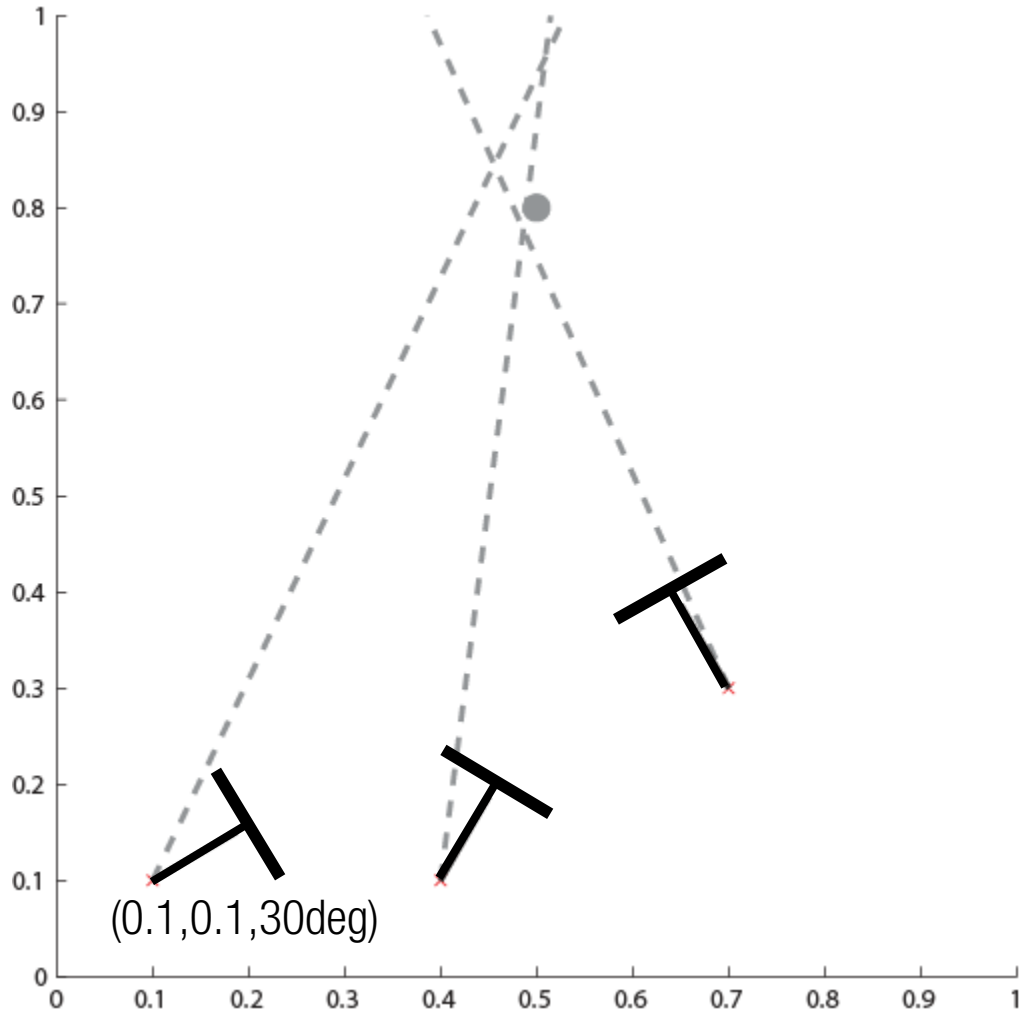
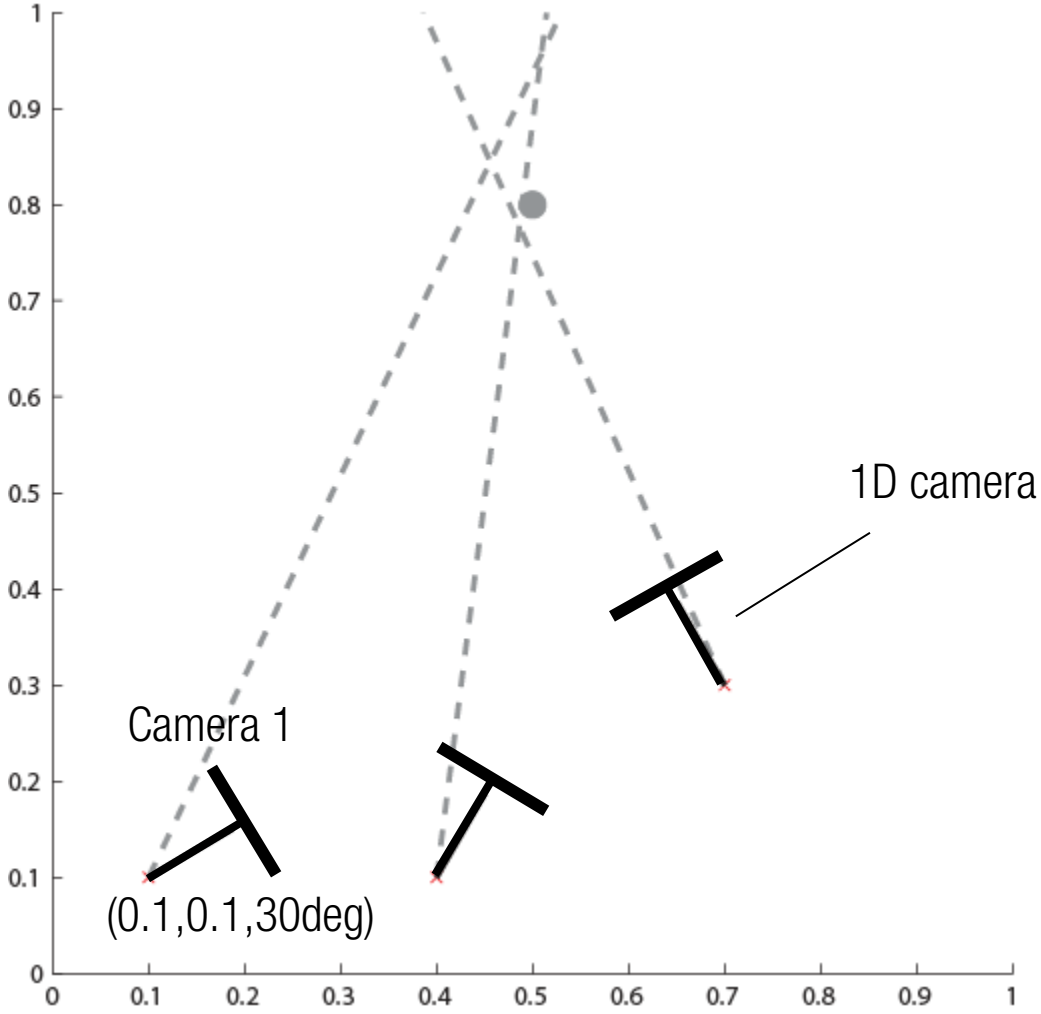
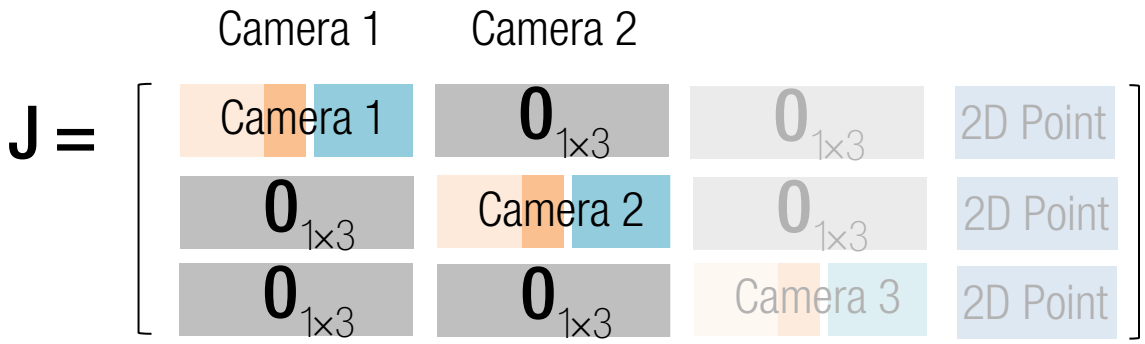
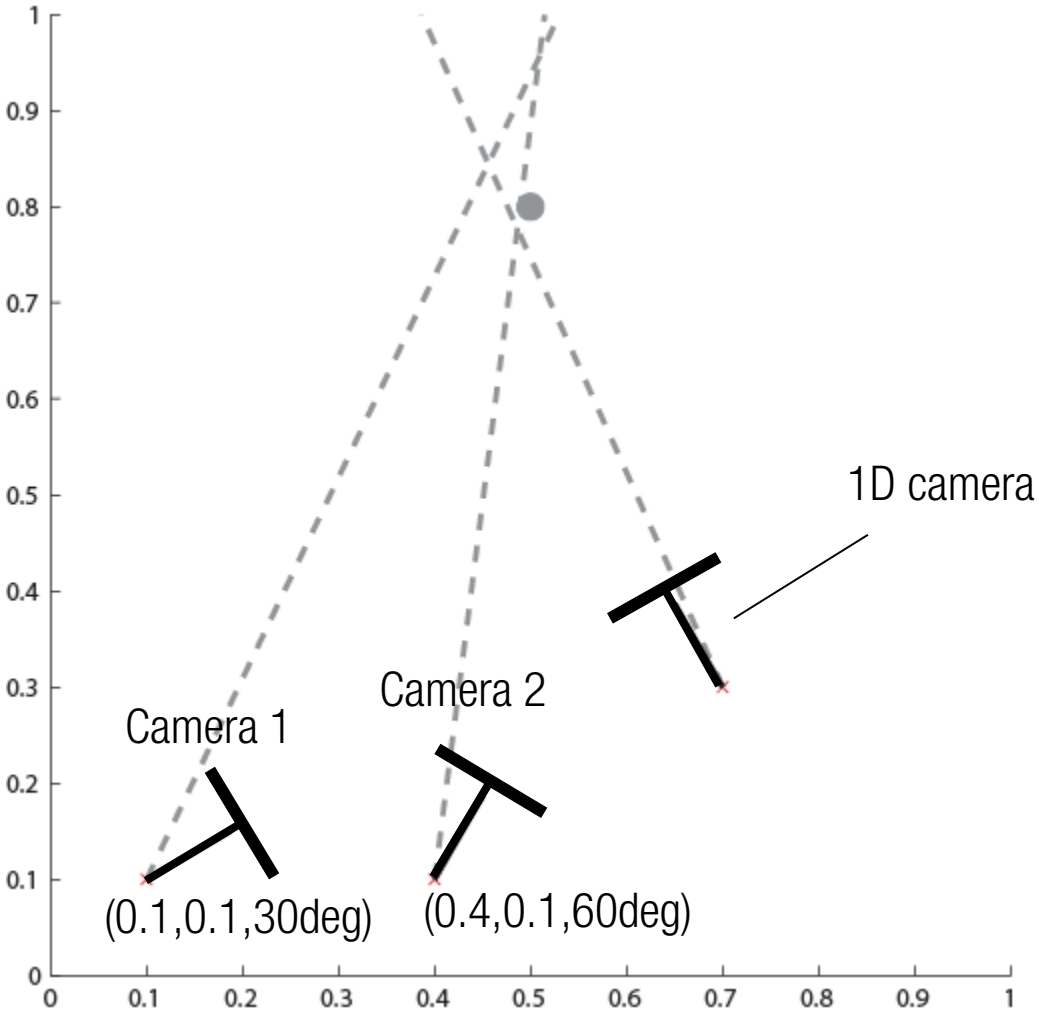

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
(0.1,0.1,30deg)

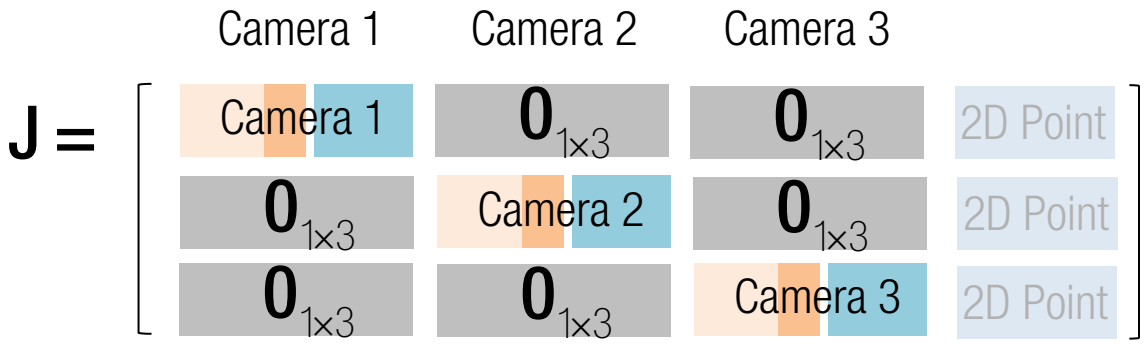
Camera 1

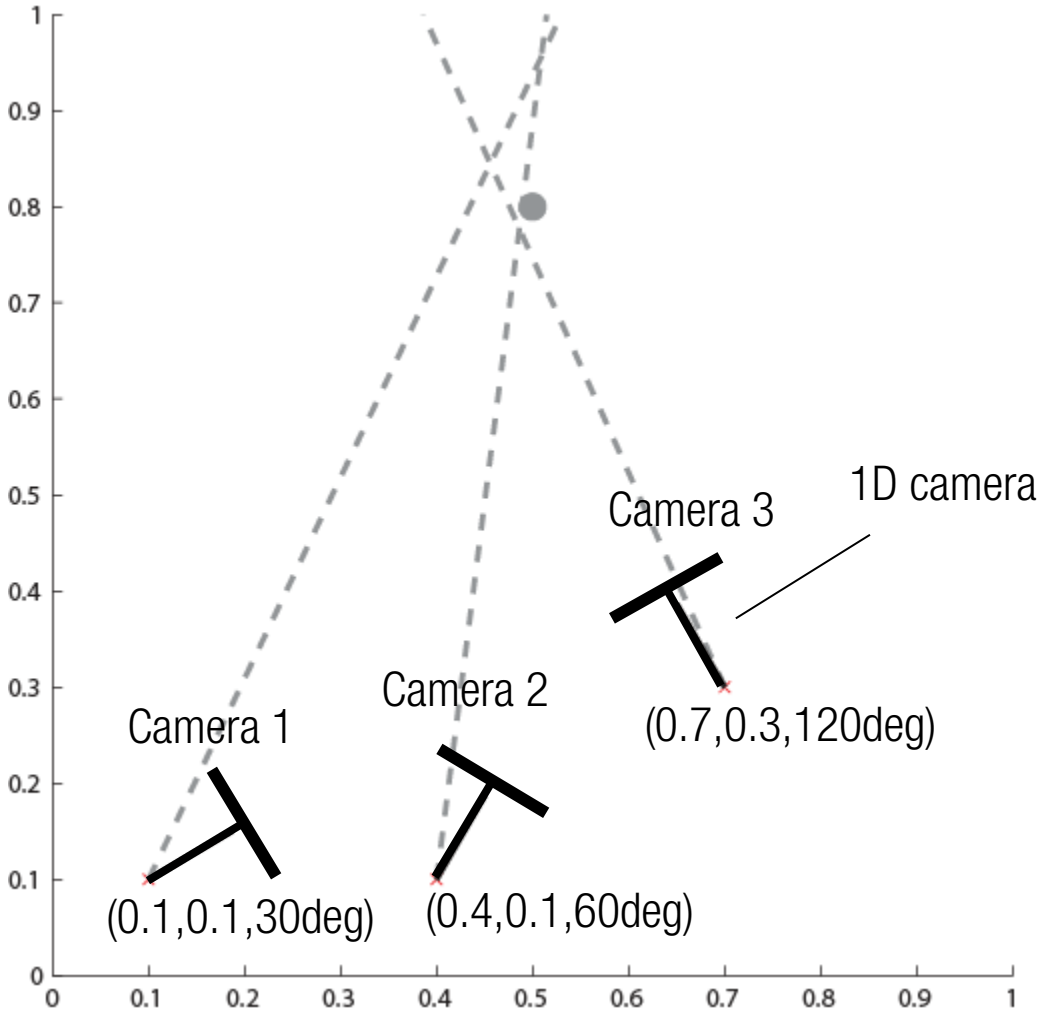
$$J = \begin{bmatrix} \text{Camera 1} & 0_{1\times3} & 0_{1\times3} & \text{2D Point} \\ 0_{1\times3} & \text{Camera 2} & 0_{1\times3} & \text{2D Point} \\ 0_{1\times3} & 0_{1\times3} & \text{Camera 3} & \text{2D Point} \end{bmatrix}$$

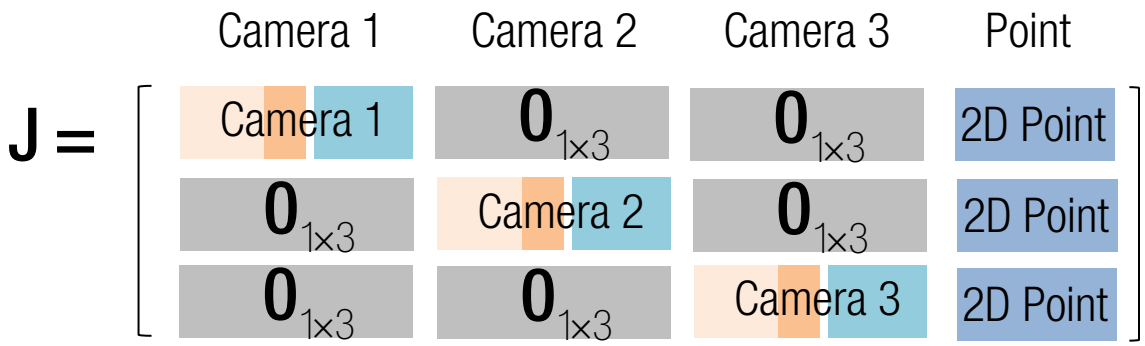



Camera 1
Camera 2
J =
Camera 1
0 1×3
0 1×3
2D Point
0 1×3
Camera 2
0 1×3
2D Point
0 1×3
0 1×3
Camera 3
2D Point

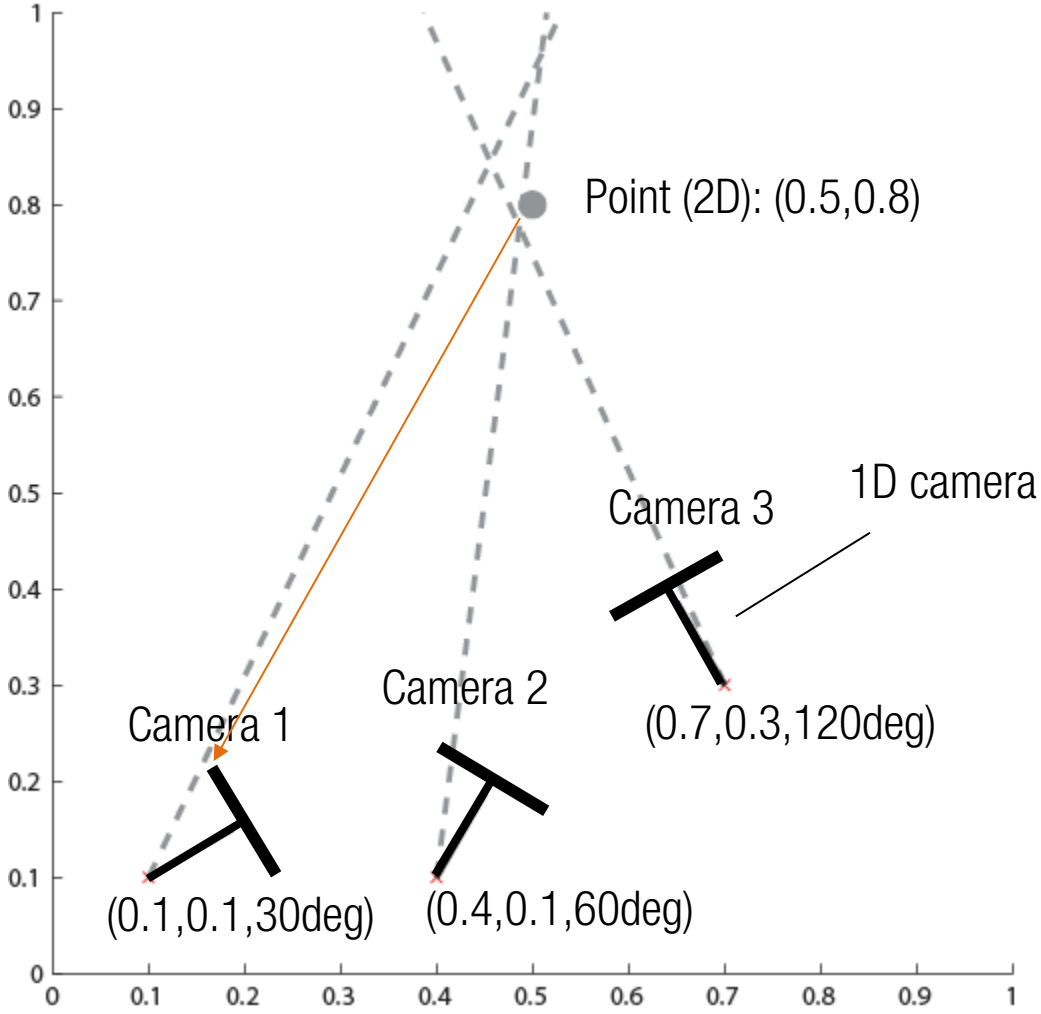

1D camera
Camera 2
Camera 1
(0.1,0.1,30deg)
(0.4,0.1,60deg)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

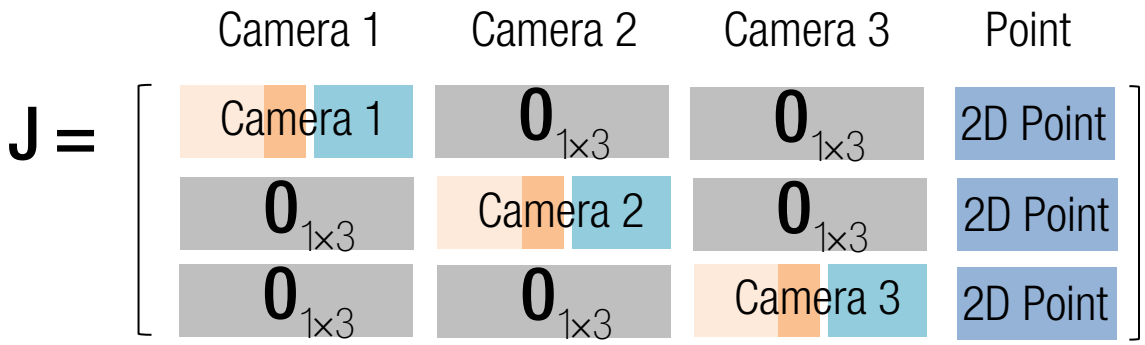

Camera 1
Camera 2
Camera 3
J =
Camera 1
0 1×3
0 1×3
2D Point
0 1×3
Camera 2
0 1×3
2D Point
0 1×3
0 1×3
Camera 3
2D Point

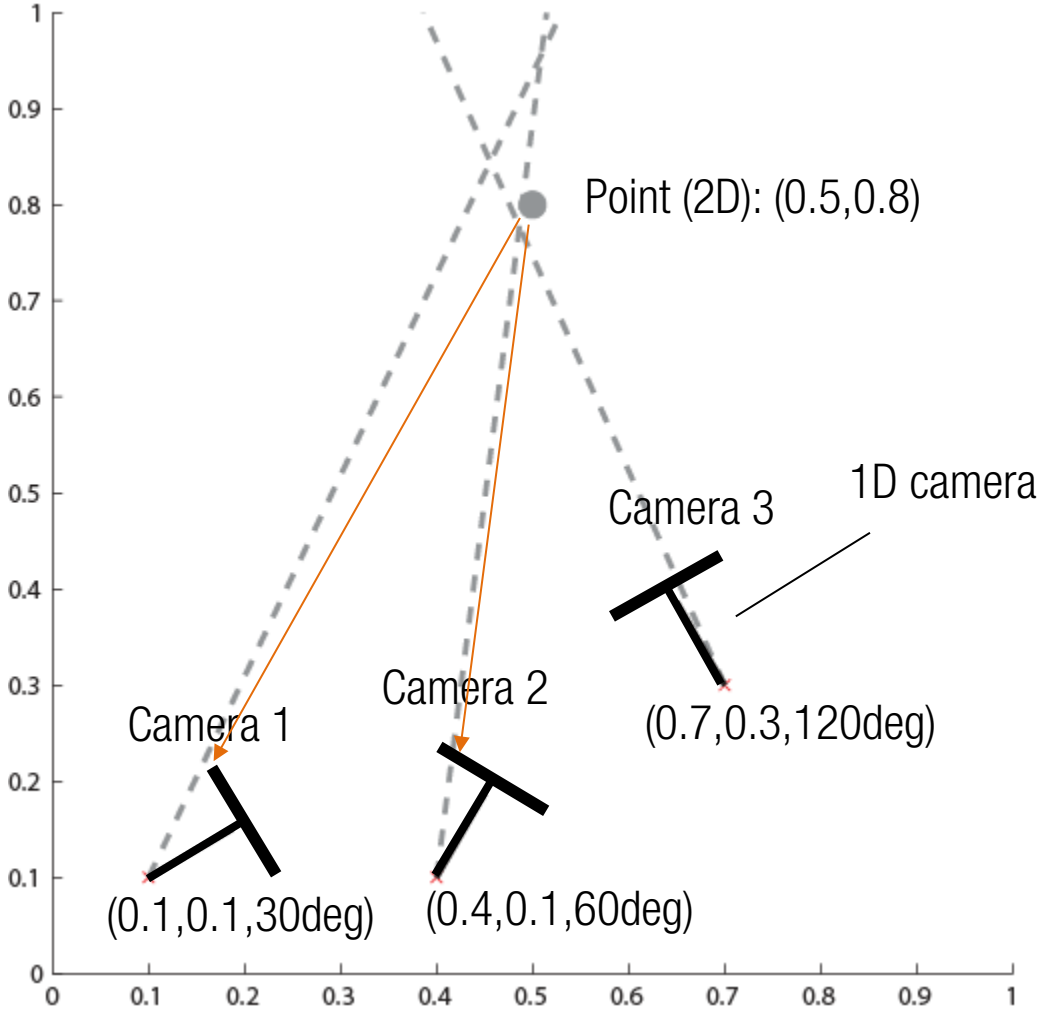

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Camera 3
1D camera
Camera 2
Camera 1
(0.7,0.3,120deg)
(0.1,0.1,30deg)
(0.4,0.1,60deg)

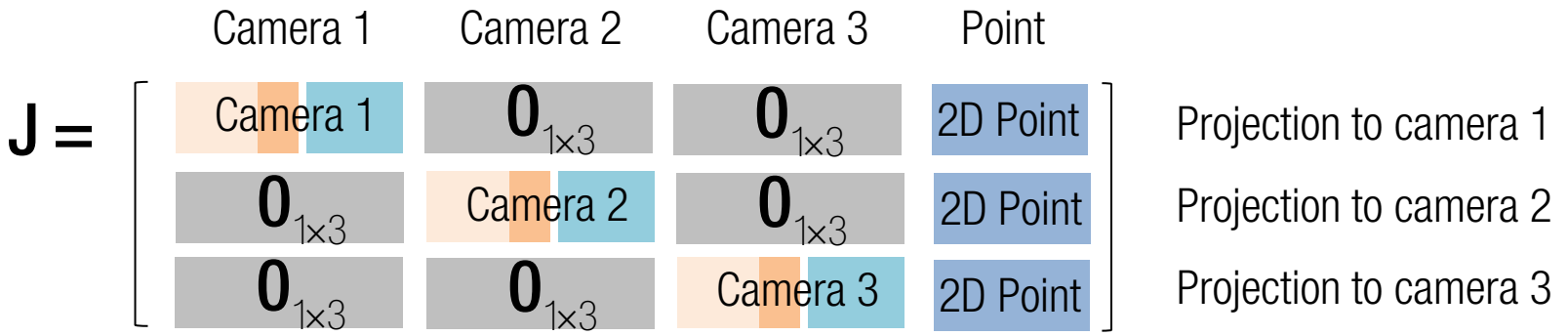


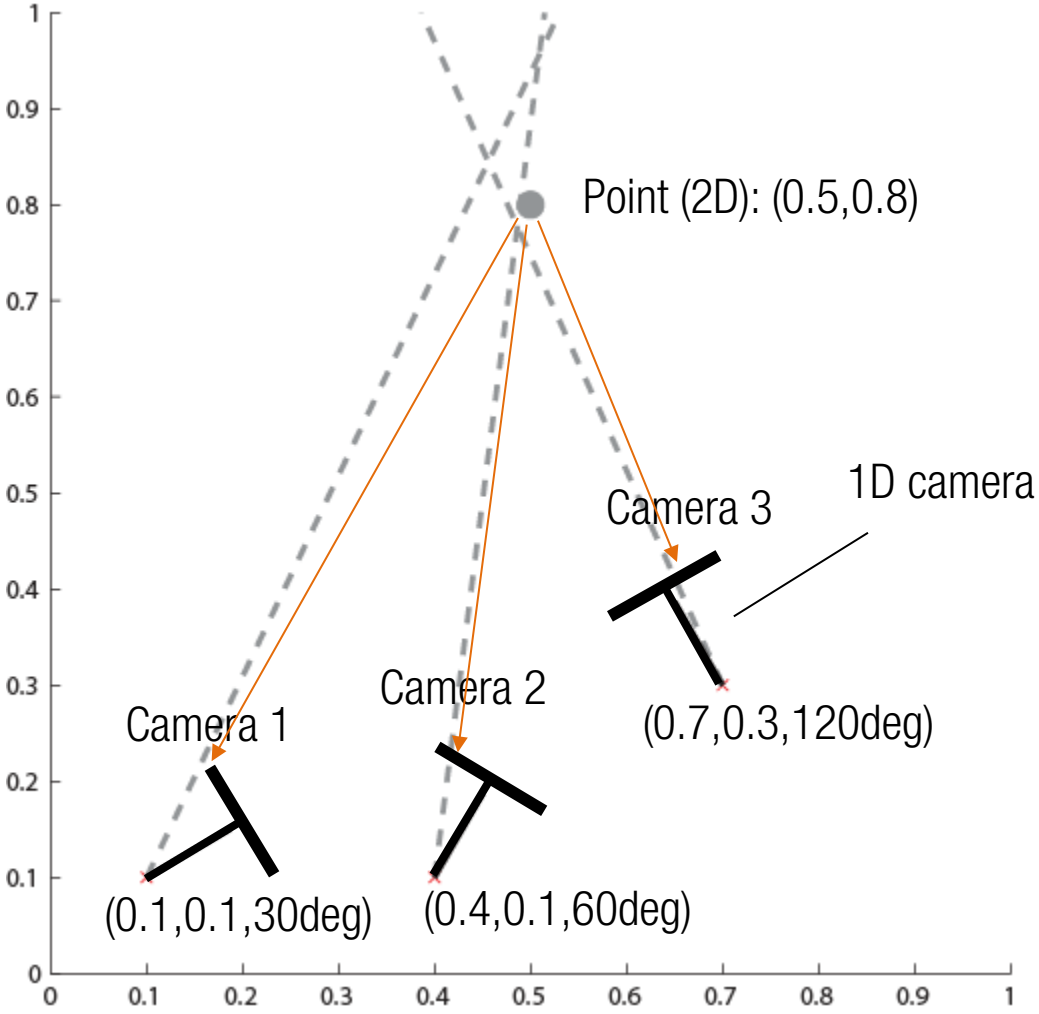
Projection to camera 1




Camera 1
Camera 2
Camera 3
Point
J =
Camera 1
0 1×3
0 1×3
2D Point
0 1×3
Camera 2
0 1×3
2D Point
0 1×3
0 1×3
Camera 3
2D Point
Projection to camera 1
Projection to camera 2


Point (2D): (0.5,0.8)
1D camera
Camera 3
Camera 2
Camera 1
(0.7,0.3,120deg)
(0.4,0.1,60deg)
(0.1,0.1,30deg)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Camera 1
Camera 2
Camera 3
Point
J =
Camera 1
0 1×3
0 1×3
2D Point
Projection to camera 1
0 1×3
Camera 2
0 1×3
2D Point
Projection to camera 2
0 1×3
0 1×3
Camera 3
2D Point
Projection to camera 3


Point (2D): (0.5,0.8)
1D camera
Camera 3
Camera 2
Camera 1
(0.7,0.3,120deg)
(0.4,0.1,60deg)
(0.1,0.1,30deg)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

$$J = \begin{bmatrix} \text{Camera 1} & 0_{1\times3} & 0_{1\times3} & \text{2D Point} \\ 0_{1\times3} & \text{Camera 2} & 0_{1\times3} & \text{2D Point} \\ 0_{1\times3} & 0_{1\times3} & \text{Camera 3} & \text{2D Point} \end{bmatrix}$$

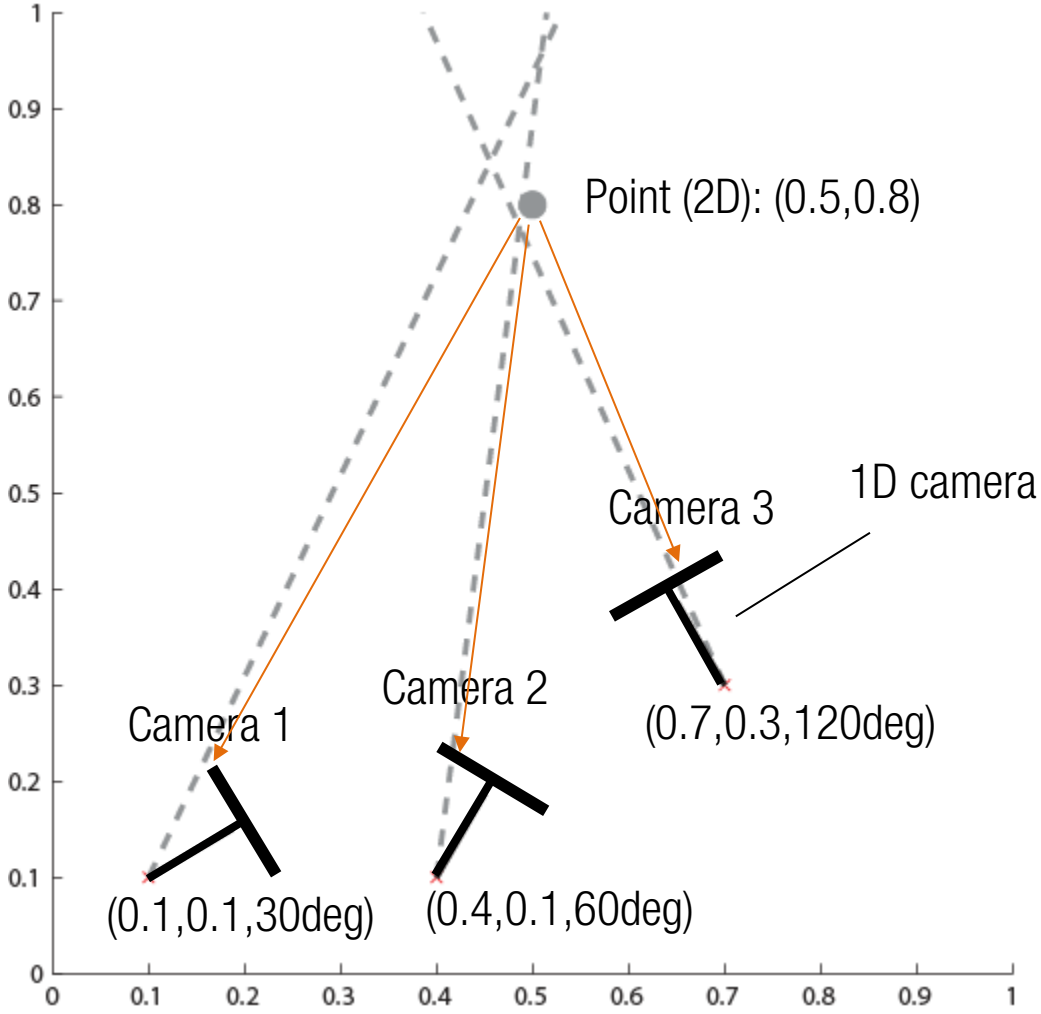
Camera 1 Camera 2 Camera 3 Point

Projection to camera 1

Projection to camera 2

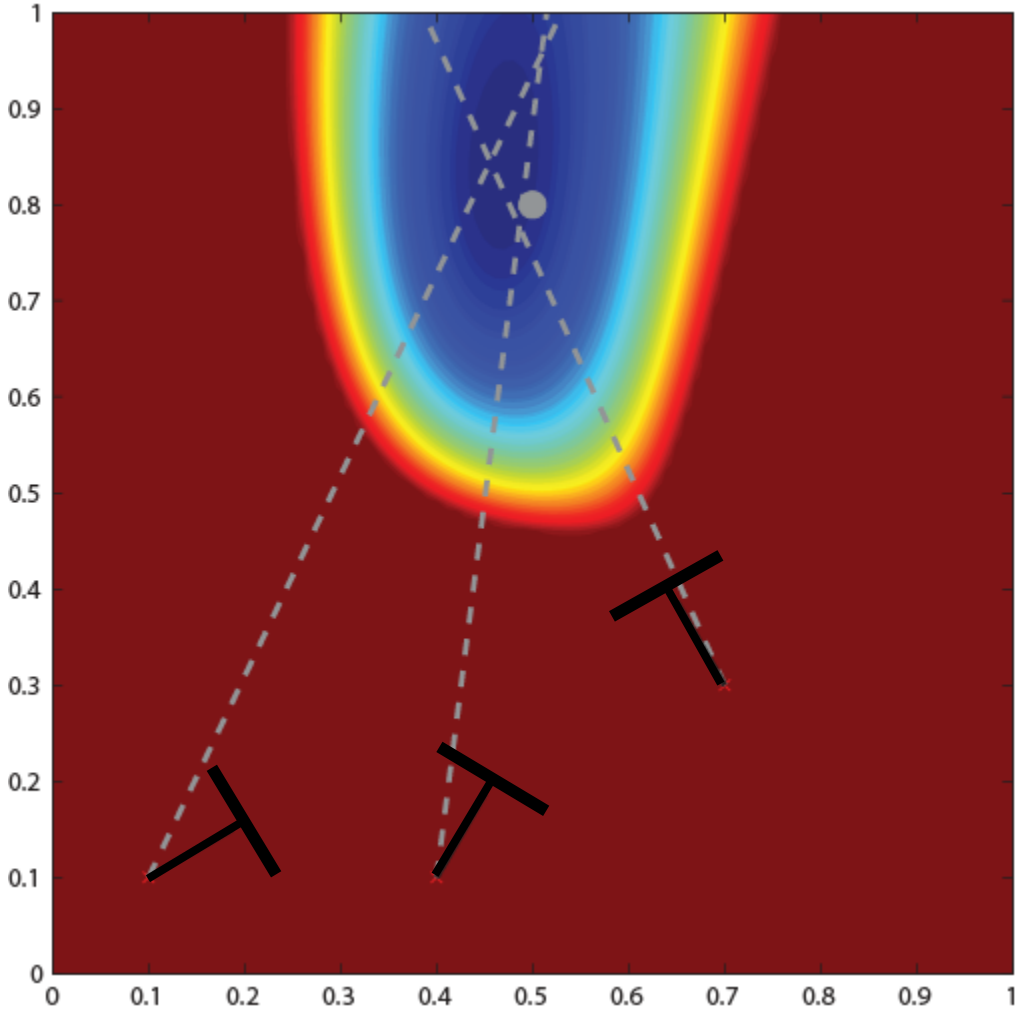
Projection to camera 3

Jacobian for point



	Camera 1	Camera 2	Camera 3	Point	
J =	Camera 1	$0_{1\times3}$	$0_{1\times3}$	2D Point	Projection to camera 1
	$0_{1\times3}$	Camera 2	$0_{1\times3}$	2D Point	Projection to camera 2
	$0_{1\times3}$	$0_{1\times3}$	Camera 3	2D Point	Projection to camera 3

Jacobian for point

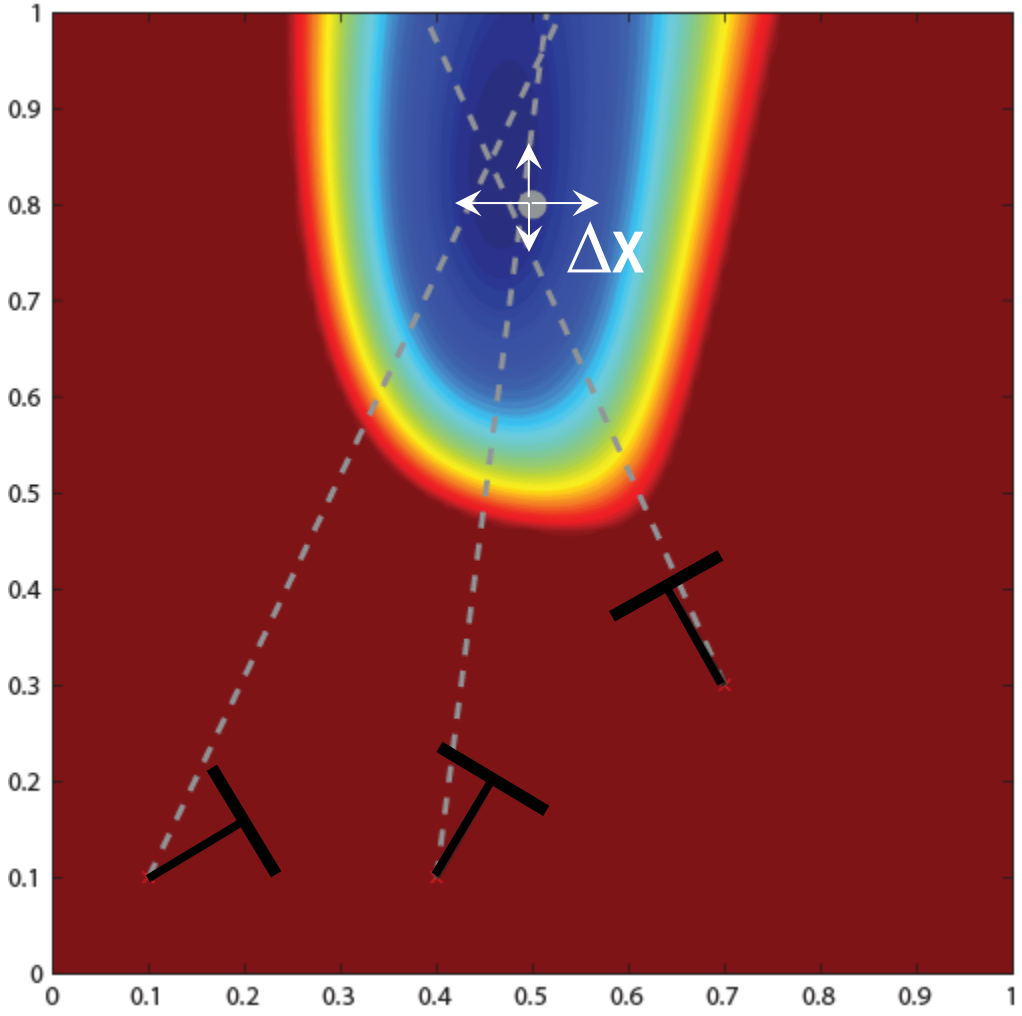


Cost:

$$\sum_{i=1}^{3} \left\| \tilde{x} - f_i(\mathbf{X}) \right\|^2 = \sum_{i=1}^{3} \left\| \tilde{x} - u_i / w_i \right\|^2$$

	Camera 1	Camera 2	Camera 3	Point	
$\mathbf{J} =$	Camera 1	$\mathbf{0}_{1\times3}$	$\mathbf{0}_{1\times3}$	2D Point	Projection to camera 1
	$\mathbf{0}_{1\times3}$	Camera 2	$\mathbf{0}_{1\times3}$	2D Point	Projection to camera 2
	$\mathbf{0}_{1\times3}$	$\mathbf{0}_{1\times3}$	Camera 3	2D Point	Projection to camera 3

Jacobian for point



Cost:

$$\sum_{i=1}^{3} \left\| \tilde{x} - f_i(\mathbf{X}) \right\|^2 = \sum_{i=1}^{3} \left\| \tilde{x} - u_i / w_i \right\|^2$$

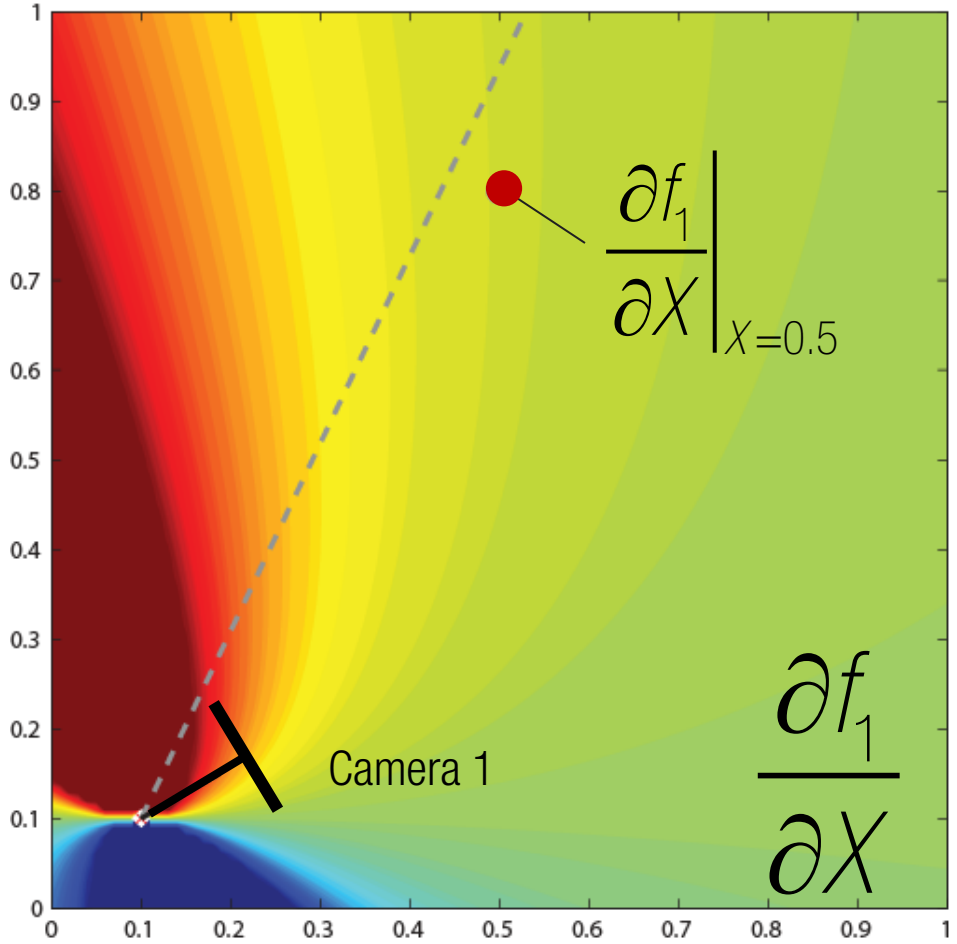
$$\mathbf{f}(\mathbf{x} + \Delta\mathbf{x}) \approx \mathbf{f}(\mathbf{x}) + \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} \Delta\mathbf{x}$$

where $\mathbf{J}^{\mathsf{T}}\mathbf{J}\Delta\mathbf{x} = \mathbf{J}^{\mathsf{T}}\left(\mathbf{b} - \mathbf{f}(\mathbf{x})\right)$

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1 & f_2 & f_3 \end{bmatrix}^{\mathsf{T}}$$

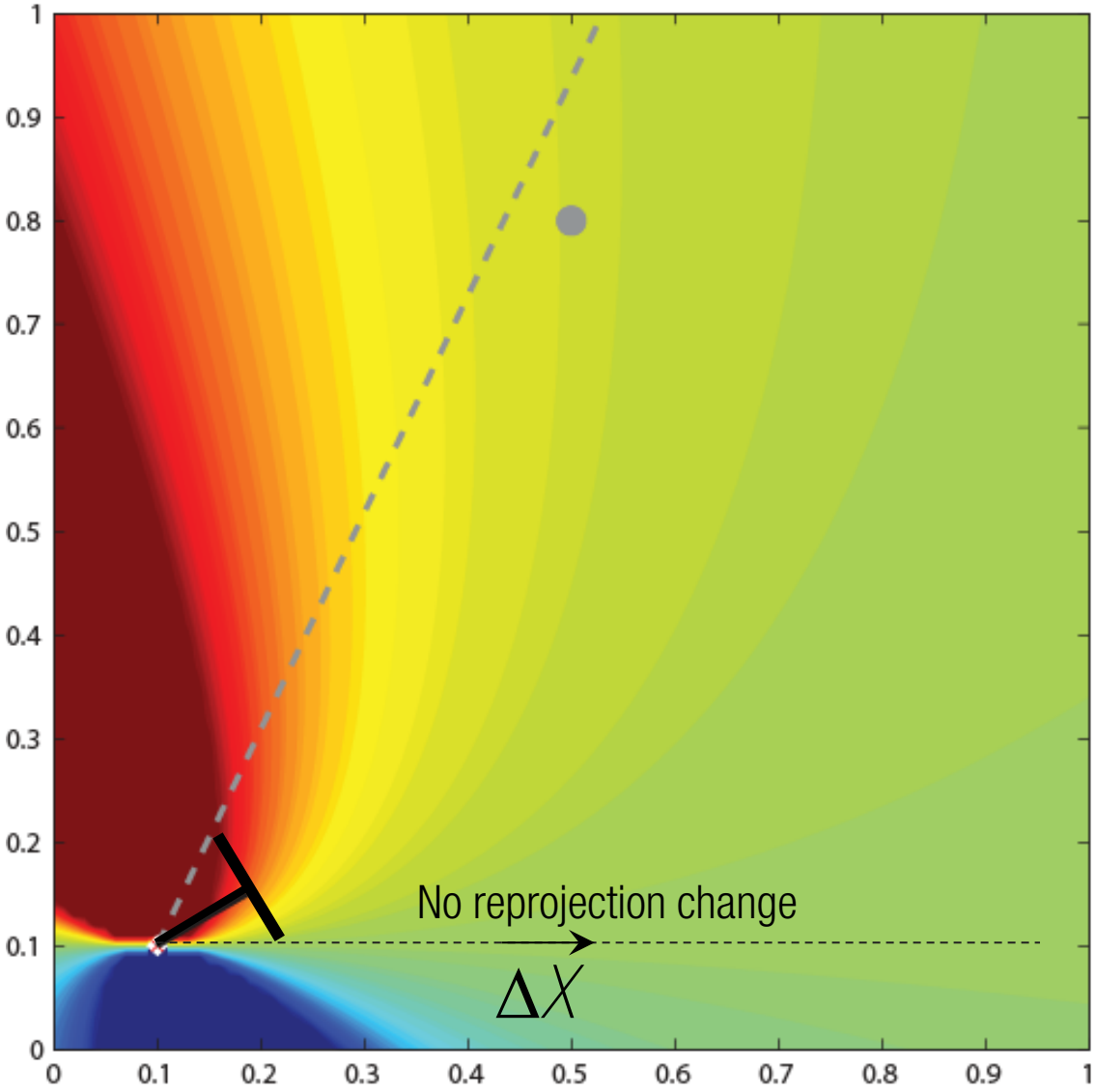
$$\mathbf{J}^{\mathsf{T}}\mathbf{J}\Delta\mathbf{x} = \mathbf{J}^{\mathsf{T}}\left(\mathbf{b} - \mathbf{f}(\mathbf{x})\right) \quad \text{where } \mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1 & f_2 & f_3 \end{bmatrix}^{\mathsf{T}}$$

$$\mathbf{J} = \begin{bmatrix} \dfrac{\partial f_1}{\partial X} & \dfrac{\partial f_1}{\partial Y} \\ \dfrac{\partial f_2}{\partial X} & \dfrac{\partial f_2}{\partial Y} \\ \dfrac{\partial f_3}{\partial X} & \dfrac{\partial f_3}{\partial Y} \end{bmatrix}$$



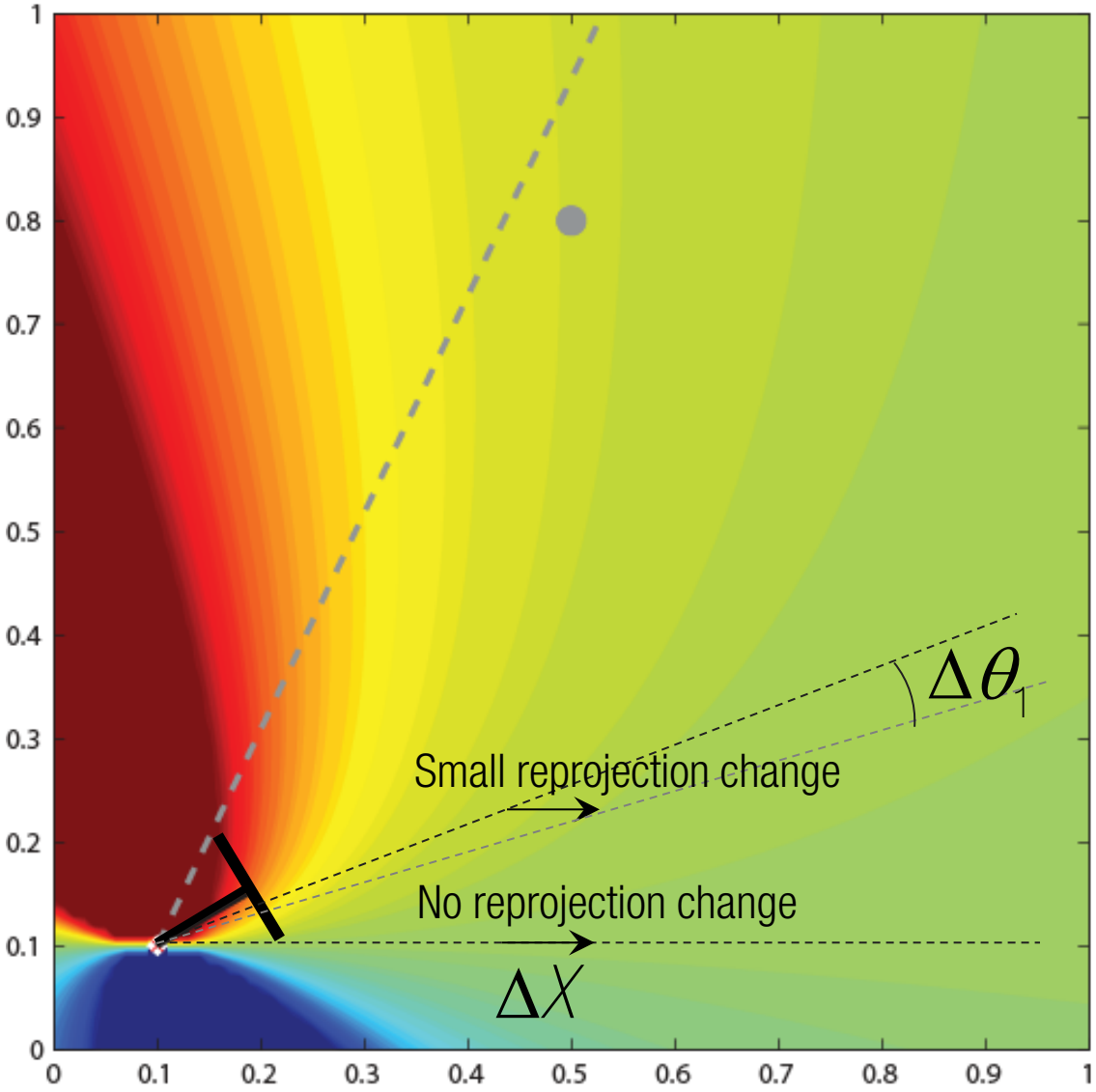
$$\mathbf{J}^{\mathsf{T}}\mathbf{J}\Delta\mathbf{x} = \mathbf{J}^{\mathsf{T}}\left(\mathbf{b} - \mathbf{f}(\mathbf{x})\right) \quad \text{where } \mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1 & f_2 & f_3 \end{bmatrix}^{\mathsf{T}}$$

$$\frac{\partial f_1}{\partial X} =$$



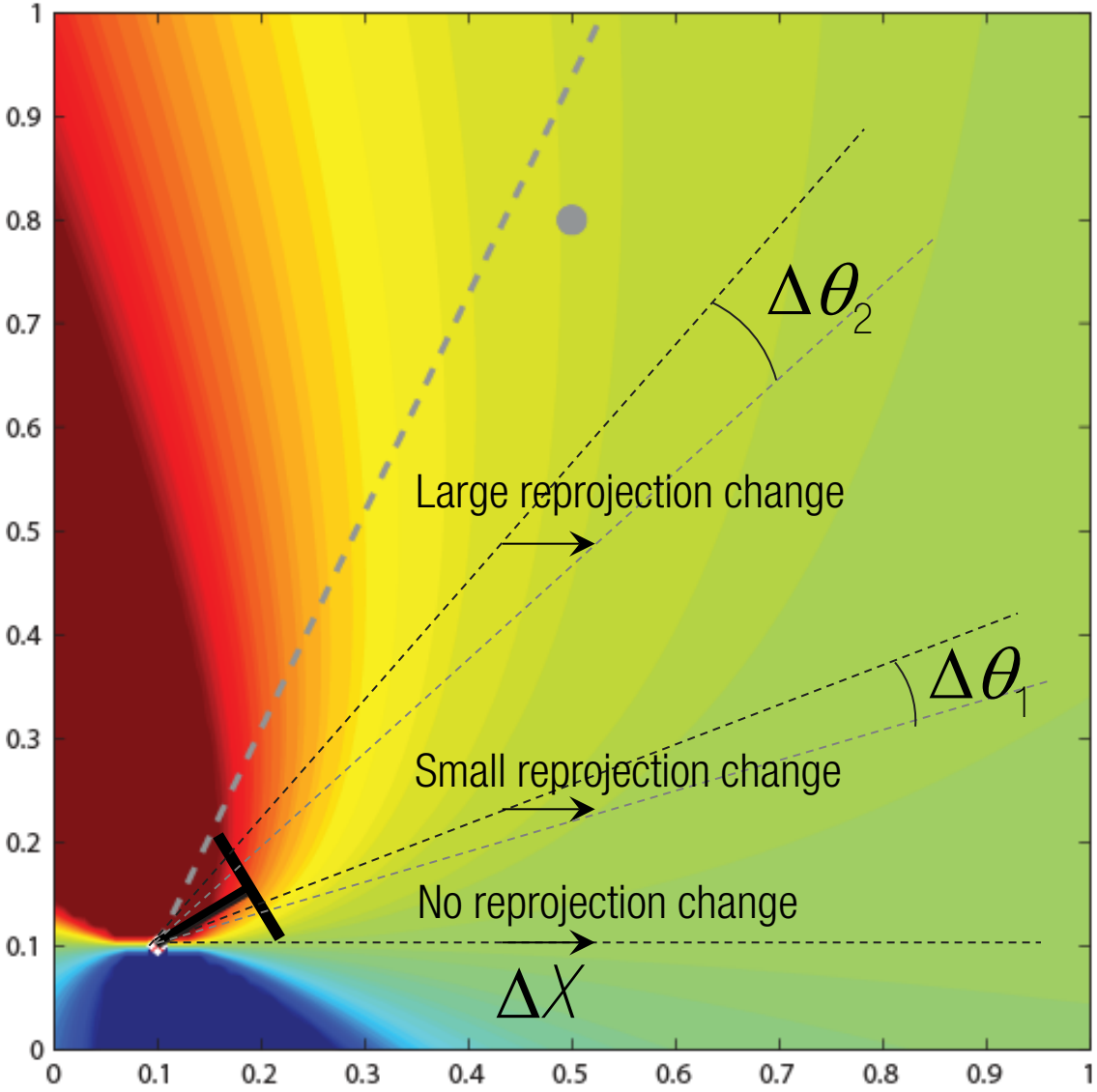
$$\mathbf{J}^{\mathsf{T}}\mathbf{J}\Delta\mathbf{x} = \mathbf{J}^{\mathsf{T}}\left(\mathbf{b} - \mathbf{f}(\mathbf{x})\right) \quad \text{where } \mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1 & f_2 & f_3 \end{bmatrix}^{\mathsf{T}}$$

$$\frac{\partial f_1}{\partial X} =$$



$$\mathbf{J}^{\mathsf{T}}\mathbf{J}\Delta\mathbf{x} = \mathbf{J}^{\mathsf{T}}\left(\mathbf{b} - \mathbf{f}(\mathbf{x})\right) \quad \text{where } \mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1 & f_2 & f_3 \end{bmatrix}^{\mathsf{T}}$$

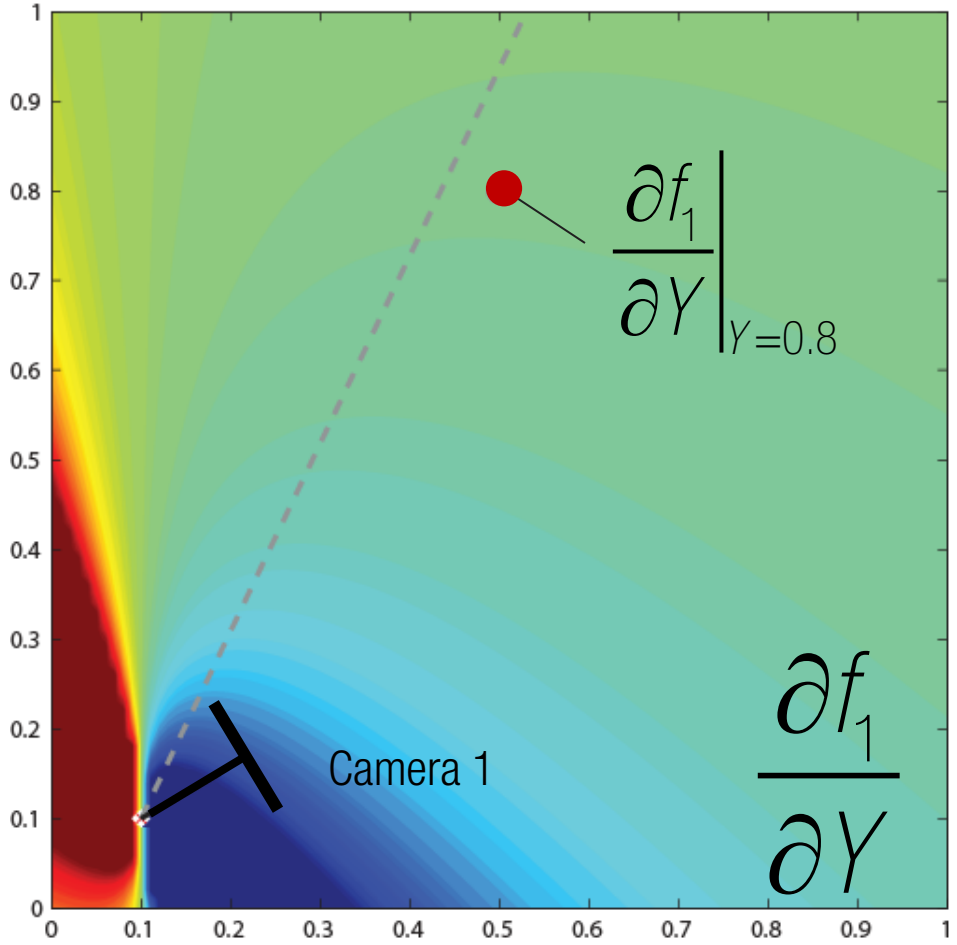
$$\frac{\partial f_1}{\partial X} =$$



$$\Delta\theta_2 > \Delta\theta_1$$

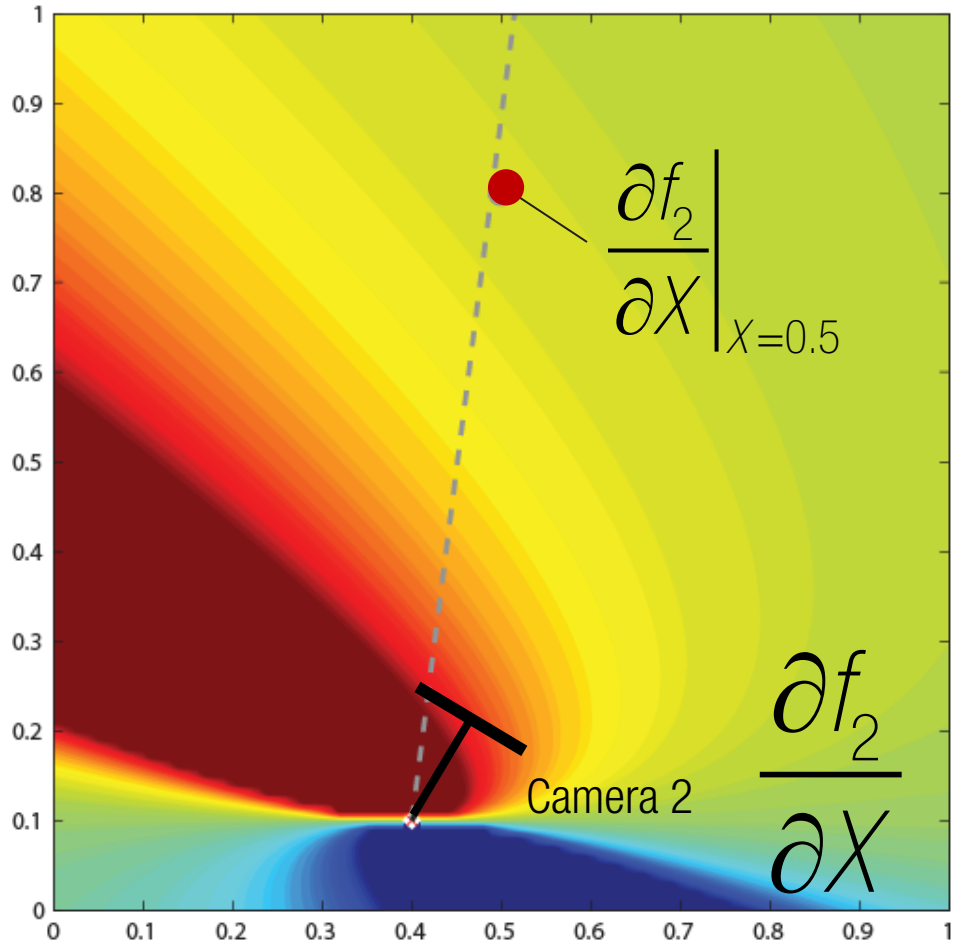
$$\mathbf{J}^{\mathsf{T}}\mathbf{J}\Delta\mathbf{x} = \mathbf{J}^{\mathsf{T}}\left(\mathbf{b} - \mathbf{f}(\mathbf{x})\right) \quad \text{where } \mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1 & f_2 & f_3 \end{bmatrix}^{\mathsf{T}}$$

$$\mathbf{J} = \begin{bmatrix} \dfrac{\partial f_1}{\partial X} & \dfrac{\partial f_1}{\partial Y} \\ \dfrac{\partial f_2}{\partial X} & \dfrac{\partial f_2}{\partial Y} \\ \dfrac{\partial f_3}{\partial X} & \dfrac{\partial f_3}{\partial Y} \end{bmatrix}$$



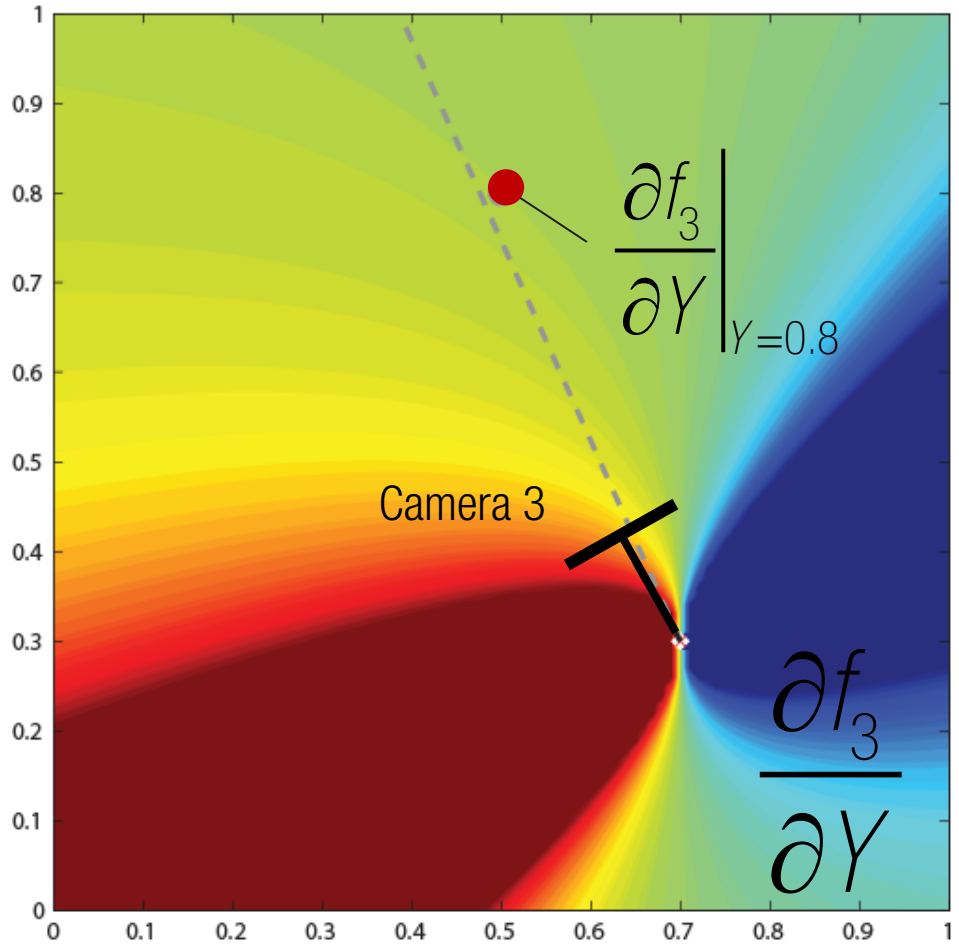
$$\mathbf{J}^{\mathsf{T}}\mathbf{J}\Delta\mathbf{x} = \mathbf{J}^{\mathsf{T}}\left(\mathbf{b} - \mathbf{f}(\mathbf{x})\right) \quad \text{where } \mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1 & f_2 & f_3 \end{bmatrix}^{\mathsf{T}}$$

$$\mathbf{J} = \begin{bmatrix} \dfrac{\partial f_1}{\partial X} & \dfrac{\partial f_1}{\partial Y} \\ \dfrac{\partial f_2}{\partial X} & \dfrac{\partial f_2}{\partial Y} \\ \dfrac{\partial f_3}{\partial X} & \dfrac{\partial f_3}{\partial Y} \end{bmatrix}$$



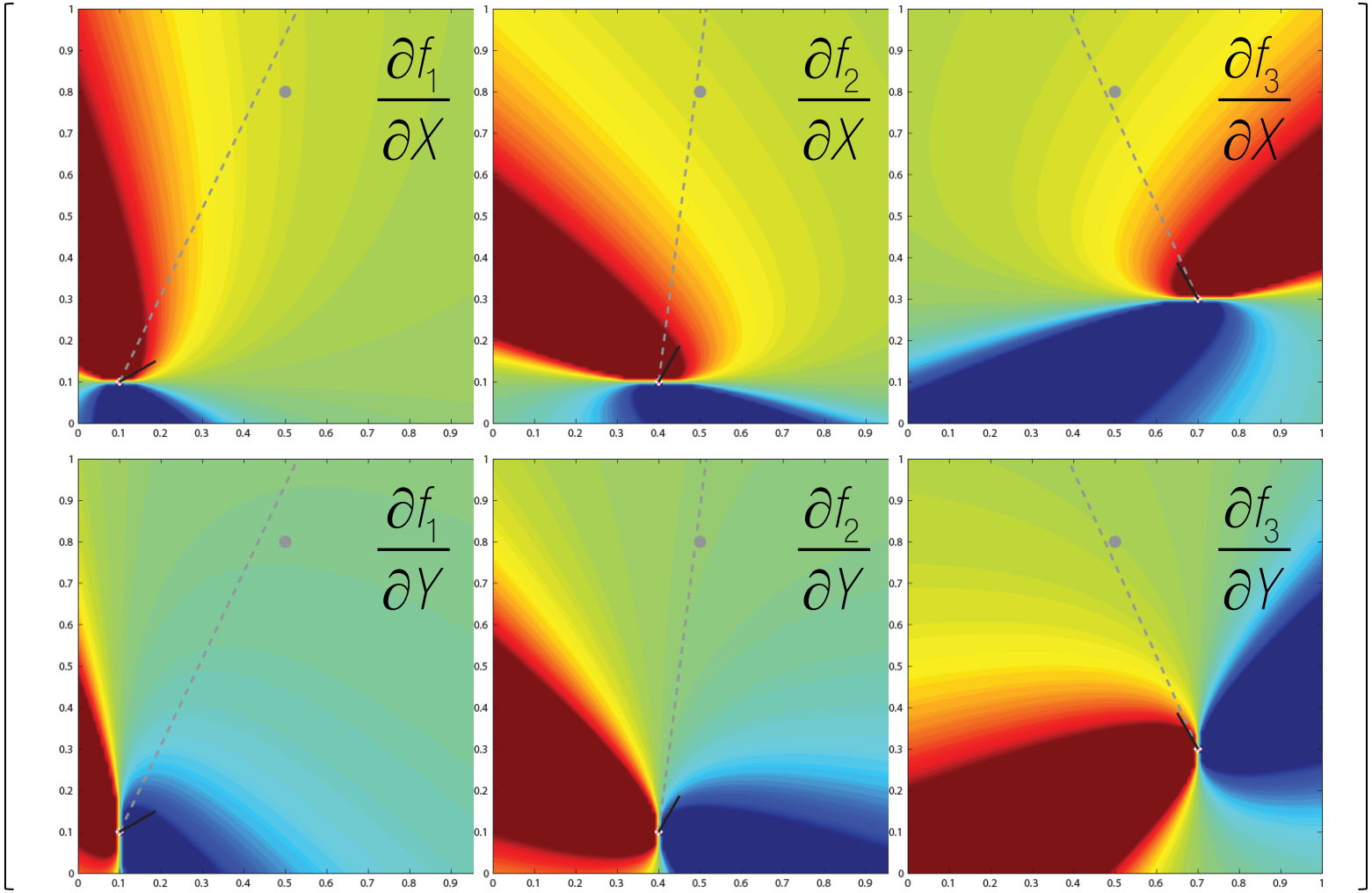
$$\mathbf{J}^{\mathsf{T}}\mathbf{J}\Delta\mathbf{x} = \mathbf{J}^{\mathsf{T}}\left(\mathbf{b} - \mathbf{f}(\mathbf{x})\right) \quad \text{where } \mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1 & f_2 & f_3 \end{bmatrix}^{\mathsf{T}}$$

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f_1}{\partial X} & \frac{\partial f_1}{\partial Y} \\ \frac{\partial f_2}{\partial X} & \frac{\partial f_2}{\partial Y} \\ \frac{\partial f_3}{\partial X} & \frac{\partial f_3}{\partial Y} \end{bmatrix}$$



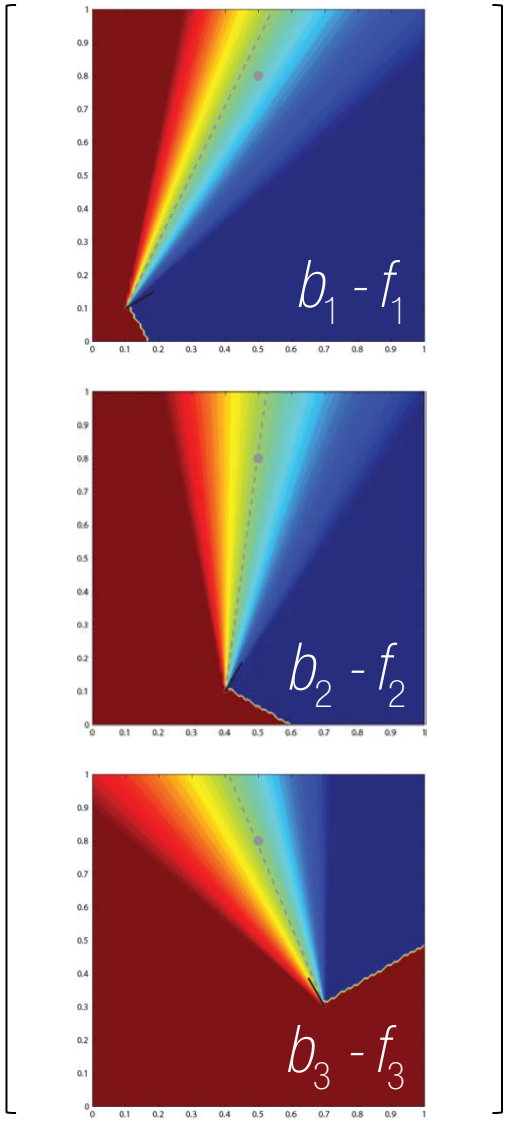
$$\mathbf{J}^\top \mathbf{J} \Delta \mathbf{x} = \mathbf{J}^\top \left(\mathbf{b} - \mathbf{f}(\mathbf{x}) \right)$$

$$\mathbf{J}^\top =$$



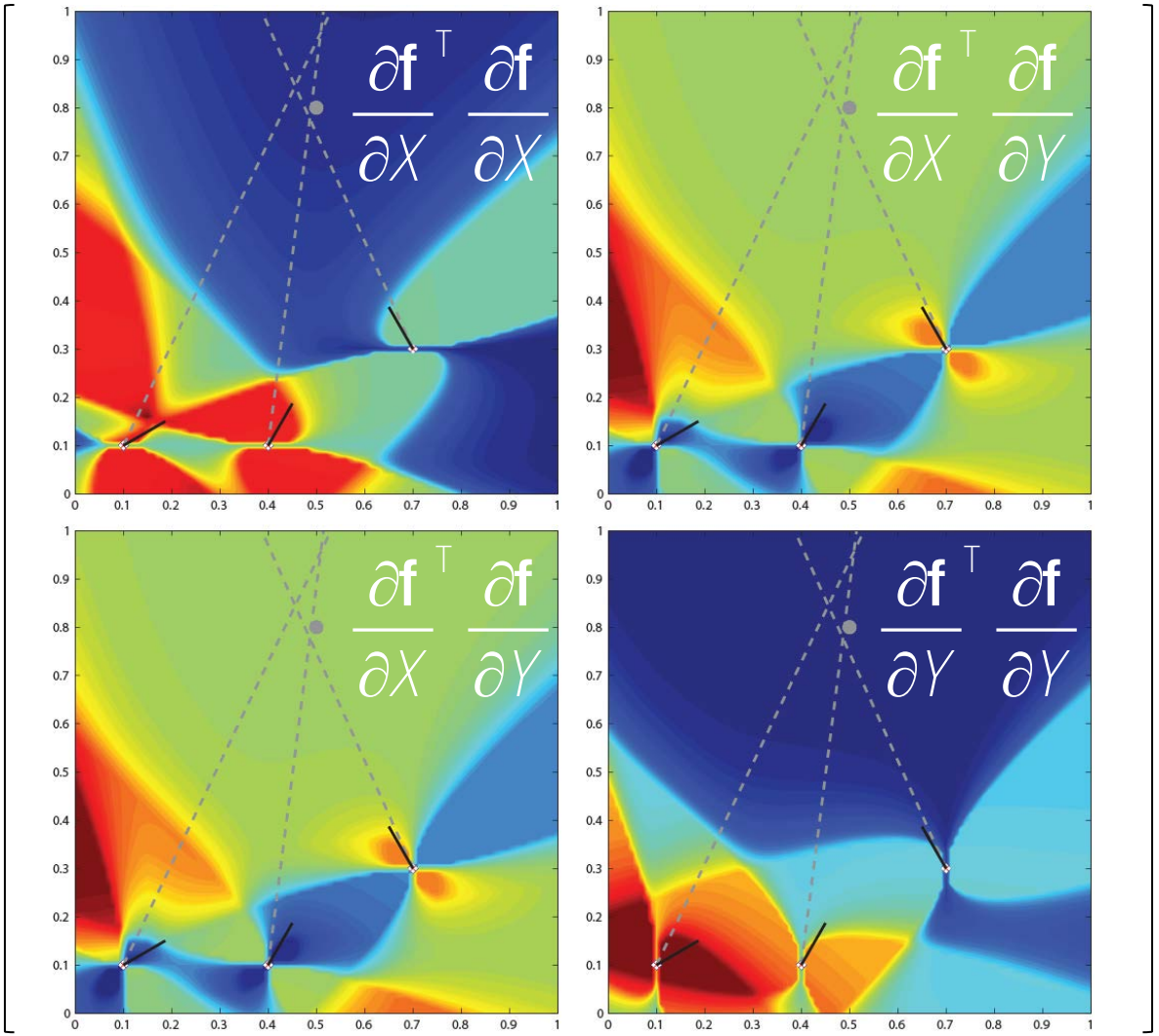
$$\mathbf{J}^\top \mathbf{J} \Delta \mathbf{x} = \mathbf{J}^\top \left(\mathbf{b} - \mathbf{f}(\mathbf{x}) \right)$$

$$\mathbf{b} - \mathbf{f}(\mathbf{x}) =$$

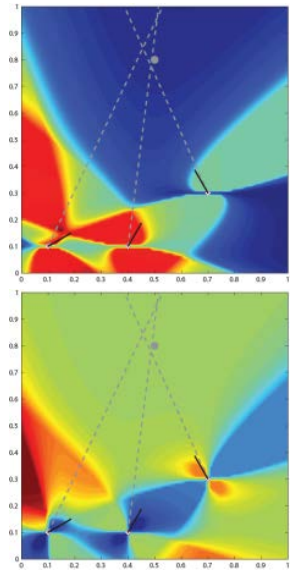


$$\mathbf{J}^\top \mathbf{J} \Delta \mathbf{x} = \mathbf{J}^\top \left(\mathbf{b} - \mathbf{f}(\mathbf{x}) \right)$$

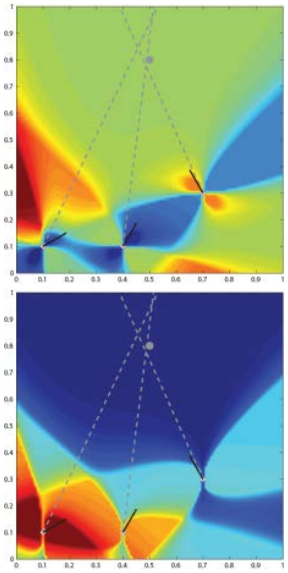
$$\mathbf{J}^\top \mathbf{J} =$$



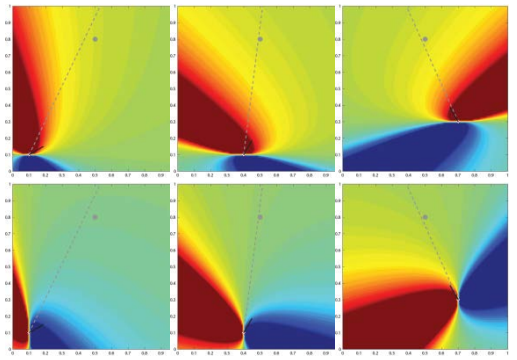
$$\mathbf{J}^\top \mathbf{J} \Delta \mathbf{x} = \mathbf{J}^\top \left(\mathbf{b} - \mathbf{f}(\mathbf{x}) \right)$$

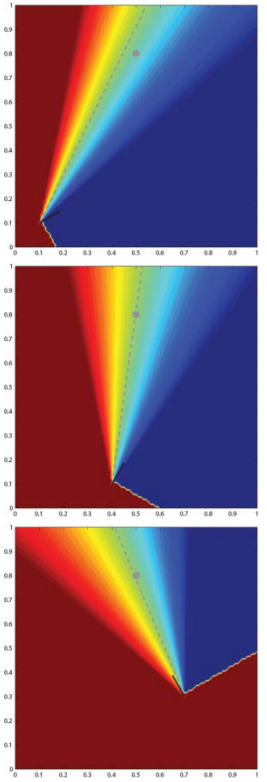


$\Delta X +$



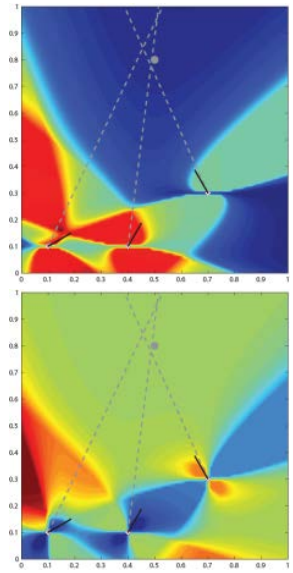
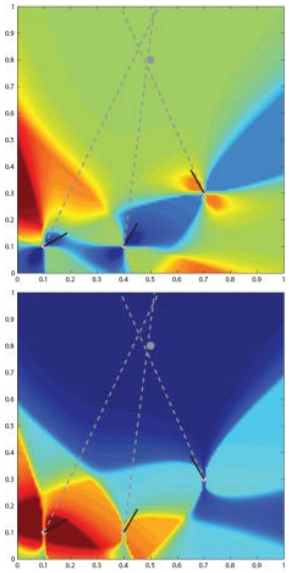
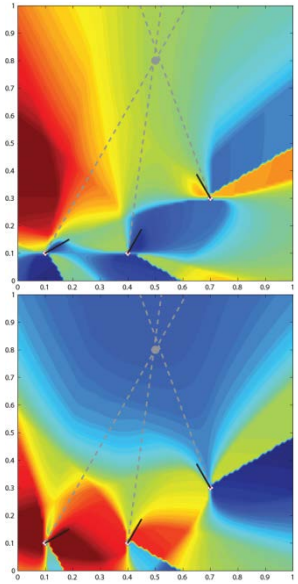
$\Delta Y =$





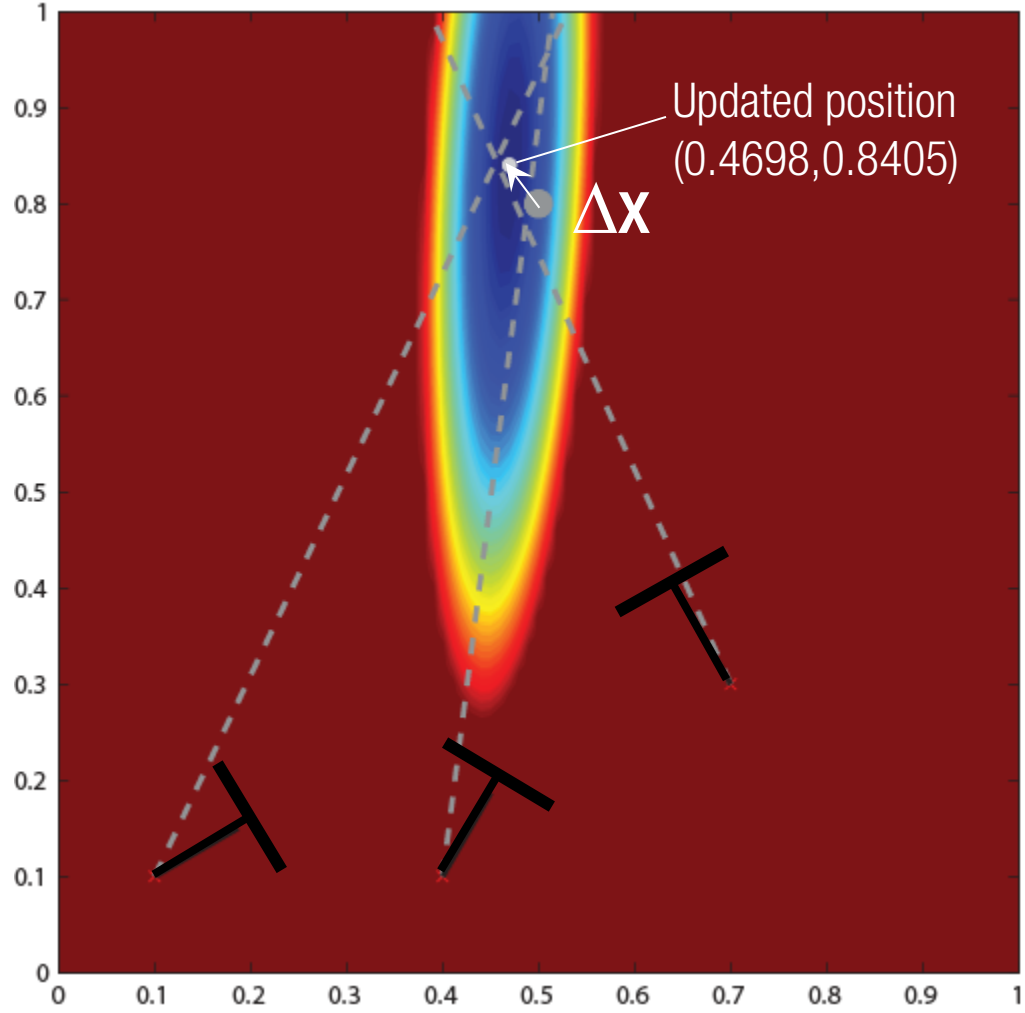
where $\Delta \mathbf{x} = \begin{bmatrix} \Delta X \\ \Delta Y \end{bmatrix}$

$$\mathbf{J}^\top \mathbf{J} \Delta \mathbf{x} = \mathbf{J}^\top \left(\mathbf{b} - \mathbf{f}(\mathbf{x}) \right)$$


$\Delta X +$

$\Delta Y =$


where $\Delta \mathbf{x} = \begin{bmatrix} \Delta X \\ \Delta Y \end{bmatrix}$

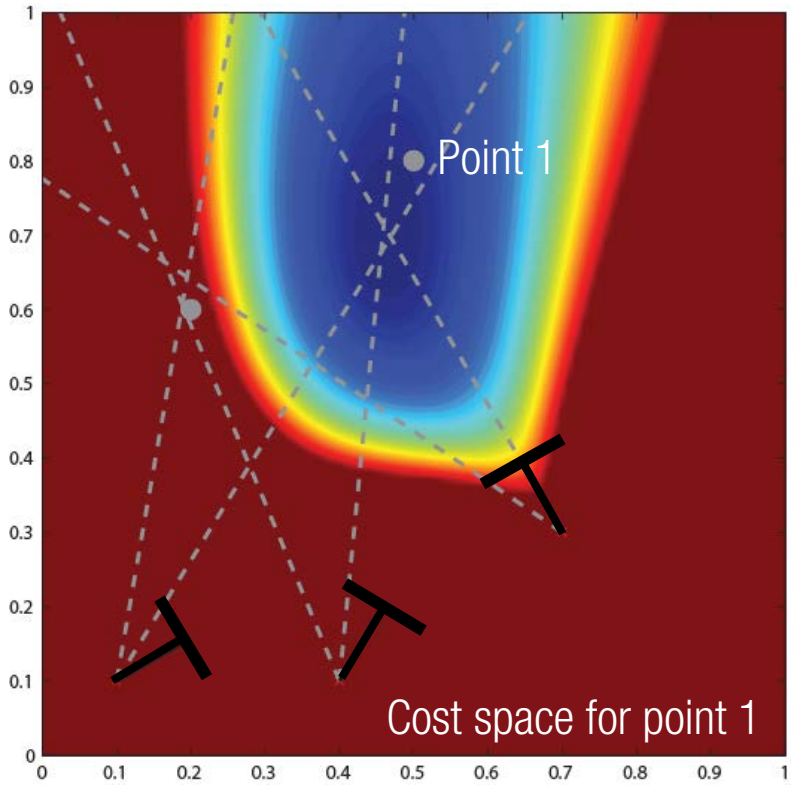
$$\Delta \mathbf{x} = \left(\mathbf{J}^\top \mathbf{J} \right)^{-1} \mathbf{J}^\top \left(\mathbf{b} - \mathbf{f}(\mathbf{x}) \right)$$



$$\left\| \mathbf{J}^\top \mathbf{J} \Delta \mathbf{x} - \mathbf{J}^\top \left(\mathbf{b} - \mathbf{f}(\mathbf{x}) \right) \right\|$$

$J =$

Camera 1	Camera 2	Camera 3	Point 1	Point 2	
Camera 1	$0_{1\times3}$	$0_{1\times3}$	2D Point	$0_{1\times2}$	Proj. point 1 to cam 1
$0_{1\times3}$	Camera 2	$0_{1\times3}$	2D Point	$0_{1\times2}$	Proj. point 1 to cam 2
$0_{1\times3}$	$0_{1\times3}$	Camera 3	2D Point	$0_{1\times2}$	Proj. point 1 to cam 3
Camera 1	$0_{1\times3}$	$0_{1\times3}$	$0_{1\times2}$	2D Point	
$0_{1\times3}$	Camera 2	$0_{1\times3}$	$0_{1\times2}$	2D Point	
$0_{1\times3}$	$0_{1\times3}$	Camera 3	$0_{1\times2}$	2D Point	



$$\sum_{i=1}^{3} \left\| \tilde{x}_{i1} - f_{i1}(\mathbf{X}) \right\|^2$$

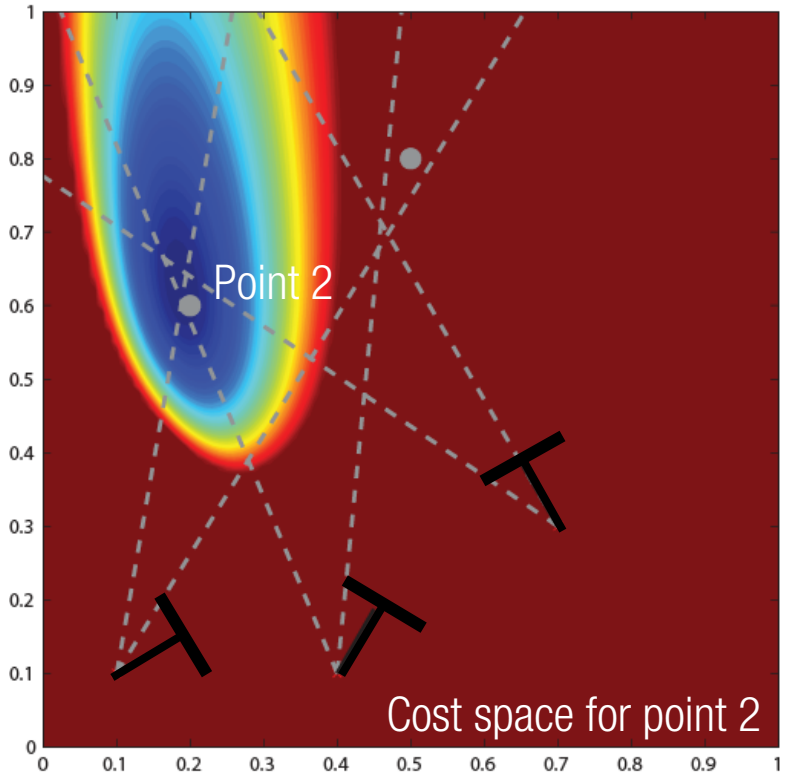
$$= \sum_{i=1}^{3} \left\| \tilde{x}_{i1} - u_{i1} / w_{i1} \right\|^2$$

$$\tilde{x}_{i1}$$

Camera index Point index

$J =$

Camera 1	Camera 2	Camera 3	Point 1	Point 2	
Camera 1	$0_{1\times3}$	$0_{1\times3}$	2D Point	$0_{1\times2}$	
$0_{1\times3}$	Camera 2	$0_{1\times3}$	2D Point	$0_{1\times2}$	
$0_{1\times3}$	$0_{1\times3}$	Camera 3	2D Point	$0_{1\times2}$	
Camera 1	$0_{1\times3}$	$0_{1\times3}$	$0_{1\times2}$	2D Point	Proj. point 2 to cam 1
$0_{1\times3}$	Camera 2	$0_{1\times3}$	$0_{1\times2}$	2D Point	Proj. point 2 to cam 2
$0_{1\times3}$	$0_{1\times3}$	Camera 3	$0_{1\times2}$	2D Point	Proj. point 2 to cam 3



$$\sum_{i=1}^{3} \left\| \tilde{x}_{i2} - f_{i2}(\mathbf{X}) \right\|^2$$

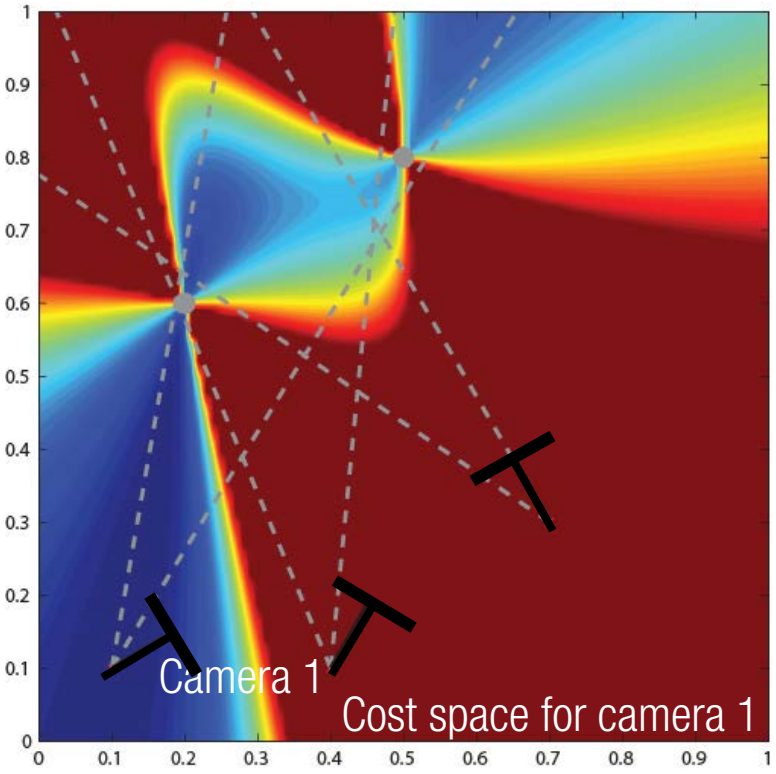
$$= \sum_{i=1}^{3} \left\| \tilde{x}_{i2} - u_{i2} / w_{i2} \right\|^2$$

$$\tilde{x}_{i2}$$

Camera index Point index

$J =$

Camera 1	Camera 2	Camera 3	Point 1	Point 2	
Camera 1	$0_{1\times3}$	$0_{1\times3}$	2D Point	$0_{1\times2}$	Proj. point 1 to cam 1
$0_{1\times3}$	Camera 2	$0_{1\times3}$	2D Point	$0_{1\times2}$	
$0_{1\times3}$	$0_{1\times3}$	Camera 3	2D Point	$0_{1\times2}$	
Camera 1	$0_{1\times3}$	$0_{1\times3}$	$0_{1\times2}$	2D Point	Proj. point 2 to cam 1
$0_{1\times3}$	Camera 2	$0_{1\times3}$	$0_{1\times2}$	2D Point	
$0_{1\times3}$	$0_{1\times3}$	Camera 3	$0_{1\times2}$	2D Point	



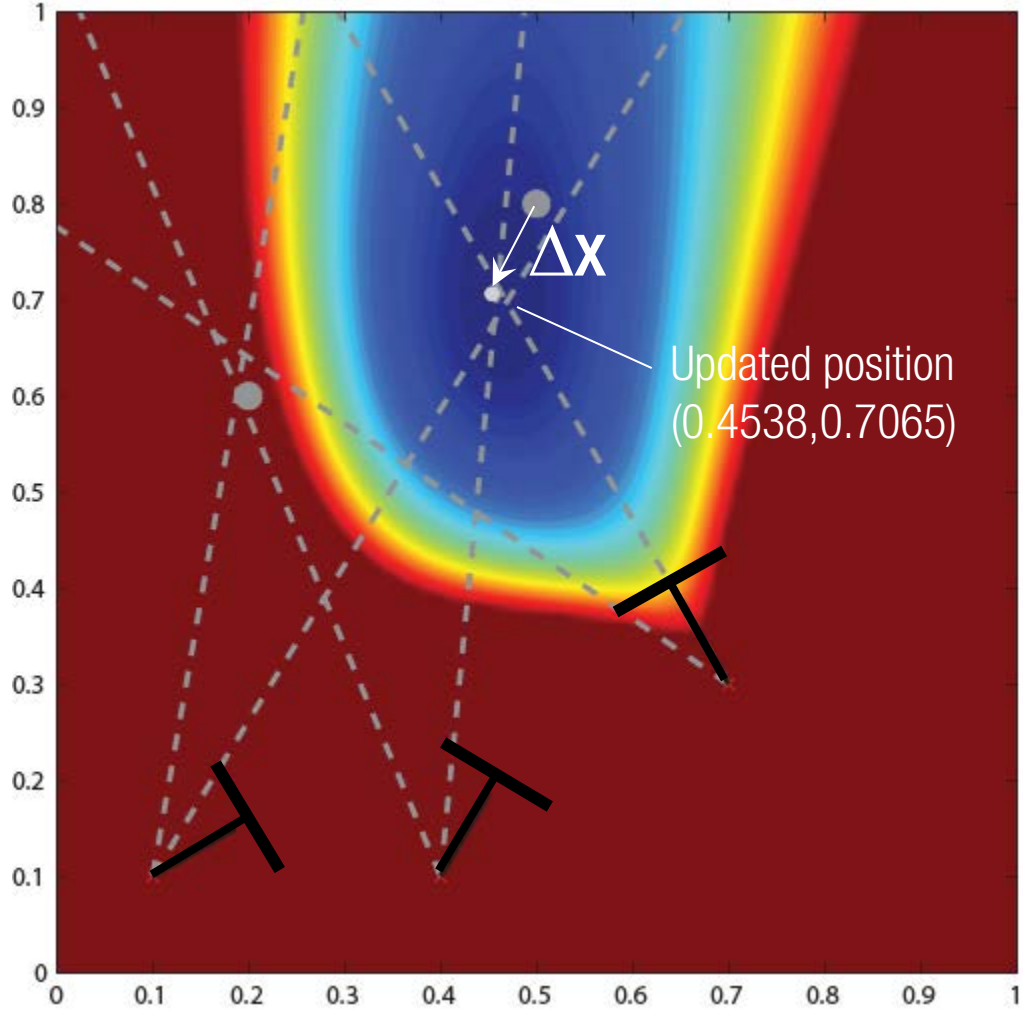
$$\sum_{j=1}^{2} \left\| \tilde{x}_{1j} - f_{1j}(\mathbf{X}) \right\|^2$$

$$= \sum_{j=1}^{2} \left\| \tilde{x}_{1j} - u_{1j} / w_{1j} \right\|^2$$

$$\tilde{x}_{1j}$$

Camera index Point index

$$\Delta \mathbf{x} = \left(\mathbf{J}^{\mathsf{T}} \mathbf{J} \right)^{-1} \mathbf{J}^{\mathsf{T}} \left(\mathbf{b} - \mathbf{f}(\mathbf{x}) \right)$$



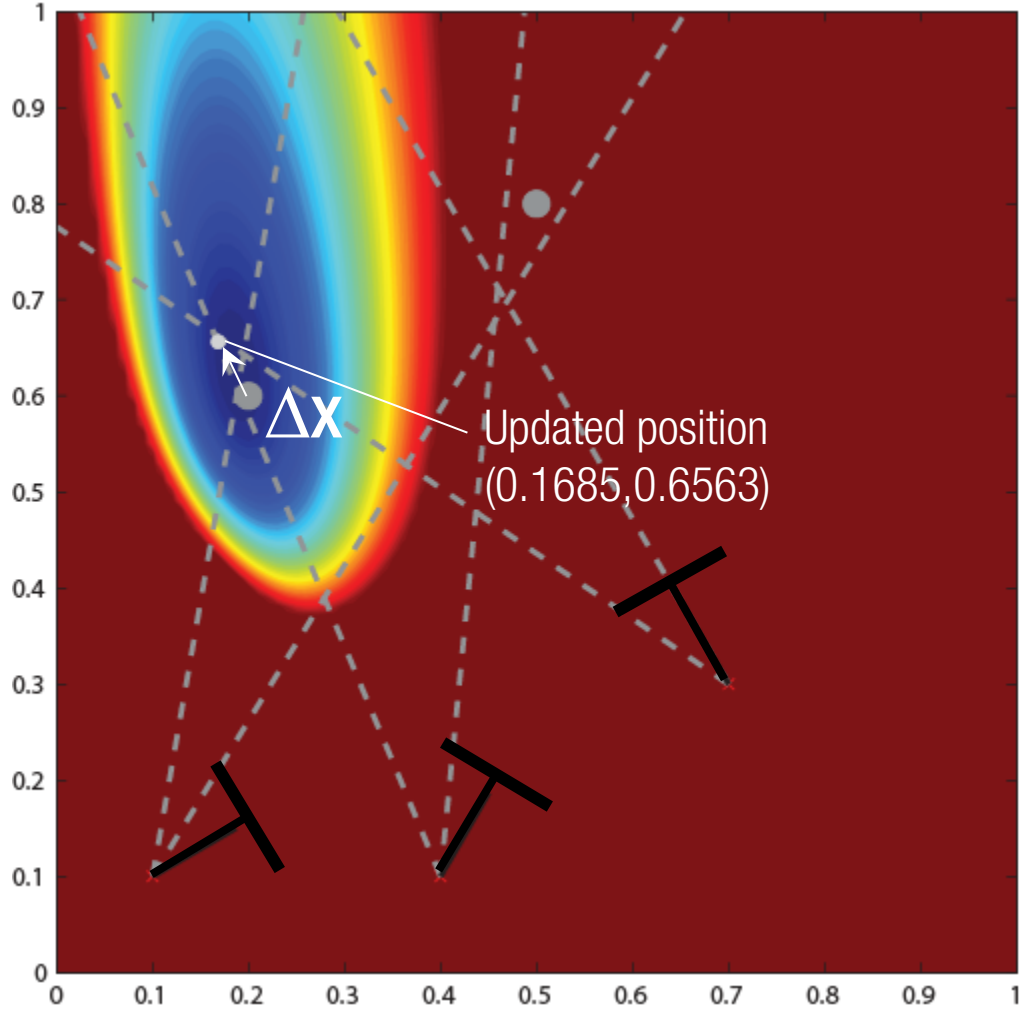
$$\sum_{i=1}^{3} \left\| \tilde{x}_{i1} - f_{i1}(\mathbf{X}) \right\|^2$$

$$= \sum_{i=1}^{3} \left\| \tilde{x}_{i1} - u_{i1} / w_{i1} \right\|^2$$

$$\tilde{x}_{i1}$$

Camera index Point index

$$\Delta \mathbf{x} = \left(\mathbf{J}^{\mathsf{T}} \mathbf{J} \right)^{-1} \mathbf{J}^{\mathsf{T}} \left(\mathbf{b} - \mathbf{f}(\mathbf{x}) \right)$$



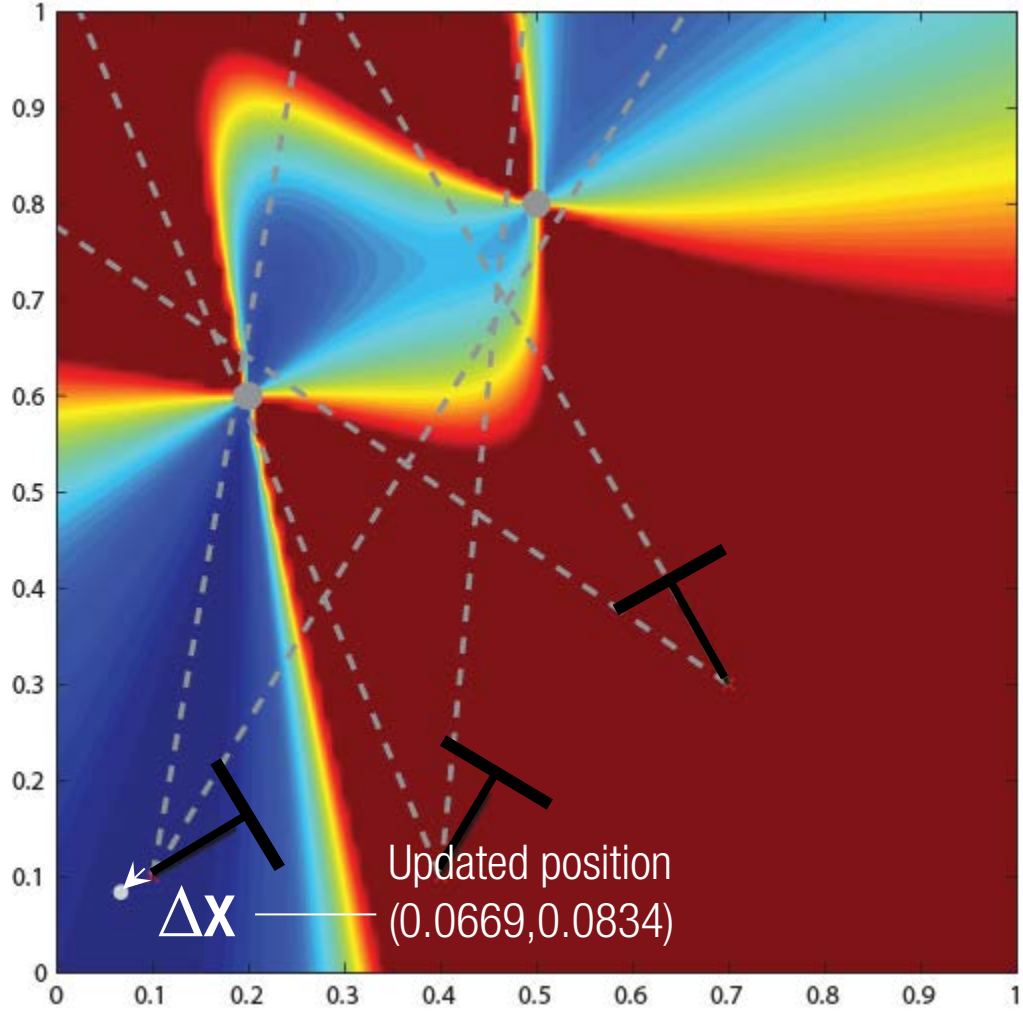
$$\sum_{i=1}^{3} \left\| \tilde{X}_{i2} - f_{i2}(\mathbf{X}) \right\|^2$$

$$= \sum_{i=1}^{3} \left\| \tilde{X}_{i2} - u_{i2} / w_{i2} \right\|^2$$

$$\tilde{X}_{i2}$$

Camera index Point index

$$\Delta \mathbf{x} = \left(\mathbf{J}^{\mathsf{T}} \mathbf{J} \right)^{-1} \mathbf{J}^{\mathsf{T}} \left(\mathbf{b} - \mathbf{f}(\mathbf{x}) \right)$$

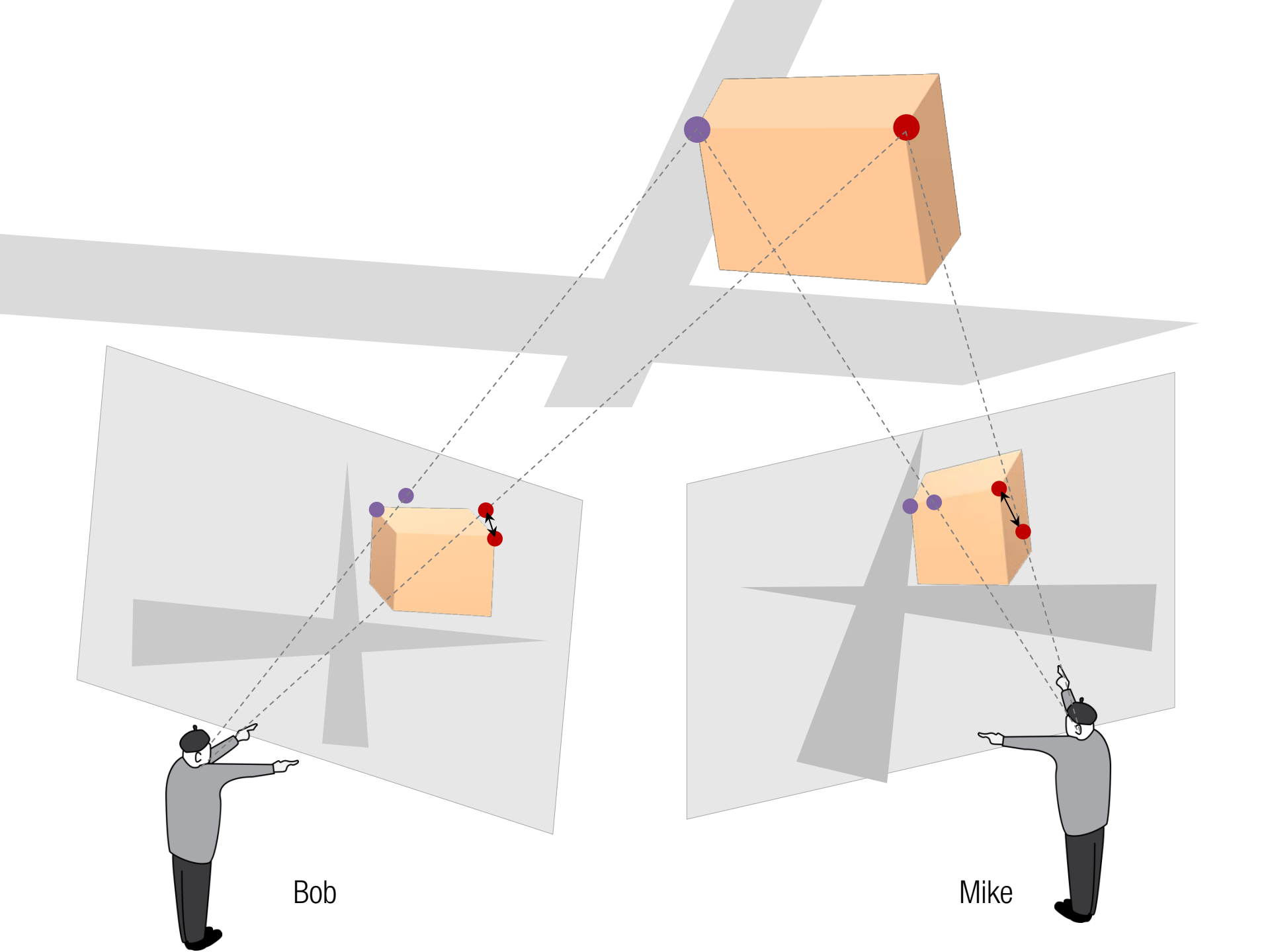


$$\sum_{j=1}^{2} \left\| \tilde{x}_{1j} - f_{1j}(\mathbf{x}) \right\|^2$$

$$= \sum_{j=1}^{2} \left\| \tilde{x}_{1j} - u_{1j} / w_{1j} \right\|^2$$

$$\tilde{x}_{1j}$$

Camera index Point index


Bob
Mike

$$J = \begin{bmatrix} \underbrace{\frac{\partial f(R(q),C,X)}{\partial R}}_{2x9} \underbrace{\frac{\partial R}{\partial q}}_{9x4} & \underbrace{\frac{\partial f(R(q),C,X)}{\partial C}}_{2x3} & \underbrace{\frac{\partial f(R(q),C,X)}{\partial X}}_{2x3} \end{bmatrix}$$

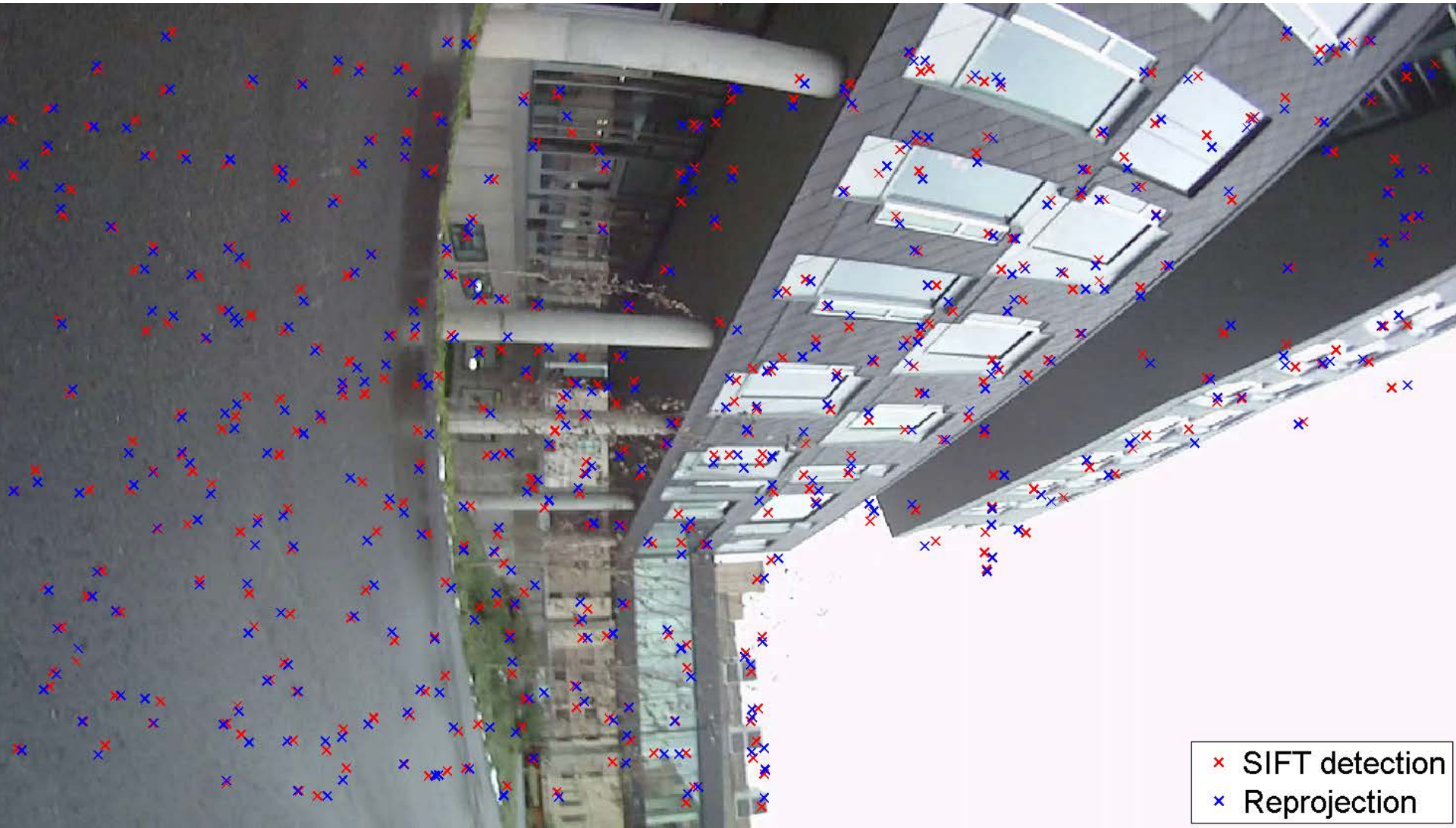
$$J = \begin{bmatrix} \text{Bob's Jacobian} & \mathbf{0}_{2\times7} & \text{3D Point} \\ \mathbf{0}_{2\times7} & \text{Mike's Jacobian} & \text{3D Point} \end{bmatrix}$$

$$J = \begin{bmatrix} \underbrace{\frac{\partial f(R(q),C,X)}{\partial R}}_{2\times 9} \underbrace{\frac{\partial R}{\partial q}}_{9\times 4} & \underbrace{\frac{\partial f(R(q),C,X)}{\partial C}}_{2\times 3} & \underbrace{\frac{\partial f(R(q),C,X)}{\partial X}}_{2\times 3} \end{bmatrix}$$

$$J = \begin{bmatrix} \text{Bob's Jacobian} & 0_{2\times 7} & \text{3D Point} & 0_{2\times 3} \\ 0_{2\times 7} & \text{Mike's Jacobian} & \text{3D Point} & 0_{2\times 3} \\ \text{Bob's Jacobian} & 0_{2\times 7} & 0_{2\times 3} & \text{3D Point} \\ 0_{2\times 7} & \text{Mike's Jacobian} & 0_{2\times 3} & \text{3D Point} \end{bmatrix}$$

Geometric Refinement

Before Bundle Adjustment



Geometric Refinement

After Bundle Adjustment