HW 16 - 7.1, 7.3

7.1) Maximize $5x+3y$

$5x-2y \geq 0$

$x+y \leq 7$

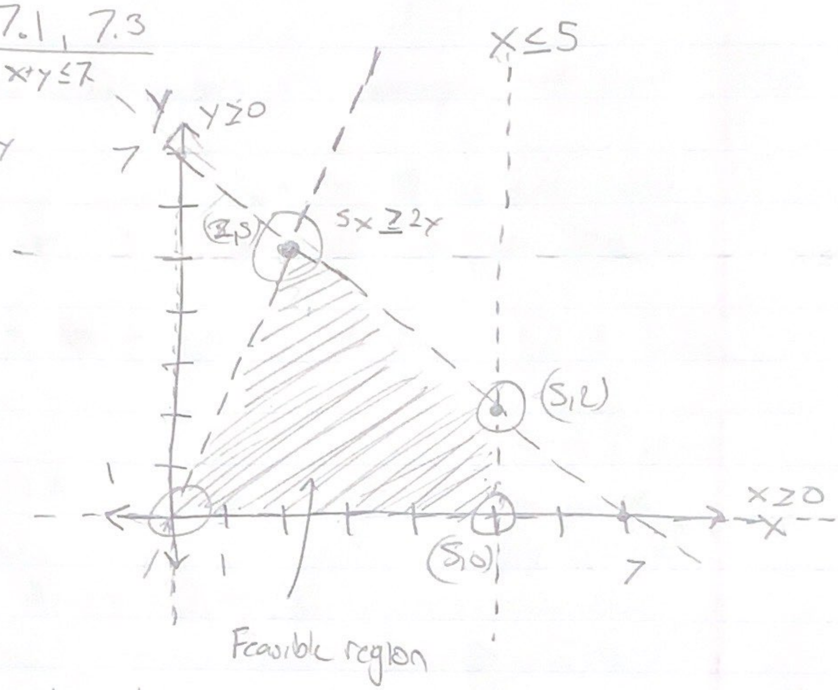
$x \leq 5$

$x \geq 0$

$y \geq 0$

$x \leq 7-y$

$y \leq 7-x$



$5x \geq 2y$

(2,5), (5,2) Border values

Max $5x+3y$

$(0,0) = 5(0)+3(0) = 0$

$(5,0) = 5(5)+3(0) = 25$

$\boxed{(5,2) = 5(5)+3(2) = 31}$ ⟸ $5x+3y$ is maxed at $x=5$, $y=2$

$(2,5) = 5(2)+3(5) = 25$

7.3) Cargo plane carries max 100 tons, max volume 60 cubic meters.

x_1: Material 1 = 2 tons, 40 cubic meters available $1,000/cubic meter
x_2: Material 2 = 1 ton, 30 is max available, $1,200/cubic meter
x_3: Material 3 = 3 ton/cubic meter max available = 20 cubic meter, $12,000/cubic meter

Maximize $1{,}000x_1 + 1200x_2 + 12{,}000x_3$

$x_1 \geq 0$
$x_2 \geq 0$
$x_3 \geq 0$
$2x_1 + x_2 + 3x_3 \leq 100$
$x_1 \leq 40$
$x_2 \leq 30$
$x_3 \leq 20$
$x_1 + x_2 + x_3 \leq 60$

Max (from solver)
$x_1 = 5$
$x_2 = 30$
$x_3 = 20$

Simplex Method Calculator

The simplex method is universal. It allows you to solve any linear programming problems.
The solution by the simplex method is not as difficult as it might seem at first glance.
This calculator only finds a general solution when the solution is a straight line segment.
You can solve your problem or see examples of solutions that this calculator has made.

Clear ► ◄ ▲

Enter integers or ordinary fractions. For example: 12, -3/4.

Find the maximum value of the function

F = 1000 x_1 + 1200 x_2 + 12000 x_3

subject to the constraints:

2	x_1 +	1	x_2 +	3	x_3	≤	100
1	x_1 +	0	x_2 +	0	x_3	≤	40
0	x_1 +	1	x_2 +	0	x_3	≤	30
0	x_1 +	0	x_2 +	1	x_3	≤	20
1	x_1 +	1	x_2 +	1	x_3	≤	60

$x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0$

Solve

Problem:

Find the maximum value of the function

$F = 1000x_1 + 1200x_2 + 12000x_3$

subject to the constraints:

$$\begin{cases} 2x_1 + x_2 + 3x_3 \leq 100 \\ x_1 \leq 40 \\ x_2 \leq 30 \\ x_3 \leq 20 \\ x_1 + x_2 + x_3 \leq 60 \end{cases}$$

$x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0$

Solution:

1.This is a necessary condition for solving the problem:
the numbers on the right parts of the constraint system must be non-negative.
This condition is done.

2. This is a necessary condition for solving the problem:
all constraints must be equations.

$$2x_1 + x_2 + 3x_3 \leq 100$$
$$x_1 \leq 40$$
$$x_2 \leq 30$$

$$\begin{cases} x_3 \le 20 \\ x_1 + x_2 + x_3 \le 60 \end{cases}$$

$$\begin{cases} 2x_1 + x_2 + 3x_3 + S_1 = 100 \\ x_1 + S_2 = 40 \\ x_2 + S_3 = 30 \\ x_3 + S_4 = 20 \\ x_1 + x_2 + x_3 + S_5 = 60 \end{cases}$$

$S_1 \ge 0$, $S_2 \ge 0$, $S_3 \ge 0$, $S_4 \ge 0$, $S_5 \ge 0$. The entered variables S_1, S_2, S_3, S_4, S_5, are called slack variables.

3. Finding the initial basis and the value of the function F which corresponds to the found initial basis.

What is a basis?
A variable is called a basic variable for an equation if it enters into this equation with a coefficient of one and does not enter into other equations system (provided that there is a non-negative number on the right side of the equation).
If each equation has a basic variable, then they say that the system has a basis.
Variables that are not basic are called non-basic.

What is the idea of the simplex method?
Each basis is corresponded to one function value. One of them is the maximum value of the function F.
We will move from one basis to another.
The next basis will be chosen in such a way that the value of the function F will be no less than we have now.
Obviously, the number of possible bases for any problem is not very large.
So sooner or later the answer will be received.

How will we move from one basis to another?
It is more convenient to record the solution in the form of tables. Each row of the table is equivalent to a system equation. The highlighted row consists of the coefficients of the function (see the table below). This allows us not to rewrite variables every time. It saves time.
In the highlighted row, select a maximum positive coefficient (we can select any positive coefficient).
This is necessary in order to get a value of the function F no less than we have.
Column is selected.
For the positive coefficients of the selected column, we count the coefficient Θ and select the minimum value.
This is necessary in order to get non-negative numbers in the right part of the equations after moving to another basis.
Row is selected.
An element is found that will be basic. Next, we will need to calculate.

Does our system have a basis?

$$\begin{cases} 2x_1 + x_2 + 3x_3 + S_1 = 100 \\ x_1 + S_2 = 40 \\ x_2 + S_3 = 30 \\ x_3 + S_4 = 20 \\ x_1 + x_2 + x_3 + S_5 = 60 \end{cases}$$

There is a basis in our system. We can begin to solve our problem.

$F = 1000x_1 + 1200x_2 + 12000x_3$

Non-basic variables are zero. In the mind, we can find the values of the basic variables. (see system)
Function F contains only non-basic variables. Therefore, the value of the function F for this basis can be found in the mind.

$x_1 = 0 \quad x_2 = 0 \quad x_3 = 0$
$S_1 = 100 \quad S_2 = 40 \quad S_3 = 30 \quad S_4 = 20 \quad S_5 = 60$ $\Rightarrow F = 0$

The initial basis was found. The value of the function F corresponding to the initial basis was found.

4. Finding a maximum value of the function F.

Step №1

x_1	x_2	x_3	S_1	S_2	S_3	S_4	S_5	const.	Θ
2	1	3	1	0	0	0	0	100	100 : 3 ≈ 33,333
1	0	0	0	1	0	0	0	40	
0	1	0	0	0	1	0	0	30	

0	0	1	0	0	0	1	0	20	20 : 1 = 20
1	1	1	0	0	0	0	1	60	60 : 1 = 60
1000	1200	12000	0	0	0	0	0	F - 0	
2	1	0	1	0	0	-3	0	40	
1	0	0	0	1	0	0	0	40	
0	1	0	0	0	1	0	0	30	
0	0	1	0	0	0	1	0	20	
1	1	0	0	0	0	-1	1	40	
1000	1200	0	0	0	0	-12000	0	F - 240000	

Non-basic variables are zero. In the mind, we can find the values of the basic variables. (see table)
Function F contains only non-basic variables. Therefore, the value of the function F for this basis can be found in the mind. (see the highlighted row in the table)

$x_1 = 0 \quad x_2 = 0 \quad S_4 = 0$

$x_3 = 20 \quad S_1 = 40 \quad S_2 = 40 \quad S_3 = 30 \quad S_5 = 40$ => F - 240000 = 0 => F = 240000

Step №2

x_1	x_2	x_3	S_1	S_2	S_3	S_4	S_5	const.	Θ
2	1	0	1	0	0	-3	0	40	40 : 1 = 40
1	0	0	0	1	0	0	0	40	
0	1	0	0	0	1	0	0	30	30 : 1 = 30
0	0	1	0	0	0	1	0	20	
1	1	0	0	0	0	-1	1	40	40 : 1 = 40
1000	1200	0	0	0	0	-12000	0	F - 240000	
2	0	0	1	0	-1	-3	0	10	
1	0	0	0	1	0	0	0	40	
0	1	0	0	0	1	0	0	30	
0	0	1	0	0	0	1	0	20	
1	0	0	0	0	-1	-1	1	10	
1000	0	0	0	0	-1200	-12000	0	F - 276000	

Non-basic variables are zero. In the mind, we can find the values of the basic variables. (see table)
Function F contains only non-basic variables. Therefore, the value of the function F for this basis can be found in the mind. (see the highlighted row in the table)

$x_1 = 0 \quad S_3 = 0 \quad S_4 = 0$

$x_2 = 30 \quad x_3 = 20 \quad S_1 = 10 \quad S_2 = 40 \quad S_5 = 10$ => F - 276000 = 0 => F = 276000

Step №3

x_1	x_2	x_3	S_1	S_2	S_3	S_4	S_5	const.	Θ
2	0	0	1	0	-1	-3	0	10	10 : 2 = 5
1	0	0	0	1	0	0	0	40	40 : 1 = 40
0	1	0	0	0	1	0	0	30	
0	0	1	0	0	0	1	0	20	
1	0	0	0	0	-1	-1	1	10	10 : 1 = 10
1000	0	0	0	0	-1200	-12000	0	F - 276000	
1	0	0	1/2	0	-1/2	-3/2	0	5	
1	0	0	0	1	0	0	0	40	
0	1	0	0	0	1	0	0	30	
0	0	1	0	0	0	1	0	20	
1	0	0	0	0	-1	-1	1	10	
1000	0	0	0	0	-1200	-12000	0	F - 276000	
1	0	0	1/2	0	-1/2	-3/2	0	5	
0	0	0	-1/2	1	1/2	3/2	0	35	
0	1	0	0	0	1	0	0	30	
0	0	1	0	0	0	1	0	20	
0	0	0	-1/2	0	-1/2	1/2	1	5	
0	0	0	-500	0	-700	-10500	0	F - 281000	

Non-basic variables are zero. In the mind, we can find the values of the basic variables. (see table)
Function F contains only non-basic variables. Therefore, the value of the function F for this basis can be found in the mind. (see the highlighted row in the table)

$S_1 = 0$ $S_3 = 0$ $S_4 = 0$
$x_1 = 5$ $x_2 = 30$ $x_3 = 20$ $S_2 = 35$ $S_5 = 5$ => F - 281000 = 0 => F = 281000

There are not any positive coefficients in the highlighted row. Therefore, the maximum value of the function F was found.

Result:

$x_1 = 5$ $x_2 = 30$ $x_3 = 20$

$F_{max} = 281000$

Up What do you think about this?

If you have any comments, please write to siteReshmat@yandex.ru