Econ Problem Set #2-2: Dynamic Programming II

OSM Lab - University of Chicago
Geoffrey Kocks

Note: All Python codes for this problem set are in the file dp2.ipynb.

Exercise 1.

Solution. Let $\sigma \in \Sigma$ be any feasible policy and let U be the operator from C to itself defined by:

$$Uw(y) = u(\sigma(y)) + \beta \int w(f(y - \sigma(y))z)\phi(dz) \qquad (y \in \mathbb{R}_+).$$

We first show that U is a contraction mapping with respect to the supremum distance. Fix a w and w'. Then:

$$\begin{aligned}
|Uw(y) - Uw'(y)| &= \beta| \int (w(f(y - \sigma(y))z) - w'(f(y - \sigma(y))z))\phi(dz)| \\
&\leq \beta sup| \int (w(f(y - \sigma(y))z) - w'(f(y - \sigma(y))z))\phi(dz)| \\
&\leq \beta sup \int |(w(f(y - \sigma(y))z) - w'(f(y - \sigma(y))z))|\phi(dz) \\
&\leq \beta sup \int |w - w'|\phi(dz) \\
&= \beta|w - w'|.
\end{aligned}$$

Therefore U is a contraction mapping. Because U is a contraction mapping, by Banach's fixed point theorem, it will have one unique fixed point, which is given by v_σ. It is reasonable that v_σ is the fixed point because the utility function is interpreted as the sum of present utility and discounted future utility. This is the same as how we define the value function.

Exercise 2.

Solution. The value functions for the two sample policies are shown below in Figure 1 and Figure 2. We note that neither one of the policies is an optimal policy which we can see by comparison with the value function presented in the lecture notes. Each of the alternative value functions have a lower value at each possible y.

Figure 1

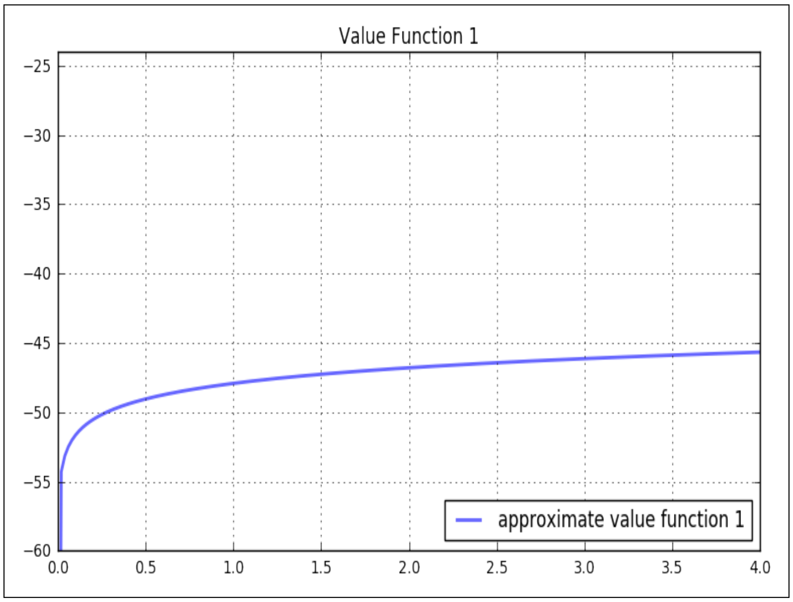

Figure 2