

Particle Acceleration Hands-On

<https://github.com/gkowal/Particle-Acceleration-tutorial>



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Motion equation for charged particle in electromagnetic field

$$\frac{d\vec{p}}{dt} = q \left(\vec{E} + \vec{v} \times \vec{B} \right)$$

$$\vec{p} = \gamma m \vec{v}$$

- particle relativistic momentum

$$m, q$$

- particle mass and charge

$$\vec{E} = -\vec{u} \times \vec{B} + \eta \vec{J}$$

- electric field

$$\vec{B}, \vec{J} = \frac{1}{\mu_0} \nabla \times \vec{B}$$

- magnetic field vector and current density

$$\vec{v}, \vec{u}$$

- particle and fluid velocity vectors

$$\gamma = \left[1 - (v/c)^2 \right]^{1/2}$$

- Lorentz factor

$$\eta, \mu_0$$

- electric resistivity coefficient and permeability

Ignoring the plasma resistivity, the motion equation can be rewritten as

$$\frac{d}{dt} (\gamma \vec{v}) = \frac{q}{m} \left[(\vec{v} - \vec{u}) \times \vec{B} \right]$$

How to find particle position?

$$\frac{d\vec{r}}{dt} = \vec{v}$$

Combining both equation together we can write the motion equation as one vector equation

$$\frac{d}{dt} \begin{Bmatrix} \gamma v_x \\ \gamma v_y \\ \gamma v_z \\ x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} \frac{q}{m} [(v_y - u_y) B_z - (v_z - u_z) B_y] \\ \frac{q}{m} [(v_z - u_z) B_x - (v_x - u_x) B_z] \\ \frac{q}{m} [(v_x - u_x) B_y - (v_y - u_y) B_x] \\ v_x \\ v_y \\ v_z \end{Bmatrix}$$

Or simply

$$\frac{d\vec{y}}{dt} = \vec{f}$$

We have to integrate the motion equation numerically.

$$\frac{d\vec{y}}{dt} = \vec{f}$$

Simple choice, 4th order Runge-Kutta method:

$$y_{n+1} = y_n + \frac{h}{6} (k_1 + 2k_2 + 2k_3 + k_4), \quad t_{n+1} = t_n + h$$

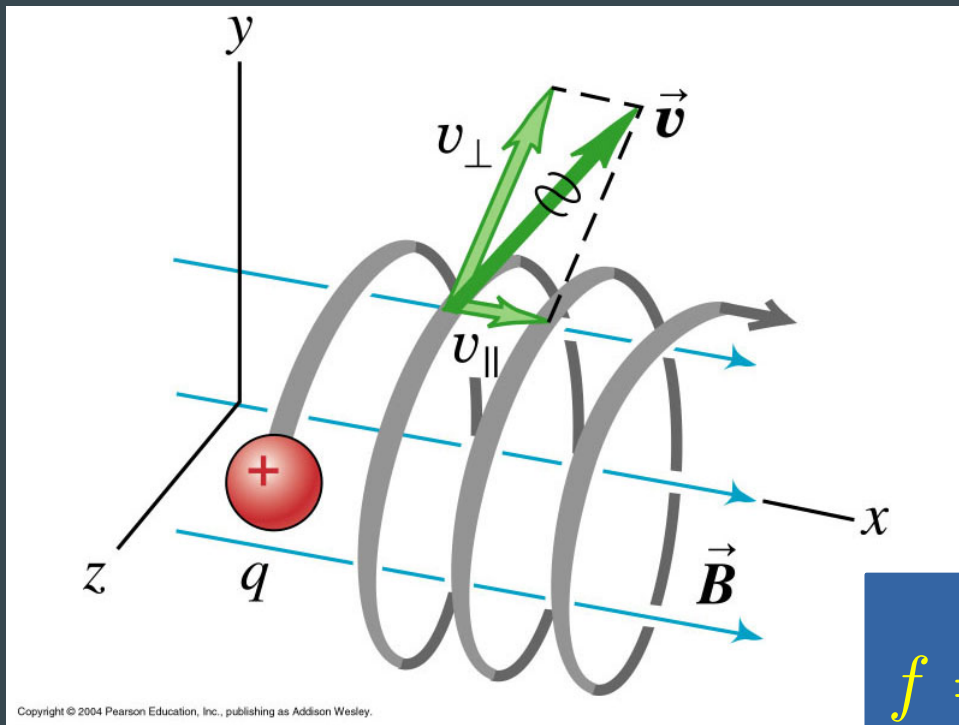
$$k_1 = f(t_n, y_n)$$

$$k_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right)$$

$$k_3 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right)$$

$$k_4 = f(t_n + h, y_n + hk_3)$$

Motion of test particle in uniform magnetic field



Gyroperiod:

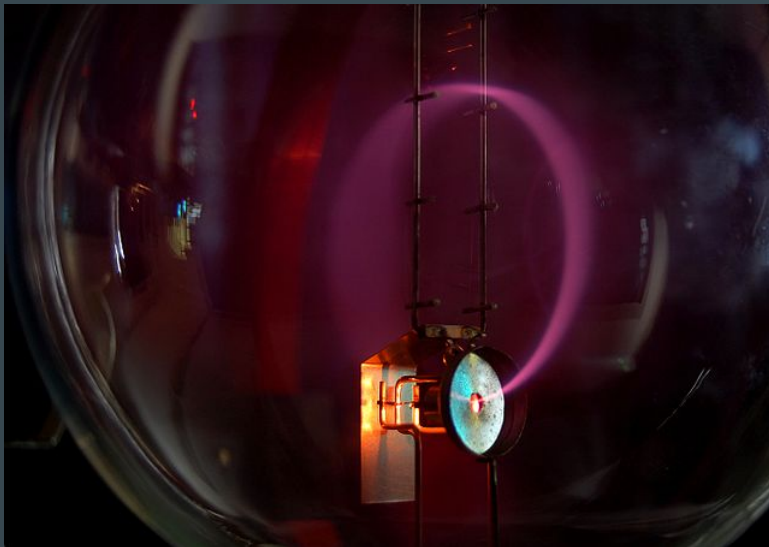
$$T = \frac{2\pi}{\omega} = \frac{2\pi r}{v} = \frac{2\pi\gamma m}{|q|B}$$

Gyrofrequency:

$$f = \frac{1}{T} = \frac{|q|B}{2\pi\gamma m} \Rightarrow \omega = 2\pi f = \frac{|q|B}{\gamma m}$$

Gyro- or Larmor radius:

$$r_L = \frac{\gamma m v_{\perp}}{|q|B}$$

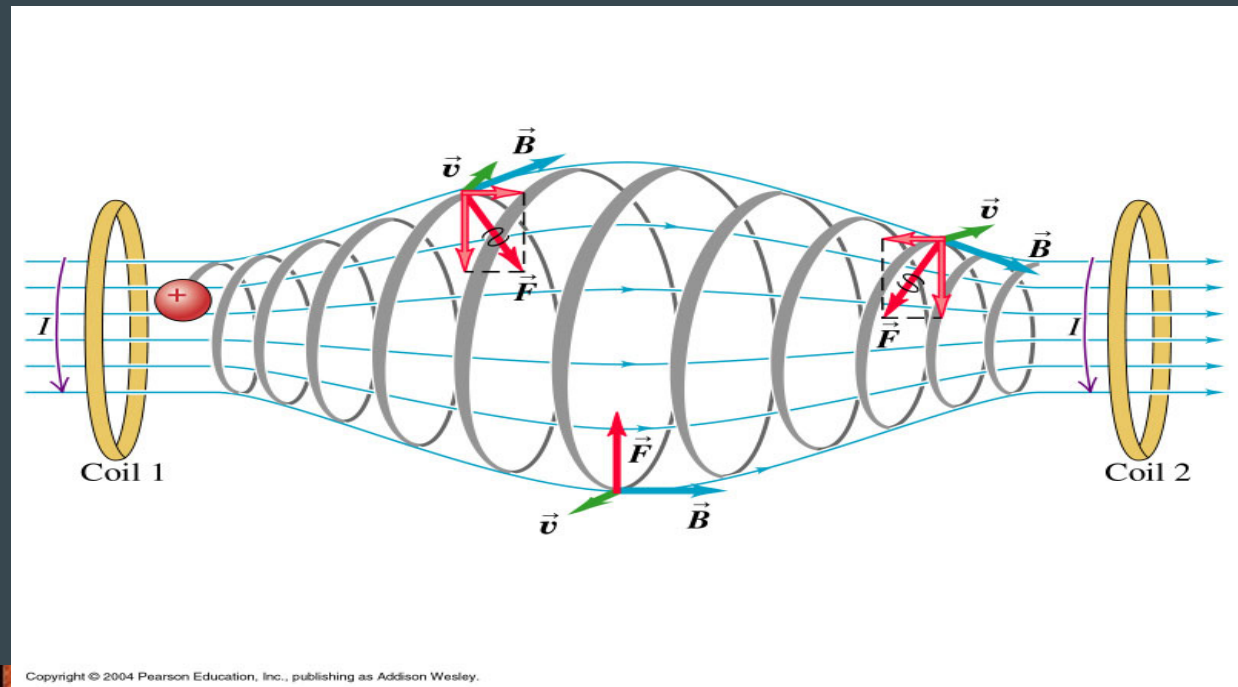


Motion of test particle in non-uniform magnetic field

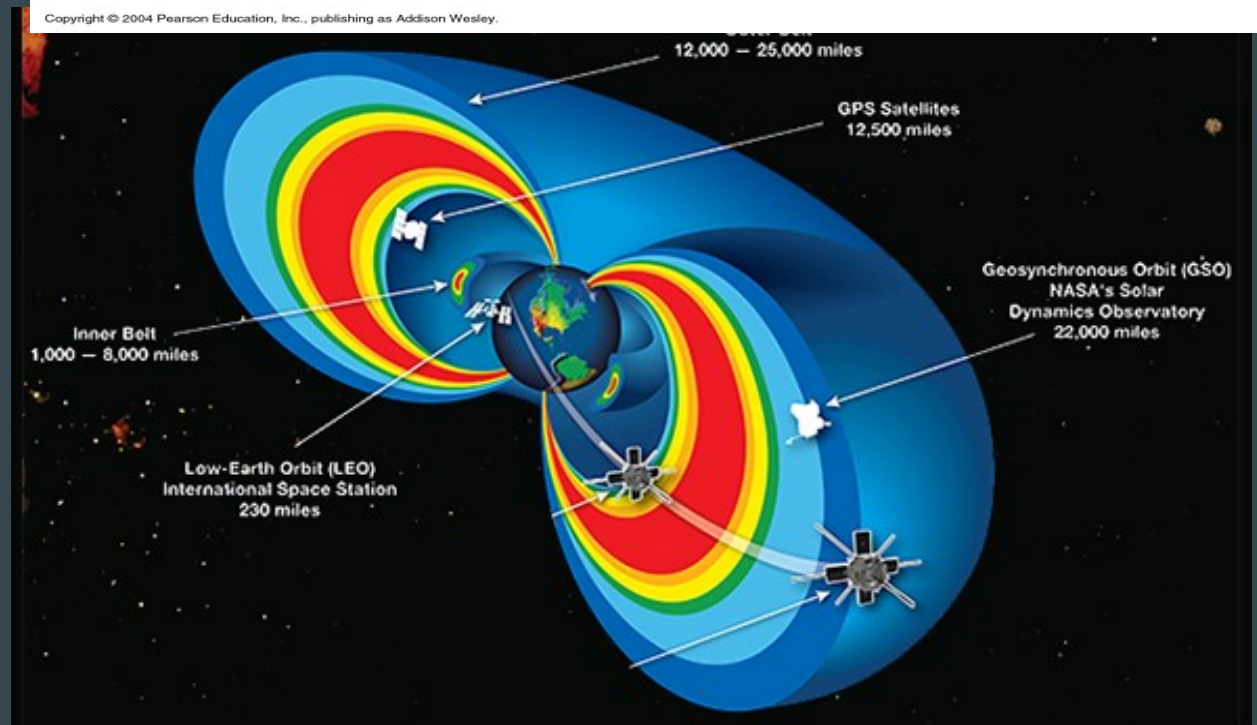
Magnetic mirror:

$$\mu = \frac{mv_{\perp}^2}{2B}$$

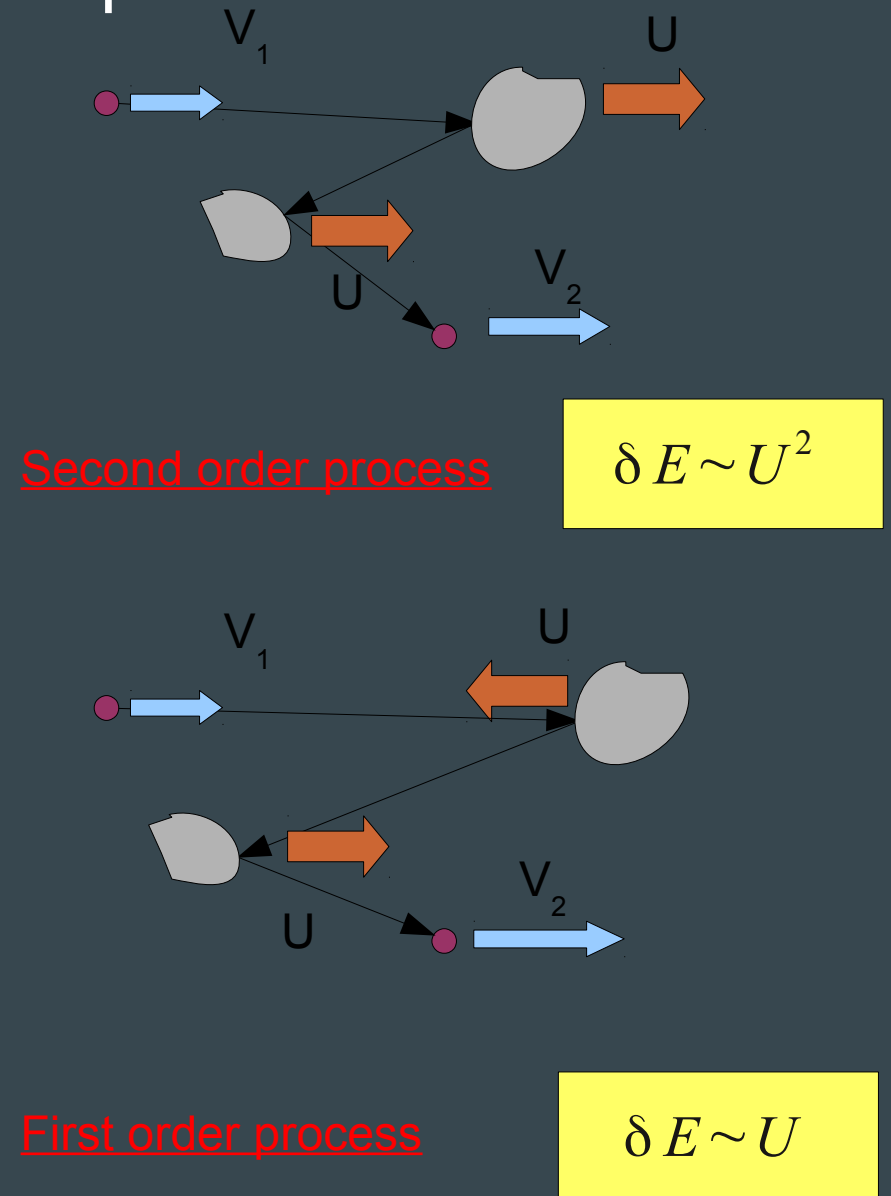
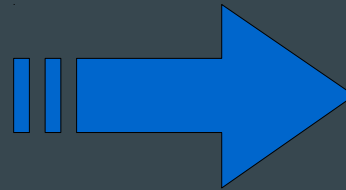
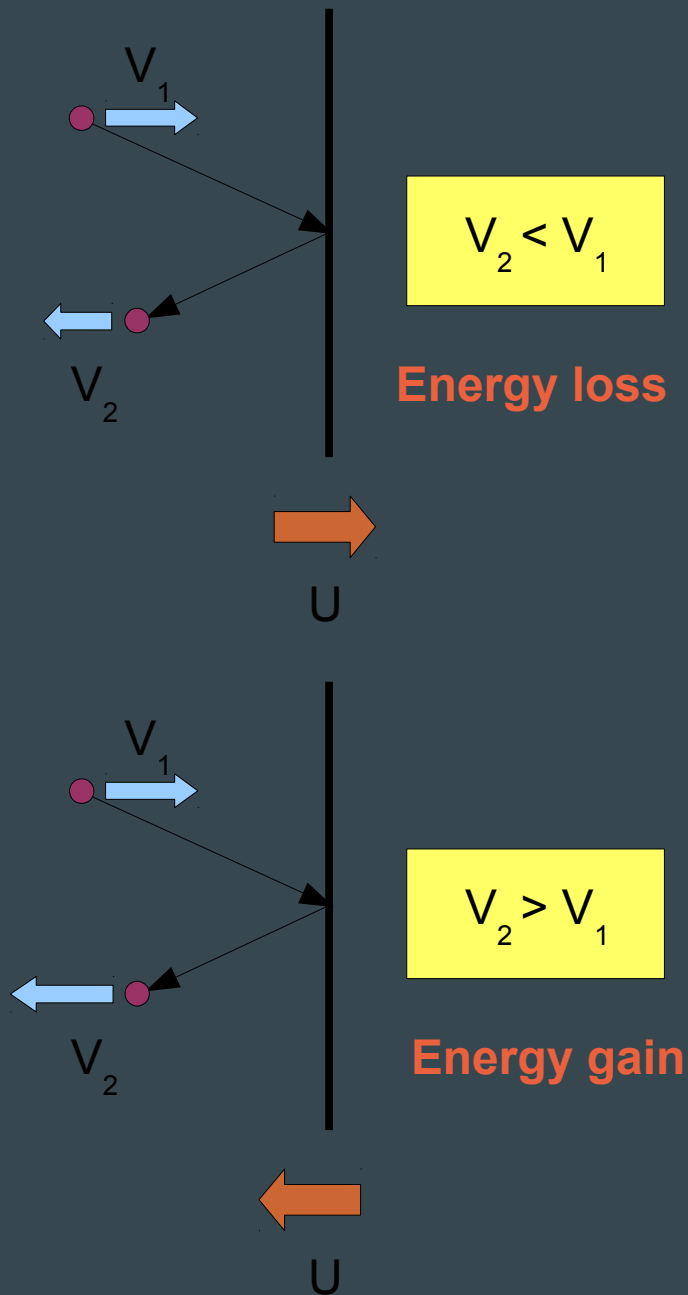
$$E_{kin} = \text{const}$$



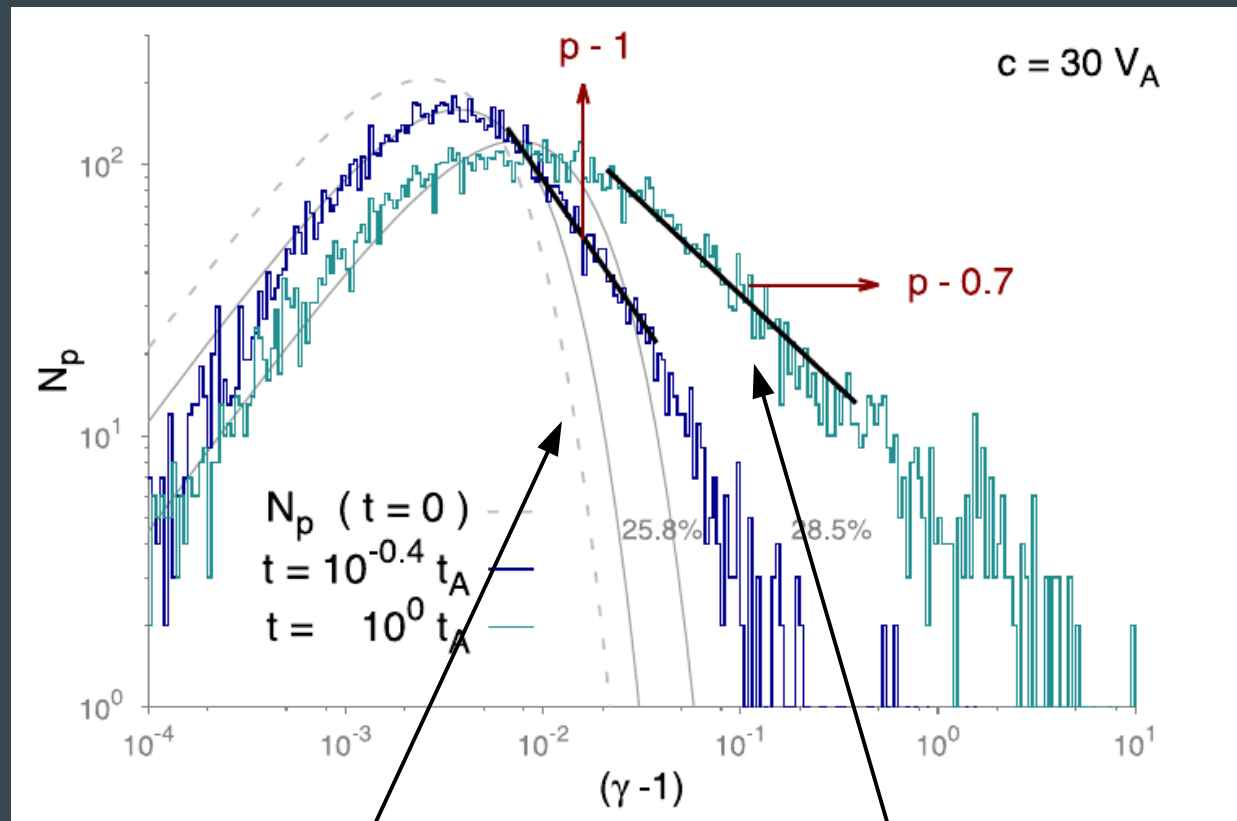
Van Allen belts



Motion of test particles in turbulent medium: 1st and 2nd order Fermi processes



Power spectra of energetic test particles



THERMAL

POWER-LAW