Perception: Projective Transformations and Vanishing Points

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Projective Transformation

- Aka Homography or Collineation
- Represents the perspective projection from a ground plane to an image plane !
- It is an invertible 3x3 matrix but has 8 independent parameters
- For example if (X,Y) measured in meters on the ground and (u,v) in pixels

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \sim H \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

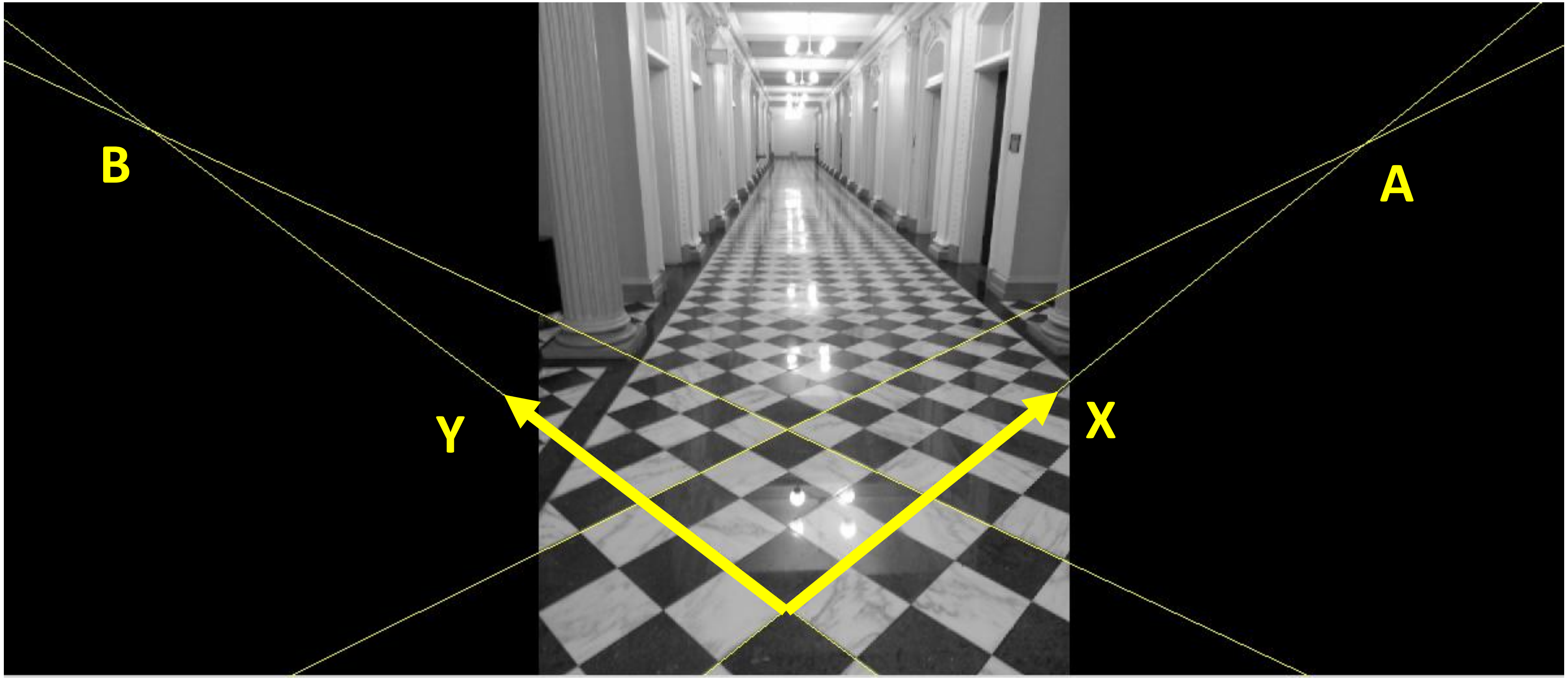
or

$$\begin{aligned} u &= \frac{H_{11}X + H_{12}Y + H_{13}}{H_{31}X + H_{32}Y + H_{33}} \\ v &= \frac{H_{21}X + H_{22}Y + H_{23}}{H_{31}X + H_{32}Y + H_{33}} \end{aligned}$$

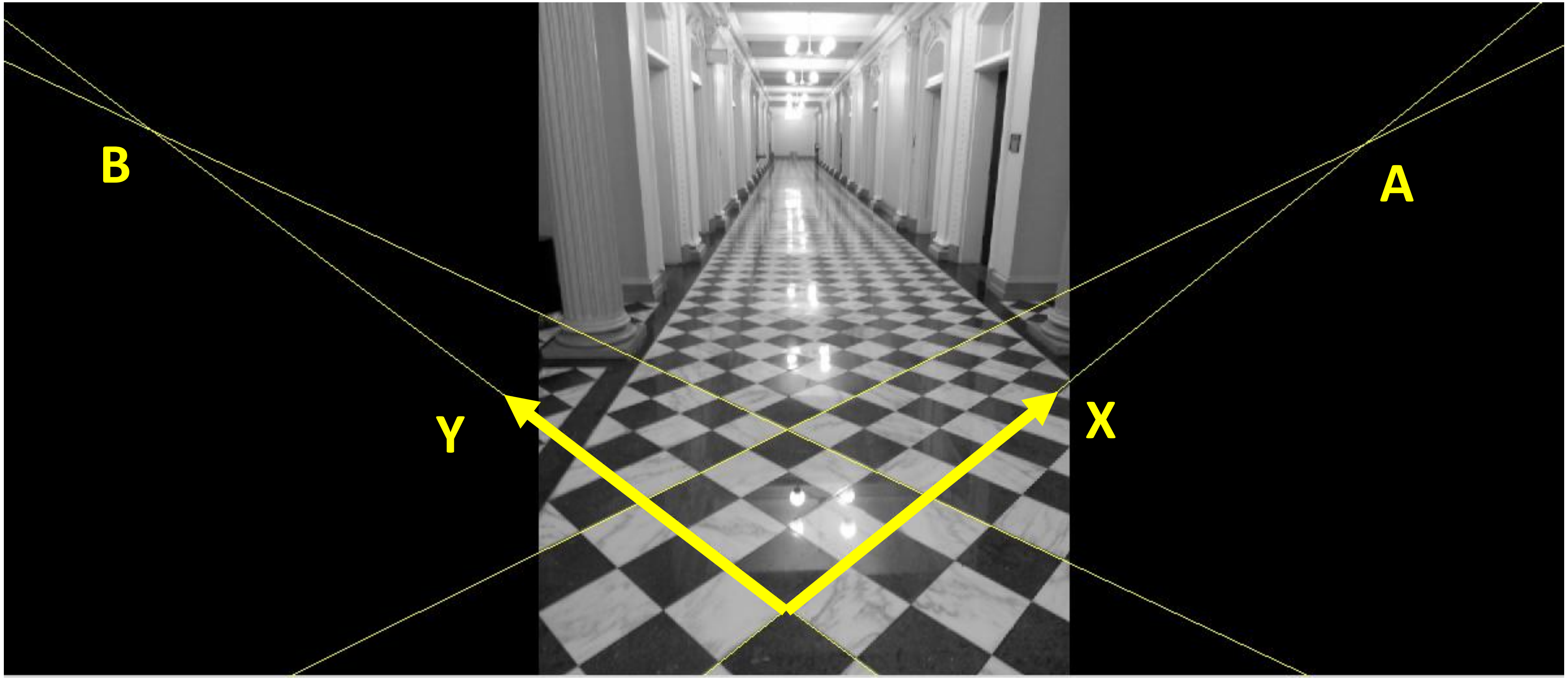
If we know H, we can make back and forth measurements:

10.2,2.3 in meters

Homography columns as vanishing points

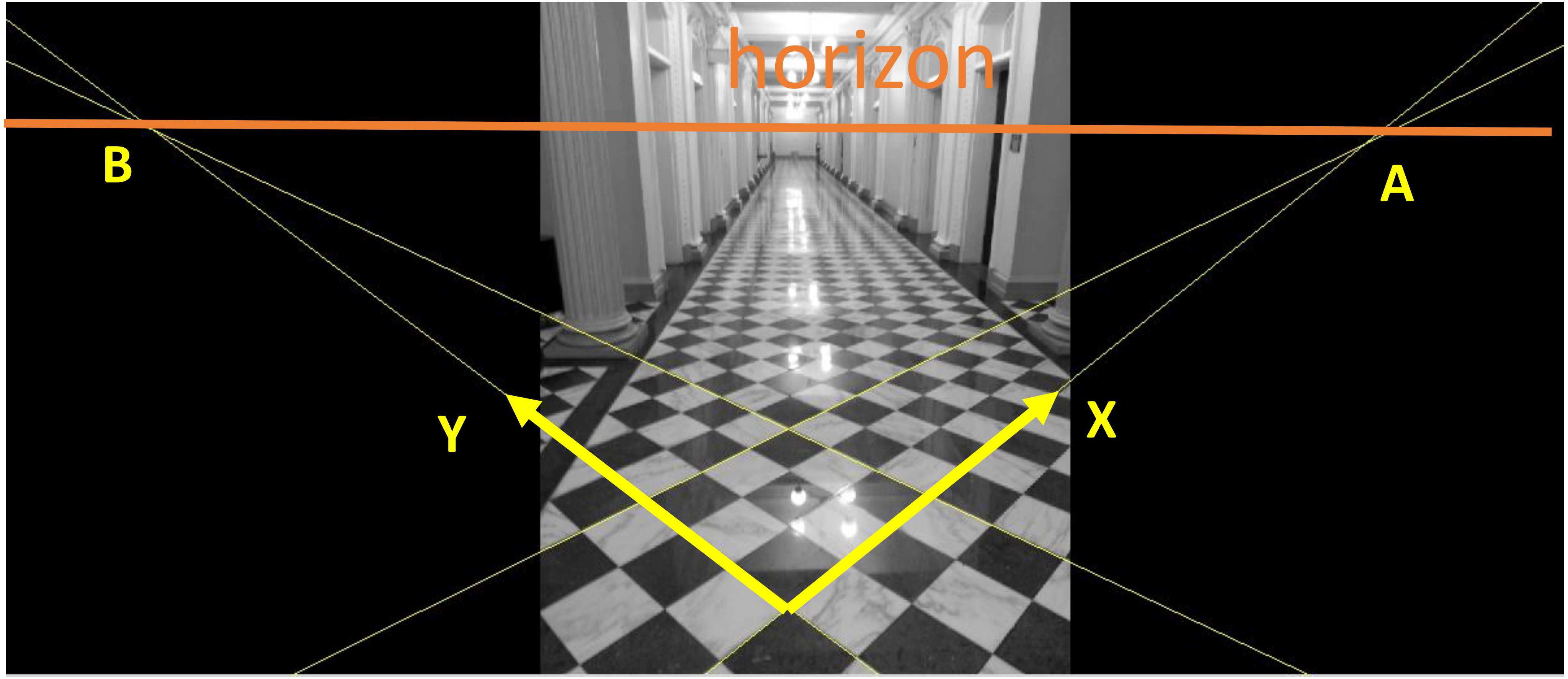


If $H = \begin{pmatrix} h_1 & h_2 & h_3 \end{pmatrix}$ then $h_1 \sim A$ and $h_2 \sim B$.

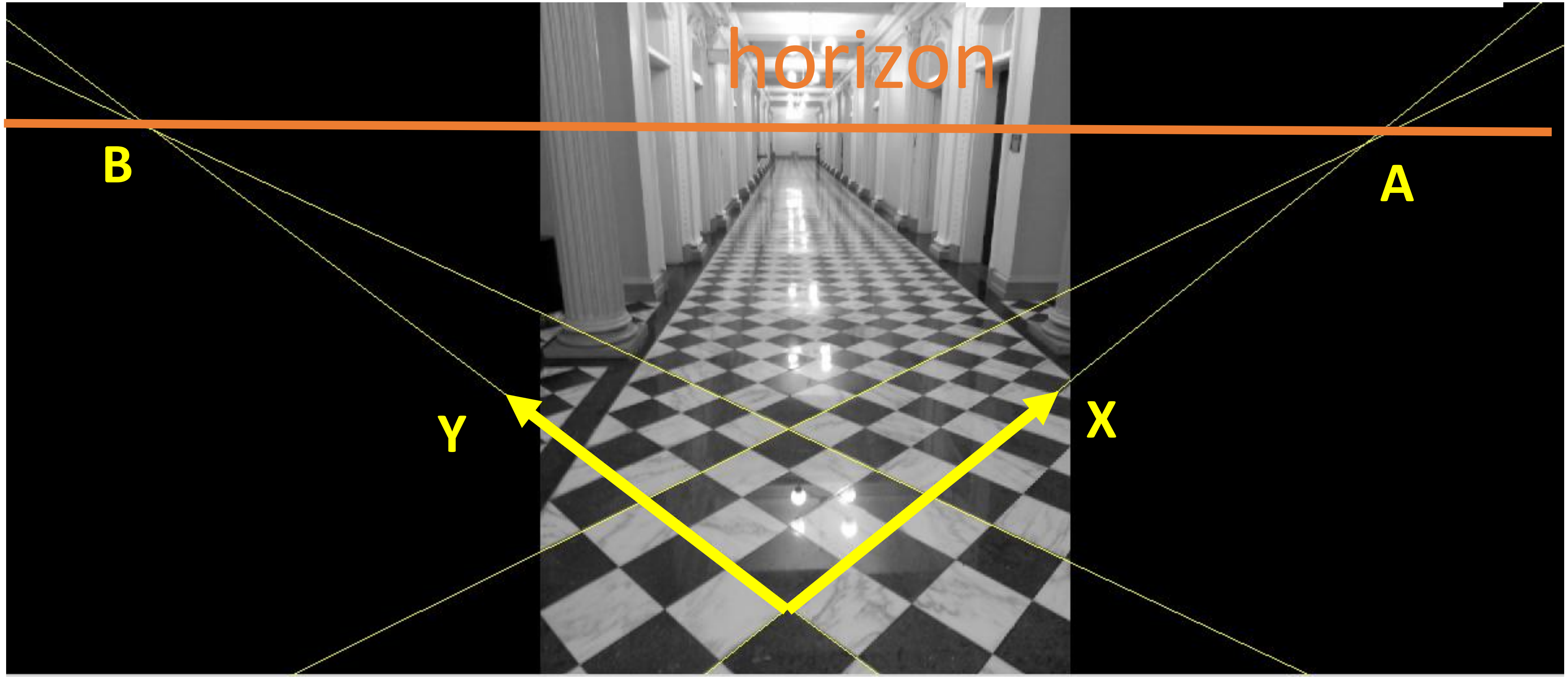


So the first two columns are two Orthogonal vanishing points

If we connect two vanishing points we obtain the horizon!

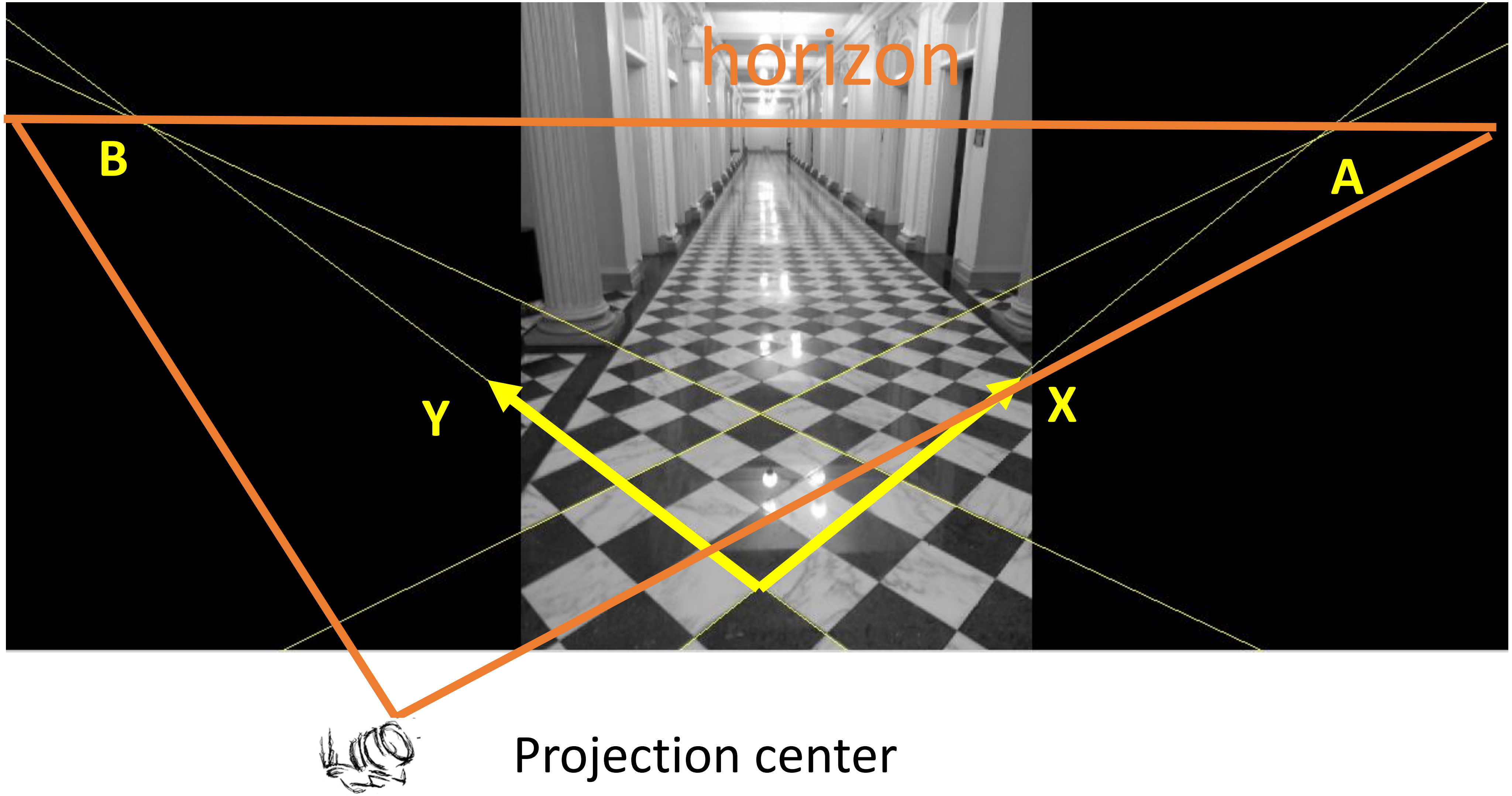


Equation of horizon: $$(h_1 \times h_2)^T \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0$$



Horizon with projection center build a horizon plane with normal

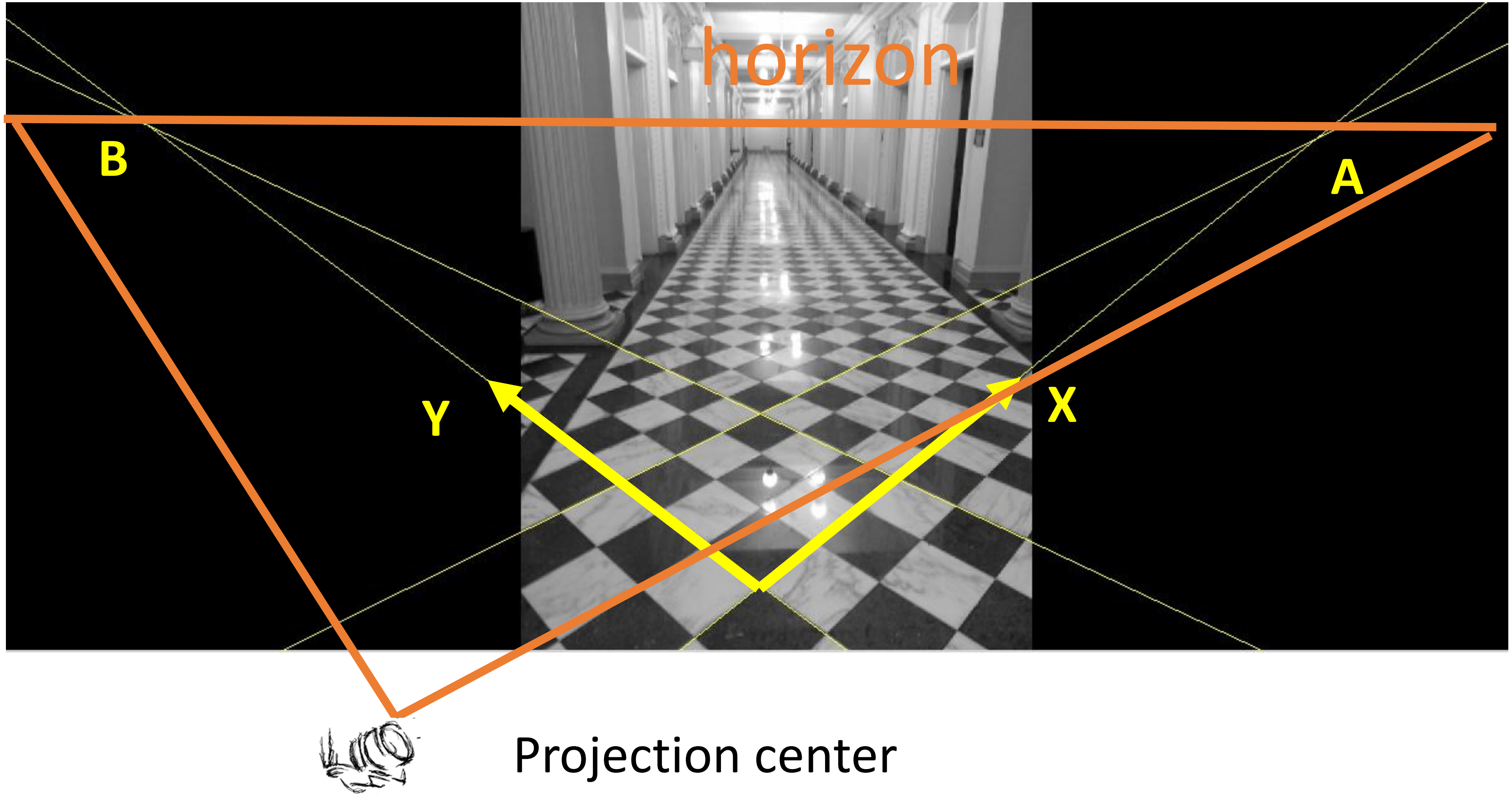
$$(h_1 \times h_2)$$



Projection center

The horizon plane is parallel to the ground plane and hence

$(h_1 \times h_2)$ is the normal to the ground plane expressed via pixels!



Horizon gives complete info about how ground plane is oriented! If horizon is horizontal the center it means that optical axis is parallel to groundplane!

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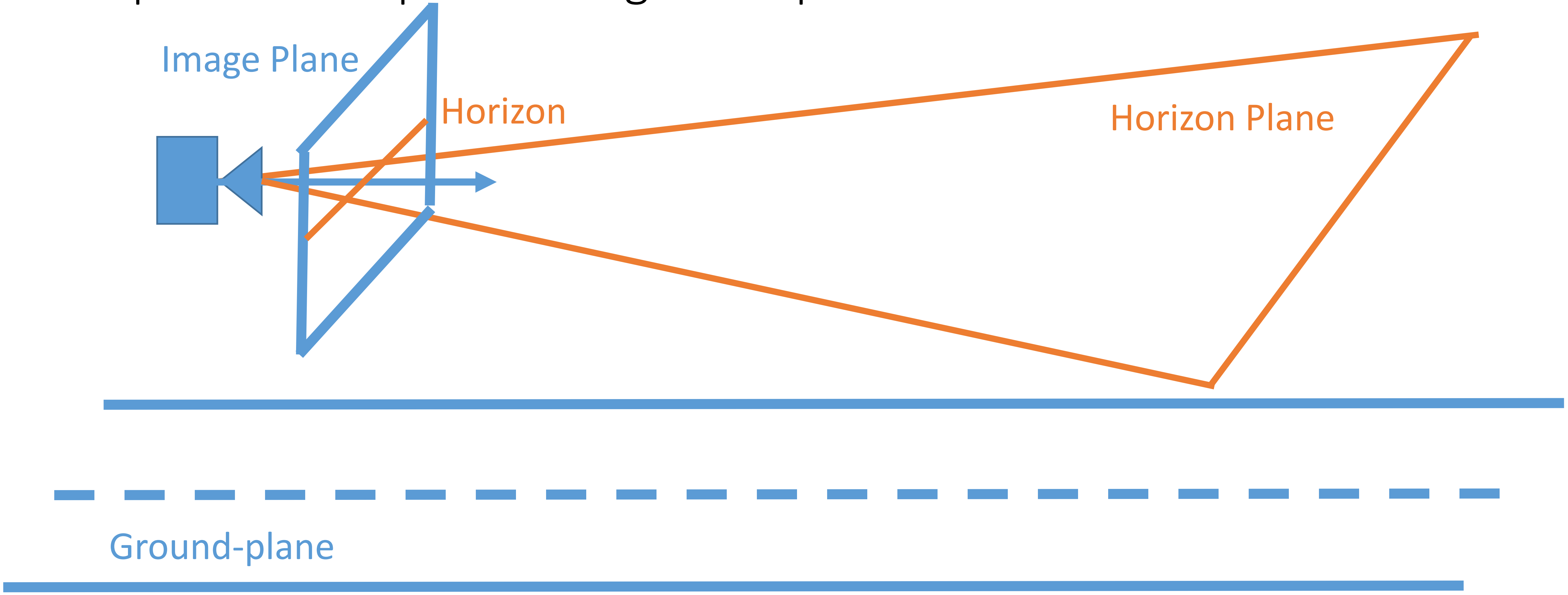
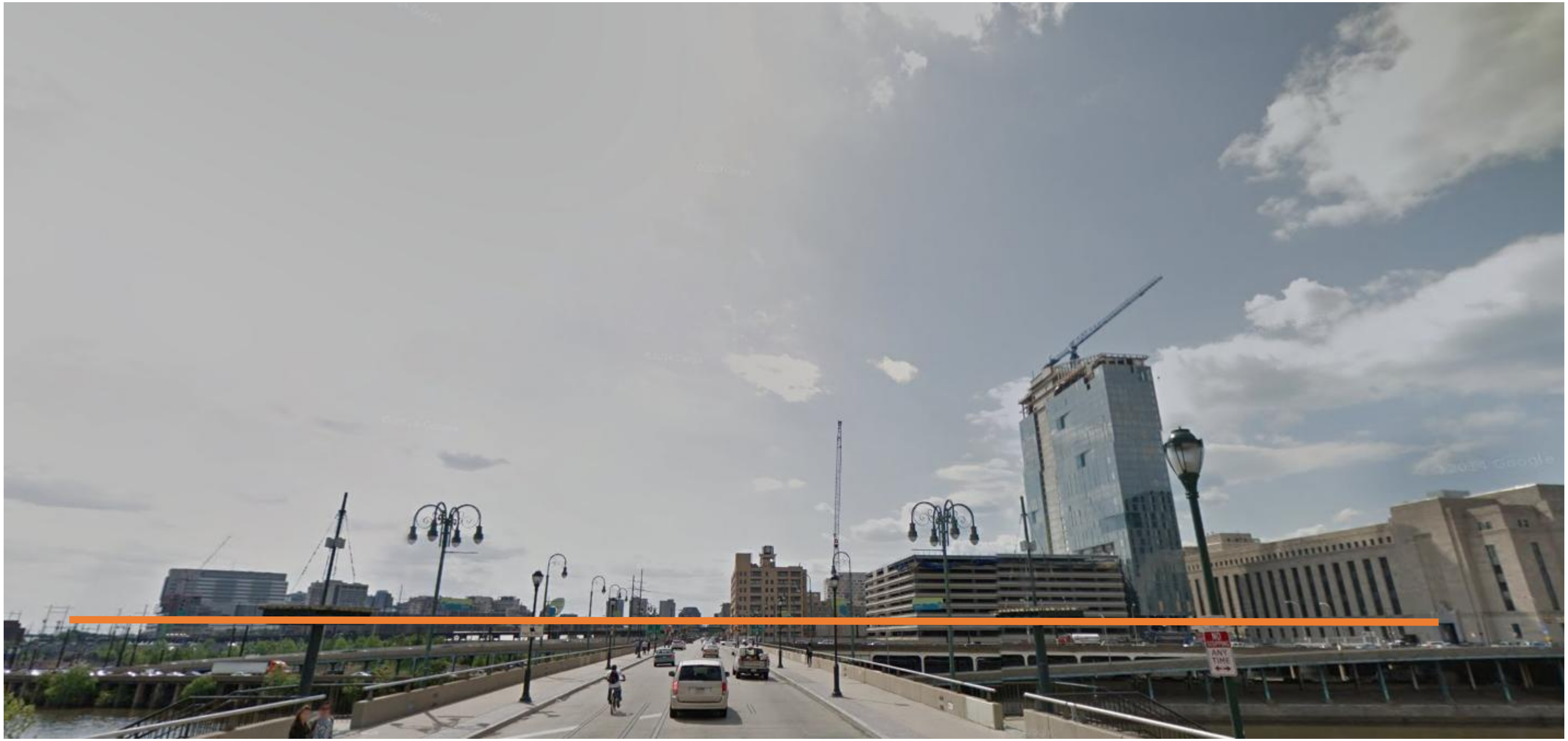
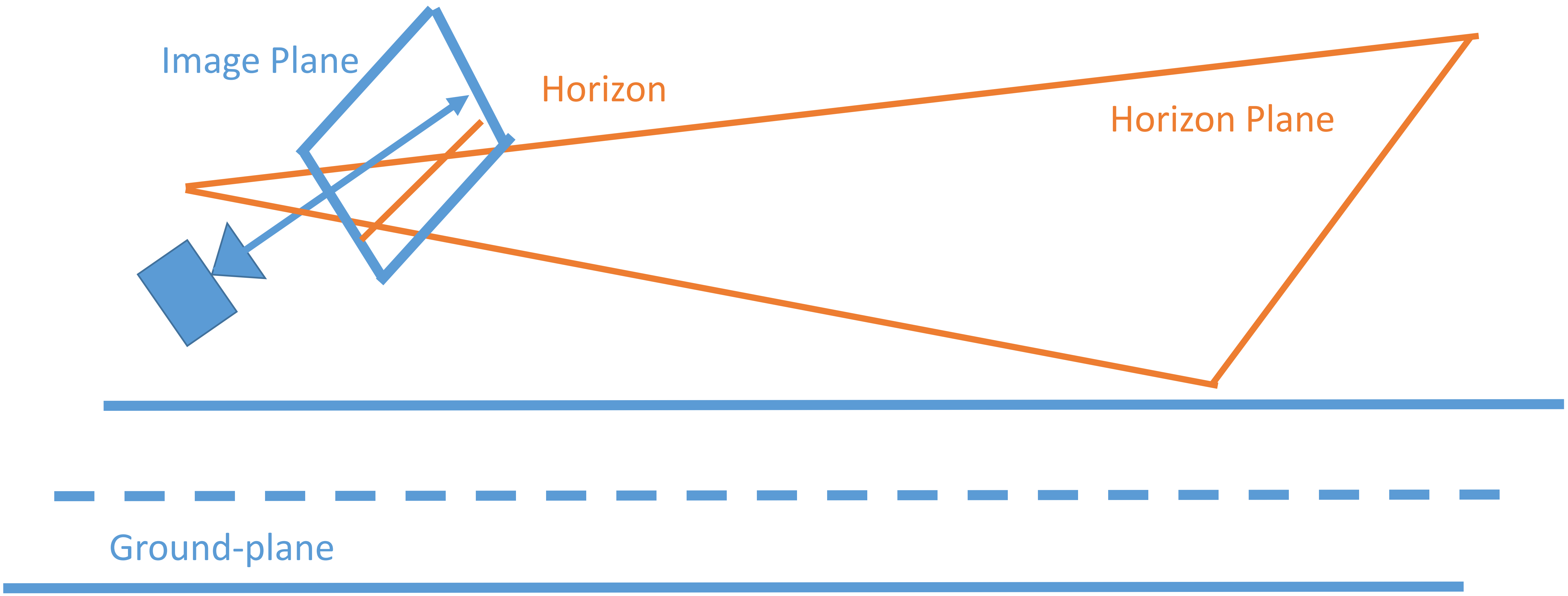


If horizon moves to the bottom it means we look upwards!

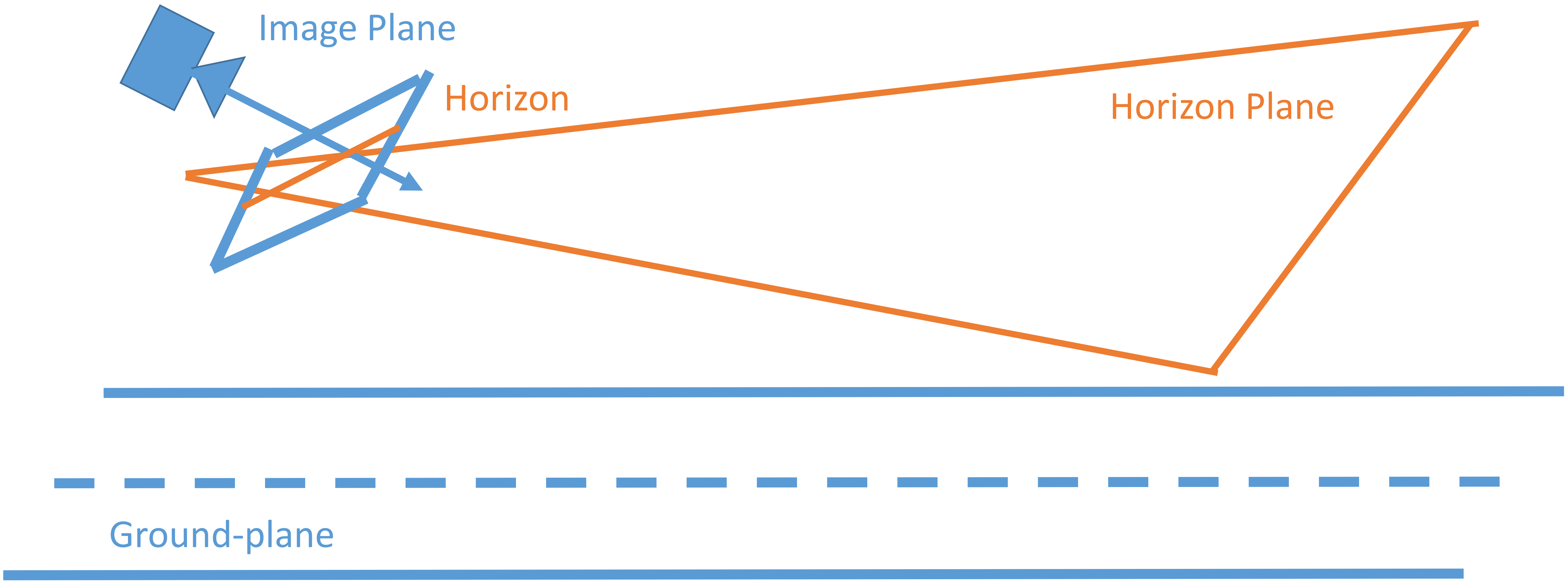


If horizon moves to the bottom it means we look upwards!



If horizon moves to the top it means we look downwards !

If horizon moves to the bottom it means we look upwards!



Horizon tells us how camera is oriented.
Constrains the collineation !