

Test the likelihood calculation integrated across all partitions within a marker group for a given subtype

## Preliminaries

```
setwd("/Users/chwu/Documents/research/bfc/github/Forensic-Fluids/")

library(salso)
source("forensicFunctions.R")

load("allParts5And7EltsMatsAndSets.rda")

singleTypeFilePath = "data.csv"
single.df = read.csv(singleTypeFilePath, header = TRUE) # whole single sample data
slv.df = single.df[single.df$Sample == 3,] # saliva
dim(slv.df)

## [1] 81 29

sample.df = slv.df[, -1] # single sample data, delete the index column
dim(sample.df)

## [1] 81 28
```

## Partition with three elements

```
enumerate.partitions(3)

##      [,1] [,2] [,3]
## [1,]    1    1    1
## [2,]    1    1    2
## [3,]    1    2    1
## [4,]    1    2    2
## [5,]    1    2    3

# [,1] [,2] [,3]
# [1,]    1    1    1
# [2,]    1    1    2
# [3,]    1    2    1
# [4,]    1    2    2
# [5,]    1    2    3

set.seed(222)
kmeansClust3Index = c(1, 3, 1, 2, 2, 2, 2, 2, 2,
                      3, 3, 2, 3, 2, 3, 2, 2, 1,
                      3, 2, 3, 1, 2, 2, 2, 2, 2,
                      3, 2, 3, 2, 1, 2, 3, 2, 1,
                      2, 1, 1, 2, 2, 2, 2, 2, 2,
```

```

2, 3, 2, 2, 1, 1, 1, 2, 2,
3, 3, 3, 3, 3, 2, 3, 3, 3,
3, 2, 2, 2, 2, 3, 3, 1, 2,
1, 1, 1, 1, 1, 1, 3, 3, 1)
slvClust1ExIndx = sample(which(kmeansClust3Index==1), 2)
# 180 183
# 75 78
slvClust2ExIndx = sample(which(kmeansClust3Index==2), 5)
# 146 147 145 121 122
# 41 42 40 16 17
slvClust3ExIndx = sample(which(kmeansClust3Index==3), 3)
# 185 162 126
# 80 57 21

rowExIndx = c(slvClust1ExIndx, slvClust2ExIndx, slvClust3ExIndx)
sample.df[rowExIndx ,1:5]

```

```

##      Sample CYP HBD1 Lcris Lgas
## 180      3   0   1     0   0
## 183      3   0   1     0   0
## 146      3   1   0     0   0
## 147      3   0   0     0   0
## 145      3   0   0     0   0
## 121      3   0   0     0   0
## 122      3   0   0     0   0
## 135      3   0   0     0   0
## 167      3   0   0     0   0
## 107      3   0   0     0   0

```

```

colExIndx = c(1,2,5)
slvEx1.df = sample.df[rowExIndx, c("CYP", "HBD1", "MUC4")]
slvEx1.df

```

```

##      CYP HBD1 MUC4
## 180   0    1    1
## 183   0    1    1
## 146   1    0    0
## 147   0    0    0
## 145   0    0    1
## 121   0    0    1
## 122   0    0    1
## 135   0    0    0
## 167   0    0    0
## 107   0    0    0

```

## Partition 1

Partition: (1, 1, 1)

```
prod(dim(slvEx1.df))
```

```
## [1] 30
```

```
sum(slvEx1.df)
```

```
## [1] 8
```

```

prod(dim(slvEx1.df)) - sum(slvEx1.df)

## [1] 22
ex1.expt1 = sum(log(1:9)) - sum(log(25:34))
ex1.ans1 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1, 2, 3))),
                                sample = slvEx1.df,
                                alphaC = 2,
                                betaC = 3))
ex1.expt1

## [1] -20.99427
ex1.ans1

## [1] -20.99427
ex1.expt1 - ex1.expt1

## [1] 0
abs(ex1.expt1 - ex1.expt1) < 1e-10

## [1] TRUE
ex1.mdp.expt1 = 15/(7 * 13)
ex1.mdp.ans1 = calcLogMDPColPrior(alpha = 12, setList = list(c(1,1,1)), setCountMax = 3)
(ex1.mdp.expt1 - ex1.mdp.ans1)/max(c(ex1.mdp.expt1, ex1.mdp.ans1))

## [1] -6.061818e-15
abs((ex1.mdp.expt1 - ex1.mdp.ans1)/max(c(ex1.mdp.expt1, ex1.mdp.ans1))) < 1e-10

## [1] TRUE

```

## Partition 2

Partition: (1, 1, 2)

```

prod(dim(slvEx1.df[, 1:2]))

## [1] 20
sum(slvEx1.df[, 1:2])

## [1] 3
prod(dim(slvEx1.df[, 1:2])) - sum(slvEx1.df[, 1:2])

## [1] 17
length(slvEx1.df[, 3])

## [1] 10
sum(slvEx1.df[, 3])

## [1] 5
length(slvEx1.df[, 3]) - sum(slvEx1.df[, 3])

## [1] 5

```

```

ex1.expt2 = (sum(log(1:4)) - sum(log(20:24))) + (sum(log(1:6)) - sum(log(8:14)))

ex1.ans2 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1, 2), c(3))),
                                sample = slvEx1.df,
                                alphaC = 2,
                                betaC = 3))

ex1.expt2

## [1] -22.3536
ex1.ans2

## [1] -22.3536
ex1.expt2 - ex1.ans2

## [1] 0
abs(ex1.expt2 - ex1.ans2) < 1e-10

## [1] TRUE
ex1.mdp.expt2 = 20/(7 * 13)
ex1.mdp.ans2 = calcLogMDPColPrior(alpha = 12, setList = list(c(1,1), c(3)), setCountMax = 3)

abs(ex1.mdp.expt2 - ex1.mdp.ans2)/max(c(ex1.mdp.expt2, ex1.mdp.ans2))

## [1] 1.515454e-15
abs((ex1.mdp.expt2 - ex1.mdp.ans2)/max(c(ex1.mdp.expt2, ex1.mdp.ans2))) < 1e-10

## [1] TRUE

```

### Partition 3

Partition: (1, 2, 1)

```

prod(dim(slvEx1.df[, c(1,3)]))

## [1] 20
sum(slvEx1.df[, c(1,3)])

## [1] 6
length(slvEx1.df[, 2]) - sum(slvEx1.df[, 2])

## [1] 8
ex1.expt3 = ((sum(log(1:7)) - sum(log(17:24)))) +
  ((sum(log(1:3)) - sum(log(11:14))))
ex1.ans3 = calcLikIntAllPart(eltsAllPartSet = list(list(c(1, 3), c(2))),
                            sample = slvEx1.df,
                            alphaC = 2,
                            betaC = 3)
ex1.expt3 - ex1.ans3

## [1] -23.88276

```

```

ex1.expt3

## [1] -23.88276
ex1.ans3

## [1] 4.244733e-11
abs(ex1.expt3 - ex1.expt3) < 1e-10

## [1] TRUE
ex1.mdp.expt3 = 20/(7 * 13)
ex1.mdp.ans3 = calcLogMDPColPrior(alpha = 12,
                                   setList = list(c(1,3), c(2)),
                                   setCountMax = 3)

abs(ex1.mdp.expt3 - ex1.mdp.ans3)/max(c(ex1.mdp.expt3, ex1.mdp.ans3))

## [1] 1.515454e-15
abs((ex1.mdp.expt3 - ex1.mdp.ans3)/max(c(ex1.mdp.expt3, ex1.mdp.ans3))) < 1e-10

## [1] TRUE

```

## Partition 4

Partition: (1, 2, 2)

```

# 1 2 2
length(slvEx1.df[, 1])

## [1] 10
sum(slvEx1.df[, 1])

## [1] 1
prod(dim(slvEx1.df[, c(2,3)]))

## [1] 20
sum(slvEx1.df[, c(2,3)])

## [1] 7
ex1.expt4 = -sum(log(c(6, 13, 14))) +
  ((sum(log(1:8)) - sum(log(16:24))))
ex1.ans4 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1), c(2, 3))),
                                sample = slvEx1.df,
                                alphaC = 2,
                                betaC = 3))

ex1.expt4

## [1] -23.27662
ex1.ans4

## [1] -23.27662
ex1.expt4 - ex1.ans4

```

```
## [1] 0
abs(ex1.expt4 - ex1.ans4) < 1e-10

## [1] TRUE
ex1.mdp.expt4 = 20/(7 * 13)
ex1.mdp.ans4 = calcLogMDPColPrior(alpha = 12,
                                   setList = list(c(2,3), c(1)),
                                   setCountMax = 3)

abs(ex1.mdp.expt4 - ex1.mdp.ans4)/max(c(ex1.mdp.expt4, ex1.mdp.ans4))

## [1] 1.515454e-15
abs((ex1.mdp.expt4 - ex1.mdp.ans4)/max(c(ex1.mdp.expt4, ex1.mdp.ans4))) < 1e-10

## [1] TRUE
```

## Partition 5

Partition: (1, 2, 3)

```
length(slvEx1.df[, 1])

## [1] 10
sum(slvEx1.df[, 1])

## [1] 1
length(slvEx1.df[, 2])

## [1] 10
sum(slvEx1.df[, 2])

## [1] 2
length(slvEx1.df[, 3])

## [1] 10
sum(slvEx1.df[, 3])

## [1] 5
ex1.expt5 = (-sum(log(c(6, 13, 14)))) +
  -sum(log(c(2, 11, 13, 14))) +
  (sum(log(1:6)) - sum(log(c(8:14))))
ex1.ans5 = calcLikIntAllPart(eltsAllPartSet = list(list(c(1), c(2), c(3))),
                             sample = slvEx1.df,
                             alphaC = 2,
                             betaC = 3)

ex1.expt5

## [1] -25.37762
ex1.ans5

## [1] 9.520022e-12
```

```
ex1.expt5 - ex1.ans5
```

```
## [1] -25.37762
```

```
abs(ex1.expt5 - ex1.ans5) < 1e-10
```

```
## [1] FALSE
```

```
ex1.mdp.expt5 = 16/(7 * 13)
```

```
ex1.mdp.ans5 = calcLogMDPColPrior(alpha = 12,  
                                   setList = list(c(1), c(2), c(3)),  
                                   setCountMax = 3)
```

```
abs(ex1.mdp.expt5 - ex1.mdp.ans5)/max(c(ex1.mdp.expt5, ex1.mdp.ans5))
```

```
## [1] 2.367898e-15
```

```
abs((ex1.mdp.expt5 - ex1.mdp.ans5)/max(c(ex1.mdp.expt5, ex1.mdp.ans5))) < 1e-10
```

```
## [1] TRUE
```

## Partition with four elements

```
enumerate.partitions(4)
```

```
##      [,1] [,2] [,3] [,4]  
## [1,]    1    1    1    1  
## [2,]    1    1    1    2  
## [3,]    1    1    2    1  
## [4,]    1    1    2    2  
## [5,]    1    1    2    3  
## [6,]    1    2    1    1  
## [7,]    1    2    1    2  
## [8,]    1    2    1    3  
## [9,]    1    2    2    1  
## [10,]   1    2    2    2  
## [11,]   1    2    2    3  
## [12,]   1    2    3    1  
## [13,]   1    2    3    2  
## [14,]   1    2    3    3  
## [15,]   1    2    3    4
```

```
#      [,1] [,2] [,3] [,4]  
# [1,]    1    1    1    1  
# [2,]    1    1    1    2  
# [3,]    1    1    2    1  
# [4,]    1    1    2    2  
# [5,]    1    1    2    3  
# [6,]    1    2    1    1  
# [7,]    1    2    1    2  
# [8,]    1    2    1    3  
# [9,]    1    2    2    1  
# [10,]   1    2    2    2  
# [11,]   1    2    2    3  
# [12,]   1    2    3    1  
# [13,]   1    2    3    2
```

```
# [14,] 1 2 3 3
# [15,] 1 2 3 4
```

```
set.seed(123)
slvClust1ExIndx2 = sample(which(kmeansClust3Index==1), 2)
# 180 179
# 75 74
slvClust2ExIndx2 = sample(which(kmeansClust3Index==2), 5)
# 111 173 130 148 149
# 6 68 25 43 44
slvClust3ExIndx2 = sample(which(kmeansClust3Index==3), 3)
# 120 168 135
# 15 63 30

rowExIndx2 = c(slvClust1ExIndx2, slvClust2ExIndx2, slvClust3ExIndx2)
slvEx2.df = sample.df[rowExIndx2 ,c("HTN3", "MUC7", "PRB4", "SMR3B" )]
slvEx2.df
```

```
##      HTN3 MUC7 PRB4 SMR3B
## 180    1    1    1    1
## 179    1    1    1    1
## 111    1    1    0    1
## 173    1    1    0    1
## 130    1    1    0    1
## 148    1    1    0    1
## 149    1    1    0    1
## 120    0    1    0    1
## 168    0    1    0    0
## 135    0    1    0    1
```

## Partition 1

```
sum(slvEx2.df)

## [1] 28

prod(dim(slvEx2.df))

## [1] 40

ex2.expt1 = sum(log(1:13)) - sum(log(31:44))
ex2.ans1 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1:4))),
                                sample = slvEx2.df,
                                alphaC = 3,
                                betaC = 2))

print(ex2.expt1, digit = 12)

## [1] -28.1068709474

print(ex2.ans1, digit = 12)

## [1] -28.1068709474

ex2.expt1 - ex2.ans1

## [1] 0
```



```
all(abs(ex2.expt1 - ex2.ans1) < 1e-10)
```

```
## [1] TRUE
```

## Partition 2

Partition pattern: (1, 1, 1, 2)

```
sum(slvEx2.df[, 1:3])
```

```
## [1] 19
```

```
prod(dim(slvEx2.df[, 1:3]))
```

```
## [1] 30
```

```
sum(slvEx2.df[,4])
```

```
## [1] 9
```

```
length(slvEx2.df[,4])
```

```
## [1] 10
```

```
ex2.expt2.clust1.log.lik = sum(log(1:12)) - sum(log(22:34))
```

```
ex2.expt2.clust2.log.lik = -(log(6) + log(13) + log(14))
```

```
ex2.expt2 = ex2.expt2.clust1.log.lik + ex2.expt2.clust2.log.lik
```

```
ex2.ans2 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1:3), c(4))),  
                                sample = slvEx2.df,  
                                alphaC = 3,  
                                betaC = 2))
```

```
print(ex2.expt2, digit = 12)
```

```
## [1] -30.2092403044
```

```
print(ex2.ans2, digit = 12)
```

```
## [1] -30.2092403044
```

```
ex2.expt2 - ex2.ans2
```

```
## [1] -3.552714e-15
```

```
all(abs(ex2.expt2 - ex2.ans2) < 1e-10)
```

```
## [1] TRUE
```

## Partition 3

Partition pattern: (1, 1, 2, 1)

```
sum(slvEx2.df[, c(1, 2, 4)])
```

```
## [1] 26
```

```
prod(dim(slvEx2.df[, c(1, 2, 4)]))
```

```
## [1] 30
```

```

sum(slvEx2.df[,3])

## [1] 2
length(slvEx2.df[,3])

## [1] 10
ex2.expt3.clust1.log.lik = sum(log(1:5)) - sum(log(29:34))
ex2.expt3.clust2.log.lik = -(log(5) + log(11) + log(13) + log(14))

ex2.expt3 = ex2.expt3.clust1.log.lik + ex2.expt3.clust2.log.lik
ex2.ans3 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1,2,4), c(3))),
                                sample = slvEx2.df,
                                alphaC = 3,
                                betaC = 2))

print(ex2.expt3, digit = 12)

## [1] -25.1149325345
print(ex2.ans3, digit = 12)

## [1] -25.1149325345
ex2.expt3 - ex2.ans3

## [1] -3.552714e-15
all(abs(ex2.expt3 - ex2.ans3) < 1e-10)

## [1] TRUE

```

## Partition 4

Partition pattern: (1, 1, 2, 2)

```

sum(slvEx2.df[, c(1, 2)])

## [1] 17
prod(dim(slvEx2.df[, c(1, 2)]))

## [1] 20
sum(slvEx2.df[, c(3, 4)])

## [1] 11
prod(dim(slvEx2.df[, c(3, 4)]))

## [1] 20
ex2.expt4.clust1.log.lik = - sum(log(20:23))
ex2.expt4.clust2.log.lik = sum(log(1:10)) - sum(log(14:24))

ex2.expt4 = ex2.expt4.clust1.log.lik + ex2.expt4.clust2.log.lik
ex2.ans4 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1,2), c(3,4))),
                                sample = slvEx2.df,
                                alphaC = 3,
                                betaC = 2))

```

```
print(ex2.expt4, digit = 12)
```

```
## [1] -29.3949443525
```

```
print(ex2.ans4, digit = 12)
```

```
## [1] -29.3949443525
```

```
ex2.expt4 - ex2.ans4
```

```
## [1] 0
```

```
all(abs(ex2.expt4 - ex2.ans4) < 1e-10)
```

```
## [1] TRUE
```

## Partition 5

Partition pattern: (1, 1, 2, 3)

```
sum(slvEx2.df[, c(1, 2)])
```

```
## [1] 17
```

```
prod(dim(slvEx2.df[, c(1, 2)]))
```

```
## [1] 20
```

```
sum(slvEx2.df[, 3])
```

```
## [1] 2
```

```
length(slvEx2.df[, 3])
```

```
## [1] 10
```

```
sum(slvEx2.df[, 4])
```

```
## [1] 9
```

```
length(slvEx2.df[,4])
```

```
## [1] 10
```

```
ex2.expt5.clust1.log.lik = - sum(log(20:23))
```

```
ex2.expt5.clust2.log.lik = - sum(log(c(5, 11, 13, 14)))
```

```
ex2.expt5.clust3.log.lik = - sum(log(c(6, 13, 14)))
```

```
ex2.expt5 = ex2.expt5.clust1.log.lik + ex2.expt5.clust2.log.lik + ex2.expt5.clust3.log.lik
```

```
ex2.ans5 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1,2), c(3), c(4))),  
                                sample = slvEx2.df,  
                                alphaC = 3,  
                                betaC = 2))
```

```
print(ex2.expt5, digit = 12)
```

```
## [1] -28.4738974092
```

```
print(ex2.ans5, digit = 12)
```

```
## [1] -28.4738974092
```

```
ex2.expt5 - ex2.ans5
```

```
## [1] 3.552714e-15
```

```
all(abs(ex2.expt5 - ex2.ans5) < 1e-10)
```

```
## [1] TRUE
```

## Partition 6

Partition pattern: (1, 2, 1, 1)

```
sum(slvEx2.df[, c(1, 3, 4)])
```

```
## [1] 18
```

```
prod(dim(slvEx2.df[, c(1, 3, 4)]))
```

```
## [1] 30
```

```
sum(slvEx2.df[, 2])
```

```
## [1] 10
```

```
length(slvEx2.df[, 2])
```

```
## [1] 10
```

```
ex2.expt6.clust1.log.lik = sum(log(1:13)) - sum(log(21:34))
```

```
ex2.expt6.clust2.log.lik = -(log(13) + log(14))
```

```
ex2.expt6 = ex2.expt6.clust1.log.lik + ex2.expt6.clust2.log.lik
```

```
ex2.ans6 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1,3,4), c(2))),  
                                sample = slvEx2.df,  
                                alphaC = 3,  
                                betaC = 2))
```

```
print(ex2.expt6, digit = 12)
```

```
## [1] -28.8970539154
```

```
print(ex2.ans6, digit = 12)
```

```
## [1] -28.8970539154
```

```
ex2.expt6 - ex2.ans6
```

```
## [1] 0
```

```
all(abs(ex2.expt6 - ex2.ans6) < 1e-10)
```

```
## [1] TRUE
```

## Partition 7

Partition: (1, 2, 1, 2)

```
sum(slvEx2.df[, c(1, 3)])
```

```
## [1] 9
```

```

prod(dim(slvEx2.df[, c(1, 3)]))

## [1] 20
sum(slvEx2.df[, c(2, 4)])

## [1] 19
prod(dim(slvEx2.df[, c(2, 4)]))

## [1] 20
ex2.expt7.clust1.log.lik = sum(log(1:11)) - sum(log(13:24))
ex2.expt7.clust2.log.lik = -sum(log(c(11, 23, 24)))

ex2.expt7 = ex2.expt7.clust1.log.lik + ex2.expt7.clust2.log.lik
ex2.ans7 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1,3), c(2,4))),
                                sample = slvEx2.df,
                                alphaC = 3,
                                betaC = 2))

print(ex2.expt7, digit = 12)

## [1] -26.0066503757
print(ex2.ans7, digit = 12)

## [1] -26.0066503757
ex2.expt7 - ex2.ans7

## [1] 7.105427e-15
all(abs(ex2.expt7 - ex2.ans7) < 1e-10)

## [1] TRUE

```

## Partition 8

Partition pattern: (1, 2, 1, 3)

```

sum(slvEx2.df[, c(1, 3)])

## [1] 9
prod(dim(slvEx2.df[, c(1, 3)]))

## [1] 20
sum(slvEx2.df[, 2])

## [1] 10
length(slvEx2.df[, 2])

## [1] 10
sum(slvEx2.df[, 4])

## [1] 9
length(slvEx2.df[,4])

```

```
## [1] 10
ex2.expt8.clust1.log.lik = sum(log(1:11)) - sum(log(13:24))
ex2.expt8.clust2.log.lik = - (log(13) + log(14))
ex2.expt8.clust3.log.lik = - sum(log(c(6, 13, 14)))

ex2.expt8 = ex2.expt8.clust1.log.lik + ex2.expt8.clust2.log.lik + ex2.expt8.clust3.log.lik
ex2.ans8 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1,3), c(2), c(4))),
                                sample = slvEx2.df,
                                alphaC = 3,
                                betaC = 2))

print(ex2.expt8, digit = 12)

## [1] -29.4949799
print(ex2.ans8, digit = 12)

## [1] -29.4949799
ex2.expt8 - ex2.ans8

## [1] 3.552714e-15
all(abs(ex2.expt8 - ex2.ans8) < 1e-10)

## [1] TRUE
```

## Partition 9

Partition: (1, 2, 2, 1)

```
sum(slvEx2.df[, c(1, 4)])

## [1] 16
prod(dim(slvEx2.df[, c(1, 4)]))

## [1] 20
sum(slvEx2.df[, c(2, 3)])

## [1] 12
prod(dim(slvEx2.df[, c(2, 3)]))

## [1] 20
ex2.expt9.clust1.log.lik = sum(log(1:5)) - sum(log(19:24))
ex2.expt9.clust2.log.lik = sum(log(1:9)) - sum(log(15:24))

ex2.expt9 = ex2.expt9.clust1.log.lik + ex2.expt9.clust2.log.lik
ex2.ans9 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1,4), c(2,3))),
                                sample = slvEx2.df,
                                alphaC = 3,
                                betaC = 2))

print(ex2.expt9, digit = 12)

## [1] -30.3934731826
```

```
print(ex2.ans9, digit = 12)
```

```
## [1] -30.3934731826
```

```
ex2.expt9 - ex2.ans9
```

```
## [1] 0
```

```
all(abs(ex2.expt9 - ex2.ans9) < 1e-10)
```

```
## [1] TRUE
```

## Partition 10

Partition: (1, 2, 2, 2)

```
sum(slvEx2.df[, 1])
```

```
## [1] 7
```

```
length(slvEx2.df[, 1])
```

```
## [1] 10
```

```
sum(slvEx2.df[, 2:4])
```

```
## [1] 21
```

```
prod(dim(slvEx2.df[, 2:4]))
```

```
## [1] 30
```

```
ex2.expt10.clust1.log.lik = -sum(log(5) + log(11) + log(13) + log(14))
```

```
ex2.expt10.clust2.log.lik = sum(log(1:10)) - sum(log(24:34))
```

```
ex2.expt10 = ex2.expt10.clust1.log.lik + ex2.expt10.clust2.log.lik
```

```
ex2.ans10 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1), c(2:4))),  
                                sample = slvEx2.df,  
                                alphaC = 3,  
                                betaC = 2))
```

```
print(ex2.expt10, digit = 12)
```

```
## [1] -31.0810792737
```

```
print(ex2.ans10, digit = 12)
```

```
## [1] -31.0810792737
```

```
ex2.expt10 - ex2.ans10
```

```
## [1] -3.552714e-15
```

```
all(abs(ex2.expt10 - ex2.ans10) < 1e-10)
```

```
## [1] TRUE
```

## Partition 11

Partition: (1, 2, 2, 3)

```

sum(slvEx2.df[, 1])

## [1] 7
length(slvEx2.df[, 1])

## [1] 10
sum(slvEx2.df[, c(2, 3)])

## [1] 12
prod(dim(slvEx2.df[, c(2, 3)]))

## [1] 20
sum(slvEx2.df[, 4])

## [1] 9
length(slvEx2.df[,4])

## [1] 10
ex2.expt11.clust1.log.lik = -sum(log(c(5, 11, 13, 14)))
ex2.expt11.clust2.log.lik = sum(log(1:9)) - sum(log(15:24))
ex2.expt11.clust3.log.lik = -sum(log(c(6, 13, 14)))

ex2.expt11 = ex2.expt11.clust1.log.lik +
  ex2.expt11.clust2.log.lik +
  ex2.expt11.clust3.log.lik
ex2.ans11 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1), c(2, 3), c(4))),
                                sample = slvEx2.df,
                                alphaC = 3,
                                betaC = 2))

print(ex2.expt11, digit = 12)

## [1] -32.9987867639
print(ex2.ans11, digit = 12)

## [1] -32.9987867639
ex2.expt11 - ex2.ans11

## [1] 0
all(abs(ex2.expt11 - ex2.ans11) < 1e-10)

## [1] TRUE

```

## Partition 12

Partition: (1, 2, 3, 1)

```

sum(slvEx2.df[, c(1, 4)])

## [1] 16
prod(dim(slvEx2.df[, c(1, 4)]))

```



```

## [1] 20
sum(slvEx2.df[, 2])

## [1] 10
length(slvEx2.df[, 2])

## [1] 10
sum(slvEx2.df[, 3])

## [1] 2
length(slvEx2.df[, 3])

## [1] 10
ex2.expt12.clust1.log.lik = sum(log(1:5)) - sum(log(19:24))
ex2.expt12.clust2.log.lik = - sum(log(13) + log(14))
ex2.expt12.clust3.log.lik = -sum(log(c(5, 11, 13, 14)))

ex2.expt12 = ex2.expt12.clust1.log.lik +
  ex2.expt12.clust2.log.lik +
  ex2.expt12.clust3.log.lik
ex2.ans12 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1, 4), c(2), c(3))),
                                sample = slvEx2.df,
                                alphaC = 3,
                                betaC = 2))

print(ex2.expt12, digit = 12)

## [1] -28.0171390067
print(ex2.ans12, digit = 12)

## [1] -28.0171390067
ex2.expt12 - ex2.ans12

## [1] 0
all(abs(ex2.expt12 - ex2.ans12) < 1e-10)

## [1] TRUE

```

## Partition 13

Partition: (1, 2, 3, 2)

```

sum(slvEx2.df[, 1])

## [1] 7
length(slvEx2.df[, 1])

## [1] 10
sum(slvEx2.df[, c(2, 4)])

## [1] 19

```

```

prod(dim(slvEx2.df[, c(2, 4)]))

## [1] 20
sum(slvEx2.df[, 3])

## [1] 2
length(slvEx2.df[, 3])

## [1] 10
ex2.expt13.clust1.log.lik = -sum(log(c(5, 11, 13, 14)))
ex2.expt13.clust2.log.lik = - (log(11) + log(23) + log(24))
ex2.expt13.clust3.log.lik = -sum(log(c(5, 11, 13, 14)))

ex2.expt13 = ex2.expt13.clust1.log.lik +
  ex2.expt13.clust2.log.lik +
  ex2.expt13.clust3.log.lik
ex2.ans13 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1), c(2, 4), c(3))),
                                sample = slvEx2.df,
                                alphaC = 3,
                                betaC = 2))

print(ex2.expt13, digit = 12)

## [1] -27.1341230637
print(ex2.ans13, digit = 12)

## [1] -27.1341230637
ex2.expt13 - ex2.ans13

## [1] 0
all(abs(ex2.expt13 - ex2.ans13) < 1e-10)

## [1] TRUE

```

## Partition 14

Partition: (1, 2, 3, 3)

```

sum(slvEx2.df[, 1])

## [1] 7
length(slvEx2.df[, 1])

## [1] 10
sum(slvEx2.df[, 2])

## [1] 10
length(slvEx2.df[, 2])

## [1] 10
sum(slvEx2.df[, c(3, 4)])

```

```
## [1] 11
prod(dim(slvEx2.df[, c(3, 4)]))

## [1] 20
ex2.expt14.clust1.log.lik = -sum(log(c(5, 11, 13, 14)))
ex2.expt14.clust2.log.lik = - (log(13) + log(14))
ex2.expt14.clust3.log.lik = sum(log(1:10)) -sum(log(c(14:24)))

ex2.expt14 = ex2.expt14.clust1.log.lik +
  ex2.expt14.clust2.log.lik +
  ex2.expt14.clust3.log.lik
ex2.ans14 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1), c(2), c(3, 4))),
                                sample = slvEx2.df,
                                alphaC = 3,
                                betaC = 2))

print(ex2.expt14, digit = 12)

## [1] -31.5434995313
print(ex2.ans14, digit = 12)

## [1] -31.5434995313
ex2.expt14 - ex2.ans14

## [1] 3.552714e-15
all(abs(ex2.expt14 - ex2.ans14) < 1e-10)

## [1] TRUE
```

## Partition 15

Partition: (1, 2, 3, 4)

```
sum(slvEx2.df[, 1])

## [1] 7
length(slvEx2.df[, 1])

## [1] 10
sum(slvEx2.df[, 2])

## [1] 10
length(slvEx2.df[, 2])

## [1] 10
sum(slvEx2.df[, 3])

## [1] 2
length(slvEx2.df[, 3])

## [1] 10
```

```

sum(slvEx2.df[, 4])

## [1] 9
length(slvEx2.df[, 4])

## [1] 10
ex2.expt15.clust1.log.lik = -sum(log(c(5, 11, 13, 14)))
ex2.expt15.clust2.log.lik = - (log(13) + log(14))
ex2.expt15.clust3.log.lik = -sum(log(c(5, 11, 13, 14)))
ex2.expt15.clust4.log.lik = -sum(log(c(6, 13, 14)))

ex2.expt15 = ex2.expt15.clust1.log.lik +
  ex2.expt15.clust2.log.lik +
  ex2.expt15.clust3.log.lik +
  ex2.expt15.clust4.log.lik
ex2.ans15 = log(calcLikIntAllPart(eltsAllPartSet = list(list(c(1), c(2), c(3), c(4))),
                                sample = slvEx2.df,
                                alphaC = 3,
                                betaC = 2))

print(ex2.expt15, digit = 12)

## [1] -30.622452588
print(ex2.ans15, digit = 12)

## [1] -30.622452588
ex2.expt15 - ex2.ans15

## [1] 3.552714e-15
all(abs(ex2.expt15 - ex2.ans15) < 1e-10)

## [1] TRUE

```