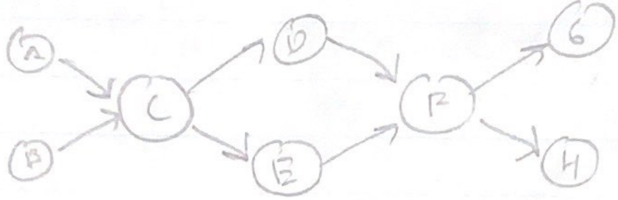
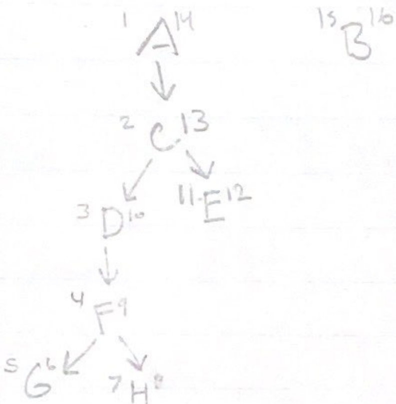
Homework 7 - 3.3, & 3.4

3)



a)



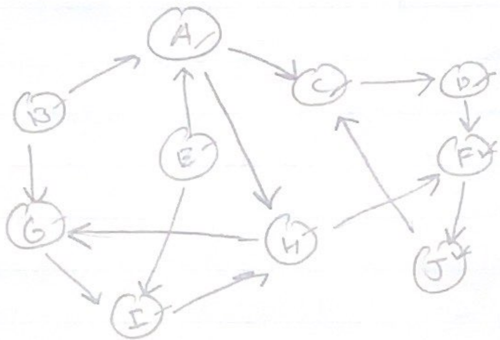
b) Sources: A, B
Sinks: G, H

c) $\underbrace{B, A}_{\text{could change}}, C, \underbrace{E, D}_{\text{could change}}, F, \underbrace{H, G}_{\text{could change}}$

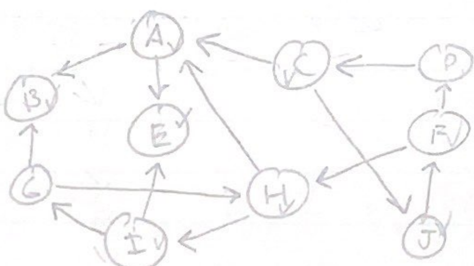
d) With the traversal by alphabetical order rule, there is only one ordering. If we discount this rule, we have 3 places where the ordering could change; B&A, E&D, and H&G. If we change the ordering of these nodes we get 2 x 2 x 2 possible orderings, or 8 possible topographical orders of this graph

3.4 i)

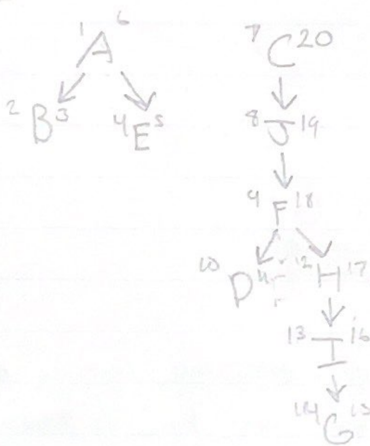
G



G^R



$DFS(G^R) =$



Post Order

~~C, J, F, H, I, G, D~~, A, E, B

DFS(G, post)

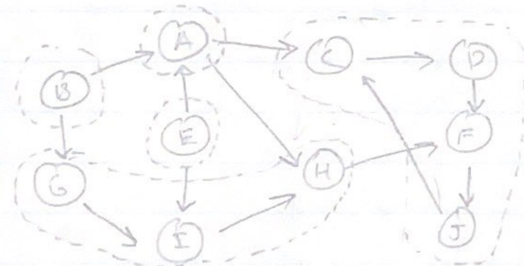
${}^{1}C^{8}$ ${}^{9}H^{14}$ ${}^{15}A^{16}$ ${}^{17}E^{18}$ ${}^{19}B^{20}$

$$\begin{array}{ccc} {}^{1}C^{8} & & {}^{9}H^{14} \\ \downarrow & & \downarrow \\ {}^{2}D^{7} & & {}^{10}G^{13} \\ \downarrow & & \downarrow \\ {}^{3}F^{6} & & {}^{11}I^{12} \\ \downarrow & & \\ {}^{4}J^{5} & & \end{array}$$

a) SSC = {C, D, F, J}, {H, G, I}, {A}, {E}, {B}

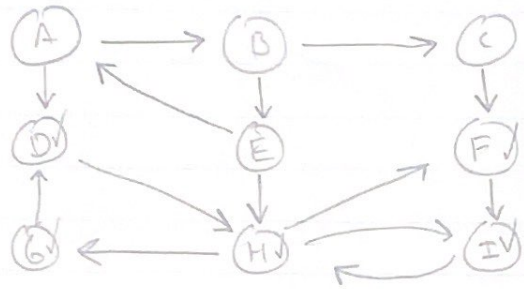
b) Sources: {B}, {E}
Sinks: {C, D, F, J}

c)

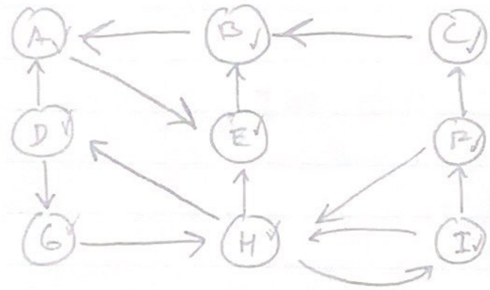


d) The minimum # of edges would be <u>2.</u> One edge to connect a source & the only sink SCC, then one other to connect the remaining source to the other source. With <u>2 edges</u>, we can eliminate all sources & sinks & make the entire graph strongly connected.

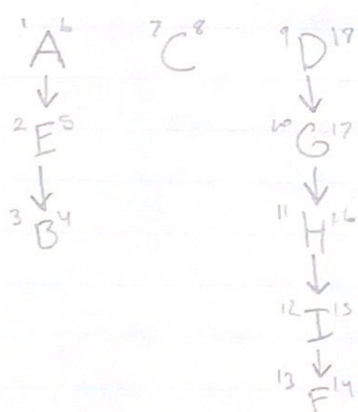
3.4 ii) G



G^R



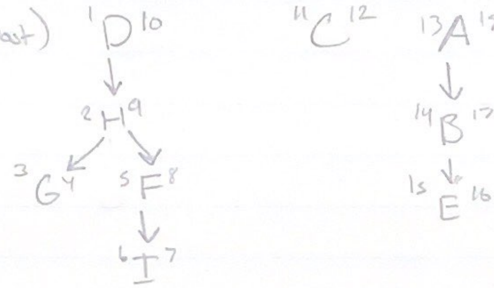
DFS(G^R)



Post Order

~~D~~, ~~G~~, ~~H~~, ~~I~~, ~~F~~, ~~C~~, A, E, B

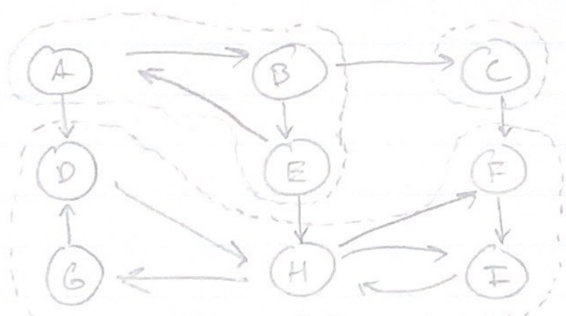
DFS(G, post)



a) SCC = {D, H, G, F, I}, {C}, {A, B, E}

b) Sources: {A, B, E}
Sinks: {D, H, G, F, I}

c)



d) With one edge connecting the sink SSC to the source SSC, the entire graph will become connected as there will be no sink or source SSC's left in it.