given that we know Z

when is a convergent valve closed?



Valve $E \rightarrow A \leftarrow B$ is closed iff

neither the value of variable A nor the value of C are known, otherwise, a burglary may change our belief in an earthquake.

A convergent valve $\rightarrow W \leftarrow$ is closed iff neither variable W nor any of its descendants appears in $\mathbf{Z}$

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$\mathbf{X}$ and $\mathbf{Y}$ are d-separated by $\mathbf{Z}$, written $\text{dsep}_G(\mathbf{X}, \mathbf{Z}, \mathbf{Y})$, iff

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A path is blocked by $\mathbf{Z}$ iff

at least one valve on the path is closed given $\mathbf{Z}$

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A path is blocked by $\mathbf{Z}$ iff

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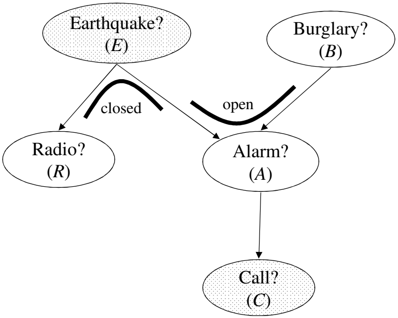
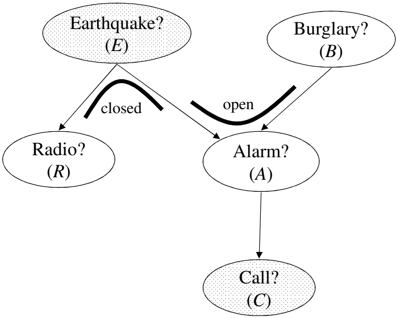
A path with no valves (i.e., $X \longrightarrow Y$) is never blocked.

d-separation



Are B and R d-separated by E and C?

d-separation

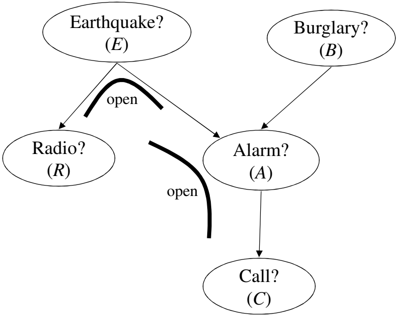


Are B and R d-separated by E and C?

Yes

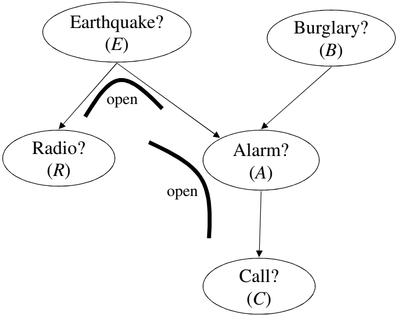
The closure of only one valve is sufficient to block the path, therefore, establishing d-separation.

d-separation



Are C and R d-separated?

d-separation

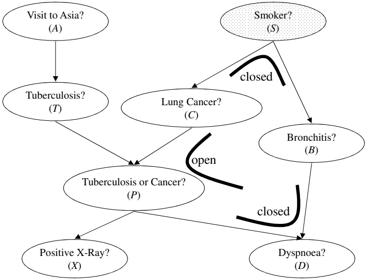


Are C and R d-separated?

No

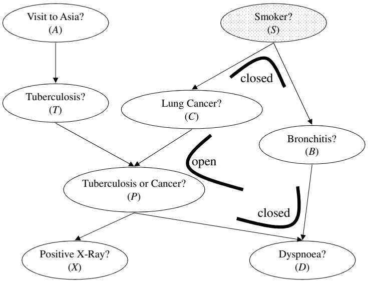
Both valves are open. Hence, the path is not blocked and d-separation does not hold.

d-separation



Are C and B d-separated by S?

d-separation



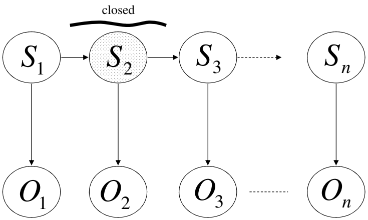
Are C and B d-separated by S?

Yes

Both paths between them are blocked by S.

d-separation

Is $I_{\Pr}(S_1, S_2, \{S_3, S_4\})$ for any probability distribution $\Pr$ induced by the DAG?



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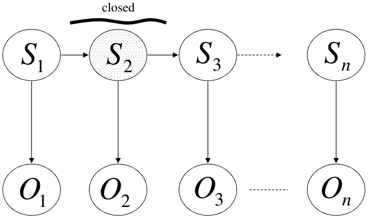

Valve $S_1 {\rightarrow} S_2 {\rightarrow} S_3$ on every path between S_1 and $\{S_3, S_4\}$

d-separation

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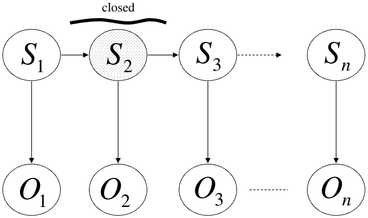

Valve $S_1 \rightarrow S_2 \rightarrow S_3$ on every path between S_1 and $\{S_3, S_4\}$

Valve is closed given S_2

d-separation

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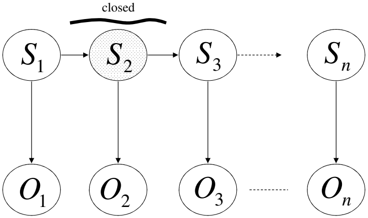

Valve $S_1 \rightarrow S_2 \rightarrow S_3$ on every path between S_1 and $\{S_3, S_4\}$

Valve is closed given S_2

Every path from S_1 to $\{S_3, S_4\}$ is blocked by S_2 and we have $\text{dsep}_G(S_1, S_2, \{S_3, S_4\})$

Blankets and Boundaries

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A minimal Markov blanket

is called a Markov Boundary.

Blankets and Boundaries

A Markov blanket for variable X

is a set of variables which, when known, will render every other variable irrelevant to X

A Markov blanket $\mathbf{B}$ is minimal iff

no strict subset of $\mathbf{B}$ is also a Markov blanket.

A minimal Markov blanket

is called a Markov Boundary.

The Markov Boundary is not unique

unless the distribution is strictly positive.

Blankets and Boundaries

If distribution $\Pr$ is induced by DAG G

then a Markov blanket for variable X with respect to $\Pr$ can be constructed using its parents, children, and spouses in DAG G

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Variable Y is a spouse of X iff

the two variables have a common child in DAG G

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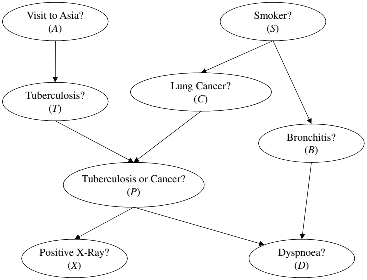
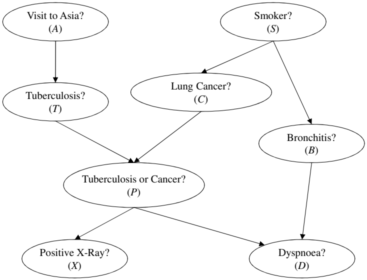


Markov blanket for C

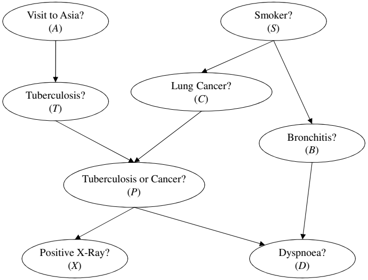
Blankets and Boundaries

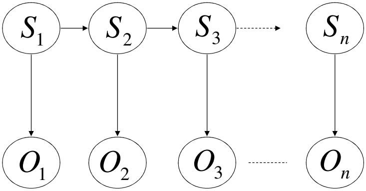


Markov blanket for C

S, P, T

Blankets and Boundaries



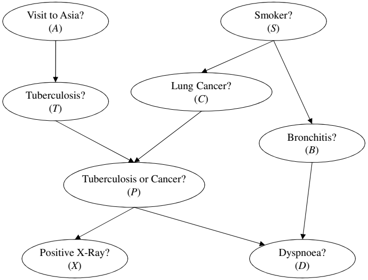


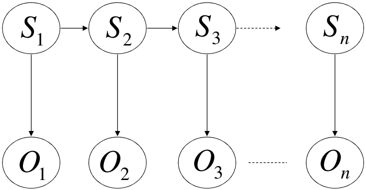
Markov blanket for C

S, P, T

Markov blanket for S_t, $t > 1$

Blankets and Boundaries





Markov blanket for S_t, $t > 1$

S_{t-1}, S_{t+1}, O_t

Markov blanket for C

S, P, T