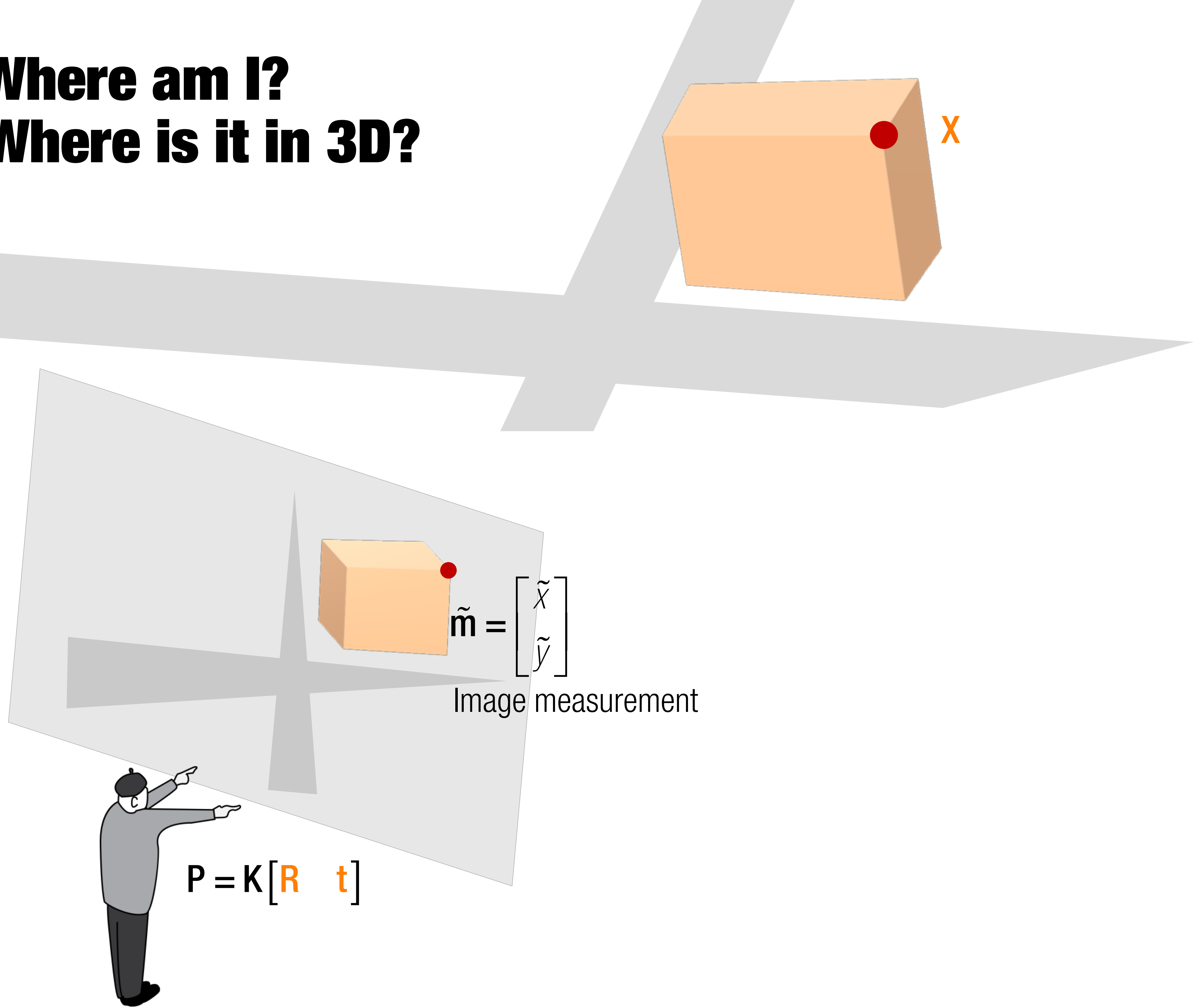
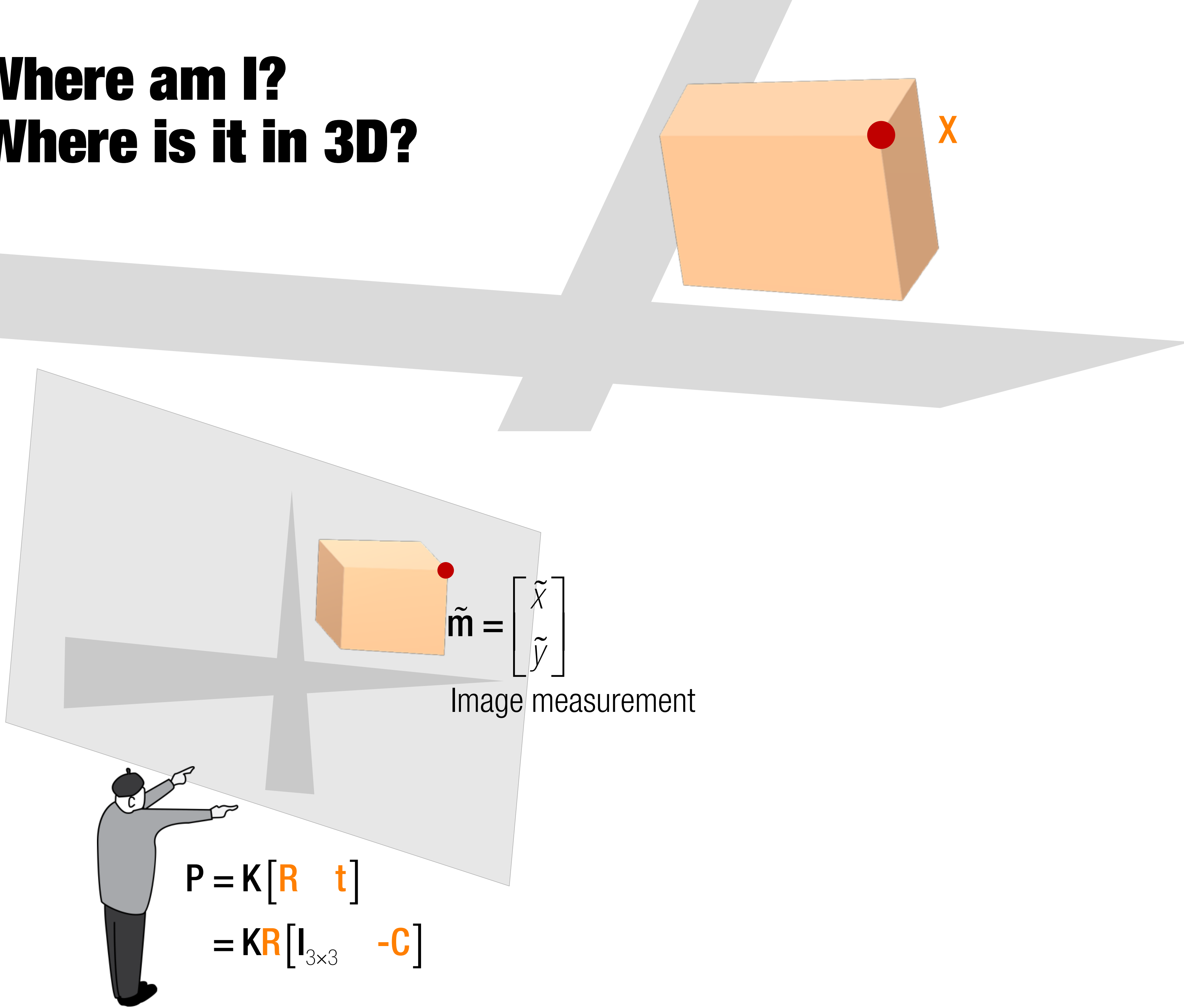
# Where am I?
# Where is it in 3D?



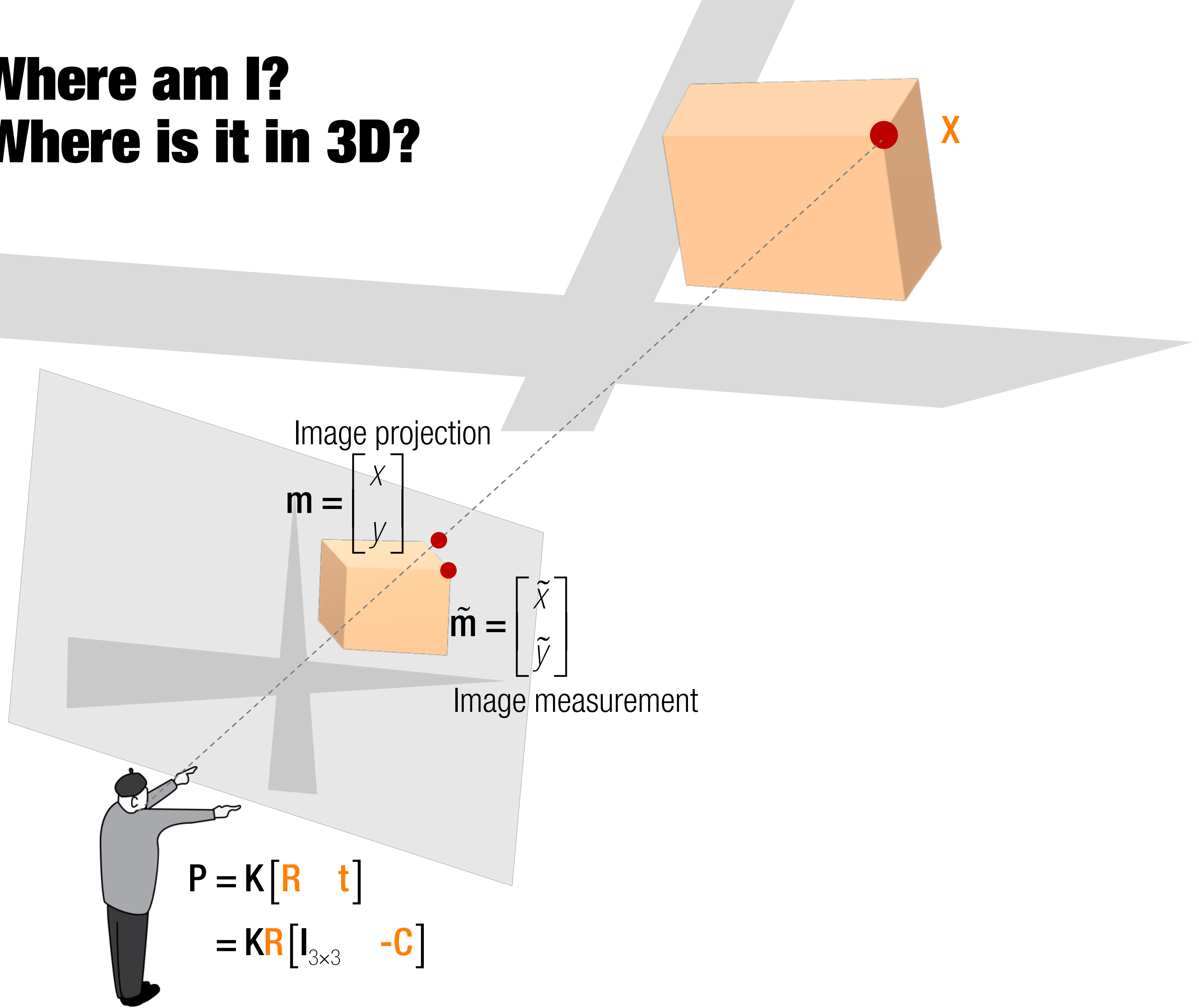
# Where am I?
# Where is it in 3D?



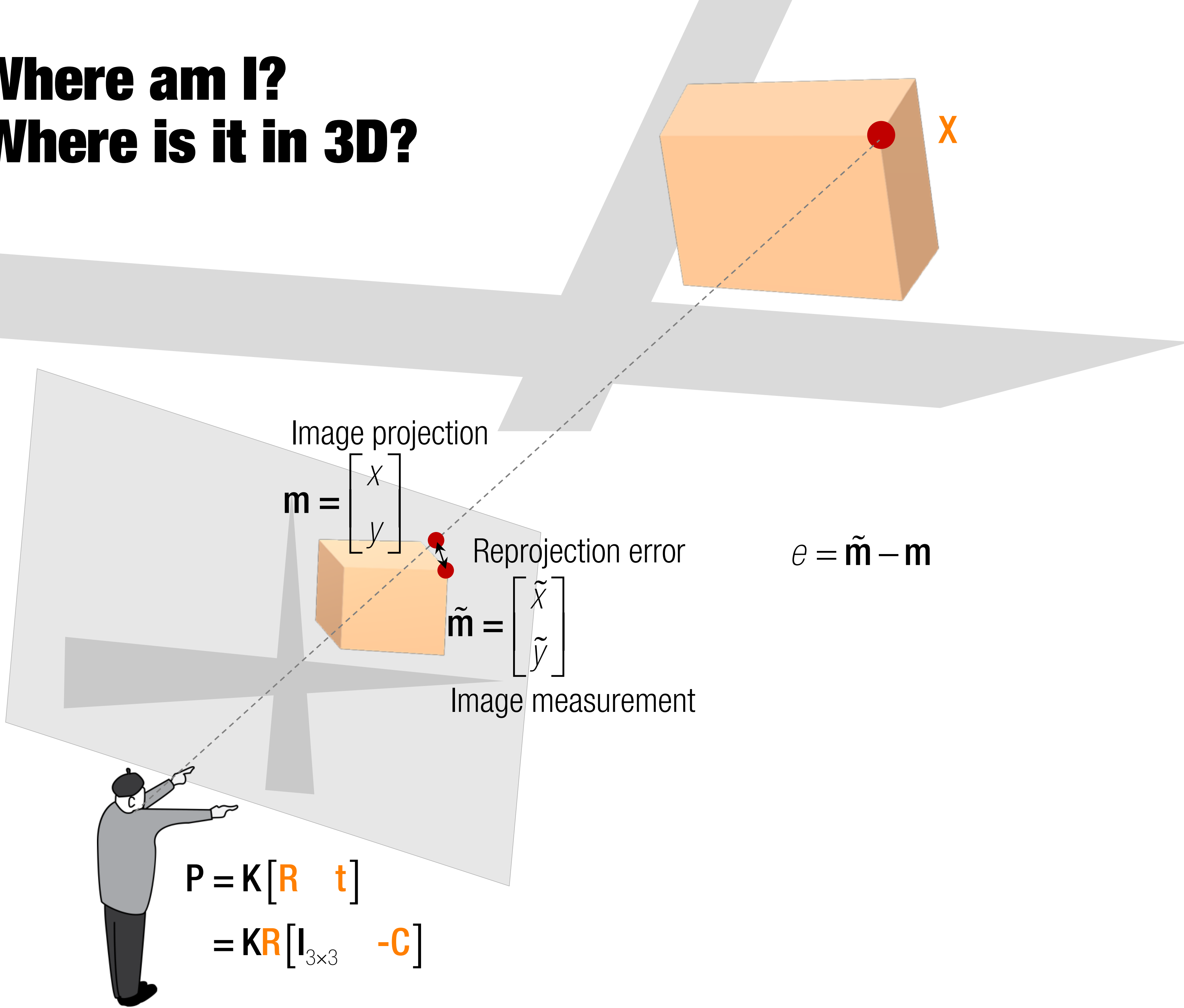

$$\tilde{\mathbf{m}} = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix}$$

$$\mathbf{P} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{I}_{3\times3} & -\mathbf{C} \end{bmatrix}$$

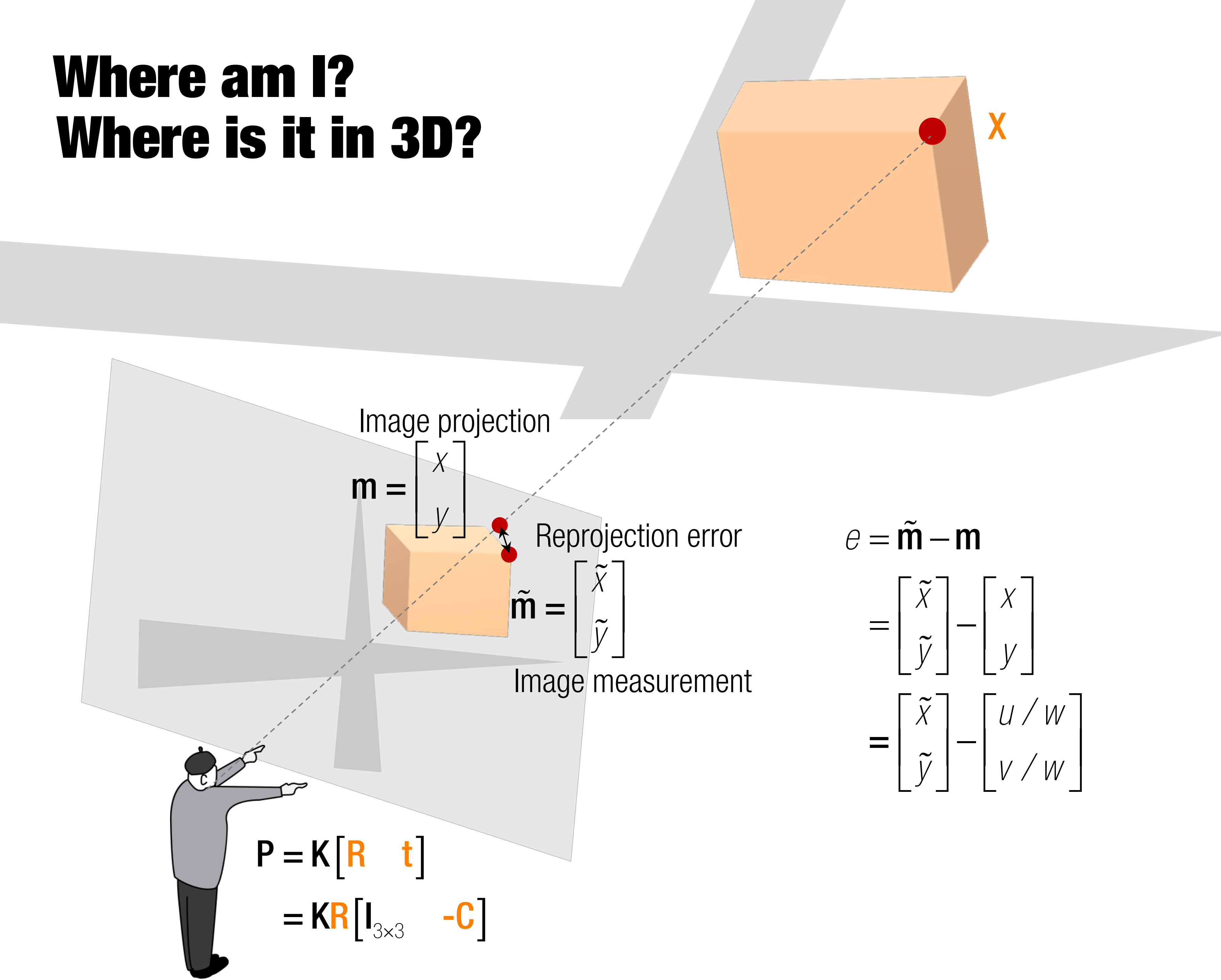
# Where am I?
# Where is it in 3D?



# Where am I?
# Where is it in 3D?



# Where am I?
# Where is it in 3D?



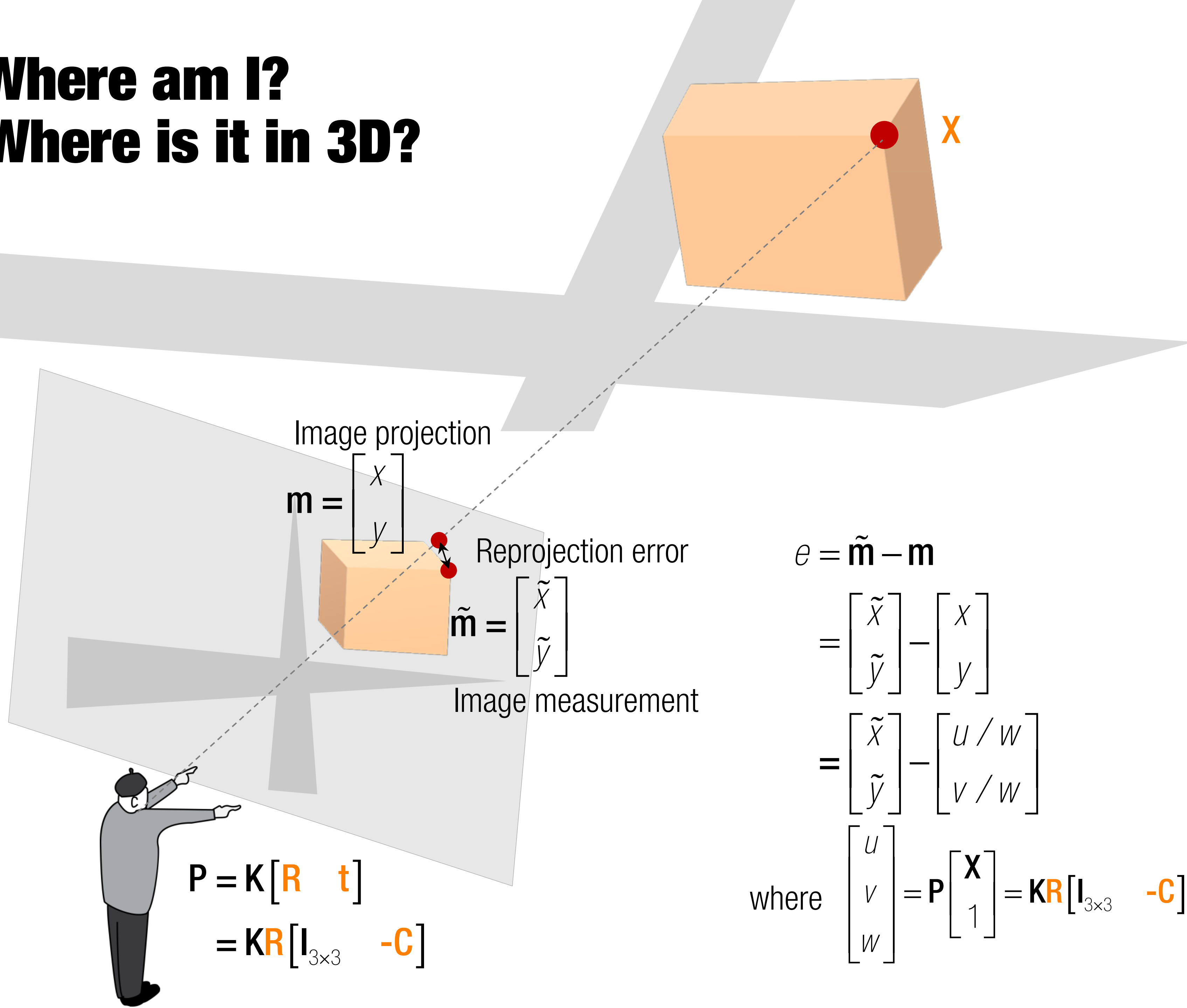

$$\mathbf{m} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\tilde{\mathbf{m}} = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix}$$

$$e = \tilde{\mathbf{m}} - \mathbf{m} = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u / w \\ v / w \end{bmatrix}$$

$$\mathbf{P} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{I}_{3\times3} & -\mathbf{C} \end{bmatrix}$$

# Where am I?
# Where is it in 3D?




$$\mathbf{m} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\tilde{\mathbf{m}} = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix}$$

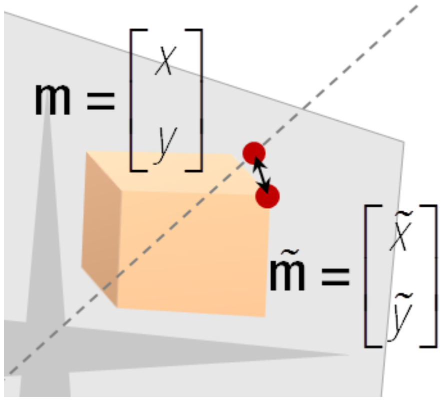
$$\mathbf{P} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{C} \end{bmatrix}$$

$$e = \tilde{\mathbf{m}} - \mathbf{m} = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u/w \\ v/w \end{bmatrix}$$

where $\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{P}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{C} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$

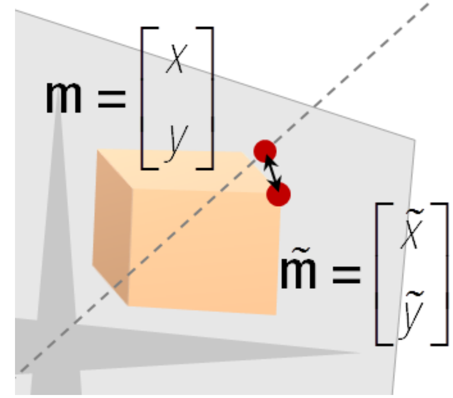
# Reprojection error

$$e = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u/w \\ v/w \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{P}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{C} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



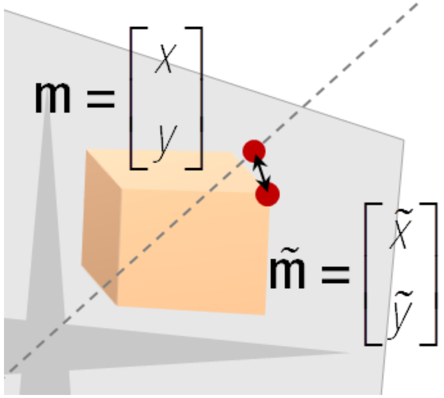
# Reprojection error

$$e = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u/w \\ v/w \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{P}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{C} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$




$$\underset{\mathbf{R},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u(\mathbf{R},\mathbf{C},\mathbf{X}) / w(\mathbf{R},\mathbf{C},\mathbf{X}) \\ v(\mathbf{R},\mathbf{C},\mathbf{X}) / w(\mathbf{R},\mathbf{C},\mathbf{X}) \end{bmatrix} \right\|^2$$

# Reprojection error




$$e = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u/w \\ v/w \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{P}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{C} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

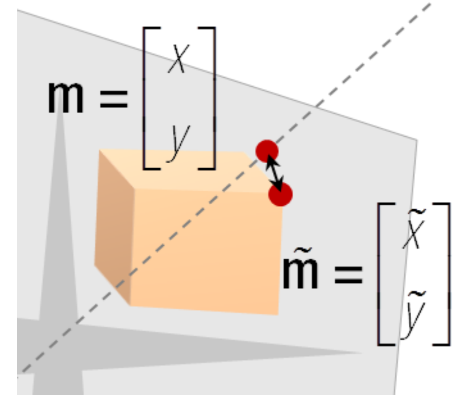
$$\underset{\mathbf{R},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u(\mathbf{R},\mathbf{C},\mathbf{X})/w(\mathbf{R},\mathbf{C},\mathbf{X}) \\ v(\mathbf{R},\mathbf{C},\mathbf{X})/w(\mathbf{R},\mathbf{C},\mathbf{X}) \end{bmatrix} \right\|^2$$

$$= \underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})/w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \\ v(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})/w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \end{bmatrix} \right\|^2$$

: Quaternion parameterization

# Reprojection error

$$e = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u/w \\ v/w \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{P}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{C} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$




$$\underset{\mathbf{R},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u(\mathbf{R},\mathbf{C},\mathbf{X})/w(\mathbf{R},\mathbf{C},\mathbf{X}) \\ v(\mathbf{R},\mathbf{C},\mathbf{X})/w(\mathbf{R},\mathbf{C},\mathbf{X}) \end{bmatrix} \right\|^2$$

$$= \underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \underbrace{\begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix}}_{\mathbf{b}} - \underbrace{\begin{bmatrix} u(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})/w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \\ v(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})/w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \end{bmatrix}}_{\mathbf{f}(\mathbf{R},\mathbf{C},\mathbf{X})} \right\|^2$$

: Quaternion parameterization

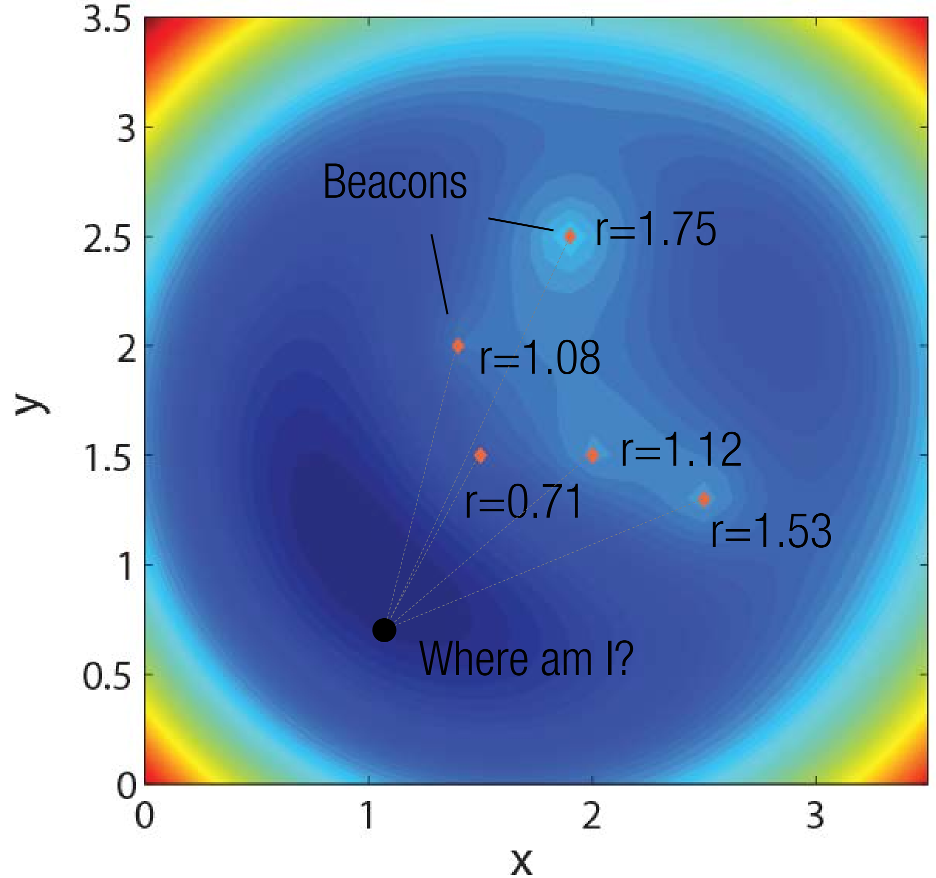
# Recall

**Nonlinear least squares:**

$$\underset{\mathbf{x}}{\text{minimize}} \left\| \mathbf{f}(\mathbf{x}) - \mathbf{b} \right\|^2 = \underset{\mathbf{x}}{\text{minimize}}\ \mathbf{f}(\mathbf{x})^{\mathsf{T}} \mathbf{f}(\mathbf{x}) - 2\mathbf{b}^{\mathsf{T}} \mathbf{f}(\mathbf{x})$$

$$\longrightarrow \left.\frac{\partial E}{\partial \mathbf{x}}\right|_{\mathbf{x}^*} = 2\frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}} \mathbf{f}(\mathbf{x}) - 2\frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}} \mathbf{b} = \mathbf{0}$$

Example:
Localization using range data from beacons



# Recall

**Nonlinear least squares:**

$$\underset{\mathbf{x}}{\text{minimize}} \left\| \mathbf{f(x)} - \mathbf{b} \right\|^2 = \underset{\mathbf{x}}{\text{minimize}}\ \mathbf{f(x)}^{\mathrm{T}}\mathbf{f(x)} - 2\mathbf{b}^{\mathrm{T}}\mathbf{f(x)}$$

$$\longrightarrow \left.\frac{\partial E}{\partial \mathbf{x}}\right|_{\mathbf{x}^*} = 2\frac{\partial \mathbf{f(x)}}{\partial \mathbf{x}}^{\mathrm{T}} \mathbf{f(x)} - 2\frac{\partial \mathbf{f(x)}}{\partial \mathbf{x}}^{\mathrm{T}} \mathbf{b} = 0$$

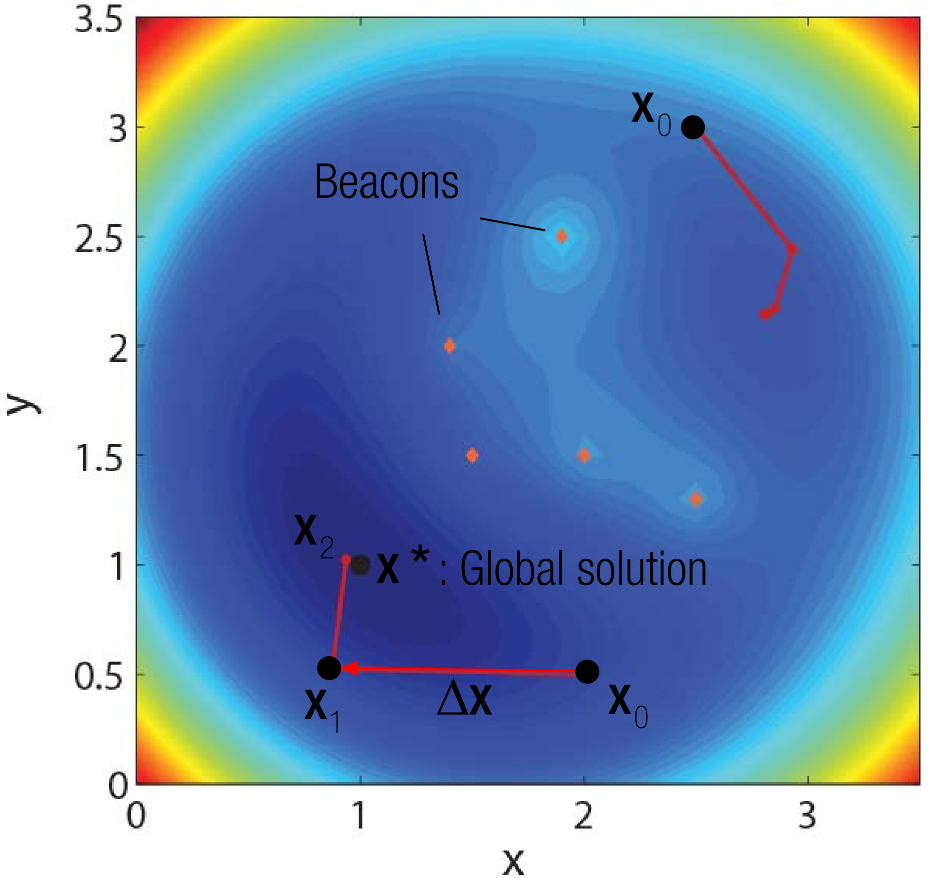
**Taylor expansion around x:**

$$\mathbf{f}(\mathbf{x}+\Delta\mathbf{x}) \approx \mathbf{f(x)} + \frac{\partial \mathbf{f(x)}}{\partial \mathbf{x}}\Delta\mathbf{x}$$

where $\boxed{\Delta\mathbf{x} = \left(\mathbf{J}^{\mathrm{T}}\mathbf{J}\right)^{-1}\mathbf{J}^{\mathrm{T}}\left(\mathbf{b} - \mathbf{f(x)}\right)}$

Normal equation

Example:
Localization using range data from beacons



# Recall

## Nonlinear least squares:

$$\underset{\mathbf{x}}{\text{minimize}} \left\| \mathbf{f}(\mathbf{x}) - \mathbf{b} \right\|^2 = \underset{\mathbf{x}}{\text{minimize}}\ \mathbf{f}(\mathbf{x})^T \mathbf{f}(\mathbf{x}) - 2\mathbf{b}^T \mathbf{f}(\mathbf{x})$$

$$\longrightarrow \left.\frac{\partial E}{\partial \mathbf{x}}\right|_{\mathbf{x}^*} = 2\frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}}^T \mathbf{f}(\mathbf{x}) - 2\frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}}^T \mathbf{b} = \mathbf{0}$$

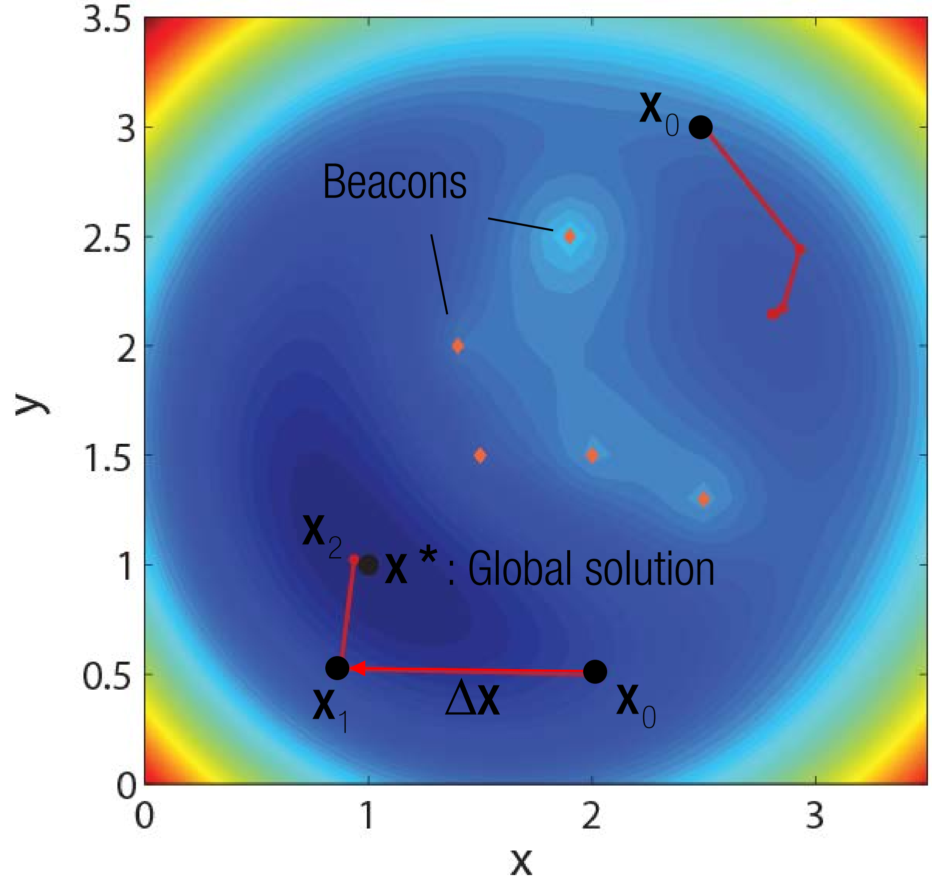
## Taylor expansion around x:

$$\mathbf{f}(\mathbf{x} + \Delta\mathbf{x}) \approx \mathbf{f}(\mathbf{x}) + \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} \Delta\mathbf{x}$$

where $$\underbrace{\boxed{\Delta\mathbf{x} = \left(\mathbf{J}^T\mathbf{J}\right)^{-1} \mathbf{J}^T \left(\mathbf{b} - \mathbf{f}(\mathbf{x})\right)}}_{\text{Normal equation}}$$

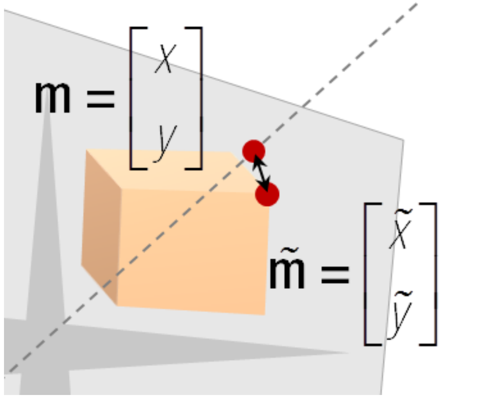
$$\mathbf{J} = \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_1} & \cdots & \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_n} \\ \vdots & & \vdots \\ \frac{\partial \mathbf{f}_m}{\partial \mathbf{x}_1} & \cdots & \frac{\partial \mathbf{f}_m}{\partial \mathbf{x}_n} \end{bmatrix}$$ : Jacobian matrix

Example:
Localization using range data from beacons



# Reprojection error

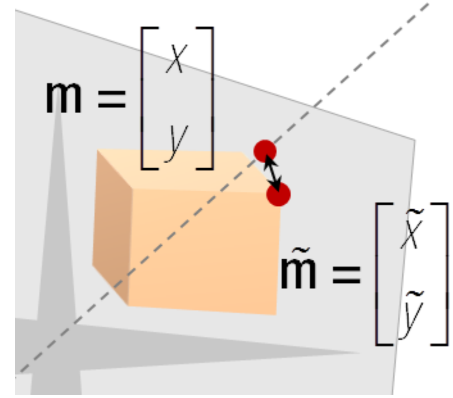
$$e = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u/w \\ v/w \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{P}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{C} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$




$$\underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) / w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \\ v(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) / w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \end{bmatrix} \right\|^2 = \underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \mathbf{b} - \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \right\|^2$$

$$\mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) = \begin{bmatrix} u/w \\ u/w \end{bmatrix}$$

# Reprojection error

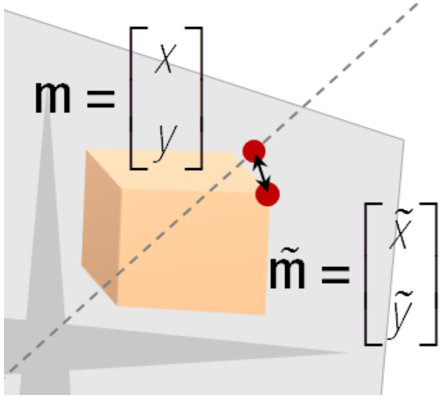



$$e = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u/w \\ v/w \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{P}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{C} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

$$\underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})/w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \\ v(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})/w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \end{bmatrix} \right\|^2 = \underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \mathbf{b} - \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \right\|^2$$

$$\mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) = \begin{bmatrix} u/w \\ u/w \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{X} - \mathbf{C} \end{bmatrix}$$

# Reprojection error



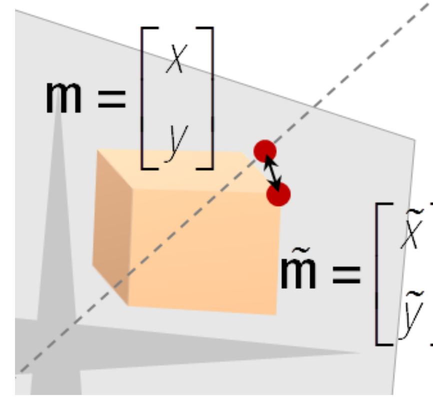

$$e = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u/w \\ v/w \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{P}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{I}_{3\times3} & -\mathbf{C} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

$$\underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})/w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \\ v(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})/w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \end{bmatrix} \right\|^2 = \underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \mathbf{b} - \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \right\|^2$$

$$\mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) = \begin{bmatrix} u/w \\ u/w \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{X} - \mathbf{C} \end{bmatrix}$$

$$= \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} \mathbf{X} - \mathbf{C} \end{bmatrix}$$

# Reprojection error




$$e = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u/w \\ v/w \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{P}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{KR}\begin{bmatrix} \mathbf{I}_{3\times3} & -\mathbf{C} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

$$\underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})/w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \\ v(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})/w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \end{bmatrix} \right\|^2 = \underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \mathbf{b} - \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \right\|^2$$

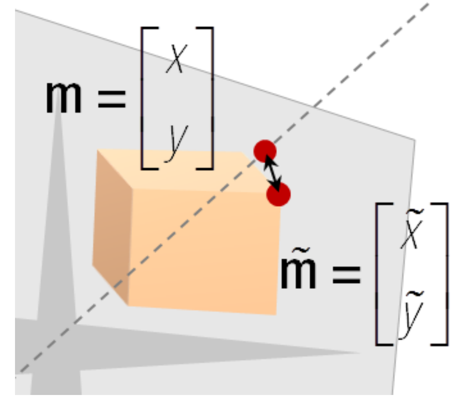
$$\mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) = \begin{bmatrix} u/w \\ u/w \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{KR}\begin{bmatrix} \mathbf{X}-\mathbf{C} \end{bmatrix}$$

$$= \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} \mathbf{X}-\mathbf{C} \end{bmatrix}$$

$$= \begin{bmatrix} fr_{11}+p_x r_{31} & fr_{12}+p_x r_{32} & fr_{13}+p_x r_{33} \\ fr_{21}+p_y r_{31} & fr_{22}+p_y r_{32} & fr_{23}+p_y r_{33} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} \mathbf{X}-\mathbf{C} \end{bmatrix}$$

# Reprojection error

$$e = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u/w \\ v/w \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{P}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{C} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



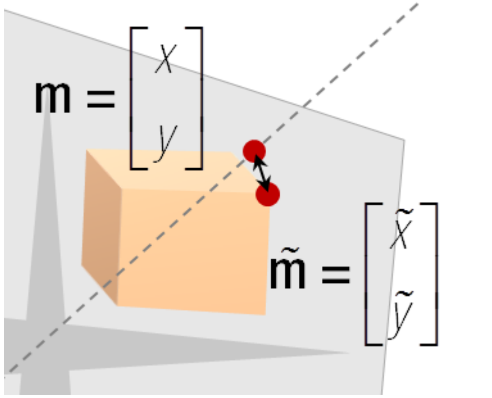

$$\underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})/w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \\ v(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})/w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \end{bmatrix} \right\|^2 = \underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \mathbf{b} - \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \right\|^2$$

$$\mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) = \begin{bmatrix} u/w \\ u/w \end{bmatrix}$$

$$\text{where} \quad \begin{aligned} u &= \begin{bmatrix} fr_{11} + p_x r_{31} & fr_{12} + p_x r_{32} & fr_{13} + p_x r_{33} \end{bmatrix}\begin{bmatrix} \mathbf{X} - \mathbf{C} \end{bmatrix} \\ v &= \begin{bmatrix} fr_{21} + p_y r_{31} & fr_{22} + p_y r_{32} & fr_{23} + p_y r_{33} \end{bmatrix}\begin{bmatrix} \mathbf{X} - \mathbf{C} \end{bmatrix} \\ w &= \begin{bmatrix} r_{31} & r_{32} & r_{33} \end{bmatrix}\begin{bmatrix} \mathbf{X} - \mathbf{C} \end{bmatrix} \end{aligned}$$

# Reprojection error

$$e = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u/w \\ v/w \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{P}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{C} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

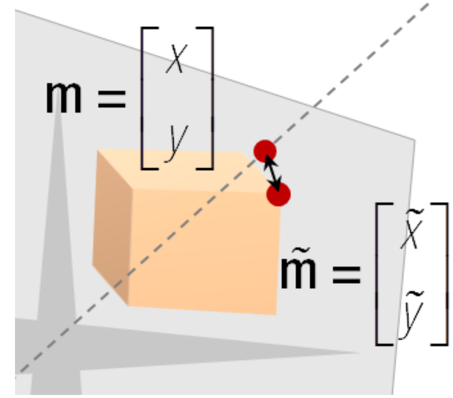



$$\underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})/w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \\ v(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})/w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \end{bmatrix} \right\|^2 = \underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \mathbf{b} - \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \right\|^2$$

$$\mathbf{J} = ?$$

# Reprojection error

$$e = \begin{bmatrix}\tilde{x}\\ \tilde{y}\end{bmatrix} - \begin{bmatrix} u/w \\ v/w\end{bmatrix} \quad \text{where} \quad \begin{bmatrix}u\\ v\\ w\end{bmatrix} = \mathbf{P}\begin{bmatrix}\mathbf{X}\\ 1\end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix}\mathbf{I}_{3\times3} & -\mathbf{C}\end{bmatrix}\begin{bmatrix}\mathbf{X}\\ 1\end{bmatrix}$$



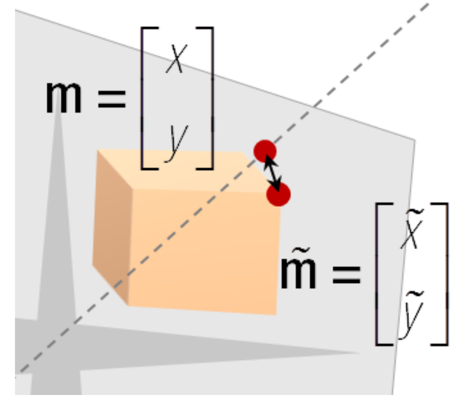

$$\underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \begin{bmatrix}\tilde{x}\\ \tilde{y}\end{bmatrix} - \begin{bmatrix} u(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) / w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \\ v(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) / w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \end{bmatrix} \right\|^2 = \underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \mathbf{b} - \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \right\|^2$$

**J =**

**q** **C** **X**

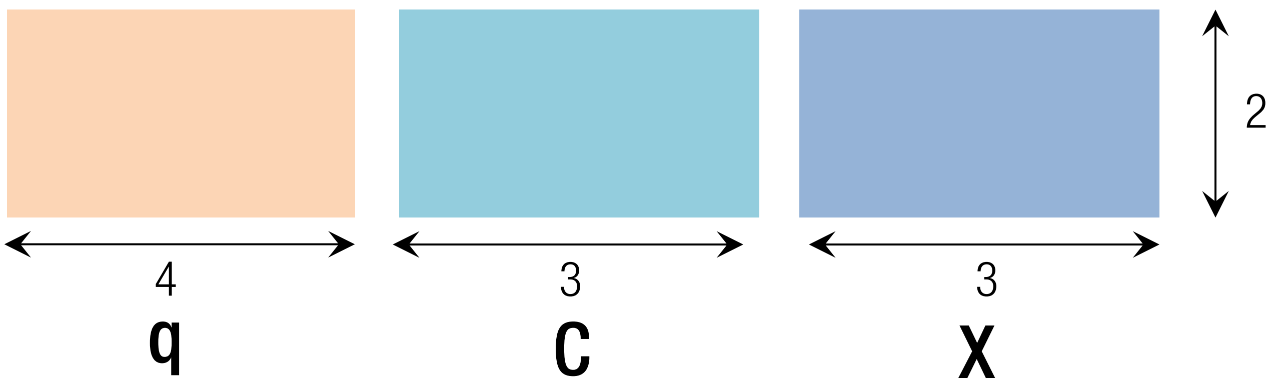
# Reprojection error

$$e = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u/w \\ v/w \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{P}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{C} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



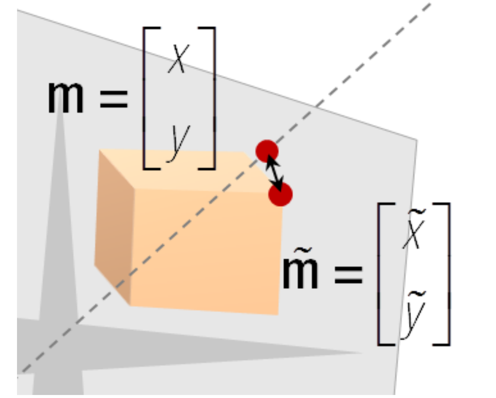

$$\underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) / w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \\ v(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) / w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \end{bmatrix} \right\|^2 = \underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \mathbf{b} - \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \right\|^2$$

$\mathbf{J} =$



# Reprojection error

$$e = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u/w \\ v/w \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{P}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{C} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

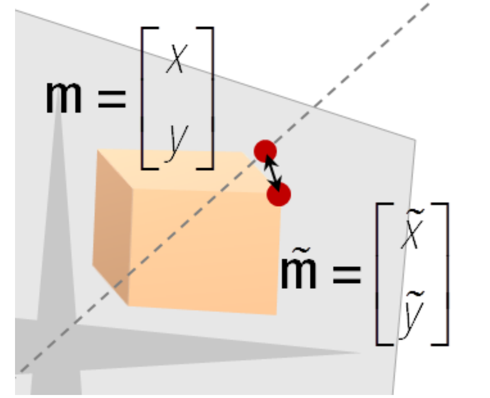



$$\underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})/w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \\ v(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})/w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \end{bmatrix} \right\|^2 = \underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \mathbf{b} - \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \right\|^2$$

$$\mathbf{J} = \begin{bmatrix} \underbrace{\dfrac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{q}}}_{\substack{4 \\ \mathbf{q}}} & \underbrace{\dfrac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{C}}}_{\substack{3 \\ \mathbf{C}}} & \underbrace{\dfrac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{X}}}_{\substack{3 \\ \mathbf{X}}} \end{bmatrix} \Big\updownarrow 2$$

# Reprojection error

$$e = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u/w \\ v/w \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{P}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{C} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$




$$\underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} - \begin{bmatrix} u(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) / w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \\ v(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) / w(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \end{bmatrix} \right\|^2 = \underset{\mathbf{q},\mathbf{C},\mathbf{X}}{\text{minimize}} \left\| \mathbf{b} - \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) \right\|^2$$

$$\mathbf{J} = \begin{bmatrix} \dfrac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{q}} & \dfrac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{C}} & \dfrac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{X}} \end{bmatrix}$$

$$= \begin{bmatrix} \underbrace{\dfrac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{R}}}_{2\text{x}9} \underbrace{\dfrac{\partial \mathbf{R}}{\partial \mathbf{q}}}_{9\text{x}4} & \underbrace{\dfrac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{C}}}_{2\text{x}3} & \underbrace{\dfrac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{X}}}_{2\text{x}3} \end{bmatrix}$$

$$\mathbf{J} = \begin{bmatrix} \underbrace{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{R}}}_{\text{2x9}} \underbrace{\frac{\partial \mathbf{R}}{\partial \mathbf{q}}}_{\text{9x4}} & \underbrace{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{C}}}_{\text{2x3}} & \underbrace{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{X}}}_{\text{2x3}} \end{bmatrix}$$

$$\mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) = \begin{bmatrix} u/w \\ u/w \end{bmatrix} \quad \text{where} \quad \begin{aligned} u &= \begin{bmatrix} fr_{11}+p_x r_{31} & fr_{12}+p_x r_{32} & fr_{13}+p_x r_{33} \end{bmatrix}\begin{bmatrix}\mathbf{X}-\mathbf{C}\end{bmatrix} \\ v &= \begin{bmatrix} fr_{21}+p_y r_{31} & fr_{22}+p_y r_{32} & fr_{23}+p_y r_{33} \end{bmatrix}\begin{bmatrix}\mathbf{X}-\mathbf{C}\end{bmatrix} \\ w &= \begin{bmatrix} r_{31} & r_{32} & r_{33} \end{bmatrix}\begin{bmatrix}\mathbf{X}-\mathbf{C}\end{bmatrix} \end{aligned}$$

$$\mathbf{J} = \begin{bmatrix} \underset{2\text{x}9}{\dfrac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{R}}} \underset{9\text{x}4}{\dfrac{\partial \mathbf{R}}{\partial \mathbf{q}}} & \underset{2\text{x}3}{\dfrac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{C}}} & \underset{2\text{x}3}{\dfrac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{X}}} \end{bmatrix}$$

$$\mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) = \begin{bmatrix} u/w \\ u/w \end{bmatrix} \quad \text{where} \quad \begin{aligned} u &= \begin{bmatrix} fr_{11}+p_x r_{31} & fr_{12}+p_x r_{32} & fr_{13}+p_x r_{33} \end{bmatrix}\begin{bmatrix}\mathbf{X}-\mathbf{C}\end{bmatrix} \\ v &= \begin{bmatrix} fr_{21}+p_y r_{31} & fr_{22}+p_y r_{32} & fr_{23}+p_y r_{33} \end{bmatrix}\begin{bmatrix}\mathbf{X}-\mathbf{C}\end{bmatrix} \\ w &= \begin{bmatrix} r_{31} & r_{32} & r_{33} \end{bmatrix}\begin{bmatrix}\mathbf{X}-\mathbf{C}\end{bmatrix} \end{aligned}$$

---

$$\underset{2\text{x}3}{\dfrac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{C}}} = \begin{bmatrix} \dfrac{w\dfrac{\partial u}{\partial \mathbf{C}} - u\dfrac{\partial w}{\partial \mathbf{C}}}{w^2} \\ \dfrac{w\dfrac{\partial v}{\partial \mathbf{C}} - v\dfrac{\partial w}{\partial \mathbf{C}}}{w^2} \end{bmatrix} \quad \text{where} \quad \begin{aligned} \frac{\partial u}{\partial \mathbf{C}} &= -\begin{bmatrix} fr_{11}+p_x r_{31} & fr_{12}+p_x r_{32} & fr_{13}+p_x r_{33} \end{bmatrix} \\ \frac{\partial v}{\partial \mathbf{C}} &= -\begin{bmatrix} fr_{21}+p_y r_{31} & fr_{22}+p_y r_{32} & fr_{23}+p_y r_{33} \end{bmatrix} \\ \frac{\partial w}{\partial \mathbf{C}} &= -\begin{bmatrix} r_{31} & r_{32} & r_{33} \end{bmatrix} \end{aligned}$$

$$\mathbf{J} = \begin{bmatrix} \underbrace{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{R}}}_{2\text{x}9} \underbrace{\frac{\partial \mathbf{R}}{\partial \mathbf{q}}}_{9\text{x}4} & \underbrace{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{C}}}_{2\text{x}3} & \underbrace{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{X}}}_{2\text{x}3} \end{bmatrix}$$

$$\mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) = \begin{bmatrix} u/w \\ u/w \end{bmatrix}$$

where

$$u = \begin{bmatrix} fr_{11} + p_x r_{31} & fr_{12} + p_x r_{32} & fr_{13} + p_x r_{33} \end{bmatrix} [\mathbf{X} - \mathbf{C}]$$

$$v = \begin{bmatrix} fr_{21} + p_y r_{31} & fr_{22} + p_y r_{32} & fr_{23} + p_y r_{33} \end{bmatrix} [\mathbf{X} - \mathbf{C}]$$

$$w = \begin{bmatrix} r_{31} & r_{32} & r_{33} \end{bmatrix} [\mathbf{X} - \mathbf{C}]$$

---

$$\underbrace{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{X}}}_{2\text{x}3} = \begin{bmatrix} \dfrac{w\dfrac{\partial u}{\partial \mathbf{X}} - u\dfrac{\partial w}{\partial \mathbf{X}}}{w^2} \\ \dfrac{w\dfrac{\partial v}{\partial \mathbf{X}} - v\dfrac{\partial w}{\partial \mathbf{X}}}{w^2} \end{bmatrix}$$

where

$$\frac{\partial u}{\partial \mathbf{X}} = \begin{bmatrix} fr_{11} + p_x r_{31} & fr_{12} + p_x r_{32} & fr_{13} + p_x r_{33} \end{bmatrix}$$

$$\frac{\partial v}{\partial \mathbf{X}} = \begin{bmatrix} fr_{21} + p_y r_{31} & fr_{22} + p_y r_{32} & fr_{23} + p_y r_{33} \end{bmatrix}$$

$$\frac{\partial w}{\partial \mathbf{X}} = \begin{bmatrix} r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$\mathbf{J} = \left[ \underbrace{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{R}}}_{2\text{x}9} \underbrace{\frac{\partial \mathbf{R}}{\partial \mathbf{q}}}_{9\text{x}4} \quad \underbrace{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{C}}}_{2\text{x}3} \quad \underbrace{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{X}}}_{2\text{x}3} \right]$$

$$\mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X}) = \begin{bmatrix} u/w \\ u/w \end{bmatrix}$$

where

$$u = \begin{bmatrix} fr_{11} + p_x r_{31} & fr_{12} + p_x r_{32} & fr_{13} + p_x r_{33} \end{bmatrix} [\mathbf{X} - \mathbf{C}]$$

$$v = \begin{bmatrix} fr_{21} + p_y r_{31} & fr_{22} + p_y r_{32} & fr_{23} + p_y r_{33} \end{bmatrix} [\mathbf{X} - \mathbf{C}]$$

$$w = \begin{bmatrix} r_{31} & r_{32} & r_{33} \end{bmatrix} [\mathbf{X} - \mathbf{C}]$$

$$\underbrace{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{R}}}_{2\text{x}9} = \begin{bmatrix} \dfrac{w\dfrac{\partial u}{\partial \mathbf{R}} - u\dfrac{\partial w}{\partial \mathbf{R}}}{w^2} \\[2ex] \dfrac{w\dfrac{\partial v}{\partial \mathbf{R}} - v\dfrac{\partial w}{\partial \mathbf{R}}}{w^2} \end{bmatrix}$$

where

$$\frac{\partial u}{\partial \mathbf{R}} = \begin{bmatrix} f(X_1 - C_1) & \mathbf{0}_{1\times 3} & p_x(X_3 - C_3) \end{bmatrix}$$

$$\frac{\partial v}{\partial \mathbf{R}} = \begin{bmatrix} \mathbf{0}_{1\times 3} & f(X_1 - C_1) & p_x(X_3 - C_3) \end{bmatrix}$$

$$\frac{\partial w}{\partial \mathbf{R}} = \begin{bmatrix} \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 3} & (X_3 - C_3) \end{bmatrix}$$

$$\mathbf{J} = \left[ \underbrace{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}), \mathbf{C}, \mathbf{X})}{\partial \mathbf{R}}}_{2\text{x}9} \underbrace{\frac{\partial \mathbf{R}}{\partial \mathbf{q}}}_{9\text{x}4} \quad \underbrace{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}), \mathbf{C}, \mathbf{X})}{\partial \mathbf{C}}}_{2\text{x}3} \quad \underbrace{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}), \mathbf{C}, \mathbf{X})}{\partial \mathbf{X}}}_{2\text{x}3} \right]$$

$$\mathbf{R} = \begin{bmatrix} 1-2q_z^2-2q_y^2 & -2q_zq_w+2q_yq_x & 2q_yq_w+2q_zq_x \\ 2q_xq_y+2q_wq_z & 1-2q_z^2-2q_x^2 & 2q_zq_y-2q_xq_w \\ 2q_xq_z-2q_wq_y & 2q_yq_z+2q_wq_x & 1-2q_y^2-2q_x^2 \end{bmatrix}$$

where

$$\mathbf{q} = \begin{bmatrix} q_w & q_x & q_y & q_z \end{bmatrix}^T$$

$$\mathbf{J} = \begin{bmatrix} \underbrace{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{R}}}_{2\text{x}9} \underbrace{\frac{\partial \mathbf{R}}{\partial \mathbf{q}}}_{9\text{x}4} & \underbrace{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{C}}}_{2\text{x}3} & \underbrace{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{X}}}_{2\text{x}3} \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} 1-2q_z^2-2q_y^2 & -2q_zq_w+2q_yq_x & 2q_yq_w+2q_zq_x \\ 2q_xq_y+2q_wq_z & 1-2q_z^2-2q_x^2 & 2q_zq_y-2q_xq_w \\ 2q_xq_z-2q_wq_y & 2q_yq_z+2q_wq_x & 1-2q_y^2-2q_x^2 \end{bmatrix}$$

where

$$\mathbf{q} = \begin{bmatrix} q_w & q_x & q_y & q_z \end{bmatrix}^{\mathsf{T}}$$

---

$$\underbrace{\frac{\partial \mathbf{R}}{\partial \mathbf{q}}}_{9\text{x}4} = \begin{bmatrix} \frac{\partial r_{11}}{\partial \mathbf{q}} \\ \frac{\partial r_{12}}{\partial \mathbf{q}} \\ \vdots \\ \frac{\partial r_{33}}{\partial \mathbf{q}} \end{bmatrix}$$

where

$$\frac{\partial \mathbf{R}_{11}}{\partial \mathbf{q}} = \begin{bmatrix} 0 & -4q_y & -4q_z & 0 \end{bmatrix}$$

$$\frac{\partial \mathbf{R}_{12}}{\partial \mathbf{q}} = \begin{bmatrix} 2q_y & 2q_x & -2q_w & -2q_z \end{bmatrix}$$

$$\frac{\partial \mathbf{R}_{13}}{\partial \mathbf{q}} = \begin{bmatrix} 2q_z & 2q_w & 2q_x & 2q_y \end{bmatrix}$$

$$\frac{\partial \mathbf{R}_{21}}{\partial \mathbf{q}} = \begin{bmatrix} 2q_y & 2q_x & 2q_w & 2q_z \end{bmatrix}$$

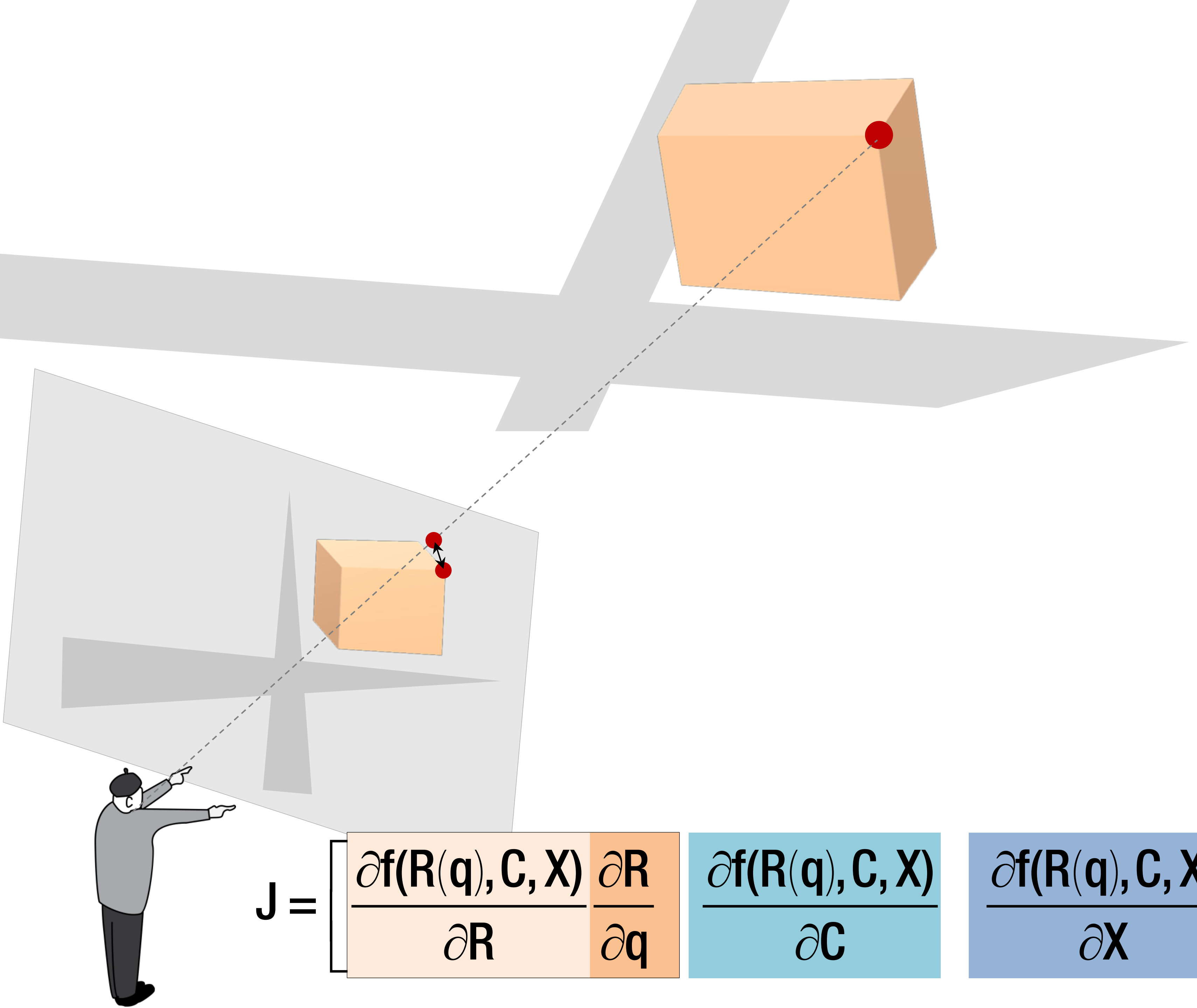
$$\frac{\partial \mathbf{R}_{22}}{\partial \mathbf{q}} = \begin{bmatrix} -4q_x & 0 & -4q_z & 0 \end{bmatrix}$$

$$\frac{\partial \mathbf{R}_{23}}{\partial \mathbf{q}} = \begin{bmatrix} -2q_w & 2q_z & 2q_y & 2q_x \end{bmatrix}$$

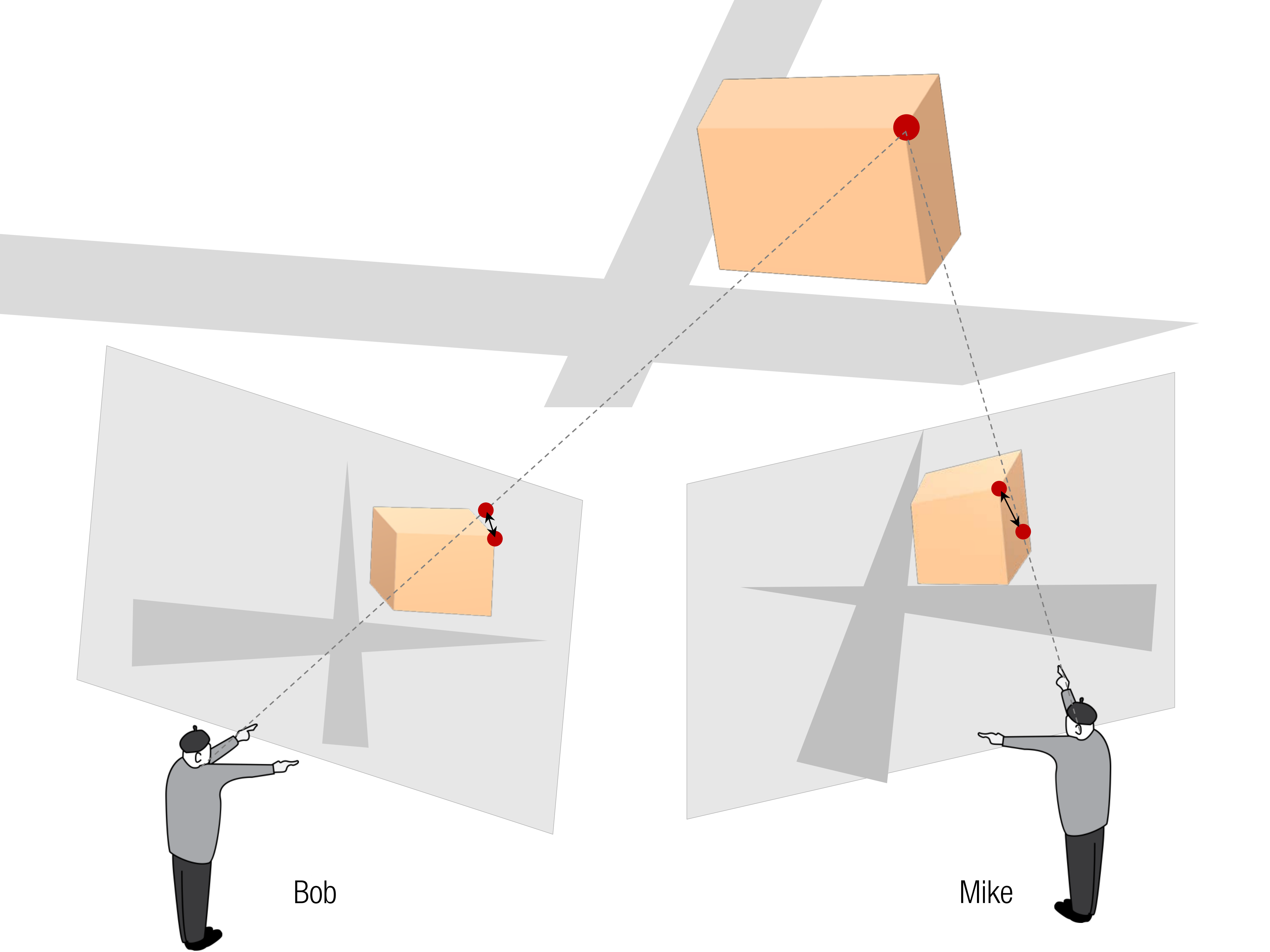
$$\frac{\partial \mathbf{R}_{31}}{\partial \mathbf{q}} = \begin{bmatrix} 2q_z & -2q_w & 2q_x & -2q_y \end{bmatrix}$$

$$\frac{\partial \mathbf{R}_{33}}{\partial \mathbf{q}} = \begin{bmatrix} -4q_x & -4q_y & 0 & 0 \end{bmatrix}$$

$$\frac{\partial \mathbf{R}_{32}}{\partial \mathbf{q}} = \begin{bmatrix} 2q_w & 2q_z & 2q_y & 2q_x \end{bmatrix}$$



$$J = \begin{bmatrix} \dfrac{\partial f(R(q), C, X)}{\partial R} \dfrac{\partial R}{\partial q} & \dfrac{\partial f(R(q), C, X)}{\partial C} & \dfrac{\partial f(R(q), C, X)}{\partial X} \end{bmatrix}$$


Bob
Mike

$$J = \begin{bmatrix} \underset{2\text{x}9}{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{R}}} \; \underset{9\text{x}4}{\frac{\partial \mathbf{R}}{\partial \mathbf{q}}} & \underset{2\text{x}3}{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{C}}} & \underset{2\text{x}3}{\frac{\partial \mathbf{f}(\mathbf{R}(\mathbf{q}),\mathbf{C},\mathbf{X})}{\partial \mathbf{X}}} \end{bmatrix}$$

$$J = \begin{bmatrix} \text{Bob's Jacobian} & \mathbf{0}_{2\times 7} & \text{3D Point} \\ \mathbf{0}_{2\times 7} & \text{Mike's Jacobian} & \text{3D Point} \end{bmatrix}$$