Economics Problem Set #5-3

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Exercise 1

Solution. Following the same procedure from the first two derivatives, we have that the third derivative is given by:

$$\frac{d}{du}(F_{xx}x_u^2 + 2F_{xu}x_u + F_x x_{uu} + F_{uu}) = 0$$

This is equivalent to:

$$(F_{xxx}x_u + F_{xxu})x_u^2 + 2F_{xx}x_u x_{uu} + 2(F_{xxu}x_u + F_{xuu})x_u + 2F_{xu}x_{uu}$$

$$+(F_{xx}x_u + F_{xu})x_{uu} + F_x x_{uuu} + F_{xuu}x_u + F_{uuu} = 0$$

$$\implies x_{uuu} = \frac{-1}{F_x}(F_{xxx}x_u^3 + 3x_u^2 F_{xxu} + 3F_{xx}x_u x_{uu} + 3F_{xuu}x_u + 3F_{xu}x_{uu} + F_{uuu})$$

where the inputs of each of the functions F are $x(u_0)$ and u_0 and the inputs of each of the functions $x(u)$ are u_0. (These arguments are omitted in the solution for purposes of readability.)

Exercise 2

Solution. See the attached Jupyter notebook `Perturbation_Exercises_2-5.ipynb` for the code for this problem. We use the first and second order approximations as defined in the notes with w in place of x and k in place of u.

Exercise 3

Solution. See the attached Jupyter notebook `Perturbation_Exercises_2-5.ipynb` for the code for this problem.

Exercise 4

Solution. See the attached Jupyter notebook `Perturbation_Exercises_2-5.ipynb` for the code for this problem.

Exercise 5

Solution. See the attached Jupyter notebook `Perturbation_Exercises_2-5.ipynb` for the code for this problem.