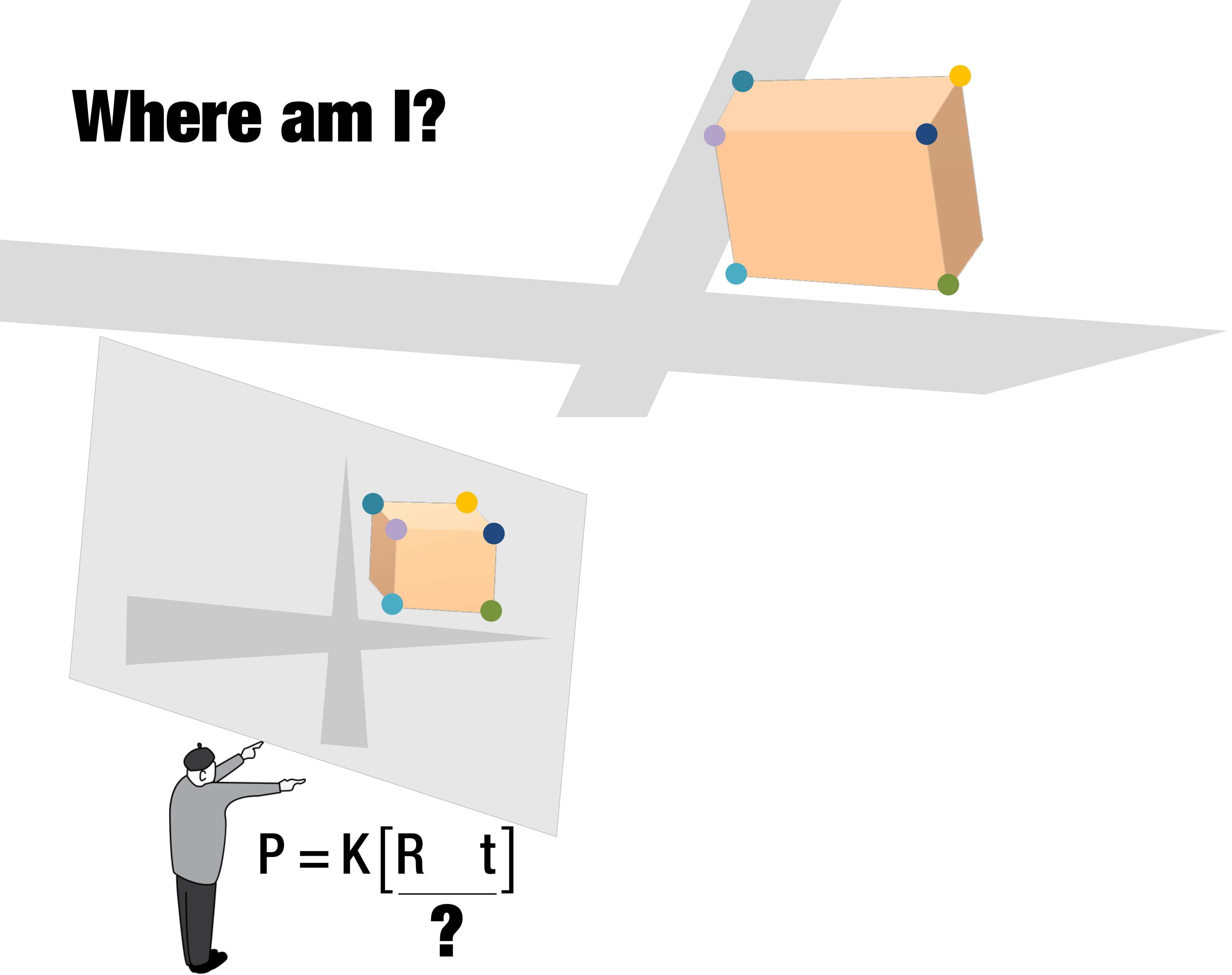
# Where am I?



$$P = K\underbrace{\begin{bmatrix} R & t \end{bmatrix}}_{?}$$

# Camera 3D Registration

## Perspective-n-Point Algorithm

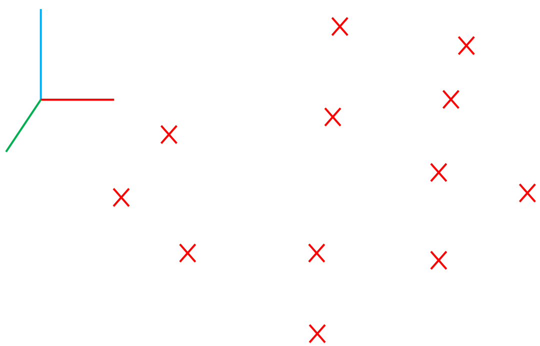
3D point cloud via triangulation

Where?

# Perspective-n-Point

3D point cloud

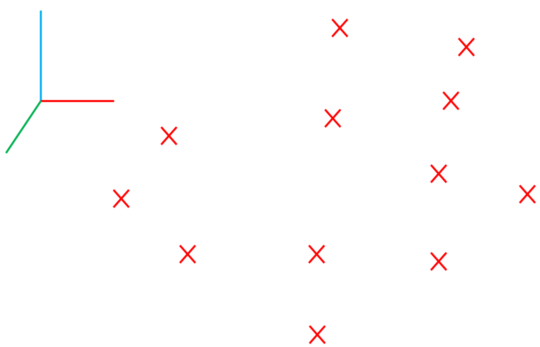
$$\mathbf{X} \in \mathbb{R}^3$$



# Perspective-n-Point

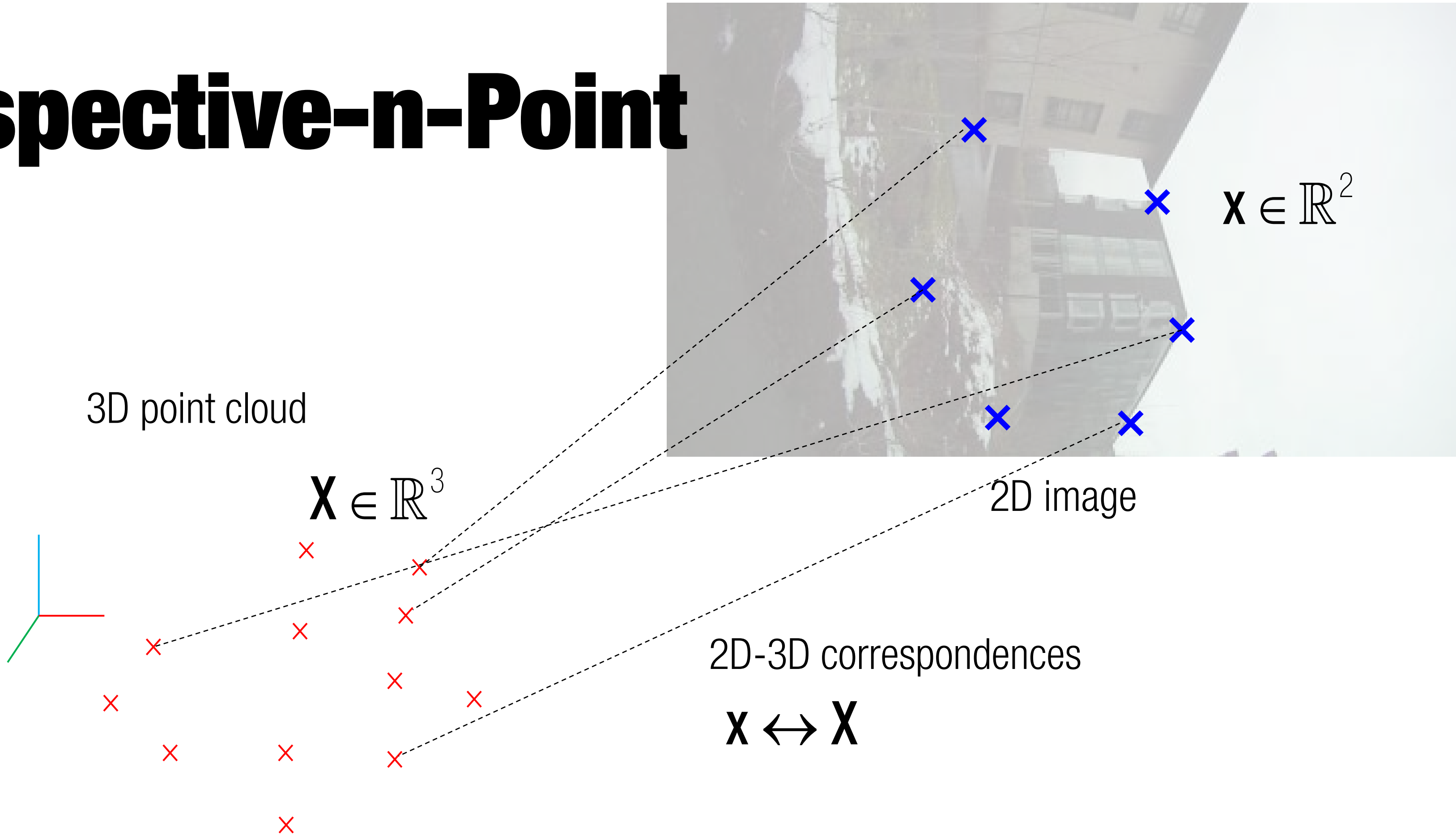
3D point cloud

$\mathbf{X} \in \mathbb{R}^3$




2D image

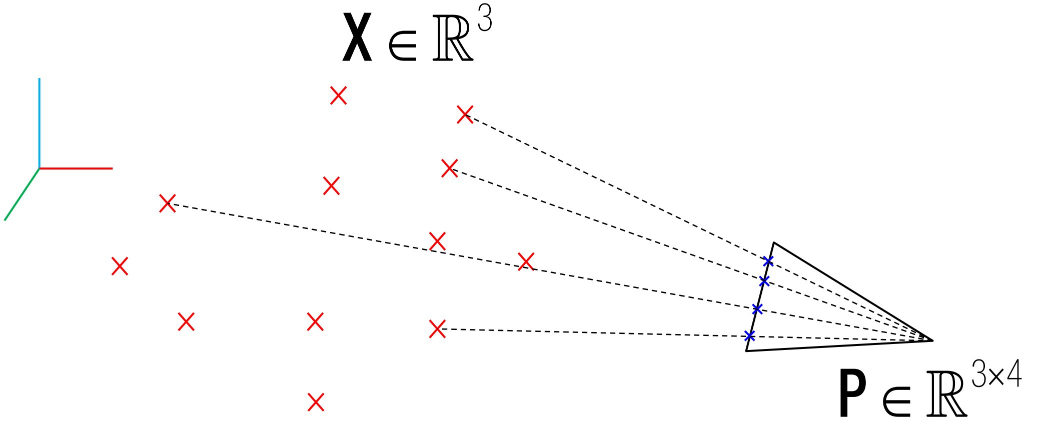
# Perspective-n-Point



# Perspective-n-Point


2D image

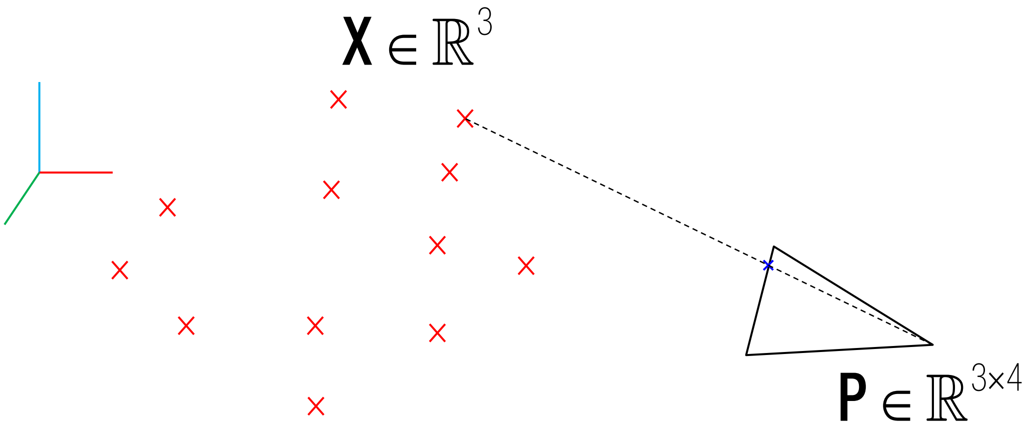
3D point cloud



# Perspective-n-Point

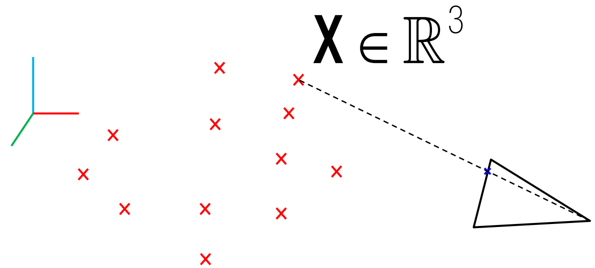

2D image

3D point cloud




$$\lambda \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix} = \mathbf{P} \begin{bmatrix} \mathbf{X}_1 \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} \mathbf{x}_1 \\ 1 \end{bmatrix}_\times \mathbf{P} \begin{bmatrix} \mathbf{X}_1 \\ 1 \end{bmatrix} = \mathbf{0}$$

# Perspective-n-Point

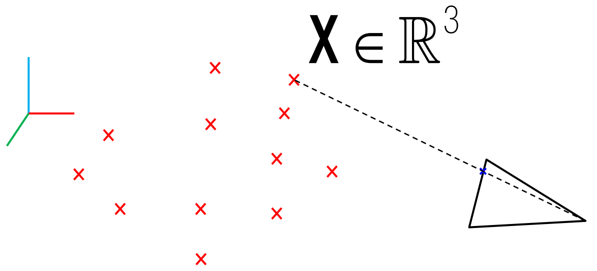




2D image

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_{\times} \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{bmatrix} \tilde{\mathbf{X}}$$

# Perspective-n-Point

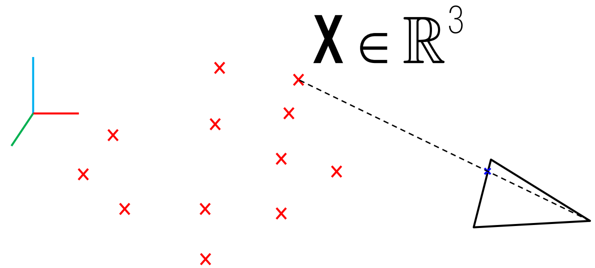




2D image

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{bmatrix} \tilde{\mathbf{X}} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{P}_1\tilde{\mathbf{X}} \\ \mathbf{P}_2\tilde{\mathbf{X}} \\ \mathbf{P}_3\tilde{\mathbf{X}} \end{bmatrix}$$

# Perspective-n-Point

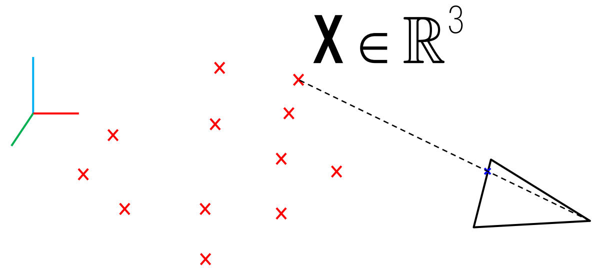




2D image

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{bmatrix} \tilde{\mathbf{X}} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{P}_1\tilde{\mathbf{X}} \\ \mathbf{P}_2\tilde{\mathbf{X}} \\ \mathbf{P}_3\tilde{\mathbf{X}} \end{bmatrix} = \begin{bmatrix} 0 & -1 & v \\ 1 & 0 & -u \\ -v & u & 0 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} & \mathbf{0}_{1\times4} \\ \mathbf{0}_{1\times4} & \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} \\ \mathbf{0}_{1\times4} & \mathbf{0}_{1\times4} & \tilde{\mathbf{X}}^\mathsf{T} \end{bmatrix} \begin{bmatrix} \mathbf{P}_1^\mathsf{T} \\ \mathbf{P}_2^\mathsf{T} \\ \mathbf{P}_3^\mathsf{T} \end{bmatrix} =$$

# Perspective-n-Point





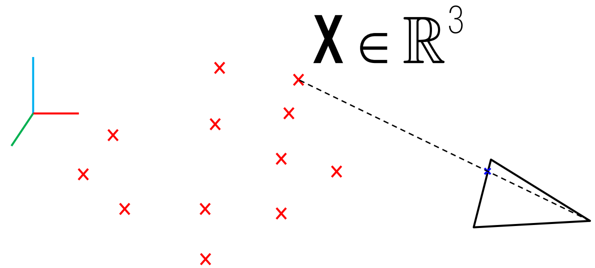
2D image

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{bmatrix} \tilde{\mathbf{X}} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{P}_1 \tilde{\mathbf{X}} \\ \mathbf{P}_2 \tilde{\mathbf{X}} \\ \mathbf{P}_3 \tilde{\mathbf{X}} \end{bmatrix} = \begin{bmatrix} 0 & -1 & v \\ 1 & 0 & -u \\ -v & u & 0 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} & \mathbf{0}_{1\times4} \\ \mathbf{0}_{1\times4} & \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} \\ \mathbf{0}_{1\times4} & \mathbf{0}_{1\times4} & \tilde{\mathbf{X}}^\mathsf{T} \end{bmatrix} \begin{bmatrix} \mathbf{P}_1^\mathsf{T} \\ \mathbf{P}_2^\mathsf{T} \\ \mathbf{P}_3^\mathsf{T} \end{bmatrix} =$$

$$\begin{bmatrix} \mathbf{0}_{1\times4} & -\tilde{\mathbf{X}}^\mathsf{T} & v\tilde{\mathbf{X}}^\mathsf{T} \\ \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} & -u\tilde{\mathbf{X}}^\mathsf{T} \\ -v\tilde{\mathbf{X}}^\mathsf{T} & u\tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} \end{bmatrix} \begin{bmatrix} \mathbf{P}_1^\mathsf{T} \\ \mathbf{P}_2^\mathsf{T} \\ \mathbf{P}_3^\mathsf{T} \end{bmatrix} = \mathbf{0}$$

3x12 matrix

# Perspective-n-Point





2D image

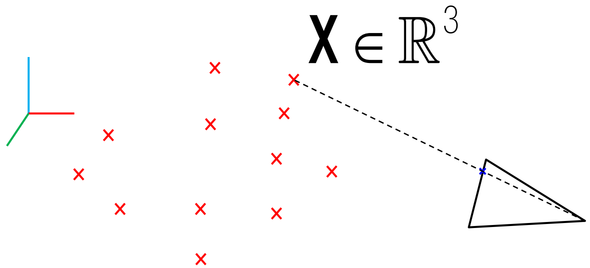
$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{bmatrix} \tilde{\mathbf{X}} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{P}_1\tilde{\mathbf{X}} \\ \mathbf{P}_2\tilde{\mathbf{X}} \\ \mathbf{P}_3\tilde{\mathbf{X}} \end{bmatrix} = \begin{bmatrix} 0 & -1 & v \\ 1 & 0 & -u \\ -v & u & 0 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times 4} & \mathbf{0}_{1\times 4} \\ \mathbf{0}_{1\times 4} & \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times 4} \\ \mathbf{0}_{1\times 4} & \mathbf{0}_{1\times 4} & \tilde{\mathbf{X}}^\mathsf{T} \end{bmatrix} \begin{bmatrix} \mathbf{P}_1^\mathsf{T} \\ \mathbf{P}_2^\mathsf{T} \\ \mathbf{P}_3^\mathsf{T} \end{bmatrix} =$$

$$\begin{bmatrix} \mathbf{0}_{1\times 4} & -\tilde{\mathbf{X}}^\mathsf{T} & v\tilde{\mathbf{X}}^\mathsf{T} \\ \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times 4} & -u\tilde{\mathbf{X}}^\mathsf{T} \\ -v\tilde{\mathbf{X}}^\mathsf{T} & u\tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times 4} \end{bmatrix} \begin{bmatrix} \mathbf{P}_1^\mathsf{T} \\ \mathbf{P}_2^\mathsf{T} \\ \mathbf{P}_3^\mathsf{T} \end{bmatrix} = \mathbf{0}$$

3x12 matrix

$$\mathbf{A} \quad \mathbf{x} = \mathbf{0}$$

# Perspective-n-Point





2D image

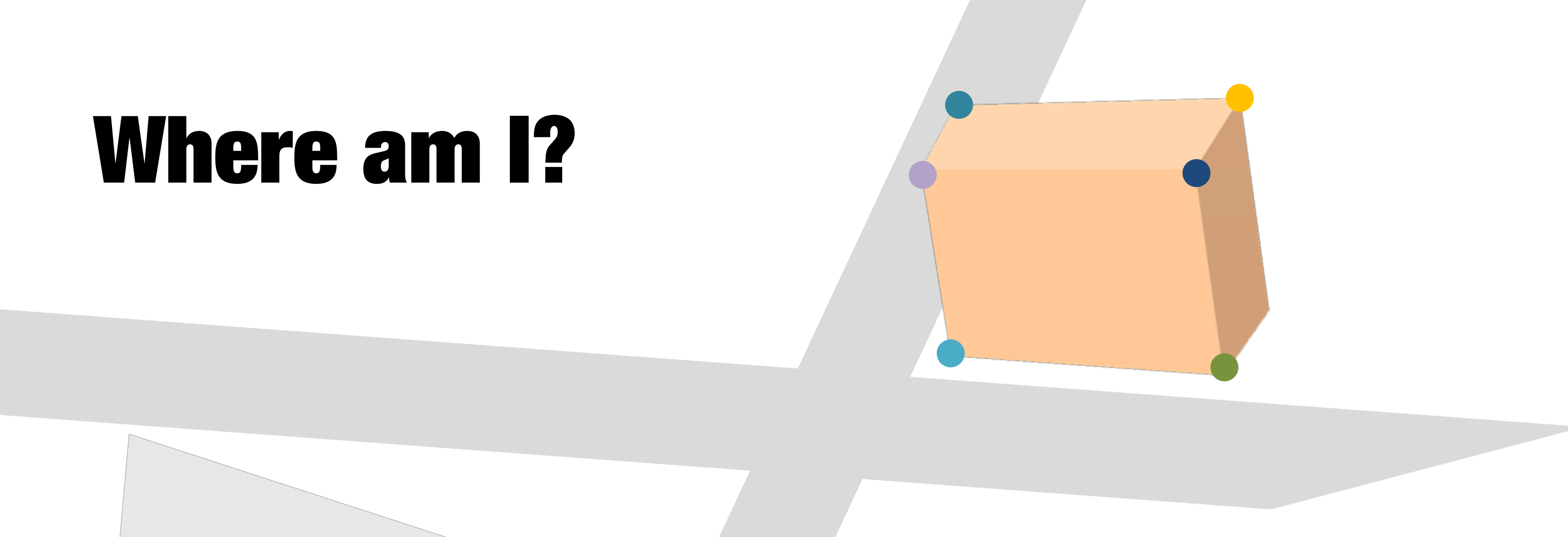
$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{bmatrix} \tilde{\mathbf{X}} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{P}_1\tilde{\mathbf{X}} \\ \mathbf{P}_2\tilde{\mathbf{X}} \\ \mathbf{P}_3\tilde{\mathbf{X}} \end{bmatrix} = \begin{bmatrix} 0 & -1 & v \\ 1 & 0 & -u \\ -v & u & 0 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} & \mathbf{0}_{1\times4} \\ \mathbf{0}_{1\times4} & \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} \\ \mathbf{0}_{1\times4} & \mathbf{0}_{1\times4} & \tilde{\mathbf{X}}^\mathsf{T} \end{bmatrix} \begin{bmatrix} \mathbf{P}_1^\mathsf{T} \\ \mathbf{P}_2^\mathsf{T} \\ \mathbf{P}_3^\mathsf{T} \end{bmatrix} =$$

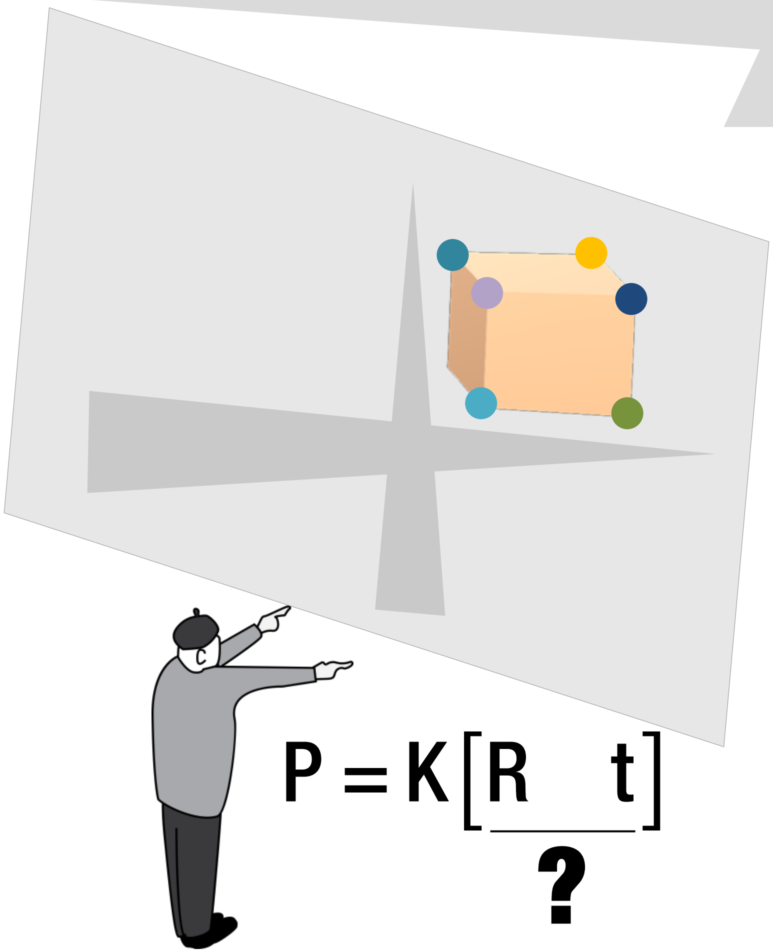
$$\begin{bmatrix} \mathbf{0}_{1\times4} & -\tilde{\mathbf{X}}^\mathsf{T} & v\tilde{\mathbf{X}}^\mathsf{T} \\ \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} & -u\tilde{\mathbf{X}}^\mathsf{T} \\ -v\tilde{\mathbf{X}}^\mathsf{T} & u\tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} \end{bmatrix} \begin{bmatrix} \mathbf{P}_1^\mathsf{T} \\ \mathbf{P}_2^\mathsf{T} \\ \mathbf{P}_3^\mathsf{T} \end{bmatrix} = \mathbf{0}$$

3x12 matrix (rank 2)

$$\begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \mathbf{A}_6 \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}$$

# Where am I?



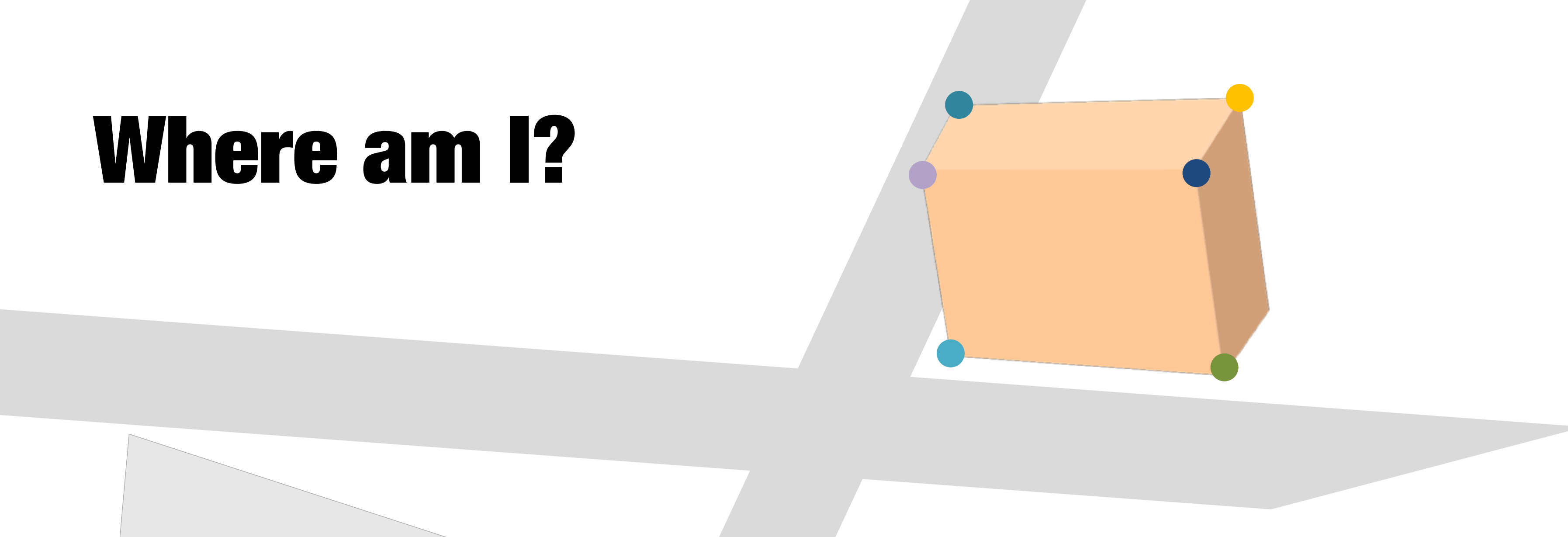


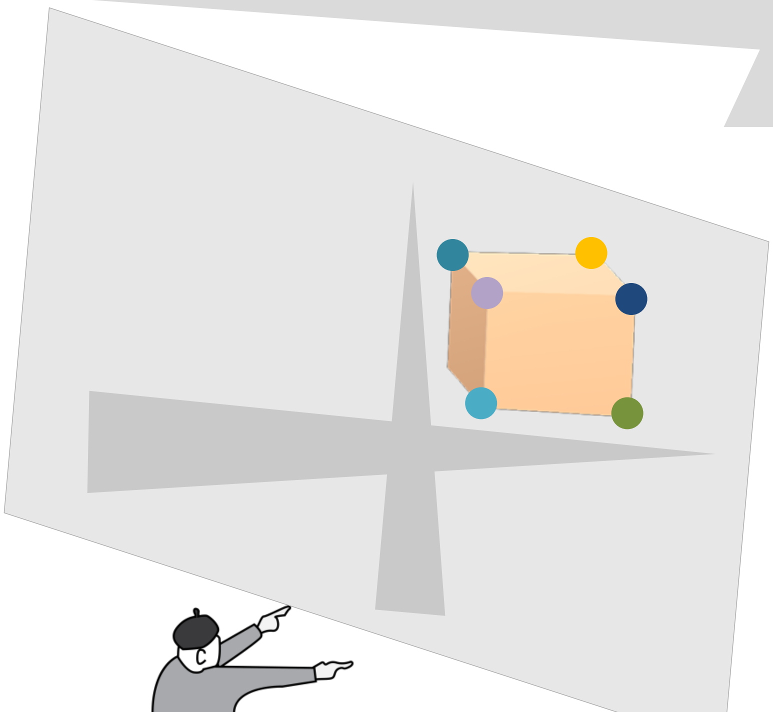
$$R = K^{-1}P_{1:3}$$

$P_{1:3}$ : First three columns of P.

$$P = K\underbrace{\begin{bmatrix} R & t \end{bmatrix}}_{?}$$

# Where am I?





$$P = K\left[\underbrace{R \quad t}_{?}\right]$$

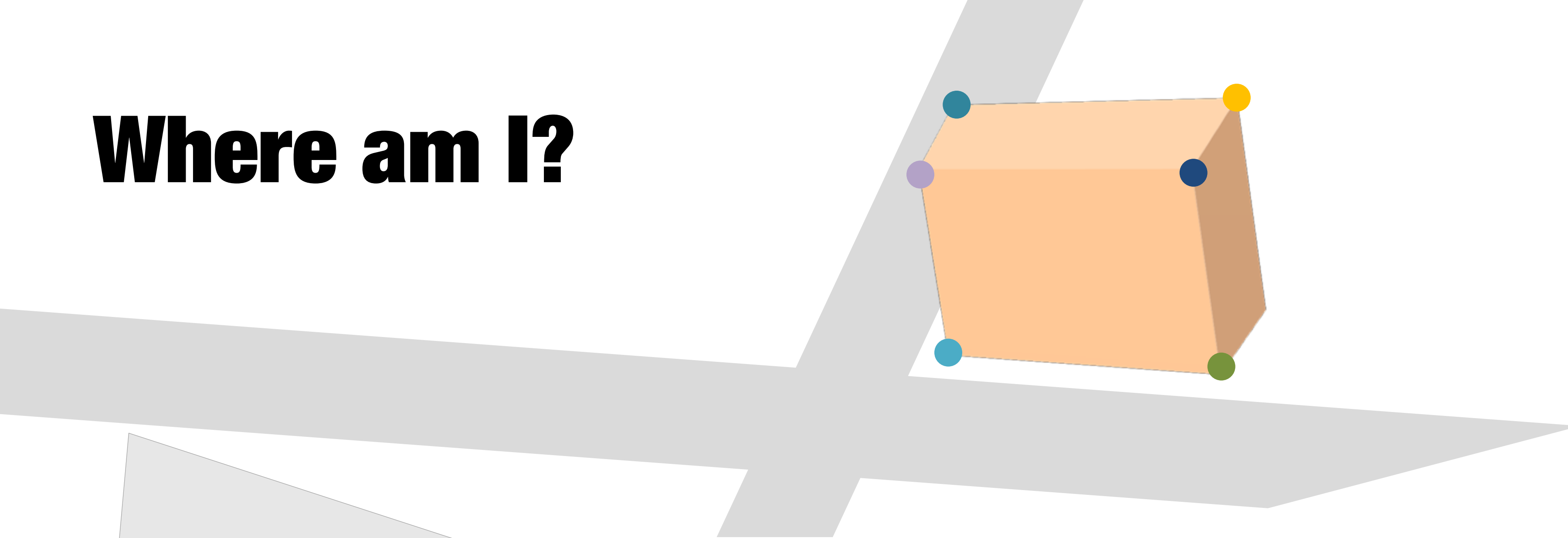
$$R = K^{-1}P_{1:3}$$

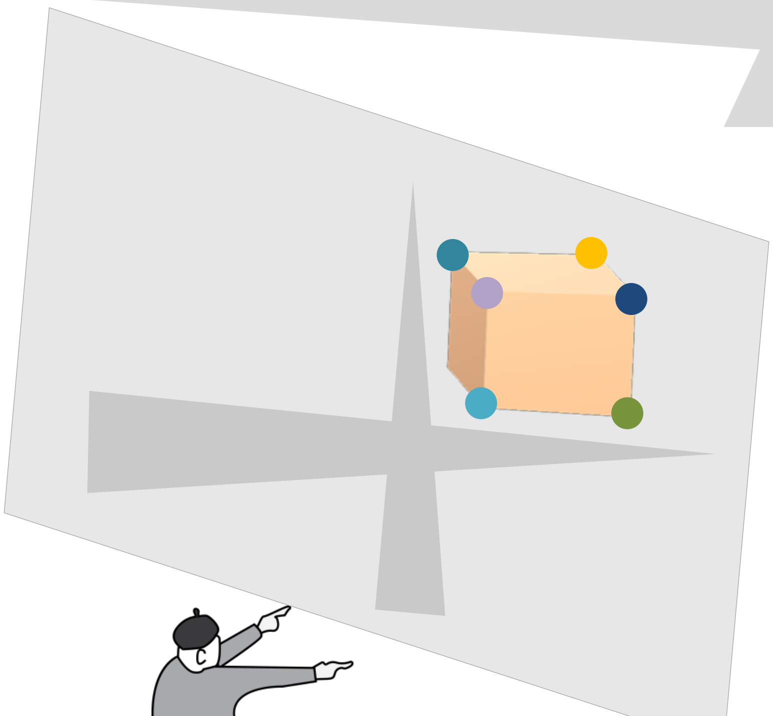
$P_{1:3}$ : First three columns of P.

$$R_{+} = UV^{\top} \quad \text{where} \quad UDV^{\top} = R$$

Enforcing orthogonal constraint of rotation matrix

# Where am I?





$$P = K\underbrace{\begin{bmatrix} R & t \end{bmatrix}}_{?}$$

$$R = K^{-1}P_{1:3}$$

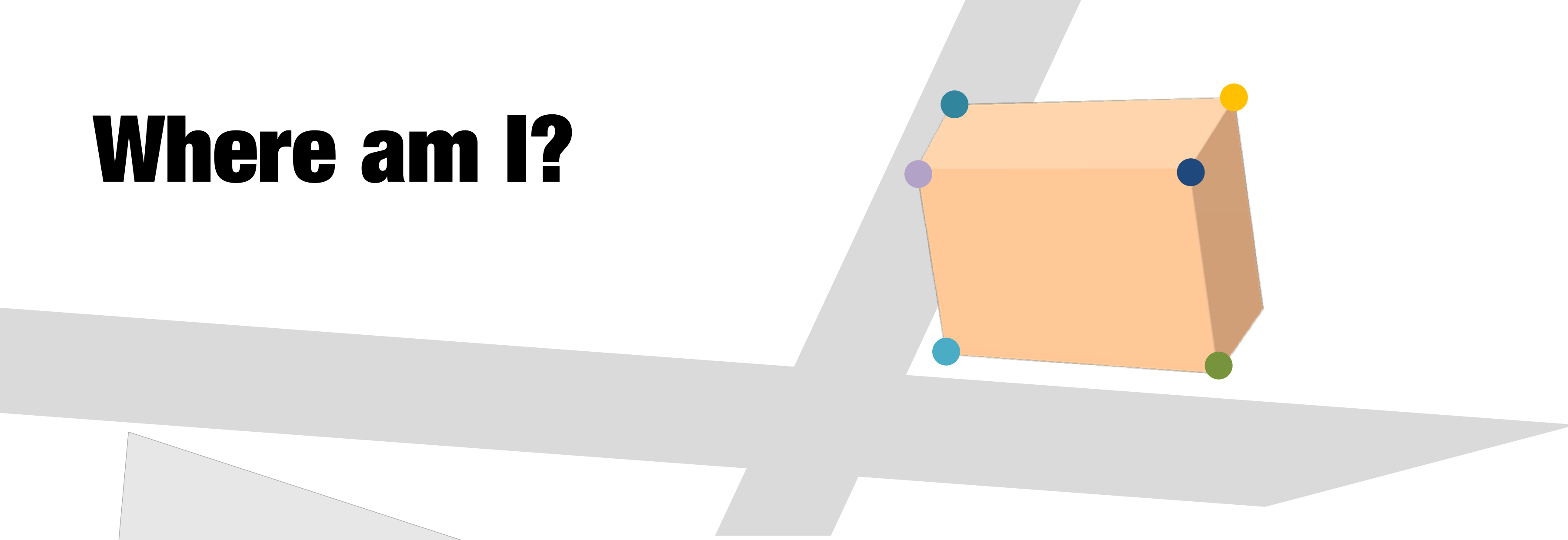
$P_{1:3}$ : First three columns of P.

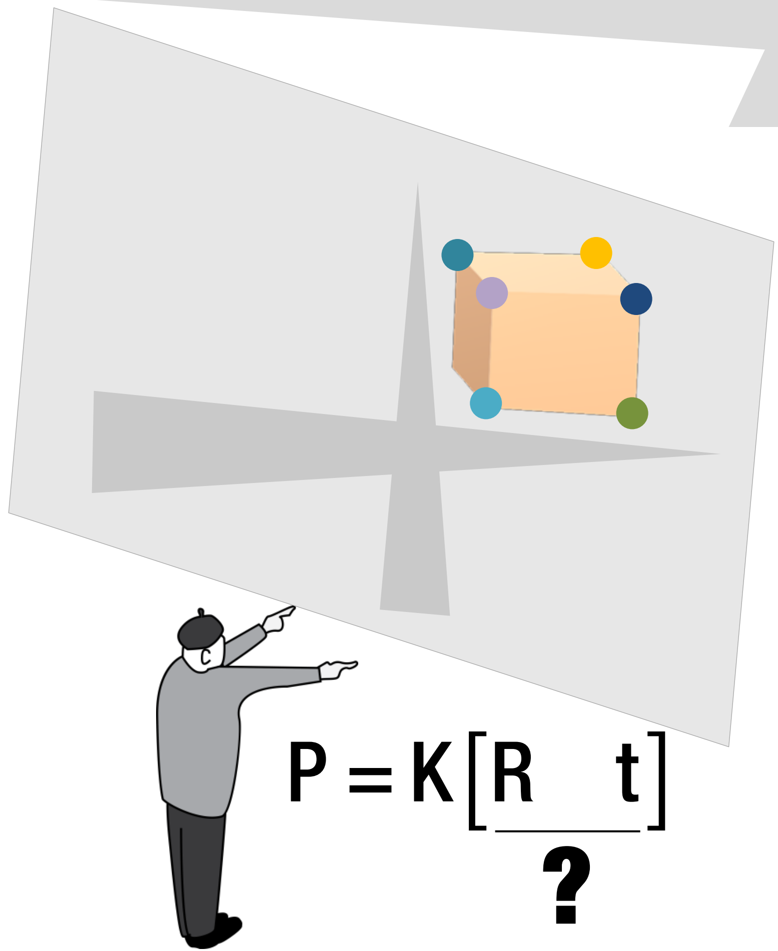
$$R_{+} = UV^{\top} \quad \text{where} \quad UDV^{\top} = R$$

Enforcing orthogonal constraint of rotation matrix

$$t = K^{-1}P_4 / \sigma_1 \quad \text{where} \quad D = \mathrm{diag}(\sigma_1, \sigma_2, \sigma_3)$$

# Where am I?






$$R = K^{-1}P_{1:3}$$

$P_{1:3}$ : First three columns of P.

$$R_{+} = UV^{\top} \quad \text{where} \quad UDV^{\top} = R$$

Enforcing orthogonal constraint of rotation matrix

$$t = K^{-1}P_{4} / \sigma_{1} \quad \text{where} \quad D = \mathrm{diag}(\sigma_{1}, \sigma_{2}, \sigma_{3})$$

$$P = K[R_{+} \quad t]$$

# Camera 3D Registration

## Perspective-n-Point Algorithm