Reviewer Report

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Manuscript Title: Trajectory-free approximation of phase space structures using the trajectory divergence rate

In this paper the authors introduce two Eulerian (instantaneous) scalar-valued functions, the trajectory divergence rate (TDR) and the trajectory divergence ratio to reveal some of the underlying geometric characteristics of a given flow in phase space. These techniques, which meaasure the local instantaneous attraction or repulsion in the normal direction of adjacent tajectories, are shown to provide useful information when applied to several examples, where slow manifolds and hyperbolic Lagrangian coherent structures are identified, and the local stability of limit cycles is addressed.

The manuscript is well structured and written, and all the details of the TDR method are nicely introduced. Moreover, the examples chosen by the authors clearly illustrate the potential of these computationally efficient diagnostics to investigate phase space structures. Another aspect I find interesting about this work is that it provides other users with a software package ManifoldID, written in Python and available at GitHub, to test the TDR method for some example flows. For all these reasons I recommend the publication of this paper in Nonlinear Dynamics after the authors deal with the following issues:

1 - There is a typo in the first paragraph of the Introduction section. It says '...of a resuting vector fields' and it should be '...of a resulting vector field'.

2 - At the end of the first paragraph of the Introduction section, applications where the computation of LCSs are of interest are cited. Reference [46] appears for describing applications to chemical reactions, comet distributions and structural mechanics. Is this correct?

3 – In the fourth paragraph of the Introduction section, several trajectory-based methods to identify coherent structures in phase space are presented by the authors. Given that the athors discuss the objectivity issue correctly, citing the work on Lagrangian Descriptors, which is a trajectory-based method, in my opinion, this method should also be cited in the paragraph where trajectory-based methods are mentioned.

4 – In the Main Result subsection the authors reference Eq. (17), but in fact that equation is the first equation that appears in this subsection. Shouldn't it be Eq. (1)? Moreover, they do not define matrix R, which is the 90º counterclockwise rotation matrix until further in the text. This should be defined at the same time it is used for the first time.

5 – In the third paragraph of the Main Result subsection the athours mention that the TDR diagnistic is easy to compute since the only data needed for that purpose is the velocity field and its gradient. Would the TDR method suffer when dealing with problems where the domain is discretized with an unstructured mesh, so that the calculation of gradient is more complicated?

6 – In the second paragraph of subsection 2.2 the dagger operator for matrix transpose is defined, but it has already been introduced in subsection 1.1.

7 – In the sentence after Eq. (7), the word tensor is missing after 'Cauchy-Green'.

8 – The first equation of Eq. (8) should be removed, or specified for k = 1.

9 – There is a typo in the next sentence to Eq. (8). It should say 'Rivlin-Ericksen tensor'.

10 – I do not understand the first sentence after Eq. (10).

11 – In Figure 2, the projection rho dT is marked in brown, which makes it difficult to distinguish. Could you change its color to improve the figure.

12 – In Eq. (13) k = 1 and k >=2 should be removed.

13 – In Eq. (14) remove the dependence on x0.

14 – In the sentence after Eq. (22) there is a typo. It should say 'the divergence ratio is dependent only...'.

15 – For the figures where the athors draw the flow streamlines, the gray color is very weak and makes visualization difficult. I recommend enhancing the color intensity of streamlines or change their color.

16 – I do not fully understand the point that the authors want to make in the last paragraph of Example 1. Could you clarify by rewriting the paragraph?

17 - I do not fully understand the point that the authors want to make in the last paragraph of subsection 3.3.2. Could you clarify by rewriting the paragraph or linking it to other parts of the discussion?

18 – In Example 2, when the authors say that the equations reduce to (29), why not put the system just after this statement?

19 - In the last paragraph of Example 2 the authors say 'The red curve represents...' but I cannot see that red curve in Figure 6.

20 – In subsection 4.1.1 the authors search for the trajectory that minimizes the TDR. gamma^{*} minimizes p^{dot}, but looking at figure 7, gamma^{*} is a local minimum isn't it?

21 – In the system introducced in Eq. (32), what does v^2 represent?

22 – In the third paragraph of Example 3 there is missing text, since the sentence finishes with 'within the'. Moreover, would it be more clear to draw the weak and strong stable submanifolds in Figure 9?

23 – In the caption of Figure 10, when describing c), this figure is representing the largest eigenvalue of S, not the minimum.

24 – The colorscale used to plot the divergence rate in figure 11 is very weak. Could you enhance the color a bit to improve the figure?

25 – In the first paragraph of Section 5, I think that the sentence 'relies on the 2-dimensionality of the system, the understanding of the results' is not complete or is written incorrectly.

26 – In the last paragraph of Example 6, the parameter mu is written with letters and not with a symbol.