$f = O(g) \rightarrow f \leq g$

$f = \Omega(g) \;/\; g = O(f) \rightarrow (f \geq g)$

$f = \Theta(g) \;/\; f = O(g) \;\&\; f = \Omega(g) \rightarrow (f = g)$

CS 312 - HW1

1) a) $f(n) = n - 100 \quad g(n) = n - 200$

$$\lim_{n\to\infty} \frac{n-100}{n-200} = \frac{\infty}{\infty} \xrightarrow{\text{L'Hôpital's Rule}} \lim_{n\to\infty} \frac{1}{1} = 1 \in R^+ \rightarrow \boxed{f(n) = \Theta(g(n))}$$

b) $f(n) = n^{1/2} \quad g(n) = n^{2/3}$

$$\lim_{n\to\infty} \frac{n^{1/2}}{n^{2/3}} = \lim_{n\to\infty} n^{-1/6} = \lim_{n\to\infty} \frac{1}{n^{1/6}} = 0 \rightarrow \boxed{f(n) = O(g(n))}$$

c) $f(n) = 100n + \log n \quad g(n) = n + (\log n)^2$

$$\lim_{n\to\infty} \frac{100n + \log n}{n + (\log n)^2} = \lim_{n\to\infty} \frac{100 + \frac{1}{n}}{1 + 2\log n(\frac{1}{n})} = \lim_{n\to\infty} \frac{100 + \frac{1}{\infty}}{1 + \frac{2\log n}{n}} = \lim_{n\to\infty} \frac{100}{1} = 100 \in R^+$$

$\boxed{f(n) = \Theta(g(n))}$

d) $f(n) = n\log n \quad g(n) = 10n \log 10n$

$$\lim_{n\to\infty} \frac{n\log n}{10n\log 10} = \frac{1}{10}\lim_{n\to\infty} \frac{\log n}{\log 10n} = \frac{1}{10}\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{n}} = \frac{1}{10}(1) = \frac{1}{10} \in R^+ \rightarrow \boxed{f(n) = \Theta(g(n))}$$

e) $f(n) = \log 2n \quad g(n) = \log 3n$

$$\lim_{n\to\infty} \frac{\log 2n}{\log 3n} = \lim_{n\to\infty} \left(\frac{\frac{1}{n}}{\frac{1}{n}}\right) = \lim_{n\to\infty} 1 = 1 \in R^+ \rightarrow \boxed{f(n) = \Theta(g(n))}$$

f) $f(n) = 10\log n \quad g(n) = n\log^2(n)$

$$\lim_{n\to\infty} \frac{10\log n}{n\log^2(n)} = \lim_{n\to\infty} \frac{0(\log n) + 10(\frac{1}{n})}{1\log^2(n) + n2\log n} = \lim_{n\to\infty} \frac{10}{\log^2(n) + 2\log n} = \frac{0}{\infty} = 0$$

$(\log^2(x))' = 2\log(x)\cdot\frac{1}{x}(1) = \frac{2\log(x)}{x}$

$\boxed{f(n) = O(g(n))}$

Closed form $g(n) = \frac{1-c^{n+1}}{1-c} = \frac{c^{n+1}-1}{c-1}$

m) $f(n) = n2^n$ $g(n) = 3^n$

Multiplicative constants can be omitted

When comparing 2^n & 3^n, 3^n dominates 2^n (a^n dominates b^n when $a > b$)

so $f(n) = O(g(n))$

2)

a. $c<1$ $\Theta(1)$

$\lim_{n\to\infty} \frac{1}{\frac{1-c^{n+1}}{1-c}} = \lim_{n\to\infty} \frac{1-c}{1-c^{n+1}} = 1-c \lim_{n\to\infty} \frac{1}{1-c^{n+1}} =$ (1-c is constant)

$1-c \frac{\lim_{n\to\infty} 1}{\lim_{n\to\infty} (1-c^{n+1})} = (1-c)\frac{1}{1} =$ $1-c$ - which is a constant value, so $g(n) = \Theta(1)$ because $(1-c) \in R^+$

when $c<1$ $\lim_{n\to\infty} c^{n+1}$ = very small = 0

b. $c=1$ $\Theta(n)$ $g(n) = 1 + c + c^2 \ldots + c^n$; when $c=1$, $g(n) = 1+1+1^2 \ldots 1^n$ or $g(n) = n$

$\lim_{n\to\infty} \frac{n}{n} = \lim_{n\to\infty} \frac{1}{1} =$ 1, because $1 \in R^+$, we can say $g(n) = \Theta(n)$

c. $c>1$ $\Theta(c^n)$ $g(n) = \frac{c^{n+1}-1}{c-1}$

Use L'Hospital's Rule

$\lim_{n\to\infty} \frac{c^n}{\frac{c^{n+1}-1}{c-1}} = \lim_{n\to\infty} \frac{c^n(c-1)}{c^{n+1}-1} = \frac{\infty}{\infty} = \lim_{n\to\infty} \frac{\ln(c)c^n(c-1)}{\ln(c)c^{n+1}}$ ← Move constant out from the limit

$(c-1)\lim_{n\to\infty} \frac{c^n}{c^{n+1}} = (c-1)\lim_{n\to\infty} c^{n-(n+1)} = (c-1)\lim_{n\to\infty} c^{-1} = (c-1)\lim_{n\to\infty} \frac{1}{c} = (c-1)\frac{1}{c}$

$(c-1)\frac{1}{c}$ is a constant value & $\in R^+$ so we can say $g(n) = \Theta(c^n)$

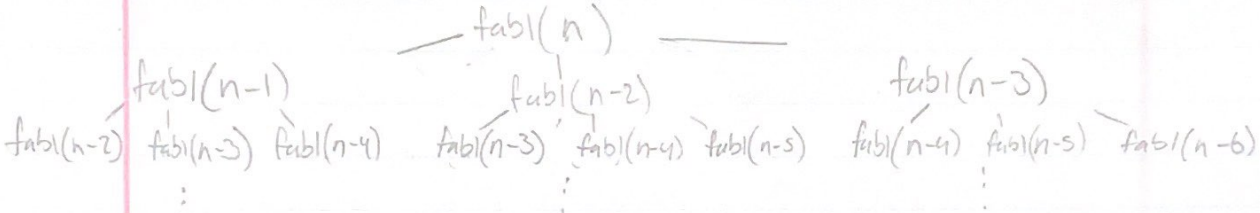
I tried my best but I'm sure there's logical/calculation issues here

HW1 (Cont.)

3 a)

```
fabl(n)
  if n < 2 then
    return 1
  return fabl(n-1) + fabl(n-2) * fabl(n-3)
```

Exponential Fab



For n levels of function calls, we get 3^n leaves worth of function calls in our tree, so we can say fabl(n) is an exponential function.

b)

```
fab2(n)
  if n < 2 then
    return 1
  create array fab[0 to n]
  f[0] f[1] f[2] = 1
  for i = 3 to n
    fab[i] = fab[n-1] + (fab[n-2] * fab[n-3])
  return fab[n]
```

Linear Fab

loop runs n-3 times performs 1 add & 1 multiply per loop

fab2(n) will perform (n-3) adds & (n-3) multiplications because the for loop runs (n-3) times. This can also be written as 2(n-3) operations total during runtime.