LINEAR REGRESSION

LEARN EASY. ENJOY EASY. EXPERTISE EASY

PREDICT APPLICANTS SALARY -

USING LINEAR REGRESSION ALGORITHM

LOAN APPLICATIONS – LET US PREDICT "SALARY" WHEN THE INPUT IS "YEARS OF EXPERIENCE"

You distribute Loans. When you distribute loans, Salary of a person or Income of an employee is an important component. Applicants will fill in their details of Income , they will also submit tax returns etc.

However you want to build a Machine Learning model that will Predict salary for years experience

Exp	$ Salary
2	15
3	28
5	42
13	64
8	50
16	90
11	58
1	8
9	54

What will be Salary if the Experience is 15?

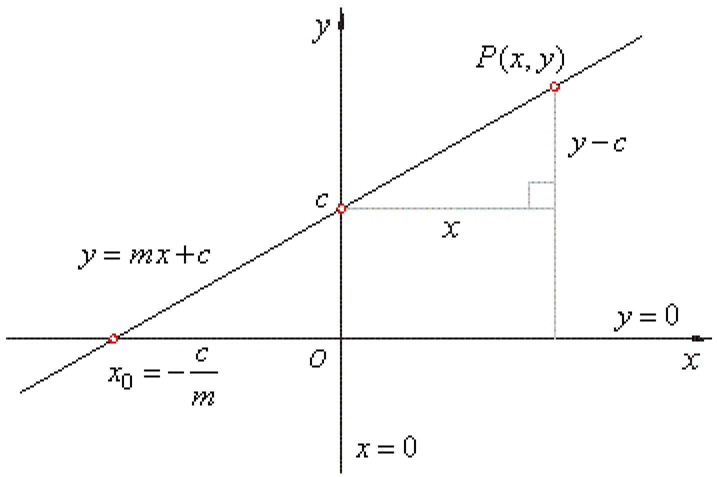
LINEAR REGRESSION

Linear regression models are used to show or predict the relationship between two variables or factors. The factor that is being predicted (the factor that the equation solves for) is called the dependent variable. The factors that are used to predict the value of the dependent variable are called the independent variables.

In linear regression, each observation consists of two values. One value is for the dependent variable and one value is for the independent variable. In this simple model, a straight line approximates the relationship between the dependent variable and the independent variable.

When two or more independent variables are used in regression analysis, the model is no longer a simple linear one. This is known as multiple regression.

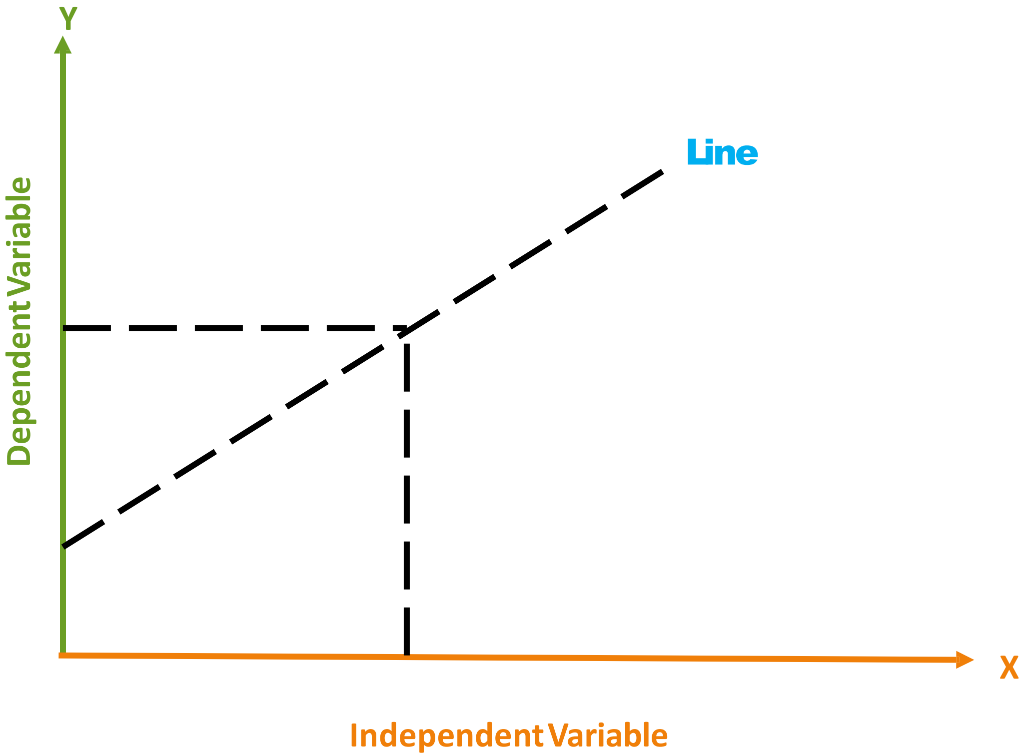
LINEAR REGRESSION - EQUATION



y = mx + c

The slope of the line is ***m***, and ***c*** is the intercept (the value of ***y*** when ***x*** = 0).

LINEAR REGRESSION – INDEPENDENT AND DEPENDENT VARIABLES



LINEAR REGRESSION – INDEPENDENT AND DEPENDENT VARIABLES

Exp	$ Salary
2	15
3	28
5	42
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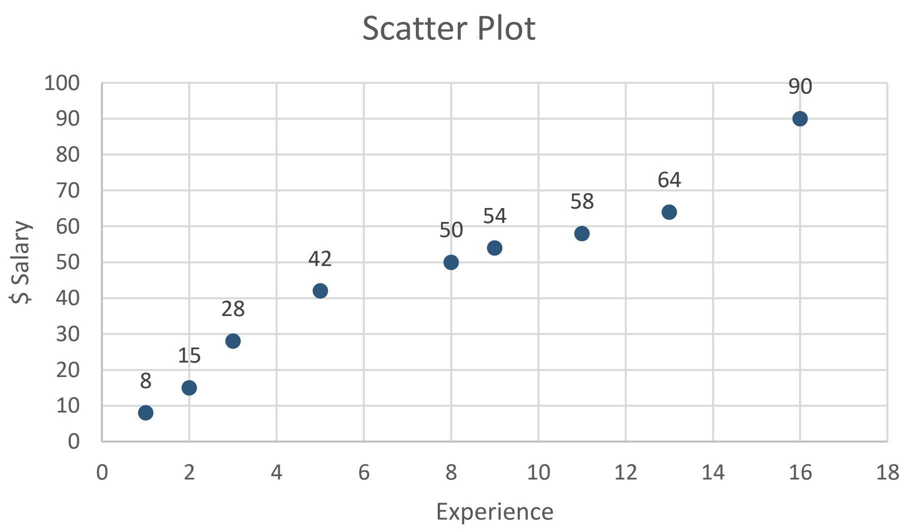
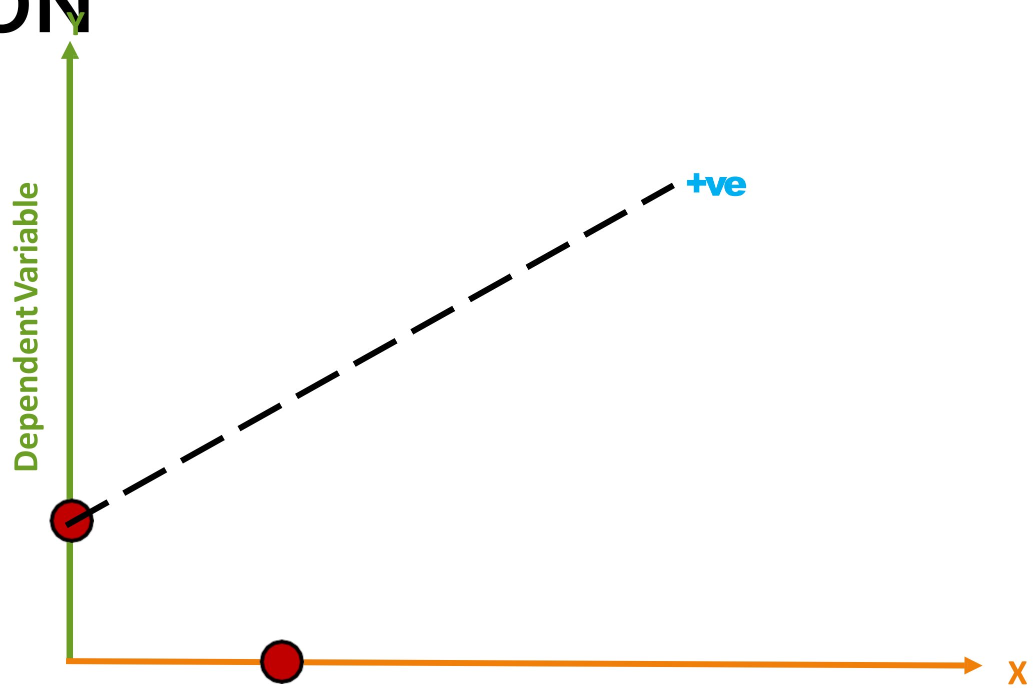
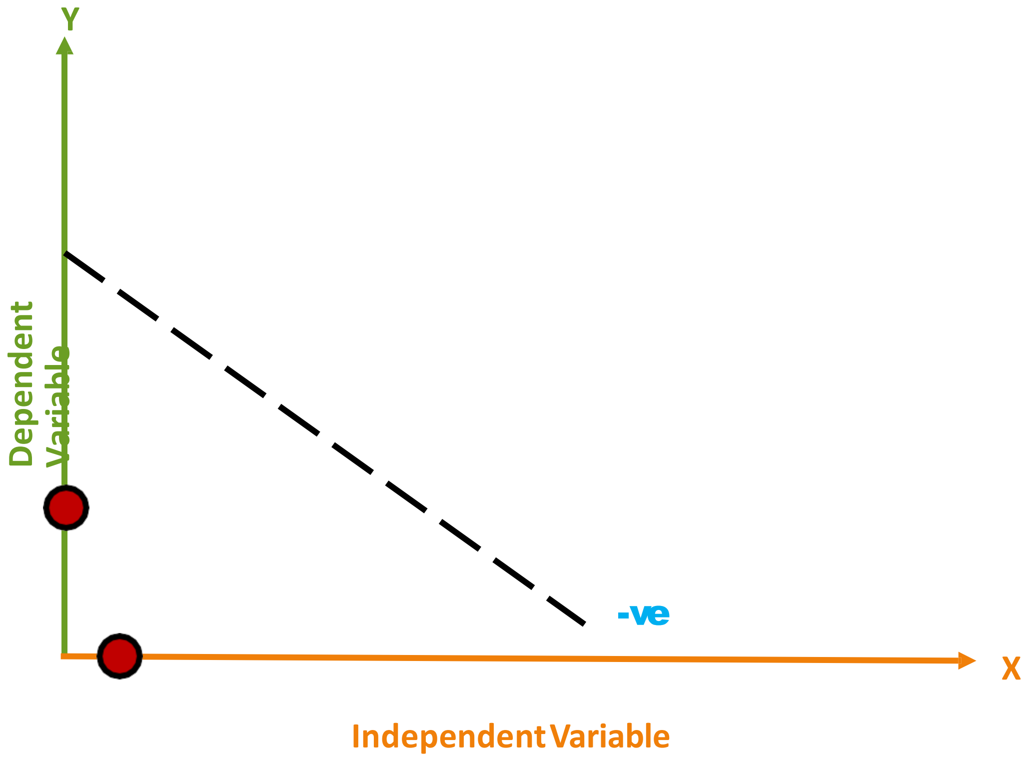


POSITIVE RELATION



NEGATIVE RELATION



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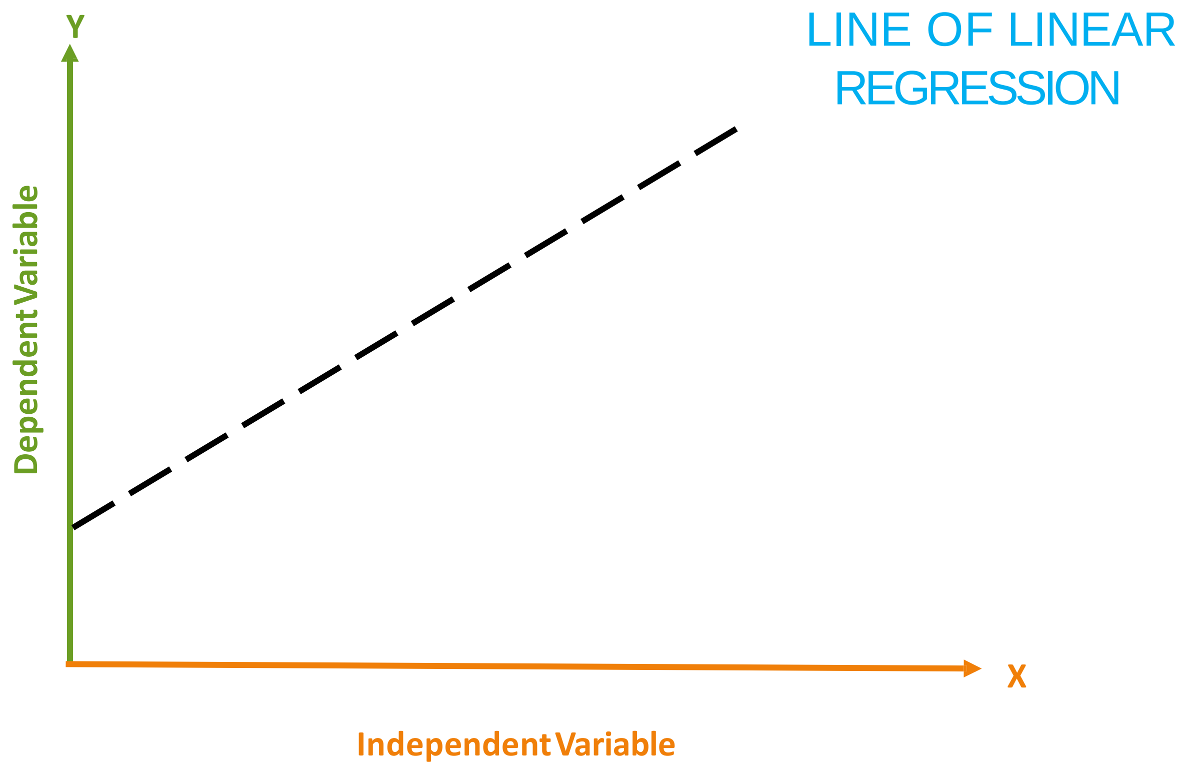
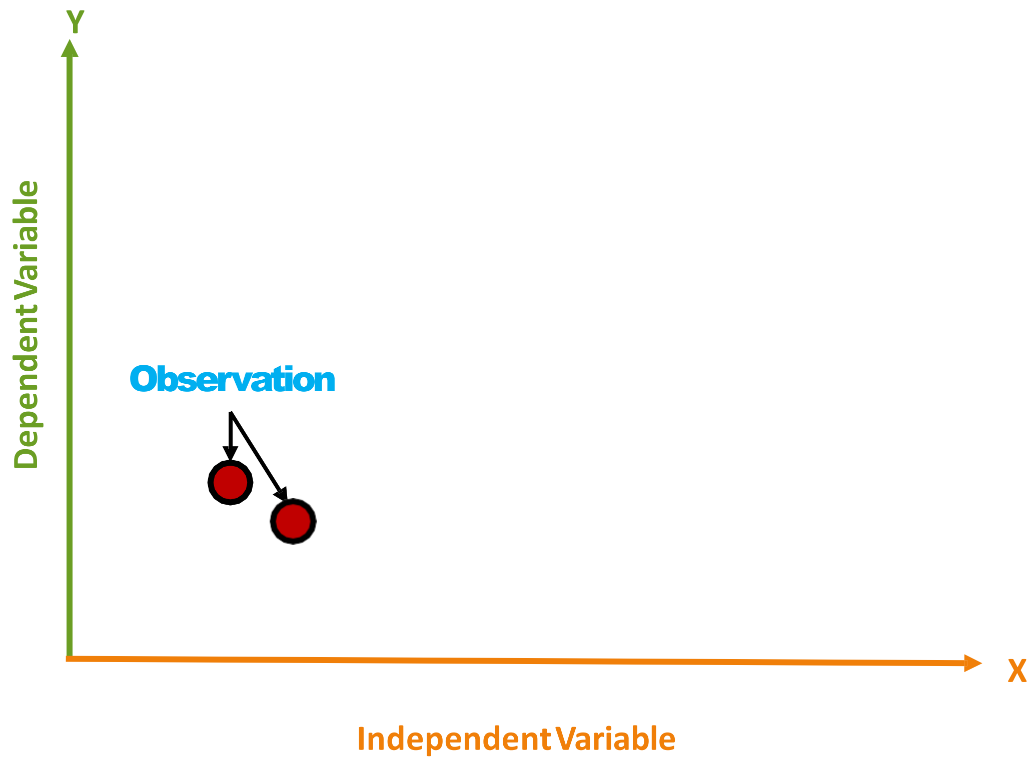
LINEAR REGRESSION



DATA POINTS - OBSERVATIONS



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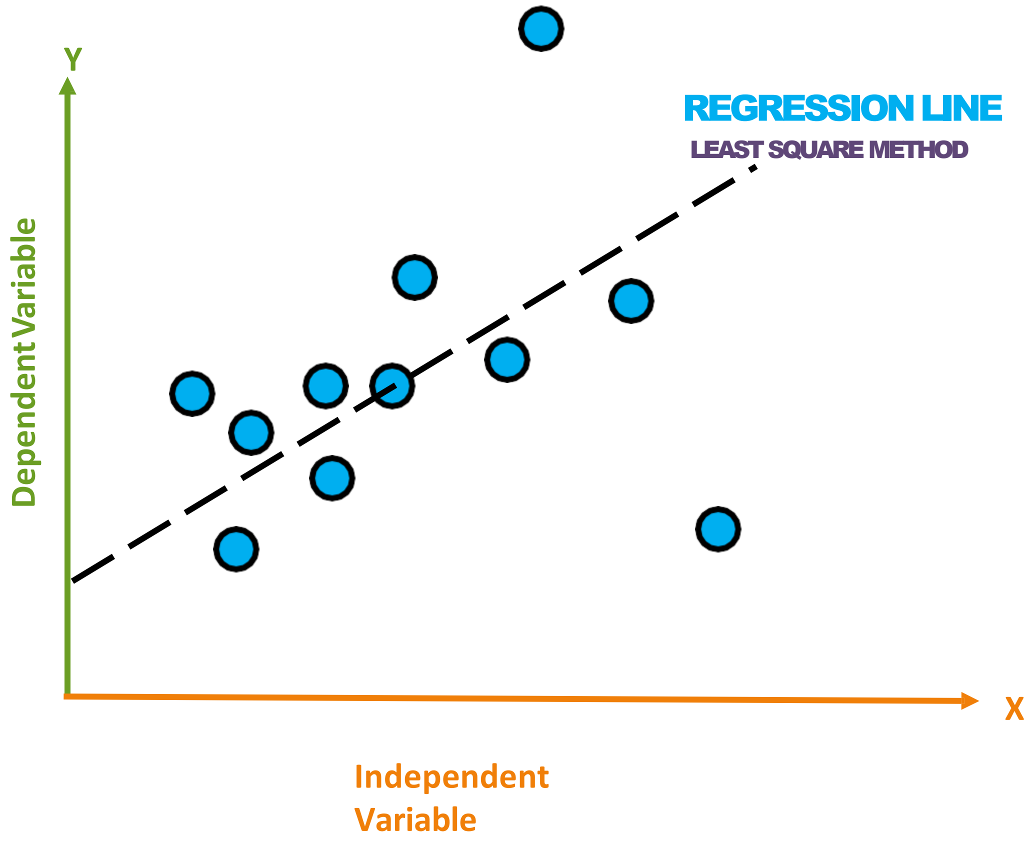
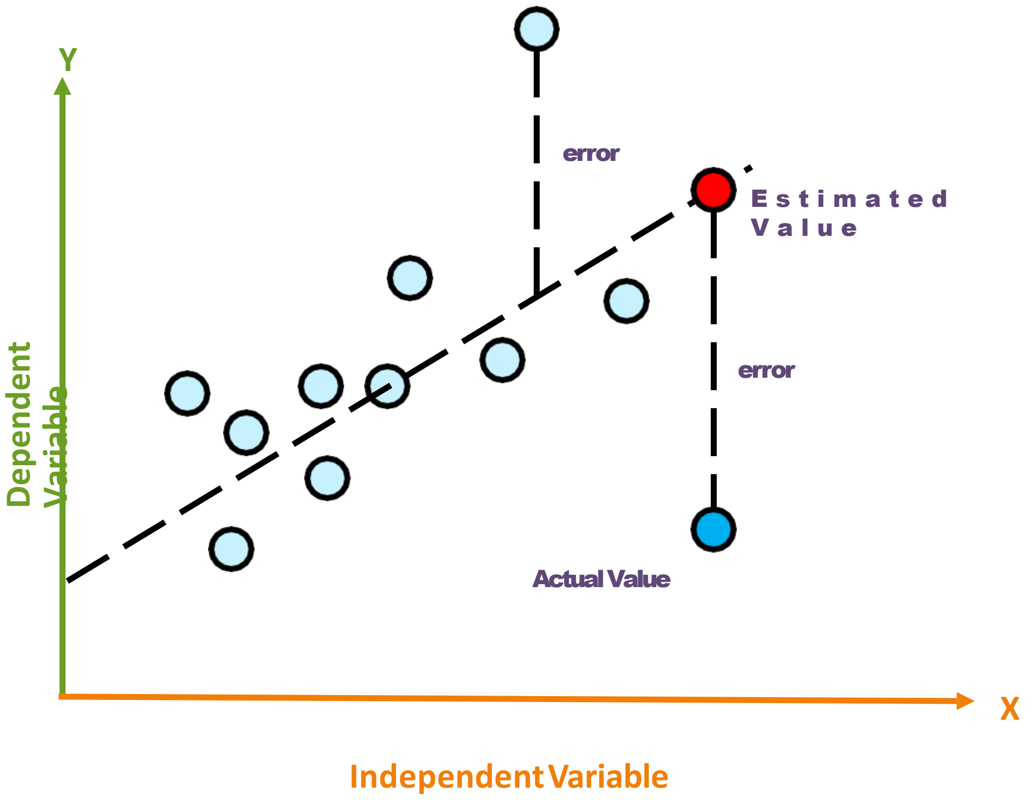
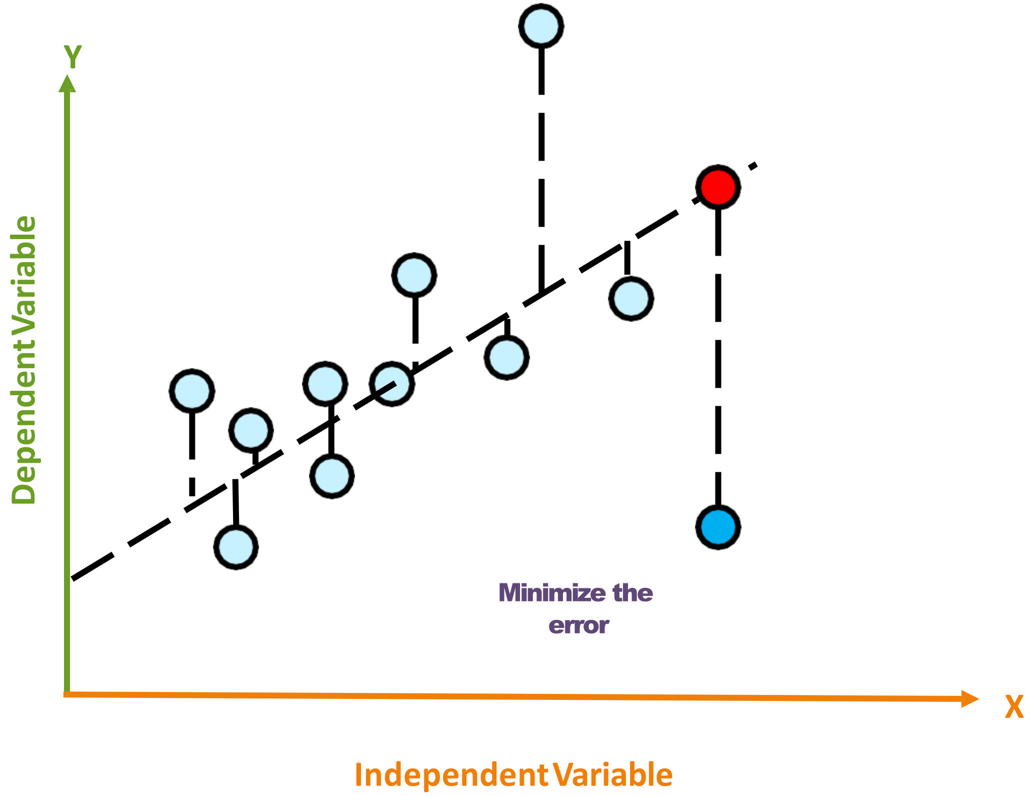
PLOTTING THE BEST FIT LINE – USING THE LEAST SQUARE METHOD (ORDINARY LEAST SQUARES)



ERROR: ESTIMATED VALUE - PREDICTED VALUE



MINIMIZE THE ERROR



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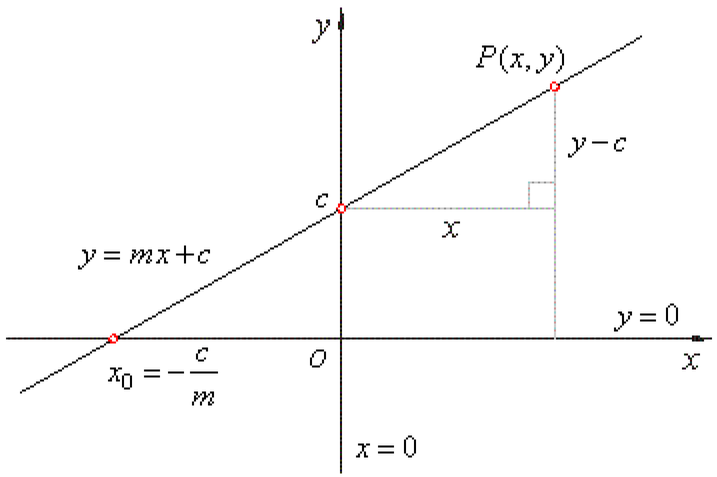
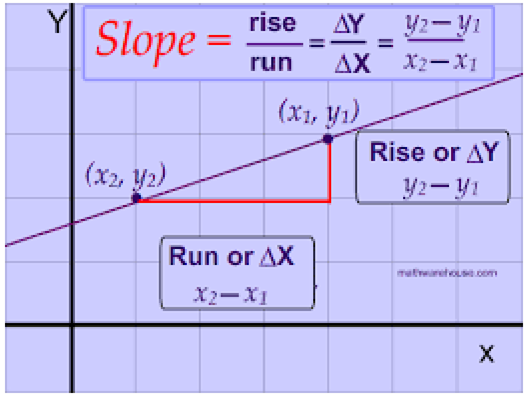
HOW TO CREATE LINEAR REGRESSION – EQUATION TO PREDICT SALARY

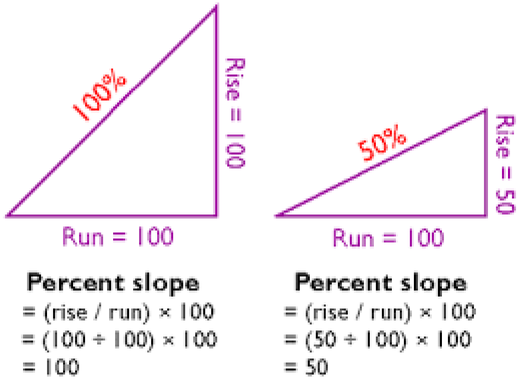


y = mx + c

The slope of the line is ***m***, and ***c*** is the intercept (the value of ***y*** when ***x*** = 0).

CALCULATING THE SLOPE





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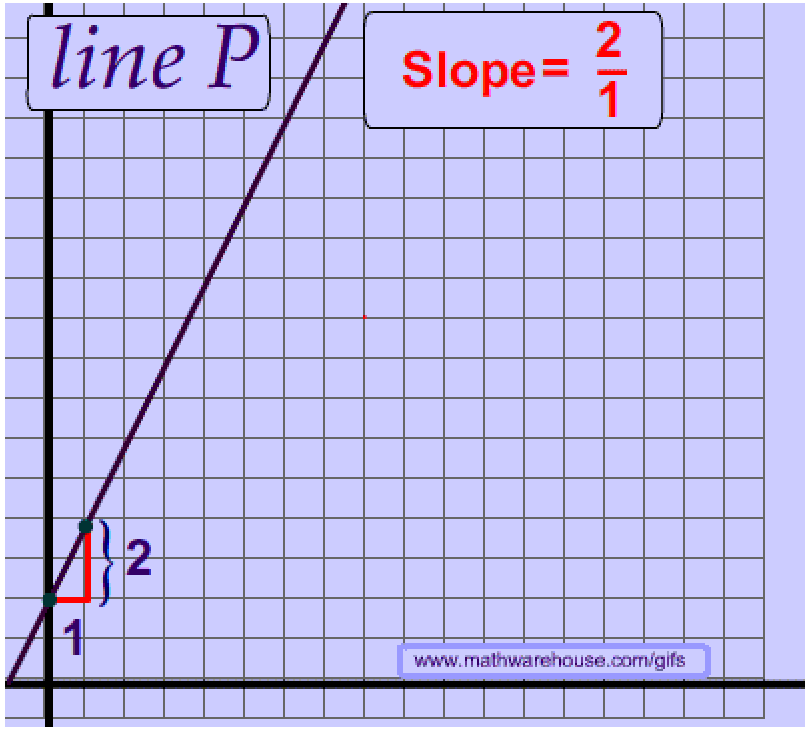
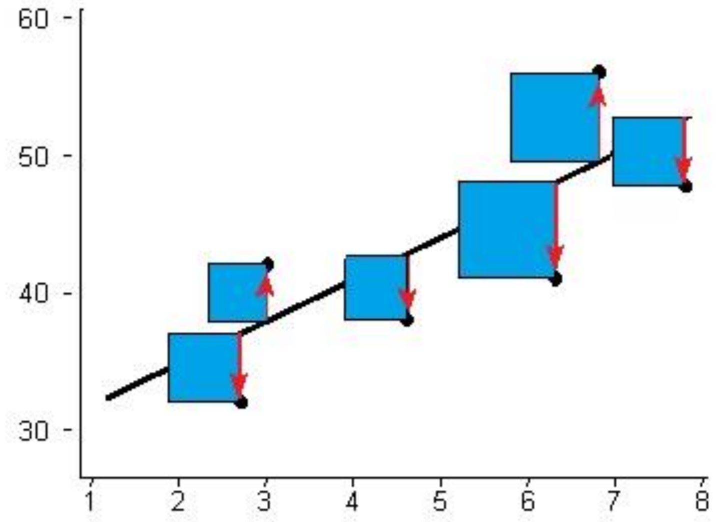
HOW A SLOPE WILL DRAW THE LINE



LEAST SQUARES REGRESSION LINE

The Least Squares Regression Line is the line that makes the vertical distance from the data points to the regression line as small as possible. It's called a "least squares" because the best line of fit is one that minimizes the variance (the sum of squares of the errors).

This can be a bit hard to visualize but the main point is you are aiming to find the equation that fits the points as closely as possible.



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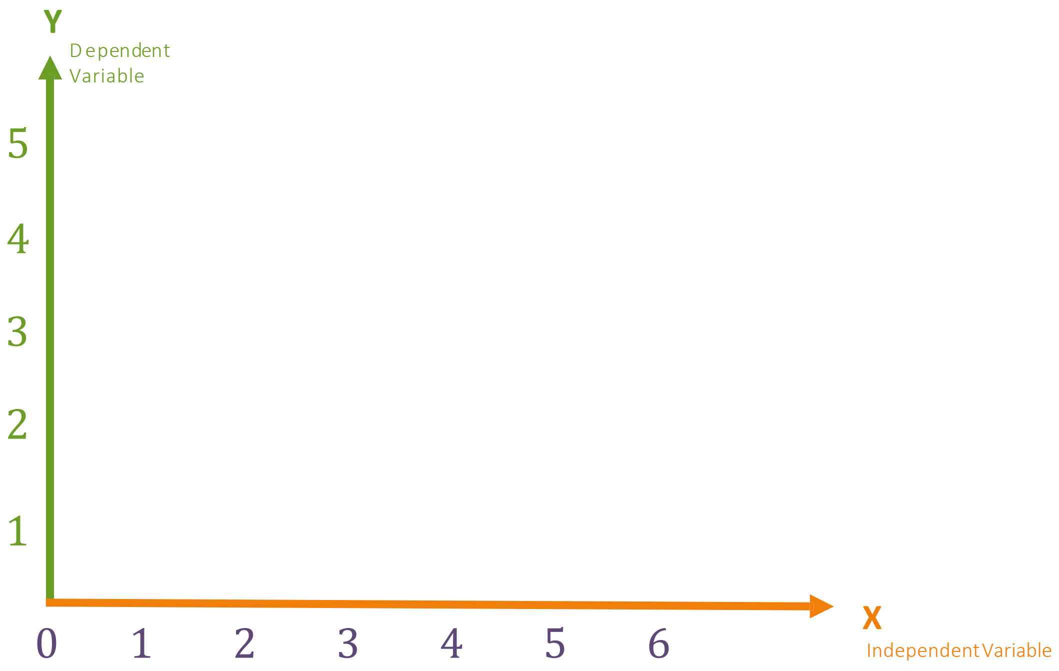
PREDICT SALARY

USING LINEAR REGRESSION EQUATION

PREDICT THE SALARY WHEN INPUT IS EXPERIENCE

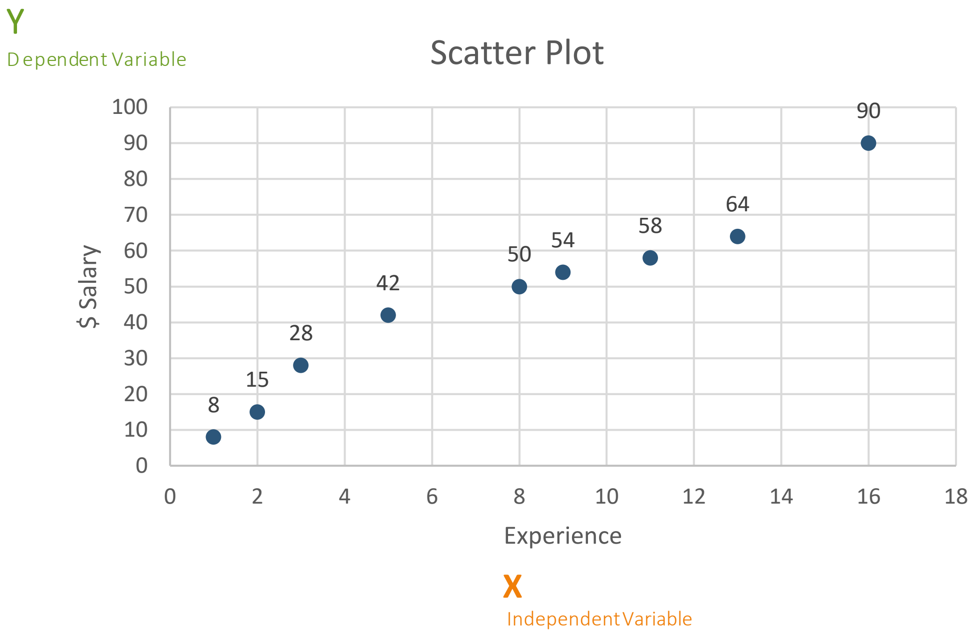


X Independent Variable Exp

Y Dependent Variable $Salary

Exp	$ Salary
2	15
3	28
5	42
13	64
8	50
16	90
11	58
1	8
9	54

PREDICT THE SALARY WHEN INPUT IS EXPERIENCE



X Independent Variable Exp

Y Dependent Variable $Salary

Exp	$ Salary
2	15
3	28
5	42
13	64
8	50
16	90
11	58
1	8
9	54

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SUM OF SQUARED ERRORS

In order to fit the best intercept line between the points in the above scatter plots, we use a metric called “Sum of Squared Errors” (SSE) and compare the lines to find out the best fit by reducing errors. The errors are sum difference between actual value and predicted value.

To find the errors for each dependent value, we need to use the formula below.

$$SSE = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

$$y_i = Dependent\ Variables\ (Salary)$$

$$\bar{y} = Average\ of\ Dependent\ Variables$$

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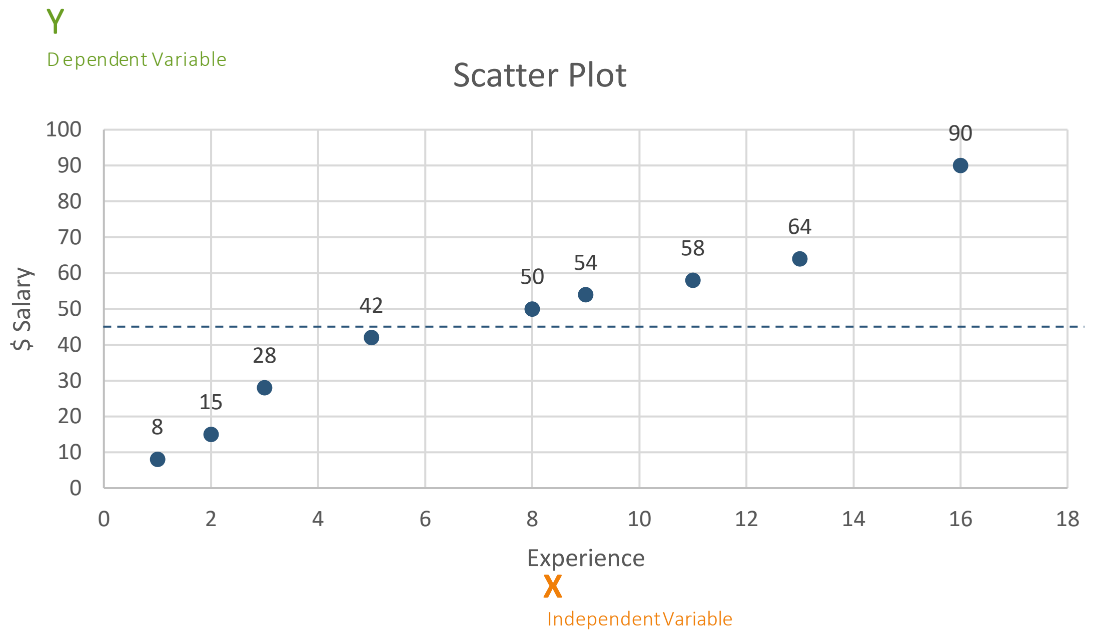
MEAN LINE OR THE WORST FIT LINE



45.4-mean line

x	y
Exp	$ Salary
2	15
3	28
5	42
13	64
8	50
16	90
11	58
1	8
9	54
	$\bar{y}$=45.444

We find the "Sum of Squared Errors" (SSE) for a Mean line

SUM OF SQUARED ERRORS

$$SSE = \sum_{i=1}^{n}(y_i - \bar{y})^2$$

y_i = *Dependent Variables (Salary)*

$\bar{y}$ = *Average of Dependent Variables*

The sum of squared errors SSE output is *5226.22* when the line is mean line.

x	y	E=y-ȳ	Error Sqaure
Exp	$ Salary	Error	Error^2
2	15	15-45.444=-30.44	926.84
3	28	28-45.444=-17.44	304.29
5	42	42-45.444=-3.44	11.86
13	64	64-45.444=18.56	344.33
8	50	50-45.444=4.56	20.76
16	90	90-45.444=44.56	1985.24
11	58	58-45.444=12.56	157.65
1	8	8-45.444=-37.44	1402.05
9	54	54-45.444=8.56	73.21
	ȳ=45.444		SSE=5226.22

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FIND THE BEST FIT OF LINE

The mean line gave us SSE *5226.22*

We have to find a line that will bring down the SSE value.

We need to find the best fit of line intercept, we need to apply a linear regression model to reduce the SSE value at minimum as possible. To get the best fit line we need to identify a slope intercept, we use the equation

$y=mx+b$

m is the slope

b is intercept

x → independent variables

y → dependent variables

x	y	$E=y-\bar{y}$	Error Sqaure
Exp	$ Salary	Error	Error^2
2	15	15-45.444=-30.44	926.84
3	28	28-45.444=-17.44	304.29
5	42	42-45.444=-3.44	11.86
13	64	64-45.444=18.56	344.33
8	50	50-45.444=4.56	20.76
16	90	90-45.444=44.56	1985.24
11	58	58-45.444=12.56	157.65
1	8	8-45.444=-37.44	1402.05
9	54	54-45.444=8.56	73.21
	$\bar{y}$=45.444		SSE=5226.22

2 METHODS TO FIND THE BEST FIT OF LINE

1. Ordinary Least Square method :will work for both univariate dataset and multi-variate dataset.

Univariate dataset which is single independent variables and single dependent variables.
Multi-variate dataset contains a single independent variables set and multiple dependent variables sets

2. **Gradient Descent** machine learning algorithm is applied on Multi-variate datasets

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ORDINARY LEAST SQUARES (OLS) METHOD

We will use Ordinary Least Squares method to find the best line intercept (b) slope (m)

y=mx+b

To use OLS method, we apply the below formula to find the equation

$$m = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$

$$b = \bar{y} - m * \bar{x}$$

x = *independent variables*

$\bar{x}$ = *average of independent variables*

y = *dependent variables*

$\bar{y}$ = *average of dependent variables*

x	y
Exp	$ Salary
2	15
3	28
5	42
13	64
8	50
16	90
11	58
1	8
9	54
$\bar{x}$=7.56	$\bar{y}$=45.444

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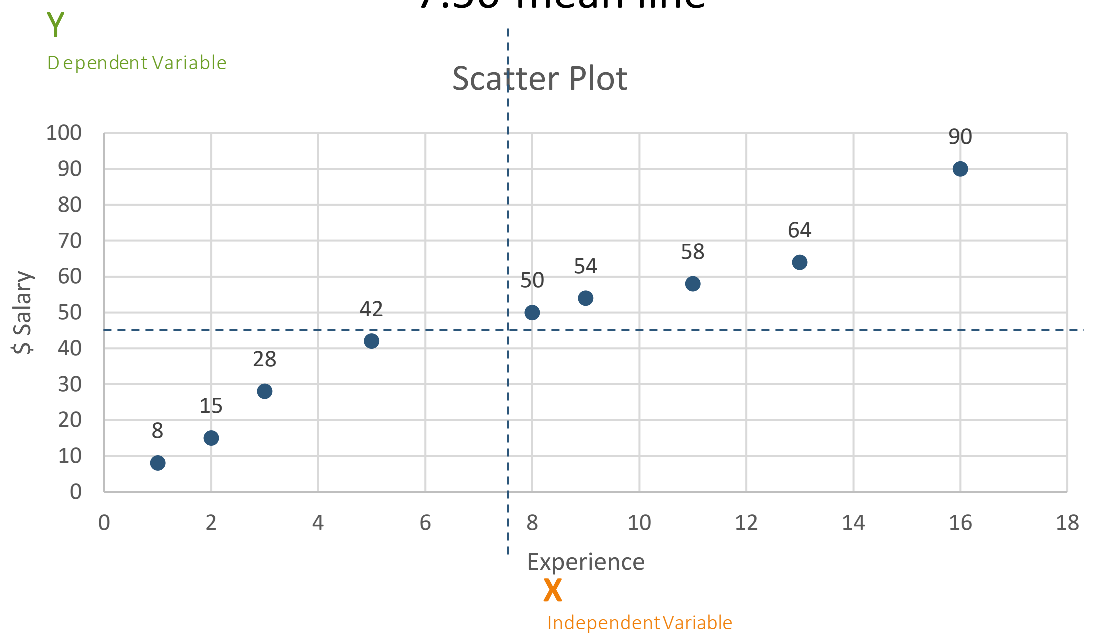
MEAN OF THE DATA POINTS

7.56-mean line



45.4-mean line

x	y
Exp	$ Salary
2	15
3	28
5	42
13	64
8	50
16	90
11	58
1	8
9	54
$\bar{x}$=7.56	$\bar{y}$=45.444

We find the "Sum of Squared Errors" (SSE) for a Mean line

ORDINARY LEAST SQUARES (OLS) METHOD

$$m = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$

$$b = \bar{y} - m * \bar{x}$$

x = *independent variables*
$\bar{x}$ = *average of independent variables*
y = *dependent variables*
$\bar{y}$ = *average of dependent variables*

$$m = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$

Calculate Slope:
m = 1037.8 / 216.19
m = 4.80

x	y	x-x̄	y-ȳ	(x-x̄)(y-ȳ)	(x-x̄)^2
Exp	$ Salary				
2	15	2-7.56=-5.56	15-45.444=-30.44	169.27	30.91
3	28	3-7.56=-4.56	28-45.444=-17.44	79.54	20.79
5	42	5-7.56=-2.56	42-45.444=-3.44	8.82	6.55
13	64	13-7.56=5.44	64-45.444=18.56	100.94	29.59
8	50	8-7.56=0.44	50-45.444=4.56	2.00	0.19
16	90	16-7.56=8.44	90-45.444=44.56	376.05	71.23
11	58	11-7.56=3.44	58-45.444=12.56	43.19	11.83
1	8	1-7.56=-6.56	8-45.444=-37.44	245.63	43.03
9	54	9-7.56=1.44	54-45.444=8.56	12.32	2.07
x̄=7.56	ȳ=45.444			1037.78	216.22

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ORDINARY LEAST SQUARES (OLS) METHOD

$$m = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$

$$b = \bar{y} - m * \bar{x}$$

x = *independent variables*
$\bar{x}$ = *average of independent variables*
y = *dependent variables*
$\bar{y}$ = *average of dependent variables*

Calculate the intercept(b)
b = 45.44 - 4.80 * 7.56 = 9.15
b=9.15
Hence:
y = mx + b
→ 4.80x + 9.15
y = 4.80x + 9.15

x	y	x-x̄	y-ȳ	(x-x̄)(y-ȳ)	(x-x̄)^2
Exp	$ Salary				
2	15	2-7.56=-5.56	15-45.444=-30.44	169.27	30.91
3	28	3-7.56=-4.56	28-45.444=-17.44	79.54	20.79
5	42	5-7.56=-2.56	42-45.444=-3.44	8.82	6.55
13	64	13-7.56=5.44	64-45.444=18.56	100.94	29.59
8	50	8-7.56=0.44	50-45.444=4.56	2.00	0.19
16	90	16-7.56=8.44	90-45.444=44.56	376.05	71.23
11	58	11-7.56=3.44	58-45.444=12.56	43.19	11.83
1	8	1-7.56=-6.56	8-45.444=-37.44	245.63	43.03
9	54	9-7.56=1.44	54-45.444=8.56	12.32	2.07
x̄=7.56	ȳ=45.444			1037.78	216.22

BEST FIT LINE USING (OLS) METHOD

$$m = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$

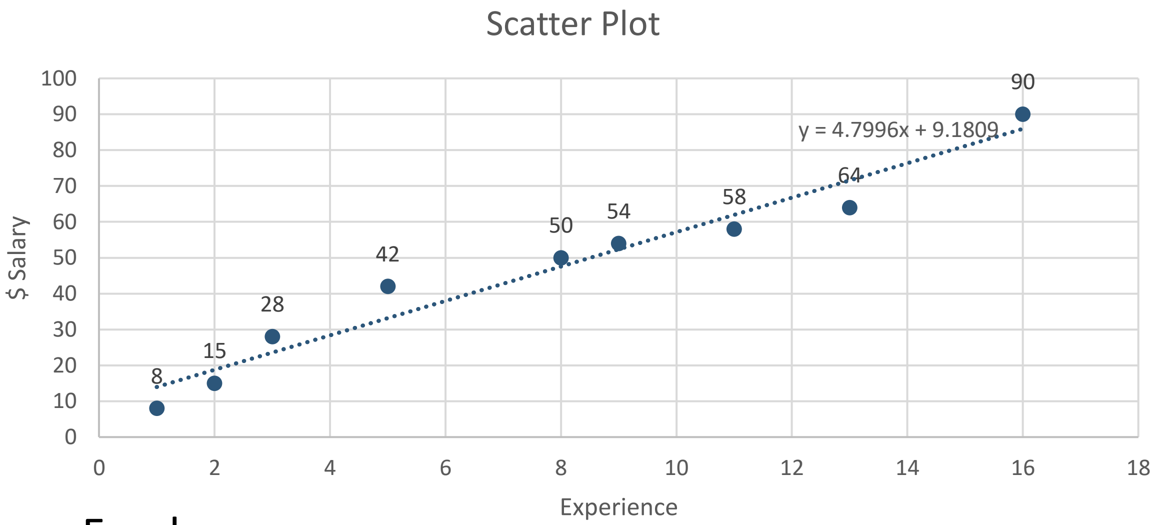
$$b = \bar{y} - m * \bar{x}$$

x = *independent variables*

$\bar{x}$ = *average of independent variables*

y = *dependent variables*

$\bar{y}$ = *average of dependent variables*



OLS Method:

m = 1037.8 / 216.19

m = 4.80

b = 45.44 - 4.80 * 7.56 = 9.15

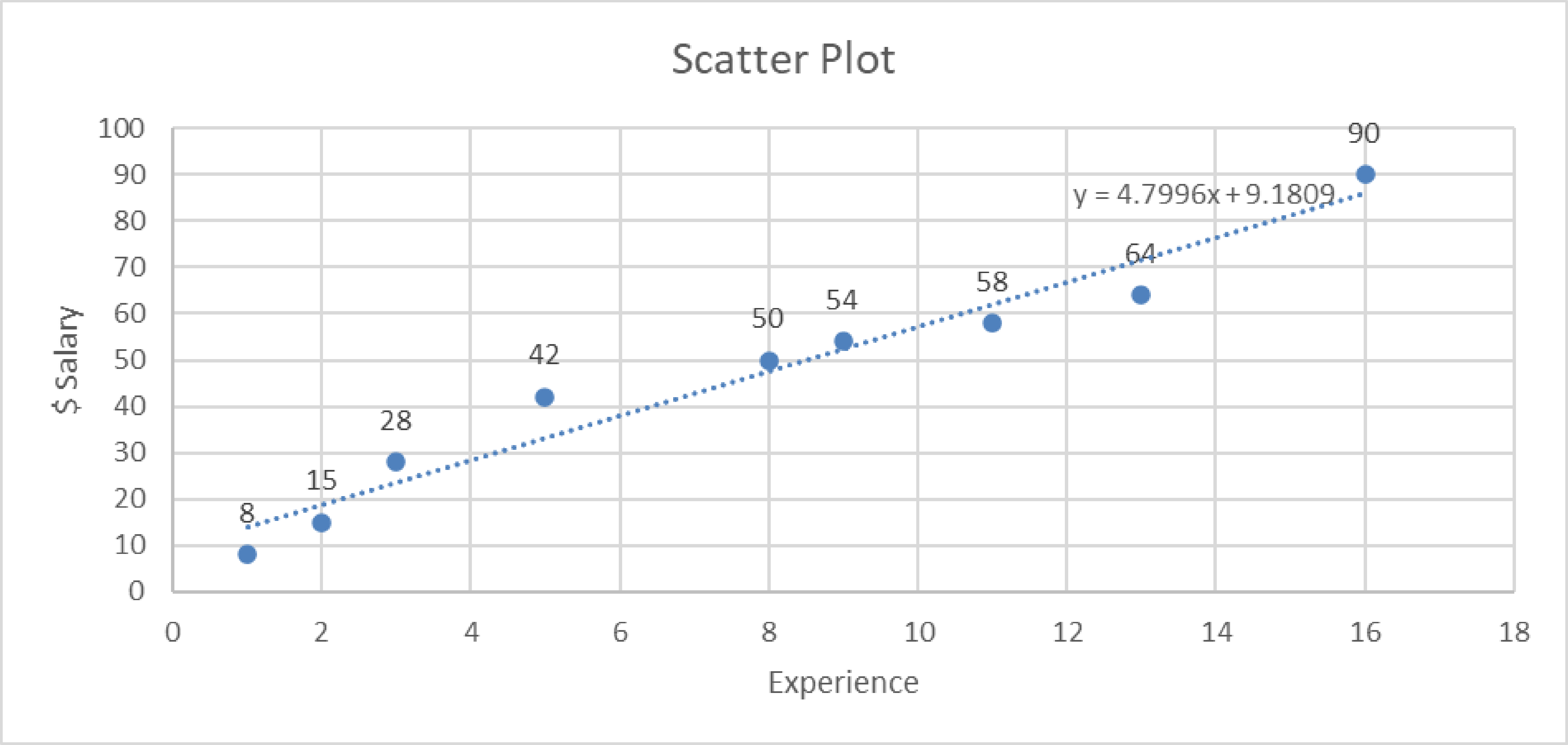
Hence, y = mx + b → 4.80x + 9.15

y = 4.80x + 9.15

Excel:

y = 4.79x + 9.18

BEST FIT LINE USING (OLS) METHOD



Excel:
$y = 4.79x + 9.18$

OLS Method:
$y = 4.80x + 9.15$

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SUM OF SQUARED ERRORS – PREDICTED OUTPUT

$$SSE = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

$$y_i = Dependent\ Variables\ (Salary)$$
$$\bar{y} = Average\ of\ Dependent\ Variables$$

The sum of squared errors SSE output is *245.38* for the predicted line.

Now Sum of Squared Error got reduced significantly from 5226.19 to 245.38.

		ŷ=mx+b		
x	y	ŷ=4.79x+9.18	y-ŷ	(y-ŷ)^2
Exp	$ Salary	Predicted (ŷ)	Error	Error^2
2	15	18.76	-3.76	14.14
3	28	23.55	4.45	19.80
5	42	33.13	8.87	78.68
13	64	71.45	-7.45	55.50
8	50	47.5	2.5	6.25
16	90	85.82	4.18	17.47
11	58	61.87	-3.87	14.98
1	8	13.97	-5.97	35.64
9	54	52.29	1.71	2.92
x̄=7.56	ȳ=45.444			245.38
				SSE

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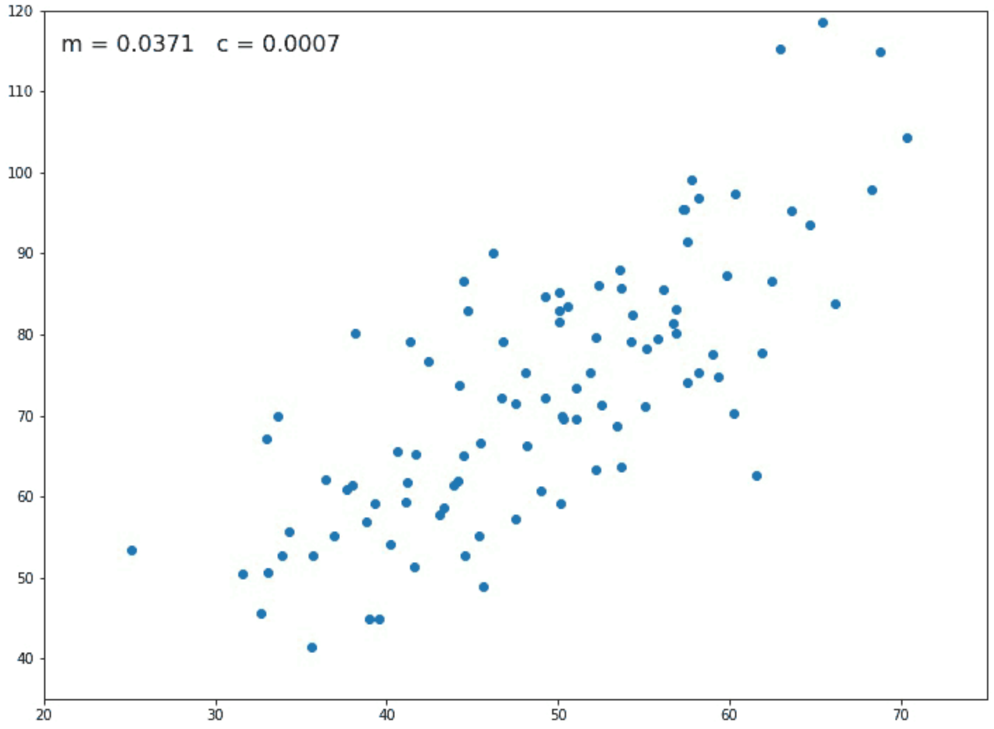
GRADIENT DESCENT

For Multi-variate dataset which contains a single independent variables set and multiple dependent variables sets we can also use a machine learning algorithm called “Gradient Descent” to get the best fit line.

FINDING THE BEST FIT LINE: USING GRADIENT DESCENT, MINIMIZING THE ERROR



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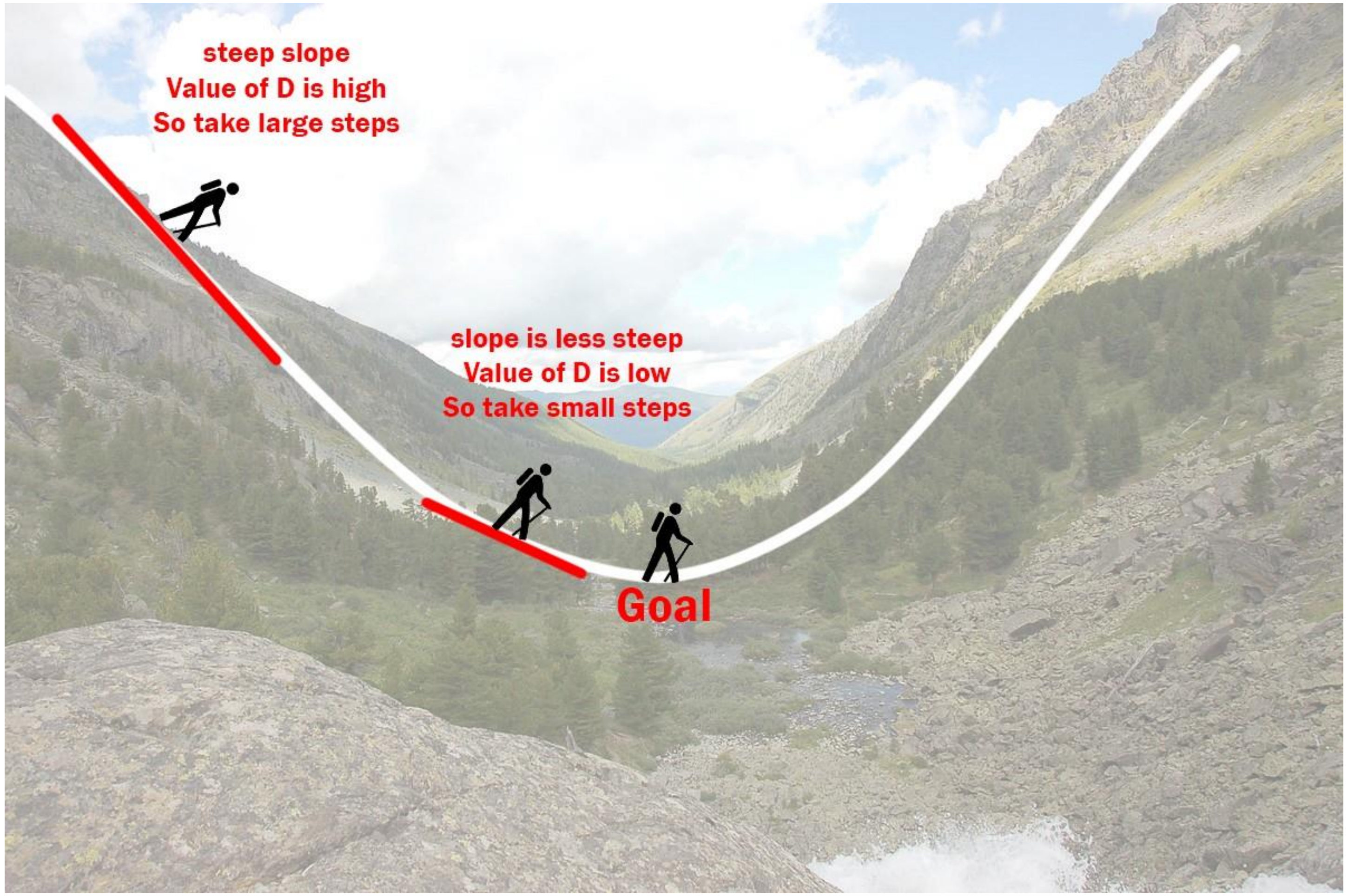
HOW GRADIENT DESCENT WORKS



R SQUARE

$$R^2 = 1 - \frac{SS_{RES}}{SS_{TOT}} = 1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \overline{y})^2}$$

(SSres) Residual sum of squared errors of our regression model

actual y value = 5 but we had **predicted y^** would be 6 then the **residual squared error** 1 and we would add that to the rest of the **residual squared errors** (SSres) for the model.

(SStot) Total sum of squared errors - This is comparing the **actual y** values to our **baseline model the mean $\overline{y}$.**

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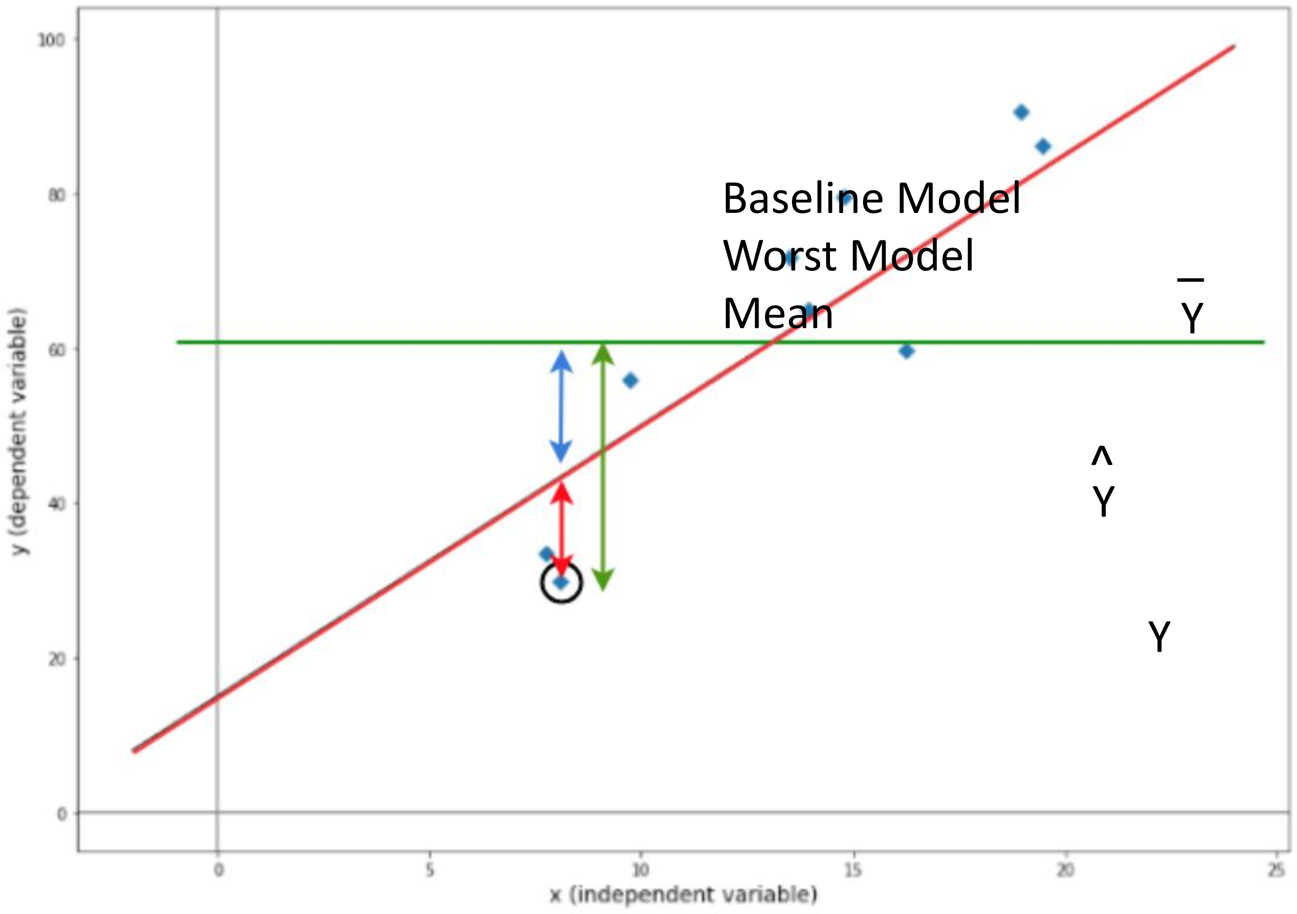
$$R^2 = 1 - \frac{SS_{RES}}{SS_{TOT}} = 1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \overline{y})^2}$$

$$R^2 = \frac{\sum (\text{Predicted Distance} - \text{Mean})^2}{\sum (\text{Actual Distance} - \text{Mean})^2}$$

$$R^2 = \frac{\Sigma \, (y_p - \overline{y})^2}{\Sigma \, (y - \overline{y})^2}$$

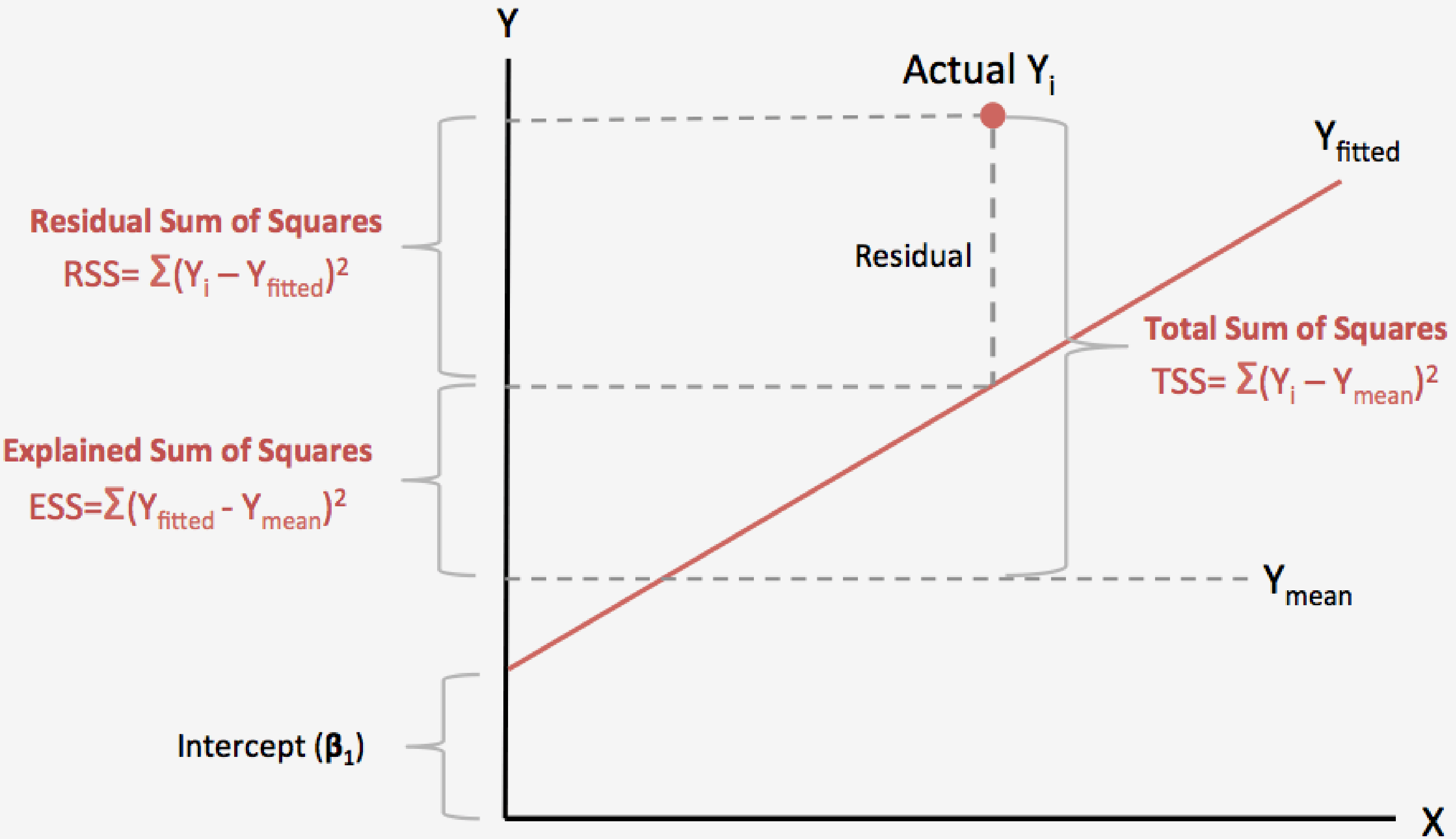


$\overline{Y}$ Mean

$\hat{Y}$ Predicted

Y Actual

R-Squared Explanation



$$R_{Sq} = 1 - \frac{RSS}{TSS}$$

GOODNESS OF FIT – R SQUARE (HOW GOOD IS OUR MODEL)

		ŷ=mx+b
x	y	ŷ=4.79x+9.18
Exp	$ Salary	Predicted (ŷ)
2	15	18.76
3	28	23.55
5	42	33.13
13	64	71.45
8	50	47.5
16	90	85.82
11	58	61.87
1	8	13.97
9	54	52.29
x̄=7.56	ȳ=45.44	

$$R^2 = \frac{\sum(\text{Predicted Distance} - \text{Mean})^2}{\sum(\text{Actual Distance} - \text{Mean})^2}$$

$$R^2 = \frac{\sum (y_p - \bar{y})^2}{\sum (y - \bar{y})^2}$$

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GOODNESS OF FIT – R SQUARE (HOW GOOD IS OUR MODEL)

		ŷ=mx+b				
x	y	ŷ=4.79x+9.18	y-ȳ	(y-ȳ)^2	ŷ-ȳ	(ŷ-ȳ)^2
Exp	$ Salary	Predicted (ŷ)				
2	15.00	18.76	15-45.44=-30.44	926.84	18.76-45.44=-26.68	712.04
3	28.00	23.55	28-45.44=-17.44	304.29	23.55-45.44=-21.89	479.35
5	42.00	33.13	42-45.44=-3.44	11.86	33.13-45.44=-12.31	151.63
13	64.00	71.45	64-45.44=18.56	344.33	71.45-45.44=26.01	676.31
8	50.00	47.5	50-45.44=4.56	20.76	50.00-45.44=2.06	4.23
16	90.00	85.82	90-45.44=44.56	1985.24	90.00-45.44=40.38	1630.22
11	58.00	61.87	58-45.44=12.56	157.65	61.87-45.44=16.43	269.81
1	8.00	13.97	8-45.44=-37.44	1402.05	13.97-45.44=-31.47	990.61
9	54.00	52.29	54-45.44=8.56	73.21	52.29-45.44=6.65	46.87
x̄=7.56	ȳ=45.44			5226.22		4961.07

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Predicted	(ŷ-ȳ)^2	4961.07
Actual	(y-ȳ)^2	5226.22

GOODNESS OF FIT – R SQUARE (HOW GOOD IS OUR MODEL)

200 150
15 15
100 101
→ 70%

10 7
100 70

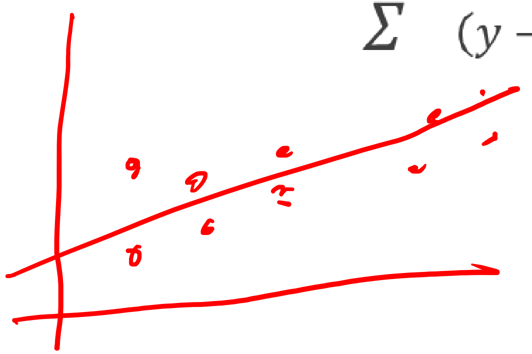
x_1 x_2 x_3
x_1

		ŷ=mx+b				
x	y	ŷ=4.79x+9.18	y-ȳ	(y-ȳ)^2	ŷ-ȳ	(ŷ-ȳ)^2
Exp	$ Salary	Predicted (ŷ)				
2	15.00	18.76	15-45.44=-30.44	926.84	18.76-45.44=-26.68	712.04
3	28.00	23.55	28-45.44=-17.44	304.29	23.55-45.44=-21.89	479.35
5	42.00	33.13	42-45.44=-3.44	11.86	33.13-45.44=-12.31	151.63
13	64.00	71.45	64-45.44=18.56	344.33	71.45-45.44=26.01	676.31
8	50.00	47.5	50-45.44=4.56	20.76	50.00-45.44=2.06	4.23
16	90.00	85.82	90-45.44=44.56	1985.24	90.00-45.44=40.38	1630.22
11	58.00	61.87	58-45.44=12.56	157.65	61.87-45.44=16.43	269.81
1	8.00	13.97	8-45.44=-37.44	1402.05	13.97-45.44=-31.47	990.61
9	54.00	52.29	54-45.44=8.56	73.21	52.29-45.44=6.65	46.87
x̄=7.56	ȳ=45.44			5226.22		4961.07

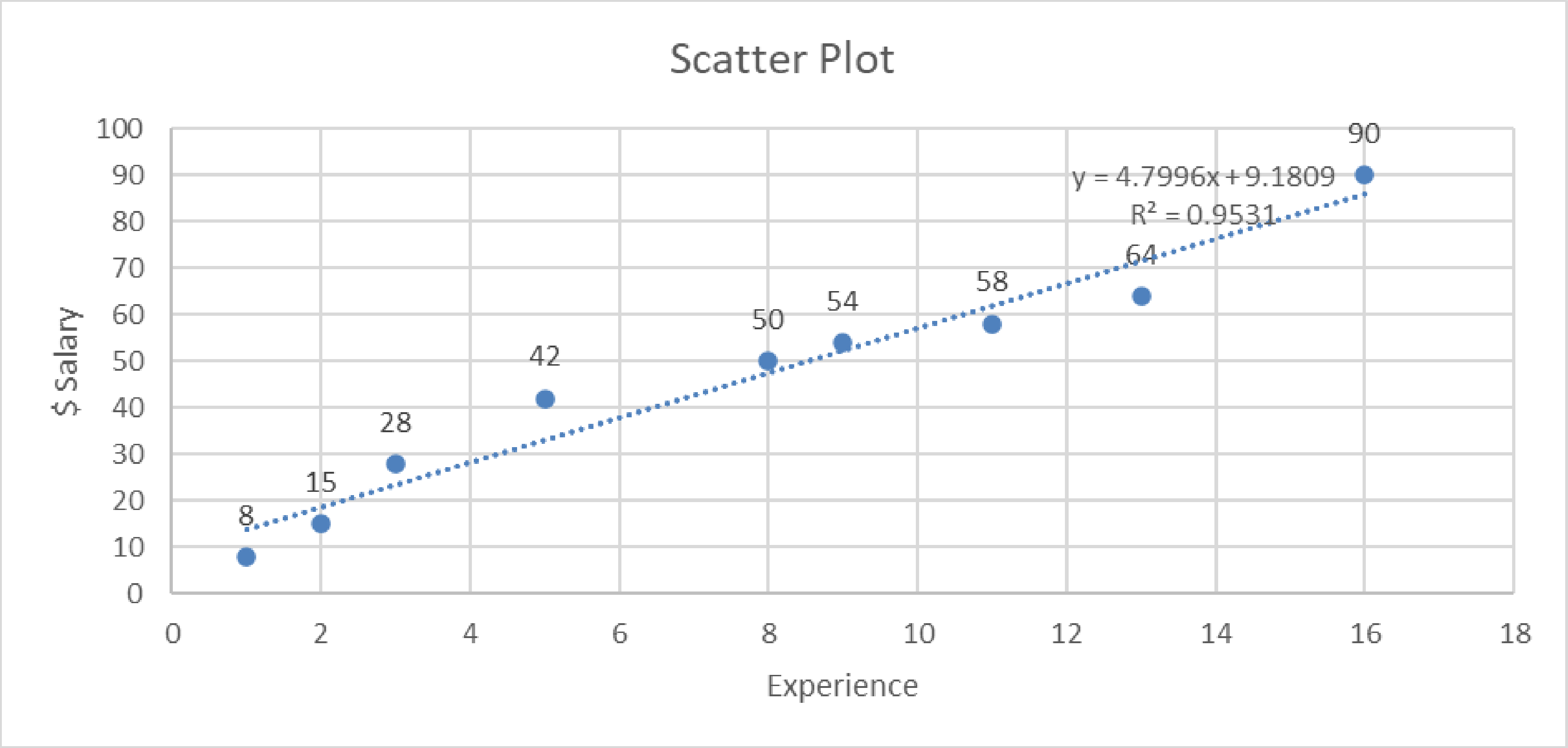
$$R^2 = \frac{\sum(\text{Predicted Distance} - \text{Mean})^2}{\sum(\text{Actual Distance} - \text{Mean})^2}$$

$$R^2 = \frac{\sum (y_p - \bar{y})^2}{\sum (y - \bar{y})^2}$$

Predicted	(ŷ-ȳ)^2	4961.07
Actual	(y-ȳ)^2	5226.22
	R Square	0.949265435



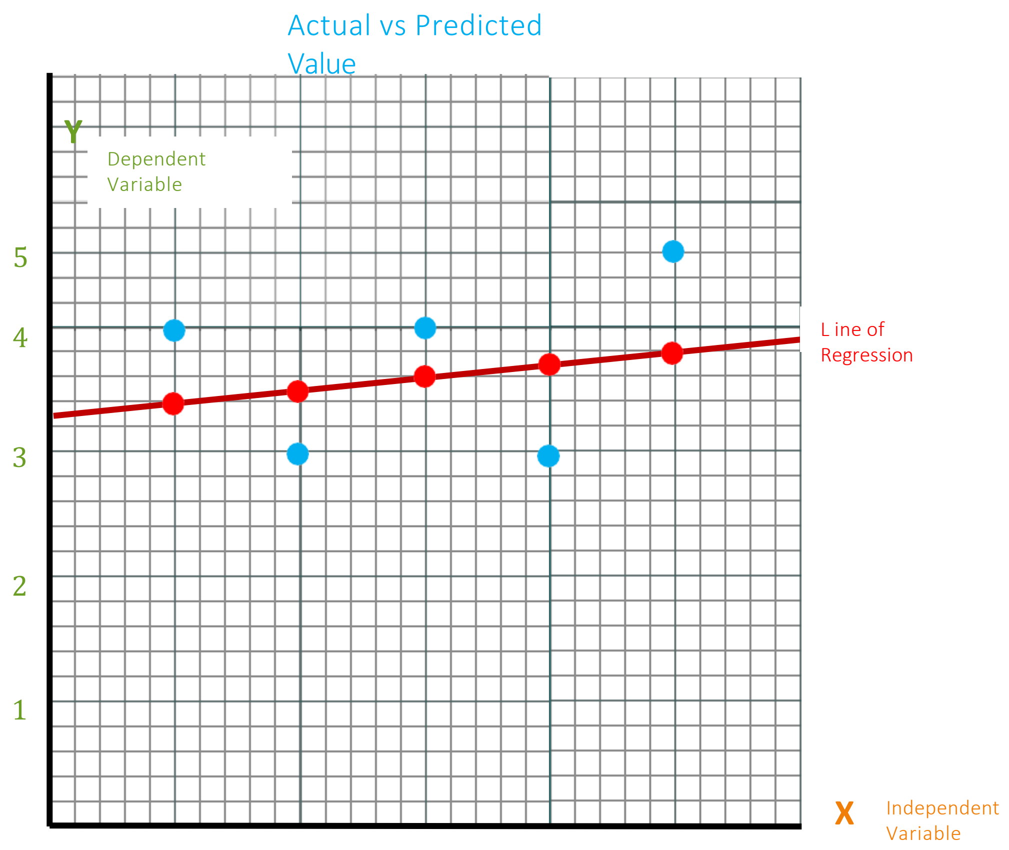
GOODNESS OF FIT – R SQUARE (HOW GOOD IS OUR MODEL)



Predicted	(ŷ-ȳ)^2	4961.07
Actual	(y-ȳ)^2	5226.22
	R Square	0.949265435

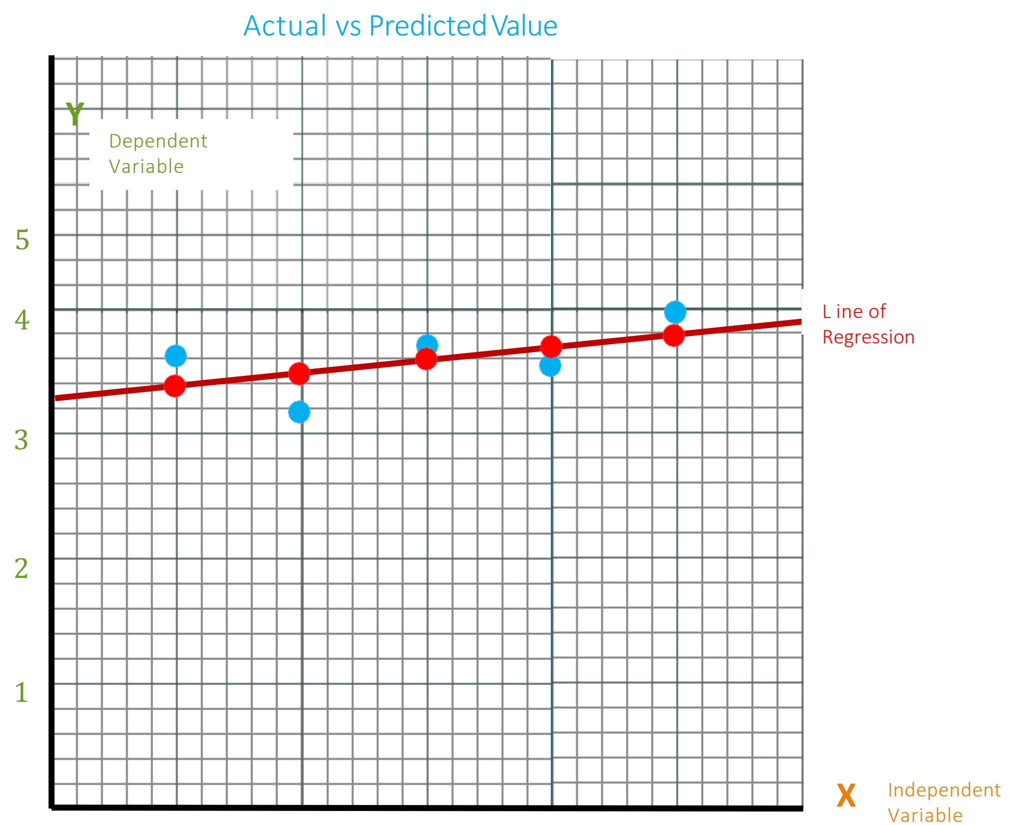
Do It Skills
do it. enjoy it.

EXAMPLES OF BETTER REGRESSION – BETTER R SQUARE



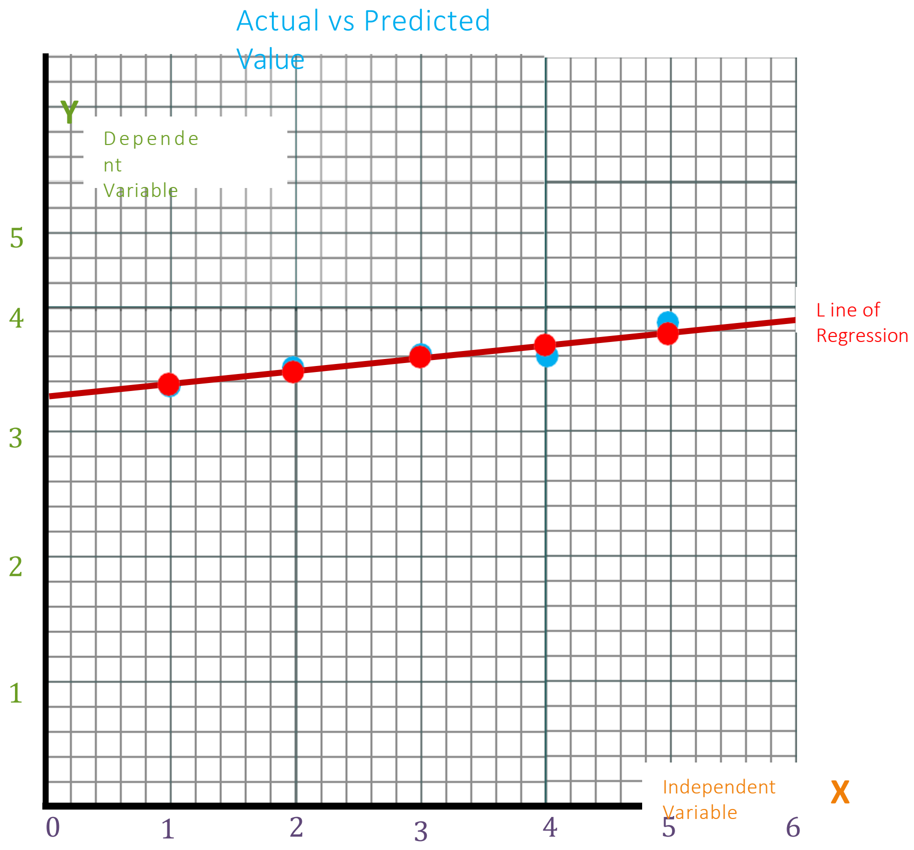
$R^2 \approx .19$

EXAMPLES OF BETTER REGRESSION – BETTER R SQUARE



$$R^2 \approx 0.9$$

EXAMPLES OF BETTER REGRESSION – BETTER R SQUARE



$$R^2 \approx 1$$