HW 17 - 7.11, 7.13

7.11) Find the dual to

$\max x+y$

$2x+y \le 3$

$x+3y \le 5$

$x, y \ge 0$

$$\max \begin{pmatrix} 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \le \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \ge \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\min \begin{pmatrix} z_1 & z_2 \end{pmatrix} \begin{pmatrix} 3 & 5 \end{pmatrix}$$

$$\begin{pmatrix} z_1 & z_2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \ge \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow$$

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \ge \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\min 3z_1 + 5z_2$

$2z_1 + z_2 \ge 1$

$z_1 + 3z_2 \ge 1$

$z_1, z_2 \ge 0$

Primal

$\max(x+y)$

$2x+y \le 3$

$y = 3-2x$

$x+3(3-2x) = 5$

$x+9-6x = 5$

$-5x = -4$

$x = \frac{4}{5}$

$2(\frac{4}{5}) + y = 3 \quad \frac{15}{5}$

$\frac{8}{5} + y = 3$

$y = \frac{7}{5}$

$\max(\frac{4}{5}, \frac{7}{5})$

Dual

$\min(3z_1 + 5z_2)$

$2z_1 + z_2 = 1$

$z_2 = 1-2z_1$

$z_1 + 3(1-2z_1) = 1$

$z_1 + 3 - 6z_1 = 1$

$-5z_1 = -2$

$z_1 = \frac{2}{5}$

$2(\frac{2}{5}) + z_2 = 1$

$\frac{4}{5} + z_2 = 1$

$z_2 = \frac{1}{5}$

$\min(\frac{2}{5}, \frac{1}{5})$

Check

$\frac{4}{5} + \frac{7}{5} = 3(\frac{2}{5}) + 5(\frac{1}{5})$

$\frac{11}{5} = \frac{6}{5} + \frac{5}{5}$

$$\boxed{\frac{11}{5} = \frac{11}{5} \checkmark}$$

$$\boxed{\begin{matrix} x = 4/5 & z_1 = \frac{2}{5} \\ y = 7/5 & z_2 = \frac{1}{5} \end{matrix}}$$

7.13) Matching pennies

a) $G =$

			C	
		h	t	
R	h	1	-1	
	t	-1	1	

Row tries to max rewards
Column tries to min penalty

b) $V(G) = \sum G_{ij} \cdot \text{prob}$

$= \sum G_{ij} p_i q_j$

$= 1(\frac{1}{2})(\frac{1}{2}) + (-1)(\frac{1}{2})(\frac{1}{2}) + 1(\frac{1}{2})(\frac{1}{2}) + (-1)(\frac{1}{2})(\frac{1}{2})$

$= \frac{1}{4} - \frac{1}{4} + \frac{1}{4} - \frac{1}{4} = 0$

The value of matching pennies is 0, a zero-sum game.

Because the value is 0 & the game is symmetrical, both players are best off with a completely random strategy. Doing so forces the expected pay off of zero on the opponent, Choosing otherwise allows for a potential positive / negative pay off (depending on which side), which is sub-optimal