

D2: Constraint Satisfaction Problems

Friday, September 28, 2018 4:45 PM



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CS188 Spring 2014 Section 2: CSPs

1 Course Scheduling

You are in charge of scheduling for computer science classes that meet Mondays, Wednesdays and Fridays. There are 5 classes that meet on these days and 3 professors who will be teaching these classes. You are constrained by the fact that each professor can only teach one class at a time.

The classes are:

1. Class 1 - Intro to Programming: meets from 8:00-9:00am
2. Class 2 - Intro to Artificial Intelligence: meets from 8:30-9:30am
3. Class 3 - Natural Language Processing: meets from 9:00-10:00am
4. Class 4 - Computer Vision: meets from 9:00-10:00am
5. Class 5 - Machine Learning: meets from 10:30-11:30am

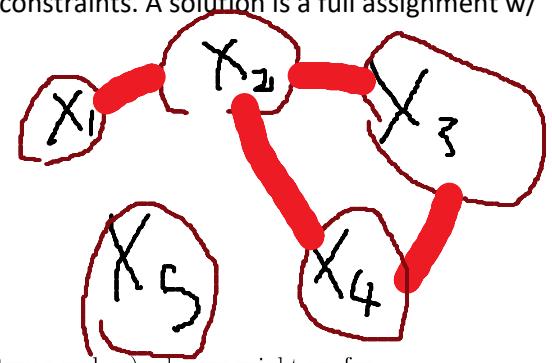
The professors are:

1. Professor A, who is qualified to teach Classes 1, 2, and 5.
 2. Professor B, who is qualified to teach Classes 3, 4, and 5.
 3. Professor C, who is qualified to teach Classes 1, 3, and 4.
1. Formulate this problem as a CSP problem in which there is one variable per class, stating the domains, and constraints. Constraints should be specified formally and precisely, but may be implicit rather than explicit.

CSP is formulated as variables, domains for variables, and constraints. A solution is a full assignment w/ no constraint conflicts.

Variables: X1: {A,C} , X2: {A}, X3: {B, C}, X4: {B,C}, X5: {A,B}
Constraints: {X1 ≠ X2, X2≠X3, X2≠X4, X3≠X4}

2. Draw the constraint graph associated with your CSP.



3. Your CSP should look nearly tree-structured. Briefly explain (one sentence or less) why we might prefer to solve tree-structured CSPs.

Variable choice can be done without backtracking

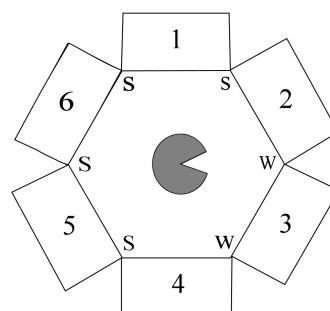
$O(nd^2)$ vs $O(d^n)$

2 CSPs: Trapped Pacman

Pacman is trapped! He is surrounded by mysterious corridors, each of which leads to either a pit (P), a ghost (G), or an exit (E). In order to escape, he needs to figure out which corridors, if any, lead to an exit and freedom, rather than the certain doom of a pit or a ghost.

The one sign of what lies behind the corridors is the wind: a pit produces a strong breeze (S) and an exit produces a weak breeze (W), while a ghost doesn't produce any breeze at all. Unfortunately, Pacman cannot measure the strength of the breeze at a specific corridor. Instead, he can stand *between* two adjacent corridors and feel the max of the two breezes. For example, if he stands between a pit and an exit he will sense a strong (S) breeze, while if he stands between an exit and a ghost, he will sense a weak (W) breeze. The measurements for all intersections are shown in the figure below.

Also, while the total number of exits might be zero, one, or more, Pacman knows that two neighboring squares will *not* both be exits.



Pacman models this problem using variables X_i for each corridor i and domains P, G, and E.

1. State the binary and/or unary constraints for this CSP (either implicitly or explicitly).

Variables:	Constraints:
$X_1: \{P\}$	$X_3 \text{ or } X_4 \text{ is exit}$
$X_2: \{G, E\}$	$X_2 \text{ or } X_3 \text{ is exit}$
$X_3: \{G, E\}$	If X_3 is exit iff X_2 is NOT exit
$X_4: \{G, E\}$	If X_3 is exit iff X_4 is NOT exit
$X_5: \{P\}$	X_3 is not exit IFF X_2 is exit
$X_6: \{P, G, E\}$	X_3 is NOT exit IFF X_4 is NOT exit

2. Cross out the values from the domains of the variables that will be deleted in enforcing arc consistency.

X_1	P	Q	E
X_2	P	G	E
X_3	P	G	E
X_4	P	G	E
X_5	P	G	E
X_6	P	G	E

For every value of X_i , there is still a valid value for X_j

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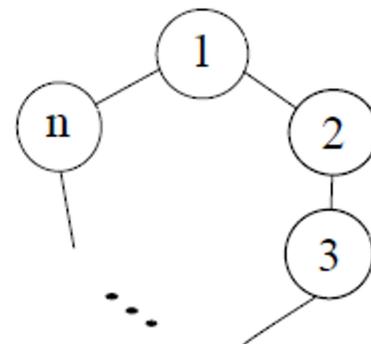
3. According to MRV, which variable or variables could the solver assign first?

Minimum Remaining Values. X_1 and X_5 should be assigned. X_1 if breaking ties by name

4. Assume that Pacman knows that $X_6 = G$. List all the solutions of this CSP or write *none* if no solutions exist.

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PGE₁GPG
PGE₂PG



5. The CSP described above has a circular structure with 6 variables. Now consider a CSP forming a circular structure that has n variables ($n > 2$), as shown below. Also assume that the domain of each variable has cardinality d . Explain precisely how to solve this general class of circle-structured CSPs efficiently (i.e. in time linear in the number of variables), using methods covered in class. Your answer should be at most two sentences.

For $i=0:n$

Pick X_i . Assign value. Apply Arc consistency(X_{i+1}, X_i)

n variables
 d values per variable
How do we solve t

- Assignment
- Pick an unassigned variable (anyone)
- For each value in domain
 - Check if arc consistent
 - If not, remove from domain

6. If standard backtracking search were run on a circle-structured graph, enforcing arc consistency at every step, what, if anything, can be said about the worst-case backtracking behavior (e.g. number of times the search could backtrack)?

We can backtrack at most once.

Arc consistency: removes all values in domain that will ensure a binary constraint violation.

- remove values from domain that have passed arc consistency check
- Return failure if no values remain

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this class of CSPs efficiently (in time linear w.r.t. number of variables).

= parameter sent in
assigned variable (Minimum remaining values, secondarily on maximum

ue in the sorted domain of the variable (sort by least constraint value
if assigning that value violates ANY of the constraints

add the value to the assignment

FORWARD CHECK via inferences

Check if inferences returns failure (an unassigned variable has empty
domain)

- save the results of the inference
 - ◆ continue backtracking with the additional assignment
 - If future searching didn't fail (solution reached): return solution
- ve the value from the assignment (this is only reached if the current value
d a consistency/inference failure at some depth)
- re (all options explored, all options violate constraint at some point)