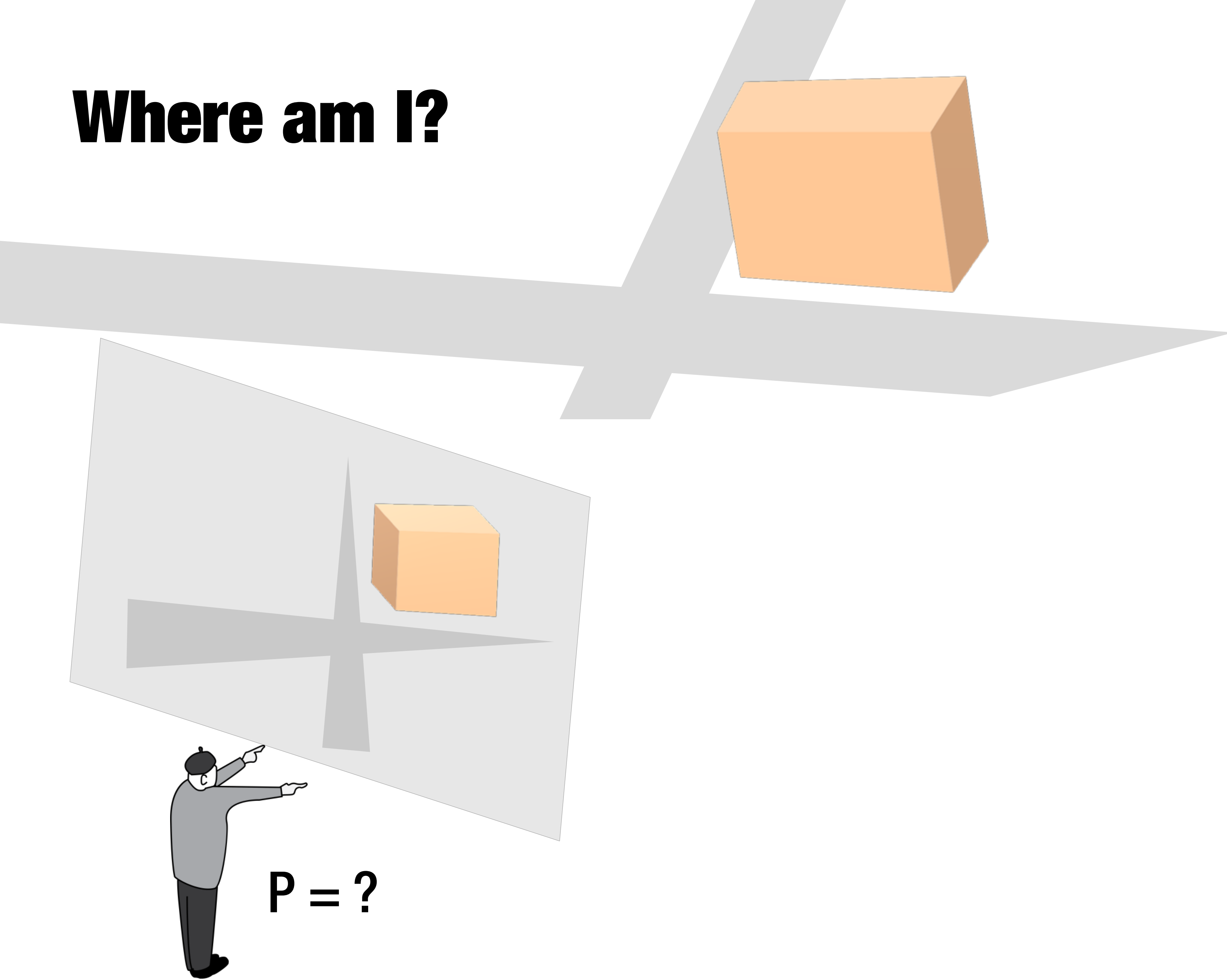
# Where am I?



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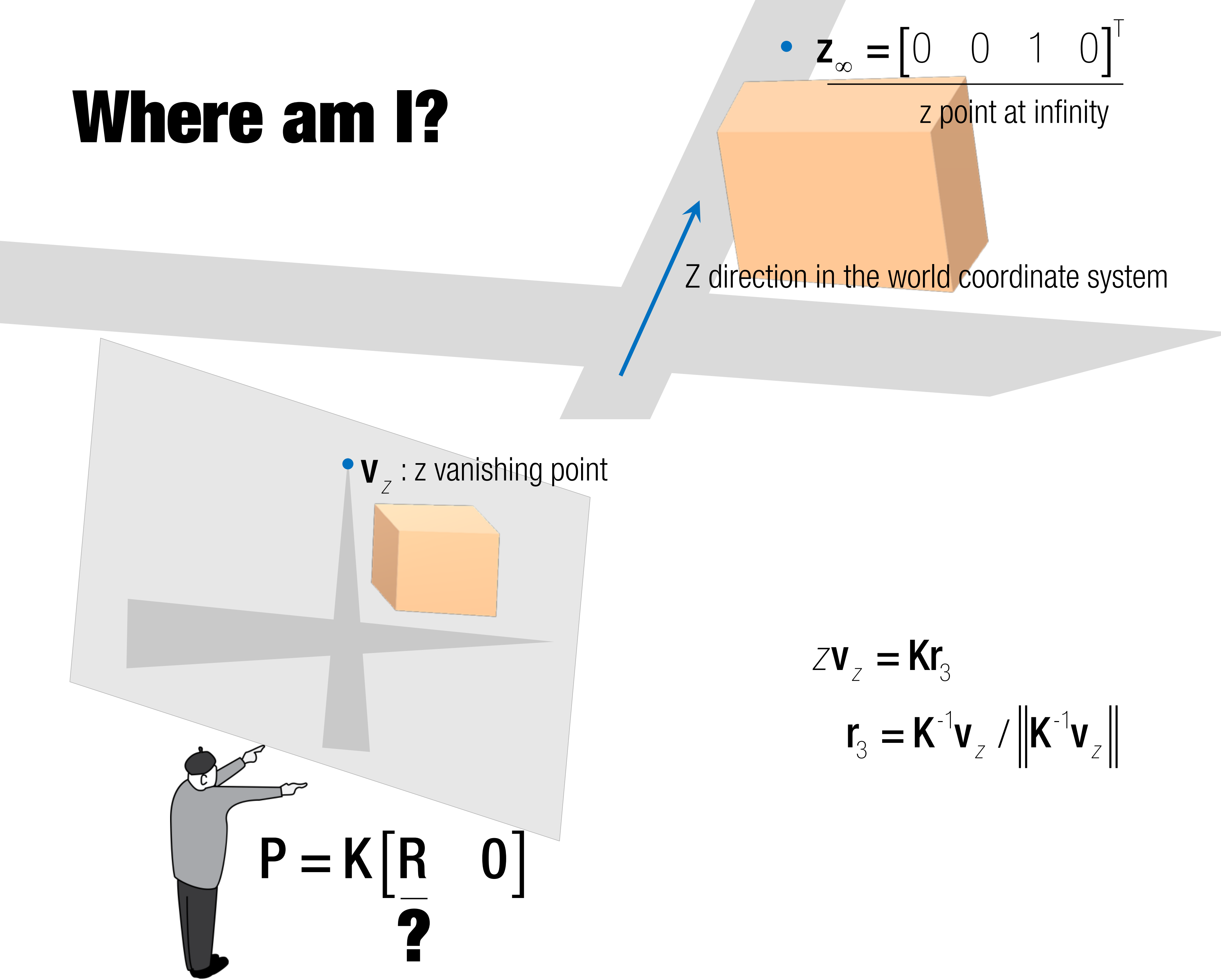


$$\mathbf{z}_\infty = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^T$$

$\mathbf{v}_Z$ : z vanishing point

$$Z\mathbf{v}_Z = \mathbf{K}\mathbf{r}_3$$

$$\mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{v}_Z / \left\|\mathbf{K}^{-1}\mathbf{v}_Z\right\|$$

$$\mathbf{P} = \mathbf{K}\begin{bmatrix} \underline{\mathbf{R}} & \mathbf{0} \end{bmatrix}$$
$$\mathbf{?}$$

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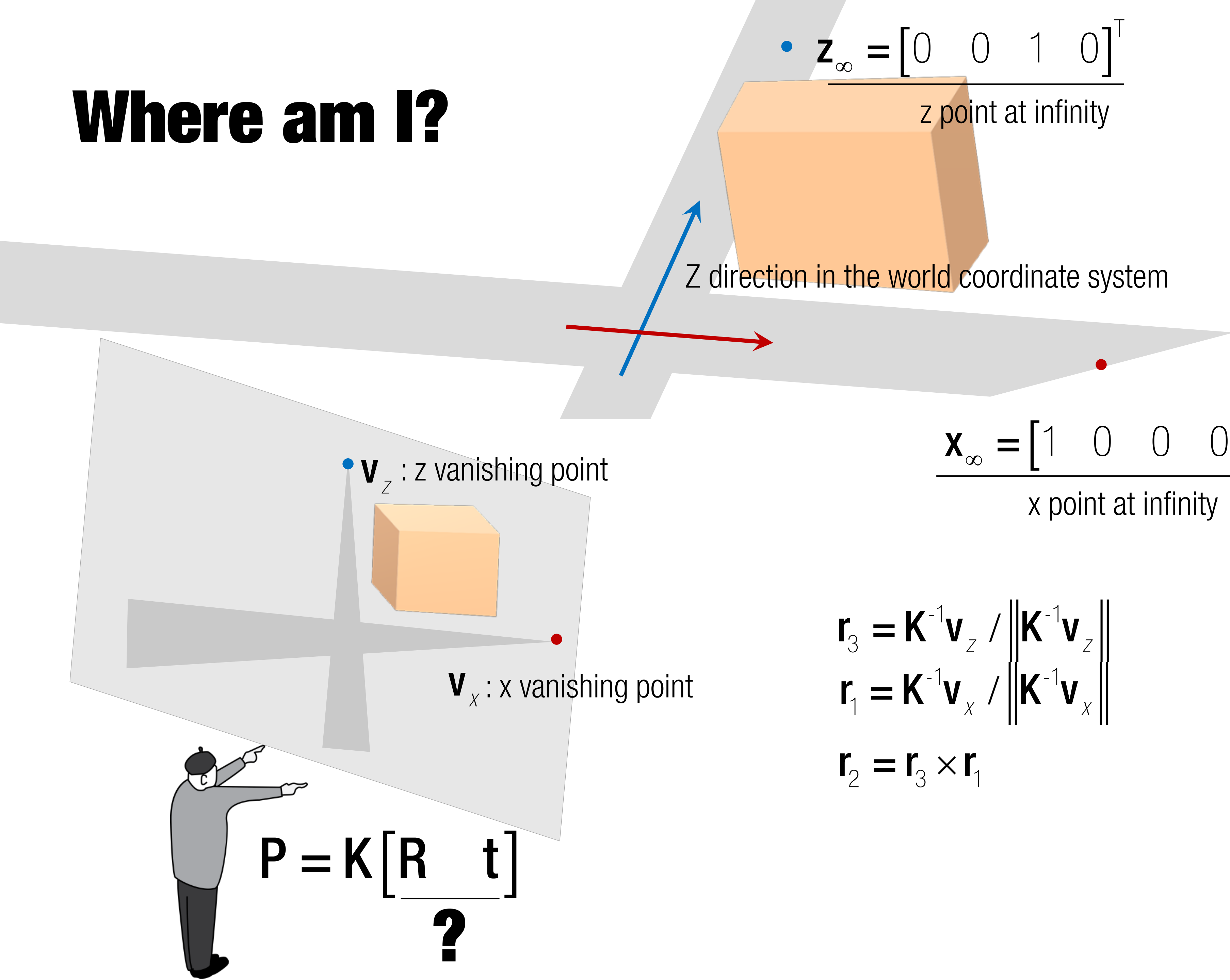


$$\underbrace{\mathbf{z}_\infty = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^\mathsf{T}}_{\text{z point at infinity}}$$

$$\underbrace{\mathbf{x}_\infty = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^\mathsf{T}}_{\text{x point at infinity}}$$

$$\mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{v}_Z \,/\, \left\|\mathbf{K}^{-1}\mathbf{v}_Z\right\|$$

$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{v}_X \,/\, \left\|\mathbf{K}^{-1}\mathbf{v}_X\right\|$$

$$\mathbf{r}_2 = \mathbf{r}_3 \times \mathbf{r}_1$$

$$\mathbf{P} = \mathbf{K}\underbrace{\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}}_{\textbf{?}}$$

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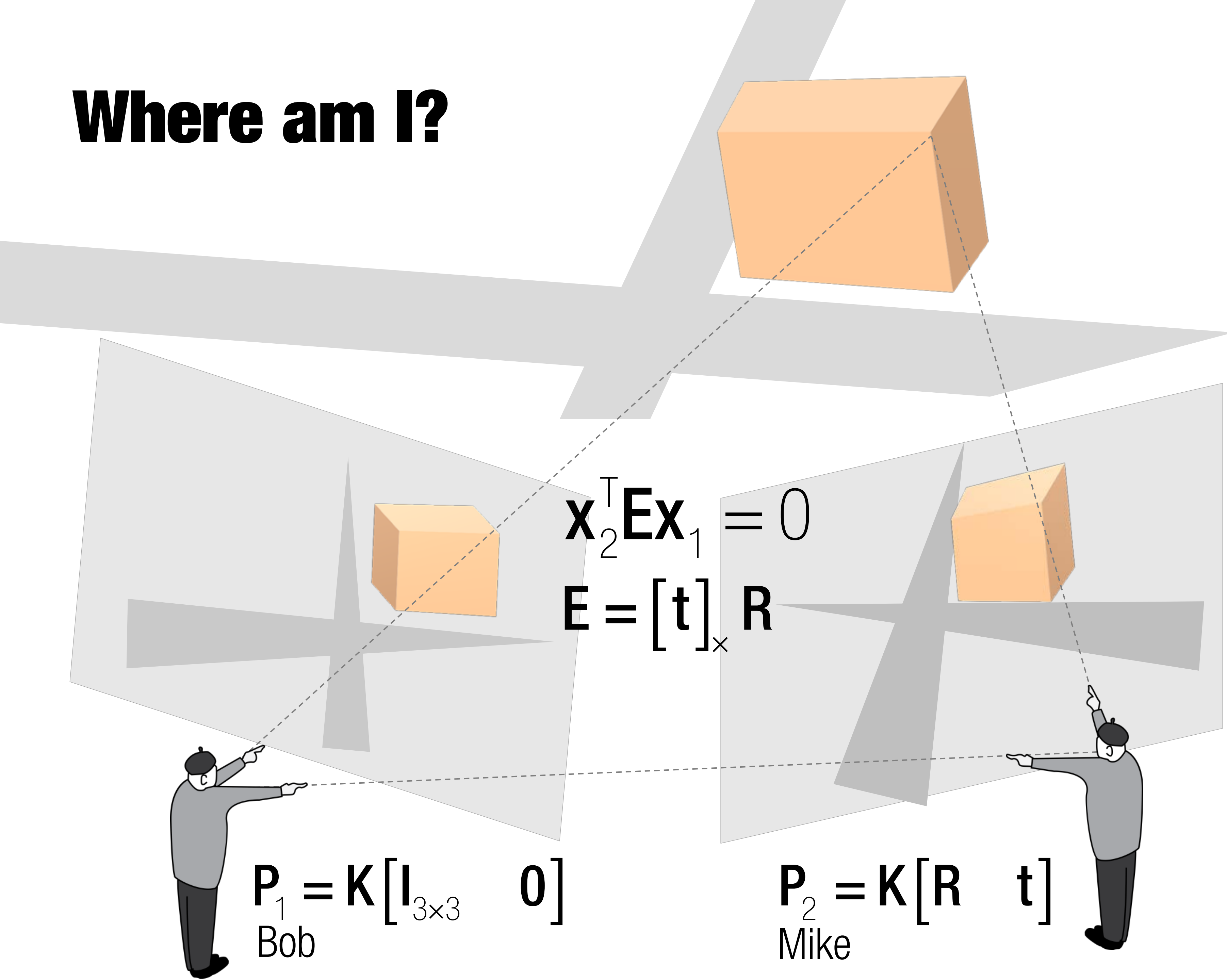


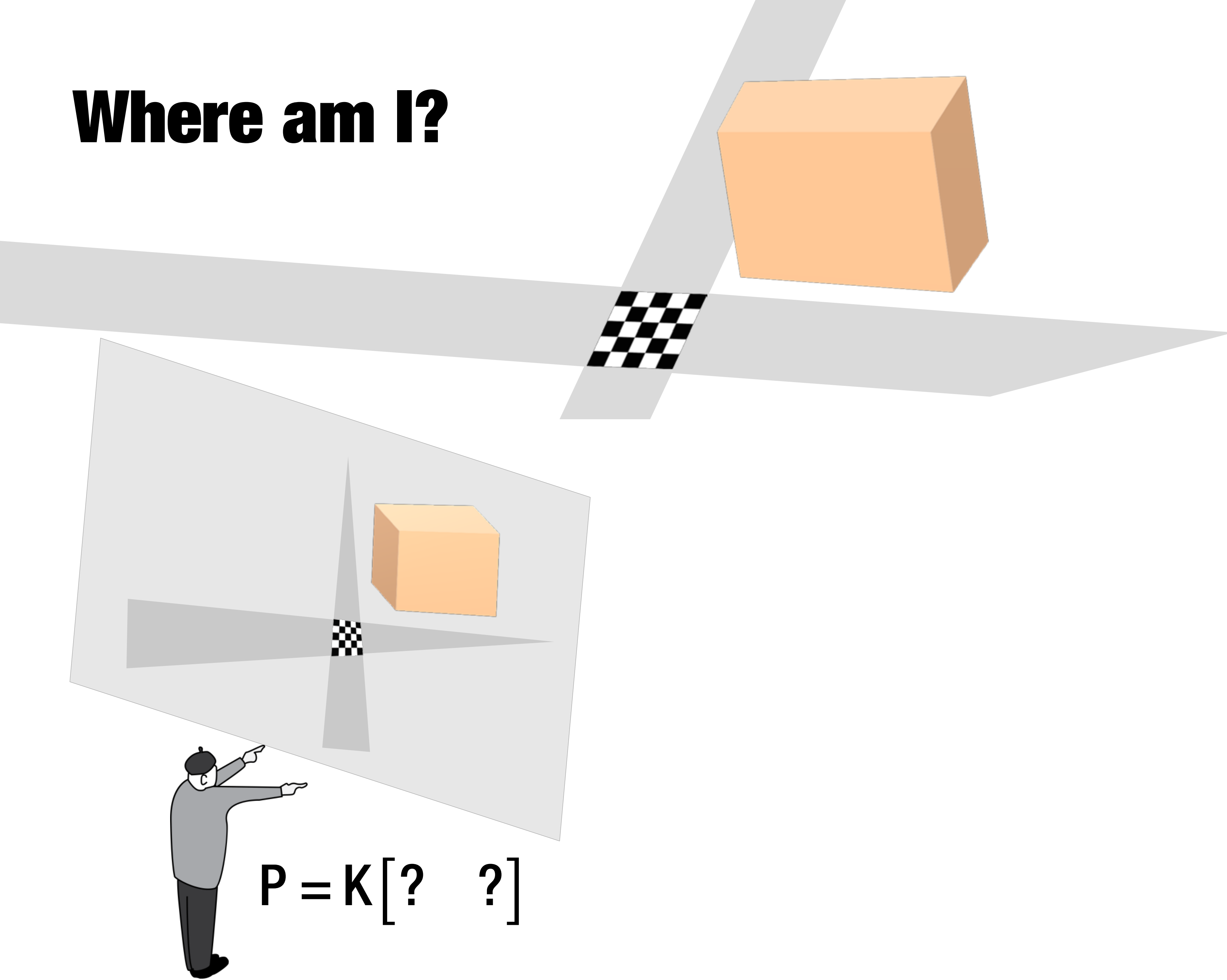

$$\mathbf{x}_2^{\mathsf{T}}\mathbf{E}\mathbf{x}_1 = 0$$

$$\mathbf{E} = [\mathbf{t}]_{\times}\mathbf{R}$$

$$\mathbf{P}_1 = \mathbf{K}\begin{bmatrix}\mathbf{I}_{3\times3} & \mathbf{0}\end{bmatrix}$$

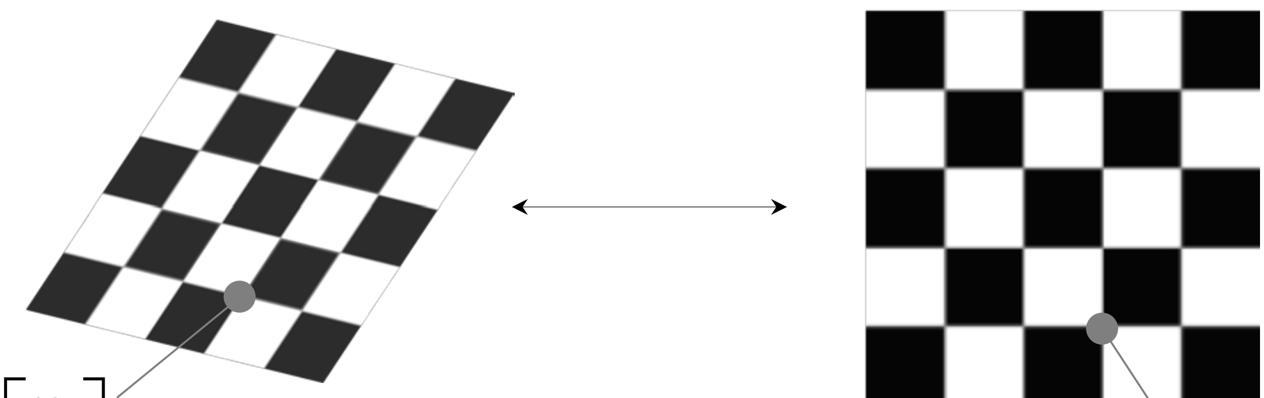
$$\mathbf{P}_2 = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$$

# Where am I?



$$P = K\left[ ? \quad ? \right]$$

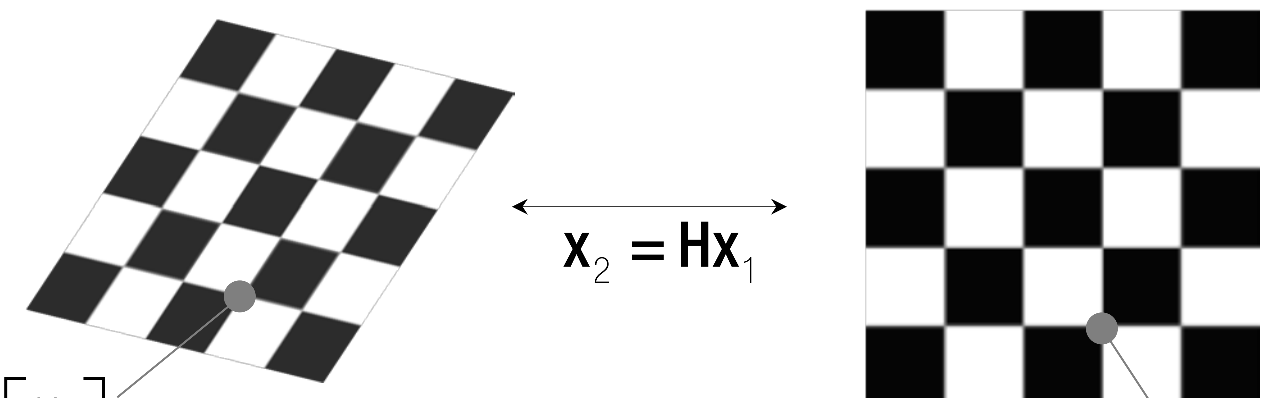
# Homography Linear Estimation



$$\mathbf{x}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}$$

$$\mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

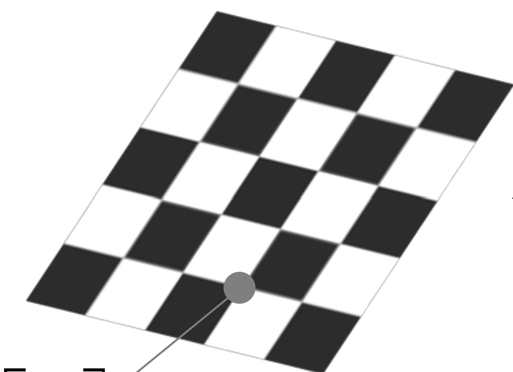
# Homography Linear Estimation

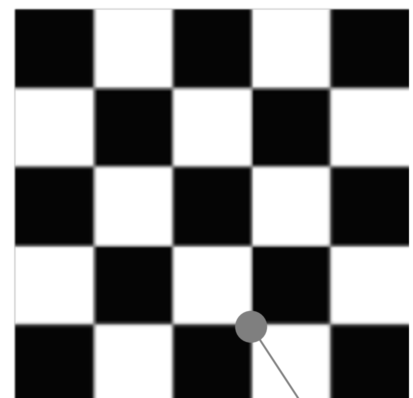


$$\mathbf{x}_2 = \mathbf{H}\mathbf{x}_1$$

$$\mathbf{x}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} \qquad \mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

# Homography Linear Estimation





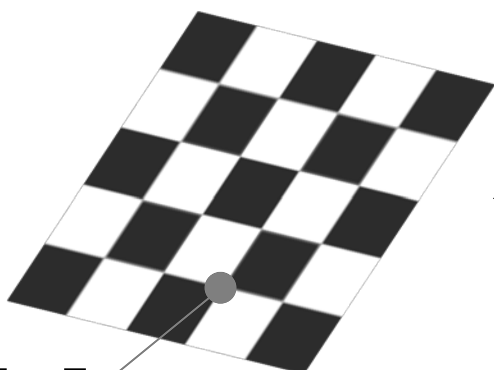
$$\mathbf{x}_2 = \mathbf{H}\mathbf{x}_1$$

$$\mathbf{x}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}$$

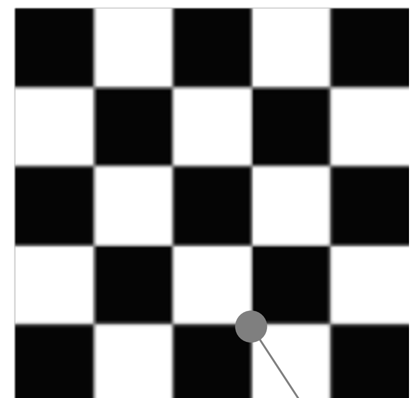
$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

$$\mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

# Homography Linear Estimation



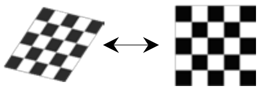
$$\mathbf{x}_2 = \mathbf{H}\mathbf{x}_1$$



$$\mathbf{x}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} \qquad \mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \qquad \mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

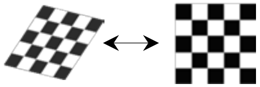
How to estimate $\mathbf{H}$?

# Homography Linear Estimation



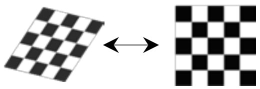
$$\mathbf{x}_2 = \mathbf{H}\mathbf{x}_1$$

# Homography Linear Estimation



$$\mathbf{x}_2 = \mathbf{H}\mathbf{x}_1 \longrightarrow [\mathbf{x}_2]_\times \mathbf{H}\mathbf{x}_1 = \mathbf{0}$$

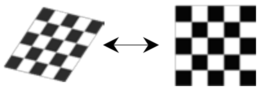
# Homography Linear Estimation



$$\mathbf{x}_2 = \mathbf{H}\mathbf{x}_1 \longrightarrow [\mathbf{x}_2]_\times \mathbf{H}\mathbf{x}_1 = \mathbf{0}$$

$$\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{bmatrix} \mathbf{x}_1$$

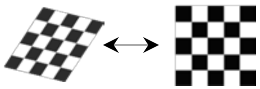
# Homography Linear Estimation



$$\mathbf{x}_2 = \mathbf{H}\mathbf{x}_1 \qquad \longrightarrow \qquad \left[\mathbf{x}_2\right]_\times \mathbf{H}\mathbf{x}_1 = 0$$

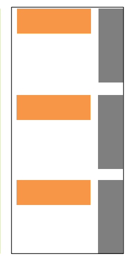
$$\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{bmatrix} \mathbf{x}_1 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{h}_1\mathbf{x}_1 \\ \mathbf{h}_2\mathbf{x}_1 \\ \mathbf{h}_3\mathbf{x}_1 \end{bmatrix}$$

# Homography Linear Estimation

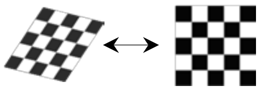


$$\mathbf{x}_2 = \mathbf{H}\mathbf{x}_1 \longrightarrow \left[\mathbf{x}_2\right]_\times \mathbf{H}\mathbf{x}_1 = \mathbf{0} \longrightarrow$$

$$\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{bmatrix} \mathbf{x}_1 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{h}_1\mathbf{x}_1 \\ \mathbf{h}_2\mathbf{x}_1 \\ \mathbf{h}_3\mathbf{x}_1 \end{bmatrix} = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times$$

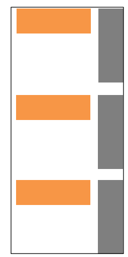


# Homography Linear Estimation

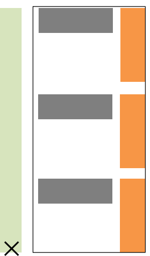


$$\mathbf{x}_2 = \mathbf{H}\mathbf{x}_1 \longrightarrow [\mathbf{x}_2]_\times \mathbf{H}\mathbf{x}_1 = \mathbf{0} \longrightarrow$$

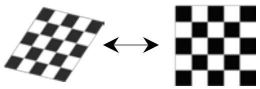
$$\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{bmatrix} \mathbf{x}_1 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{h}_1\mathbf{x}_1 \\ \mathbf{h}_2\mathbf{x}_1 \\ \mathbf{h}_3\mathbf{x}_1 \end{bmatrix} = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times$$



$$= \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times$$

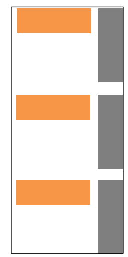


# Homography Linear Estimation

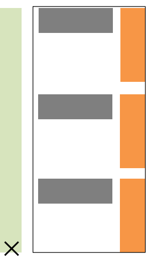


$$\mathbf{x}_2 = \mathbf{H}\mathbf{x}_1 \longrightarrow \left[\mathbf{x}_2\right]_\times \mathbf{H}\mathbf{x}_1 = \mathbf{0} \longrightarrow$$

$$\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{bmatrix} \mathbf{x}_1 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{h}_1\mathbf{x}_1 \\ \mathbf{h}_2\mathbf{x}_1 \\ \mathbf{h}_3\mathbf{x}_1 \end{bmatrix} = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times$$

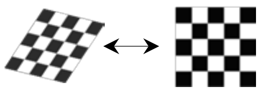


$$= \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times$$



$$= \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{x}_1^{\mathsf{T}}\mathbf{h}_3^{\mathsf{T}} \\ \mathbf{x}_2^{\mathsf{T}}\mathbf{h}_3^{\mathsf{T}} \\ \mathbf{x}_3^{\mathsf{T}}\mathbf{h}_3^{\mathsf{T}} \end{bmatrix}$$

# Homography Linear Estimation

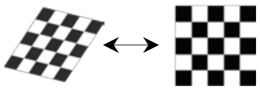


$$\mathbf{x}_2 = \mathbf{H}\mathbf{x}_1 \longrightarrow [\mathbf{x}_2]_\times \mathbf{H}\mathbf{x}_1 = \mathbf{0} \longrightarrow$$

$$\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{bmatrix} \mathbf{x}_1 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{h}_1\mathbf{x}_1 \\ \mathbf{h}_2\mathbf{x}_1 \\ \mathbf{h}_3\mathbf{x}_1 \end{bmatrix} = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{x}_1^{\mathsf{T}}\mathbf{h}_3^{\mathsf{T}} \\ \mathbf{x}_2^{\mathsf{T}}\mathbf{h}_3^{\mathsf{T}} \\ \mathbf{x}_3^{\mathsf{T}}\mathbf{h}_3^{\mathsf{T}} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & v_2 \\ 1 & 0 & -u_2 \\ -v_2 & u_2 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1^{\mathsf{T}} & \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 3} \\ \mathbf{0}_{1\times 3} & \mathbf{x}_1^{\mathsf{T}} & \mathbf{0}_{1\times 3} \\ \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 3} & \mathbf{x}_1^{\mathsf{T}} \end{bmatrix} \begin{bmatrix} \mathbf{h}_1^{\mathsf{T}} \\ \mathbf{h}_2^{\mathsf{T}} \\ \mathbf{h}_3^{\mathsf{T}} \end{bmatrix}$$

# Homography Linear Estimation



$$\mathbf{x}_2 = \mathbf{H}\mathbf{x}_1 \longrightarrow [\mathbf{x}_2]_\times \mathbf{H}\mathbf{x}_1 = \mathbf{0} \longrightarrow$$

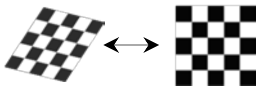
$$\begin{bmatrix}u_2\\ v_2\\ 1\end{bmatrix}_\times \begin{bmatrix}\mathbf{h}_1\\ \mathbf{h}_2\\ \mathbf{h}_3\end{bmatrix}\mathbf{x}_1 = \begin{bmatrix}u_2\\ v_2\\ 1\end{bmatrix}_\times \begin{bmatrix}\mathbf{h}_1\mathbf{x}_1\\ \mathbf{h}_2\mathbf{x}_1\\ \mathbf{h}_3\mathbf{x}_1\end{bmatrix} = \begin{bmatrix}u_2\\ v_2\\ 1\end{bmatrix}_\times = \begin{bmatrix}u_2\\ v_2\\ 1\end{bmatrix}_\times = \begin{bmatrix}u_2\\ v_2\\ 1\end{bmatrix}_\times \begin{bmatrix}\mathbf{x}_1^T\mathbf{h}_3^T\\ \mathbf{x}_2^T\mathbf{h}_3^T\\ \mathbf{x}_3^T\mathbf{h}_3^T\end{bmatrix}$$

$$= \begin{bmatrix}0 & -1 & v_2\\ 1 & 0 & -u_2\\ -v_2 & u_2 & 0\end{bmatrix} \begin{bmatrix}\mathbf{x}_1^T & \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 3}\\ \mathbf{0}_{1\times 3} & \mathbf{x}_1^T & \mathbf{0}_{1\times 3}\\ \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 3} & \mathbf{x}_1^T\end{bmatrix} \begin{bmatrix}\mathbf{h}_1^T\\ \mathbf{h}_2^T\\ \mathbf{h}_3^T\end{bmatrix}$$

$$= \begin{bmatrix}\mathbf{0}_{1\times 3} & -\mathbf{x}_1^T & v_2\mathbf{x}_1^T\\ \mathbf{x}_1^T & \mathbf{0}_{1\times 3} & -u_2\mathbf{x}_1^T\\ -v_2\mathbf{x}_1^T & u_2\mathbf{x}_1^T & \mathbf{0}_{1\times 3}\end{bmatrix} \begin{bmatrix}\mathbf{h}_1^T\\ \mathbf{h}_2^T\\ \mathbf{h}_3^T\end{bmatrix} = \mathbf{0} \longrightarrow \mathbf{A}\mathbf{x} = \mathbf{0}$$

3 x 9

# Homography Linear Estimation



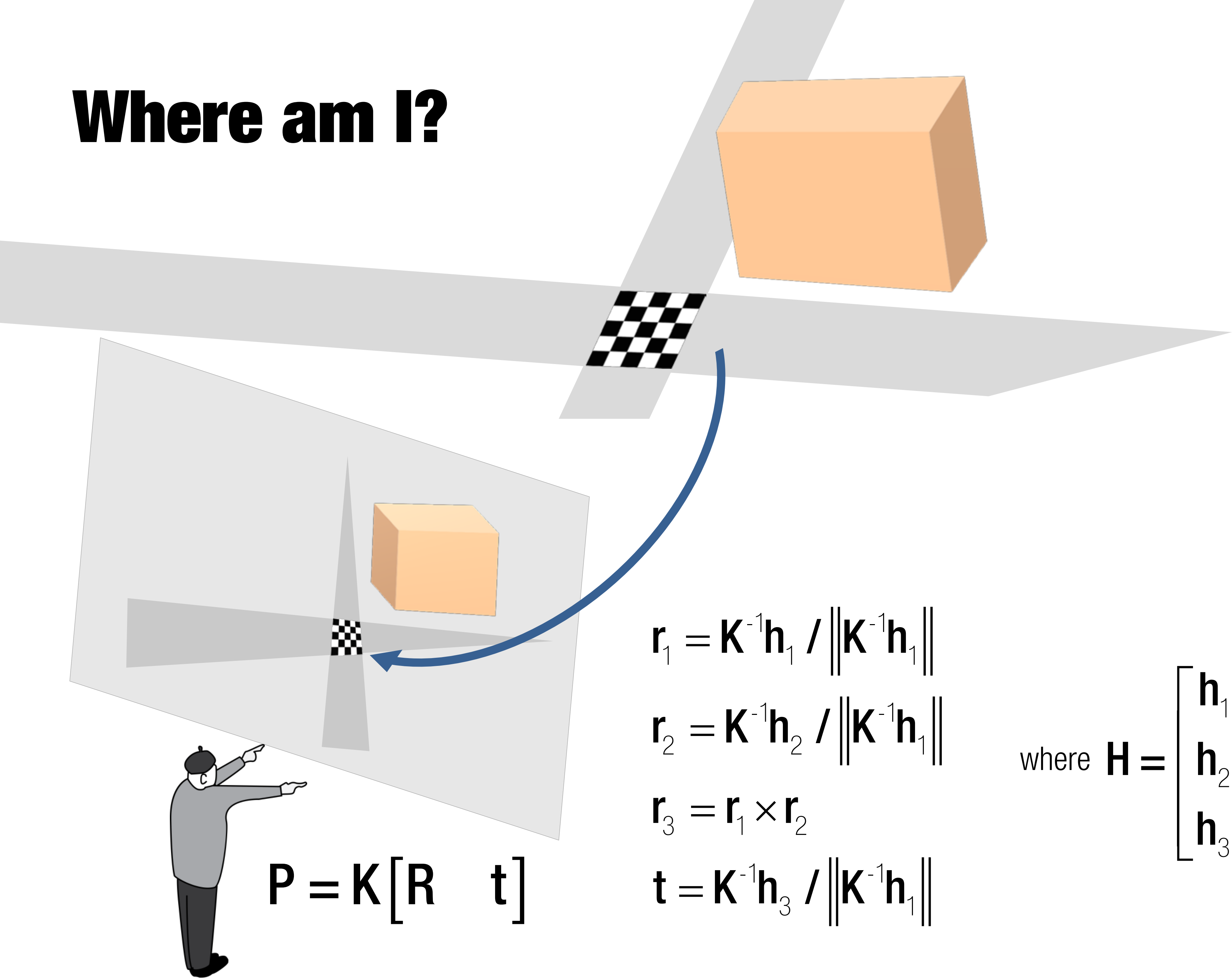
$$\mathbf{x}_2 = \mathbf{H}\mathbf{x}_1 \longrightarrow [\mathbf{x}_2]_\times \mathbf{H}\mathbf{x}_1 = \mathbf{0}$$

$$\underbrace{\begin{bmatrix} \mathbf{0}_{1\times 3} & -\mathbf{x}_1^{\mathsf{T}} & v_2\mathbf{x}_1^{\mathsf{T}} \\ \mathbf{x}_1^{\mathsf{T}} & \mathbf{0}_{1\times 3} & -u_2\mathbf{x}_1^{\mathsf{T}} \\ -v_2\mathbf{x}_1^{\mathsf{T}} & u_2\mathbf{x}_1^{\mathsf{T}} & \mathbf{0}_{1\times 3} \end{bmatrix}}_{3 \text{ x } 9} \begin{bmatrix} \mathbf{h}_1^{\mathsf{T}} \\ \mathbf{h}_2^{\mathsf{T}} \\ \mathbf{h}_3^{\mathsf{T}} \end{bmatrix} = \mathbf{0} \longrightarrow \mathbf{A}\mathbf{x} = \mathbf{0}$$

rank( ) = 2 because $[\mathbf{x}_2]_\times$ is a rank 2 matrix.

Therefore, 4 point correspondences are required to estimate a homography.

# Where am I?



$$P = K\begin{bmatrix} R & t \end{bmatrix}$$

$$r_1 = K^{-1}h_1 / \left\| K^{-1}h_1 \right\|$$

$$r_2 = K^{-1}h_2 / \left\| K^{-1}h_1 \right\|$$

$$r_3 = r_1 \times r_2$$

$$t = K^{-1}h_3 / \left\| K^{-1}h_1 \right\|$$

where $\mathbf{H} = \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{bmatrix}$