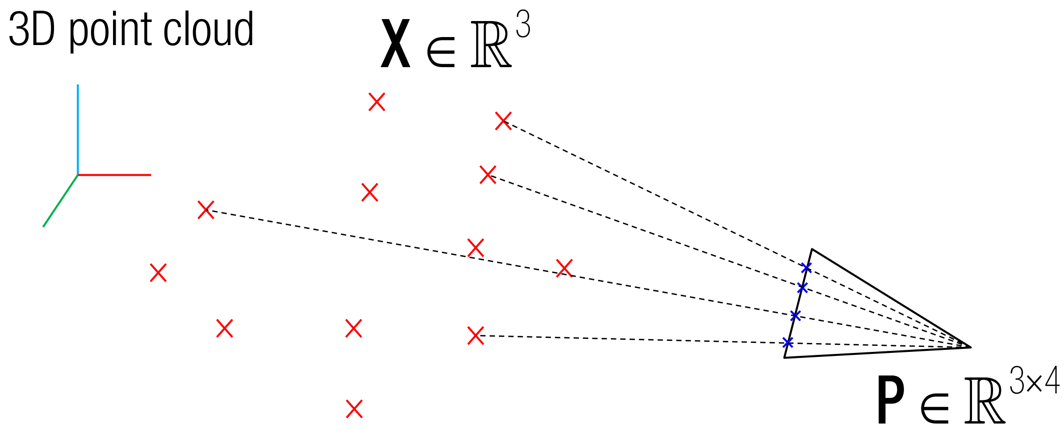
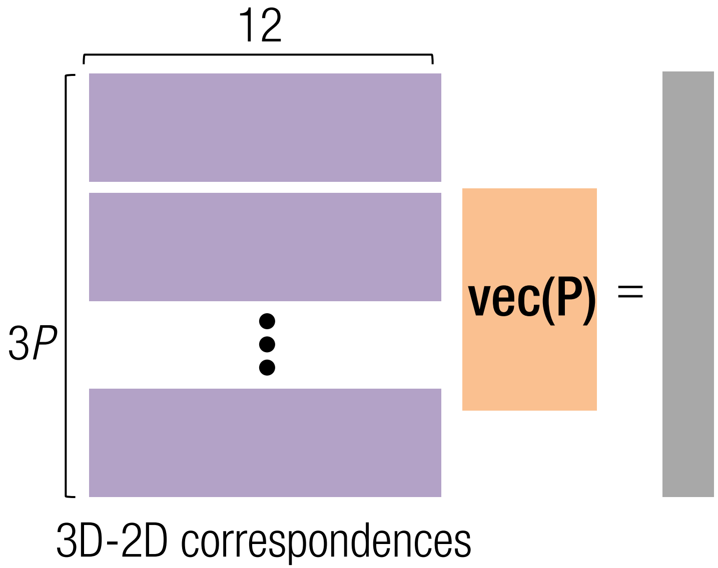
Example I: Perspective-n-Point

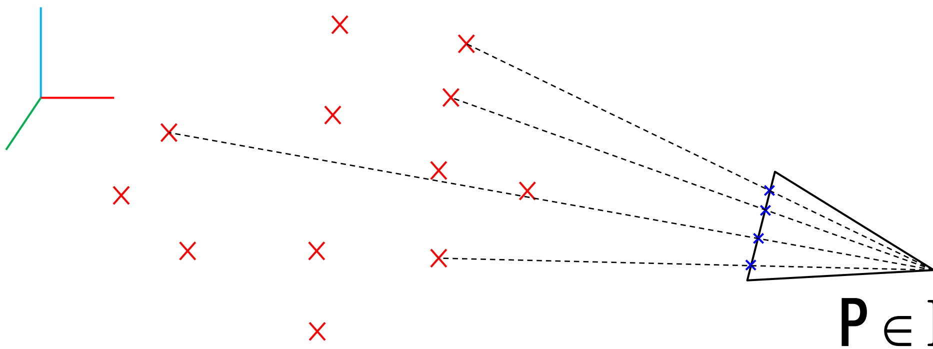




Example I: Perspective-n-Point

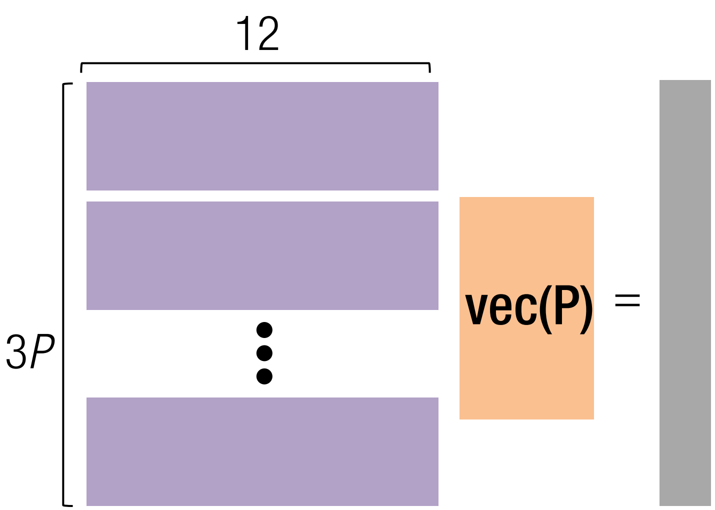
3D point cloud

$\mathbf{X} \in \mathbb{R}^3$



$\mathbf{P} \in \mathbb{R}^{3\times 4} \rightarrow \mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \in \mathbb{R}^{3\times 4}$

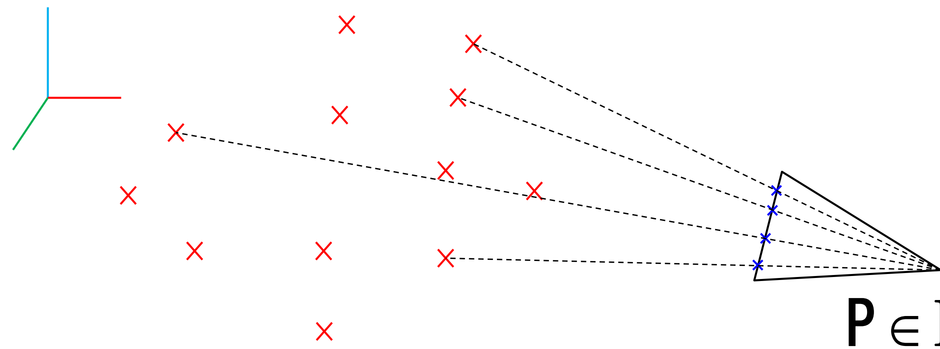
where $\mathbf{R} \in \mathbf{SO}(3)$



3D-2D correspondences

Example I: Perspective-n-Point

3D point cloud $\mathbf{X} \in \mathbb{R}^3$



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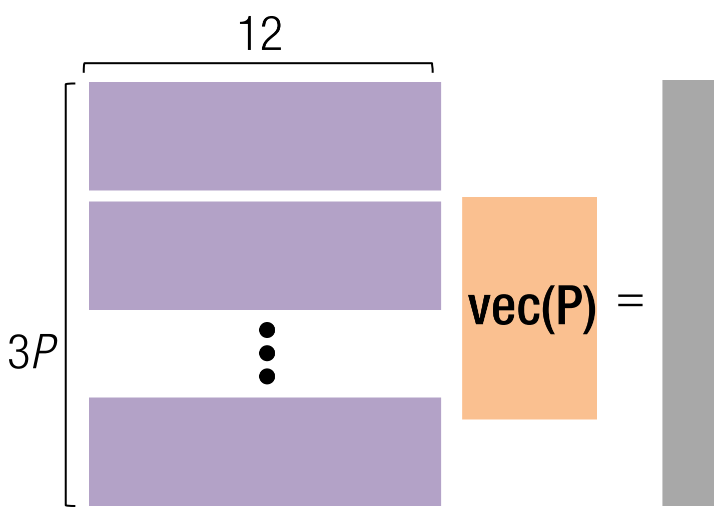
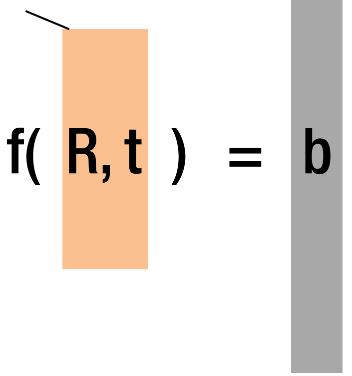


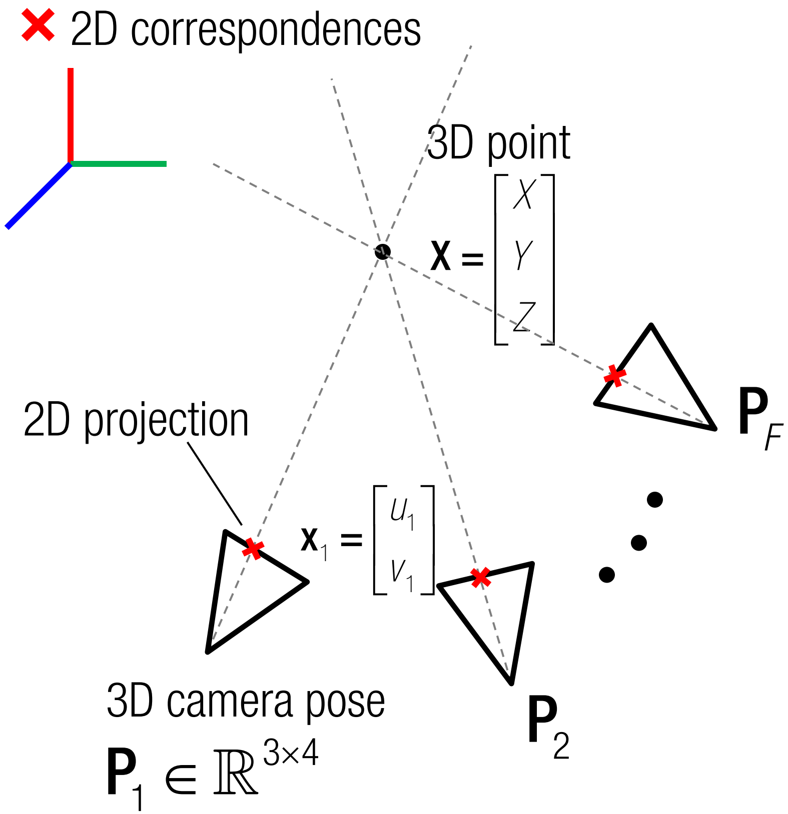
3D-2D correspondences

6 parameters: **R**(3), **t**(3)



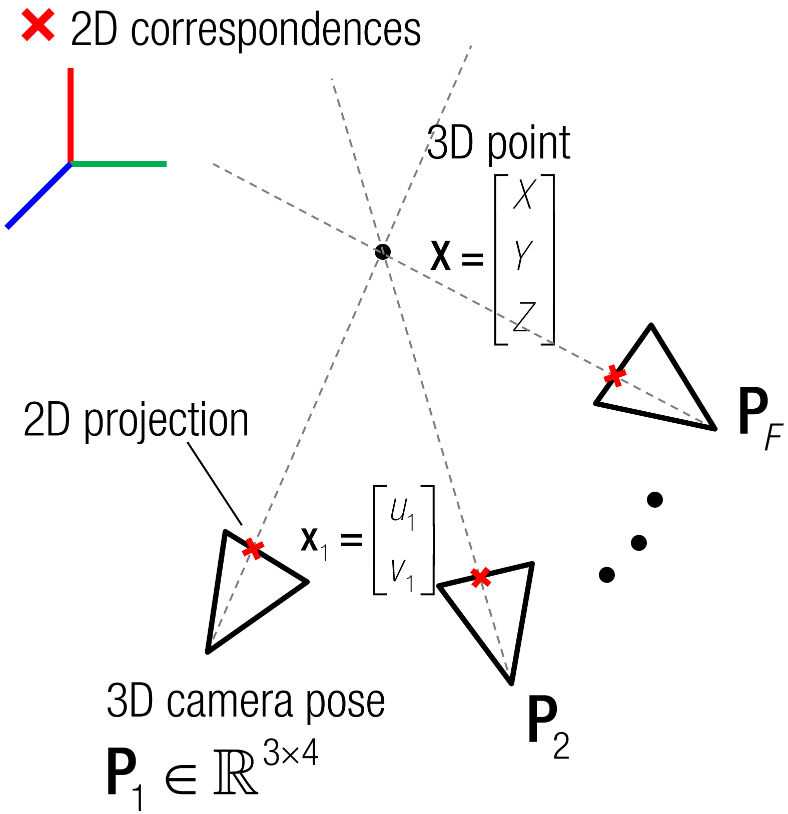
Nonlinear least squares

Example II: Triangulation



$$\begin{array}{cc} & 4 \\ 3F & \begin{bmatrix} \begin{bmatrix}\mathbf{x}_1 \\ 1\end{bmatrix}_\times \mathbf{P}_1 \\ \begin{bmatrix}\mathbf{x}_2 \\ 1\end{bmatrix}_\times \mathbf{P}_2 \\ \vdots \\ \begin{bmatrix}\mathbf{x}_F \\ 1\end{bmatrix}_\times \mathbf{P}_F \end{bmatrix} \end{array} \begin{bmatrix}\mathbf{X} \\ 1\end{bmatrix} = \mathbf{0}$$

Example II: Triangulation

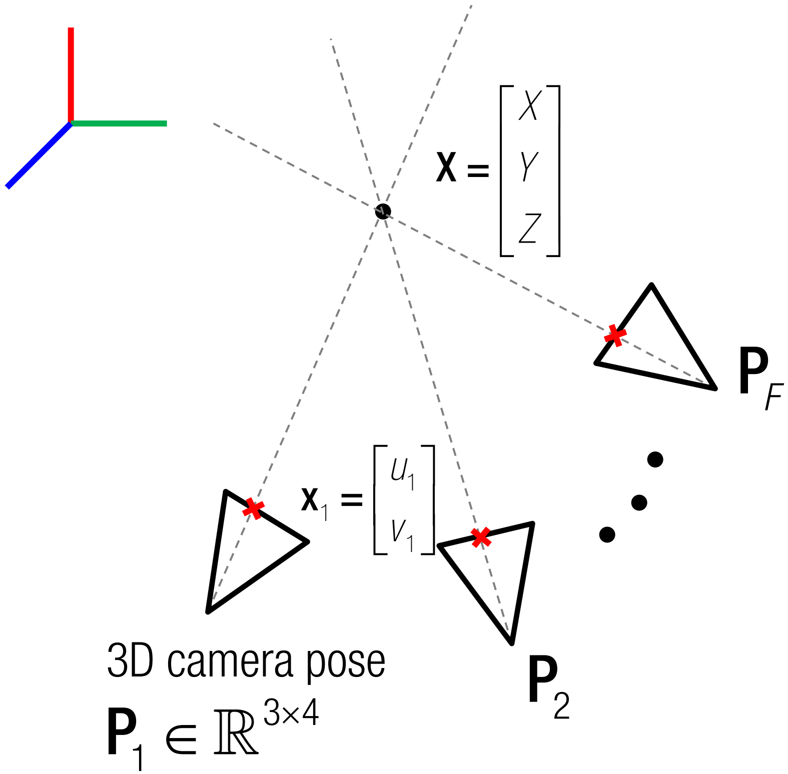


$$3F \; \overbrace{\begin{bmatrix} \begin{bmatrix}\mathbf{x}_1\\ 1\end{bmatrix}_\times \mathbf{P}_1 \\ \begin{bmatrix}\mathbf{x}_2\\ 1\end{bmatrix}_\times \mathbf{P}_2 \\ \vdots \\ \begin{bmatrix}\mathbf{x}_F\\ 1\end{bmatrix}_\times \mathbf{P}_F \end{bmatrix}}^{4} \begin{bmatrix}\mathbf{X}\\ 1\end{bmatrix} = \mathbf{0}$$

$$\min_{\mathbf{x}} \left\| \mathbf{Ax} - \mathbf{b} \right\|^2$$

Minimizes an algebraic error, i.e., there is no geometrical meaning.

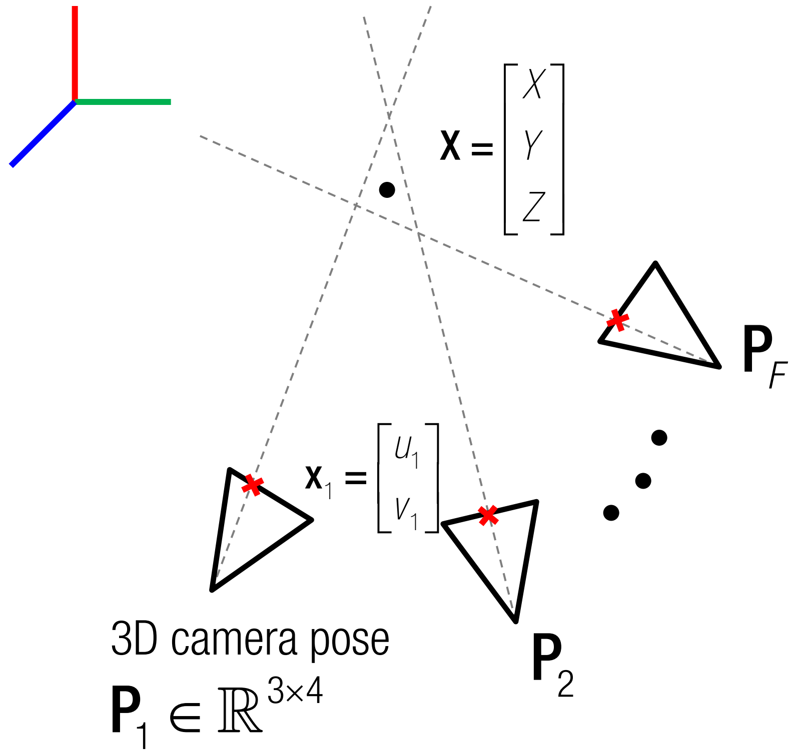
Example II: Triangulation



No noise in **x** and **P**.

✖ 2D correspondences

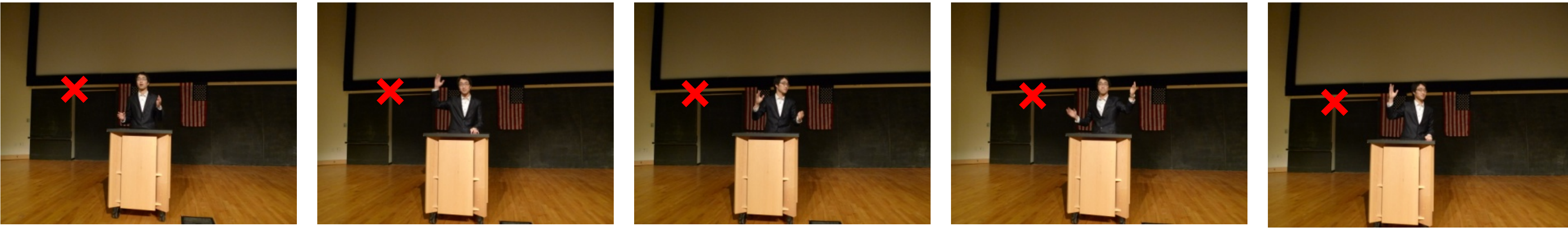
Example II: Triangulation



Noise in **x** and **P**.

→Rays do not meet at a 3D point.

✖ 2D correspondences



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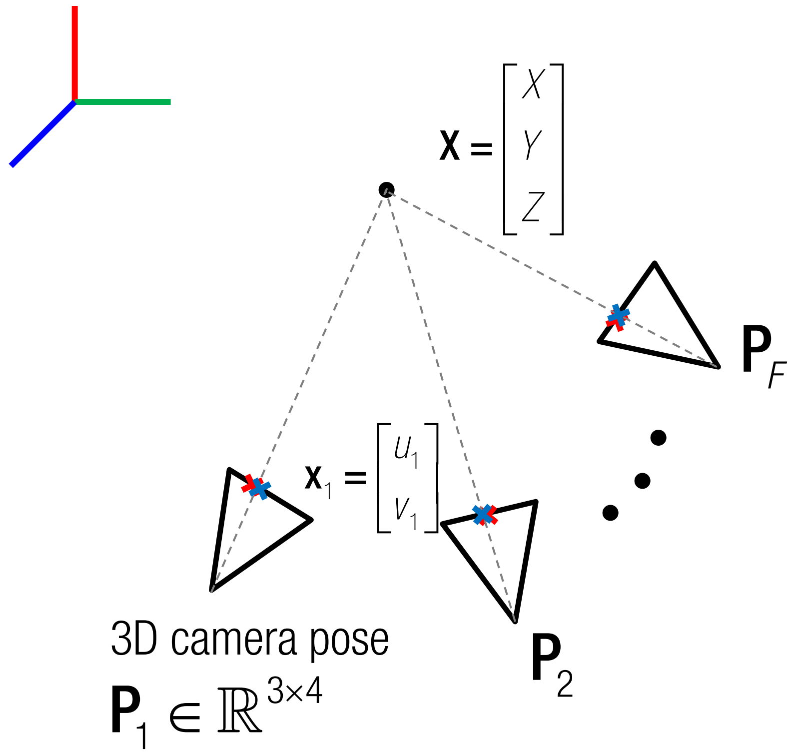

Noise in **x** and **P**.

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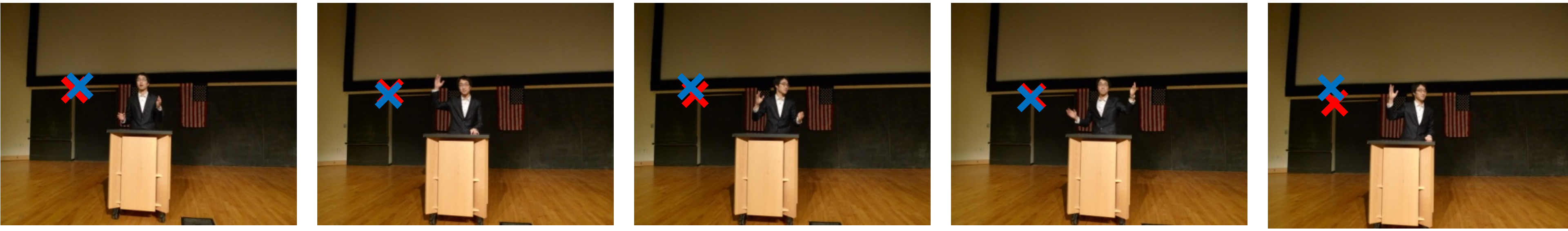
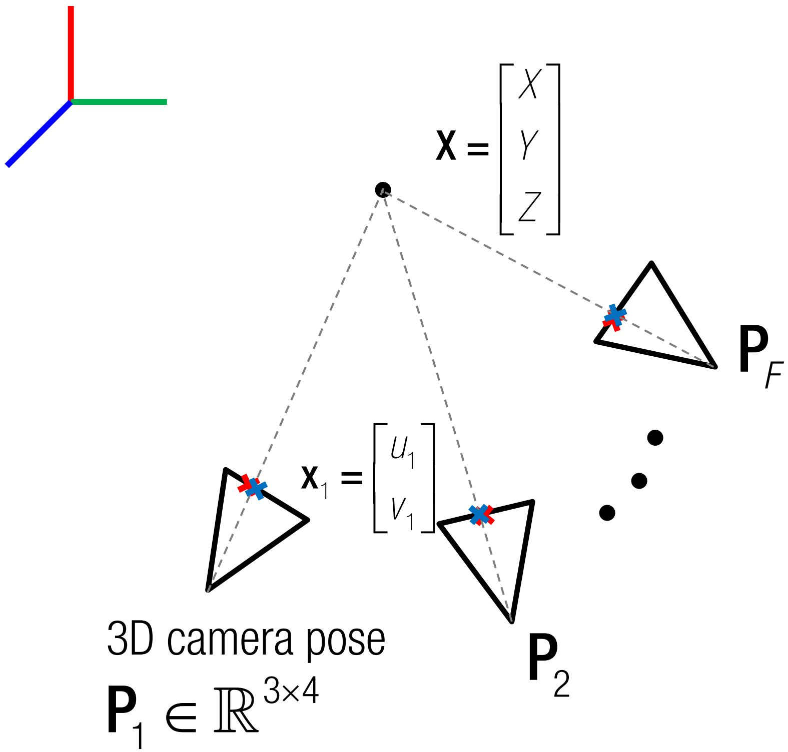
2D correspondences

Reprojection



Reprojection Error (Geometric Error)



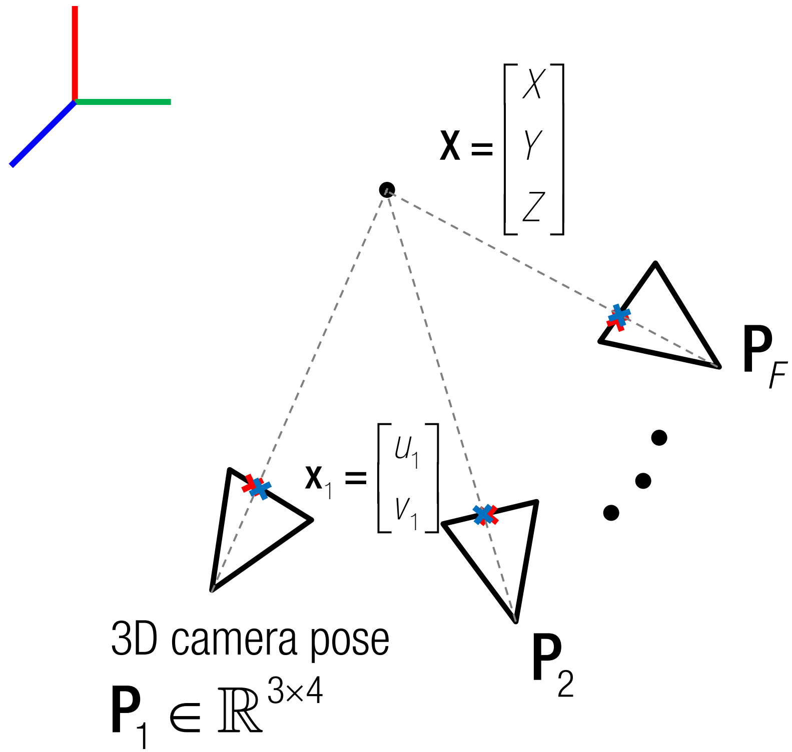
Noise in **x** and **P**.

→Rays do not meet at a 3D point.

✕ 2D correspondences

✕ Reprojection

Reprojection Error (Geometric Error)



Noise in **x** and **P**.

→Rays do not meet at a 3D point.

✕ 2D correspondences

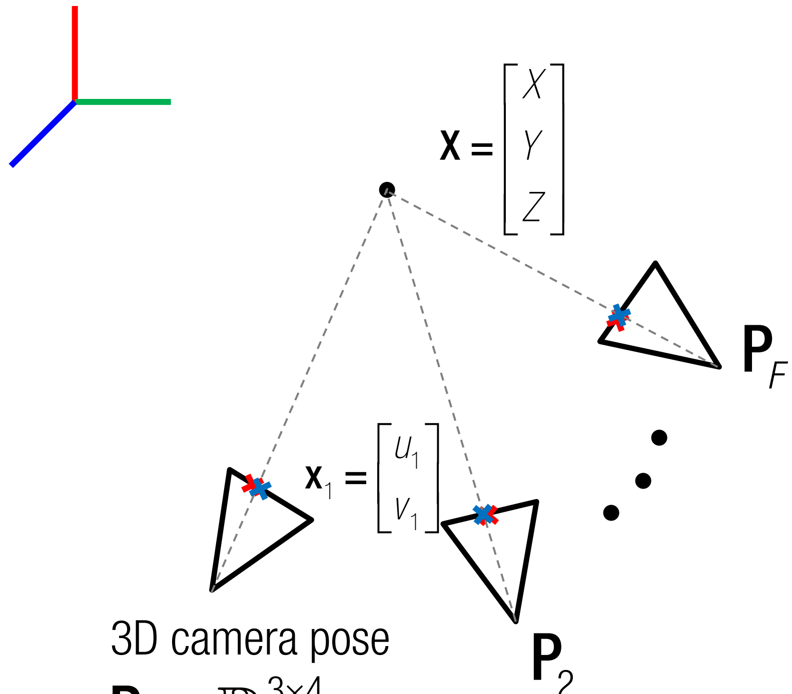
✕ Reprojection

✕ (u, v)

✕ $(u_{\mathrm{repro}}, v_{\mathrm{repro}}) = \left(\dfrac{\mathbf{P}^1\tilde{\mathbf{X}}}{\mathbf{P}^3\tilde{\mathbf{X}}}, \dfrac{\mathbf{P}^2\tilde{\mathbf{X}}}{\mathbf{P}^3\tilde{\mathbf{X}}} \right)$

where $\mathbf{P} = \begin{bmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{bmatrix}$ and $\tilde{\mathbf{X}} = \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$

Reprojection Error (Geometric Error)



3D camera pose

$\mathbf{P}_1 \in \mathbb{R}^{3\times 4}$

$$E_{\text{repro}} = \left(u - u_{\text{repro}}\right)^2 + \left(v - v_{\text{repro}}\right)^2$$

$$= \left(u - \frac{\mathbf{P}^1\tilde{\mathbf{X}}}{\mathbf{P}^3\tilde{\mathbf{X}}}\right)^2 + \left(v - \frac{\mathbf{P}^2\tilde{\mathbf{X}}}{\mathbf{P}^3\tilde{\mathbf{X}}}\right)^2$$

$$\min_{\mathbf{X}} \sum_{i=1}^{F} \left(u_i - \frac{\mathbf{P}_i^1\tilde{\mathbf{X}}}{\mathbf{P}_i^3\tilde{\mathbf{X}}}\right)^2 + \left(v_i - \frac{\mathbf{P}_i^2\tilde{\mathbf{X}}}{\mathbf{P}_i^3\tilde{\mathbf{X}}}\right)^2$$

Reprojection error minimization

✖ (u, v)

✖ $\left(u_{\text{repro}}, v_{\text{repro}}\right) = \left(\frac{\mathbf{P}^1\tilde{\mathbf{X}}}{\mathbf{P}^3\tilde{\mathbf{X}}}, \frac{\mathbf{P}^2\tilde{\mathbf{X}}}{\mathbf{P}^3\tilde{\mathbf{X}}}\right)$

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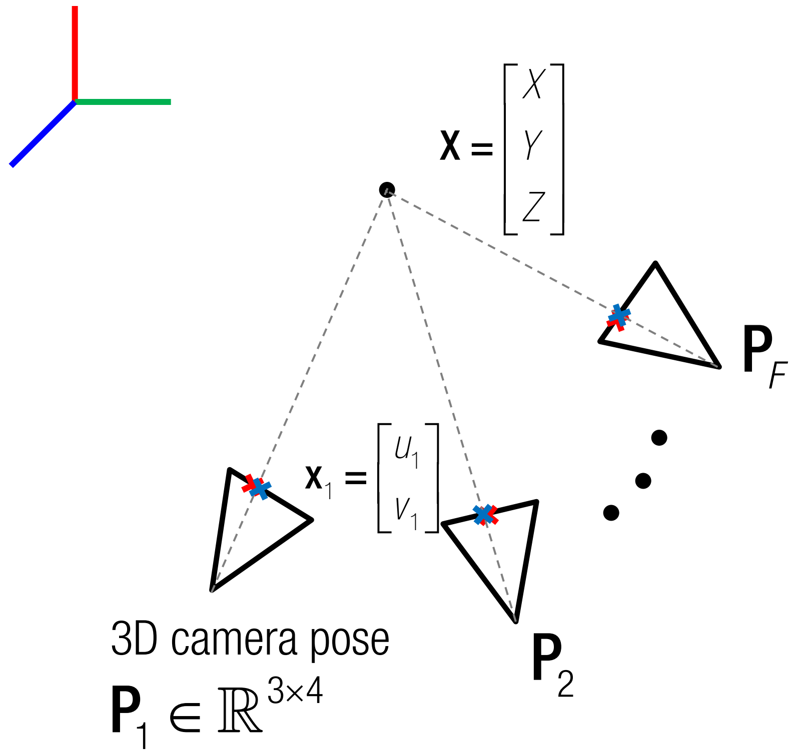

$$E_{\text{repro}} = \left(u - u_{\text{repro}}\right)^2 + \left(v - v_{\text{repro}}\right)^2$$

$$= \left(u - \frac{\mathbf{P}^1\tilde{\mathbf{X}}}{\mathbf{P}^3\tilde{\mathbf{X}}}\right)^2 + \left(v - \frac{\mathbf{P}^2\tilde{\mathbf{X}}}{\mathbf{P}^3\tilde{\mathbf{X}}}\right)^2$$

$$\mathbf{f}\left(\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}\right) = \begin{bmatrix} u_1 \\ v_1 \\ \vdots \\ u_F \\ v_F \end{bmatrix}$$

Nonlinear least squares

× (u,v)

× $\left(u_{\text{repro}}, v_{\text{repro}}\right) = \left(\frac{\mathbf{P}^1\tilde{\mathbf{X}}}{\mathbf{P}^3\tilde{\mathbf{X}}}, \frac{\mathbf{P}^2\tilde{\mathbf{X}}}{\mathbf{P}^3\tilde{\mathbf{X}}}\right)$

where $\mathbf{P} = \begin{bmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{bmatrix}$ and $\tilde{\mathbf{X}} = \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$