

# Lab 4: Lock-In Amplifier

Griffin Kowash

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## 1 Abstract

This report explores the behavior of a lock-in amplifier simulated using LT Spice. The theory of operation is summarized, and plots of the signal, noise, and output are presented and discussed. The results demonstrate that lock-in amplifiers are extremely effective at isolating a low-amplitude signal of known frequency in the presence of high-amplitude noise.

## 2 Introduction

In laboratory settings it is sometimes necessary to measure very weak signals on the order of millivolts or smaller. While conventional amplifiers are able to magnify these voltages to make them measurable, they also magnify any sources of noise, which can be much larger than the target signal.

While band-pass filters are able to filter out a particular range of frequencies from a signal, when dealing with small signals, they retain too much noise to be effective. Lock-in amplifiers combat this issue by using an external reference frequency to filter out the desired signal from the noise.

To accomplish this, a lock-in amplifier essentially multiplies the signal and the reference waves and integrates over a period of time. Because sine waves of different frequencies are orthogonal, integration of their product will tend to select out the frequency components close to the reference frequency and remove others. Frequency components very close to the reference frequency will appear in the output as DC voltages proportional to their amplitudes.

This lab simulates the behavior of a lock-in amplifier using LT Spice. The signal has an amplitude of 10mV and a frequency of 500Hz. There are three sources of noise included with amplitudes on the order of 1V: 60Hz, 120Hz, and 1/f noise. In addition, the signal is designed to simulate the effect of an experimenter periodically blocking the signal, which physically might correspond to a light source shining on a phototransistor.

### 3 Behavior without noise sources

Figures 1 and 2 show the Fourier transform of the noise-free input signal on logarithmic and linear axes. In both plots, there is a large spike at the signal frequency of 500Hz, with an amplitude roughly three orders of magnitude greater than the frequencies below it. Around 1.1kHz, the amplitude drops sharply and continues to fall up to at least 100kHz.

The linear scaling in Figure 2 is effective at highlighting the sharp peak at 500 Hz, which is less readily apparent in the log scale. On the other hand, plotting on a log scale reveals subtle details, such as the amplitude decrease beginning at 1.1 kHz, that are lost with linear scaling.

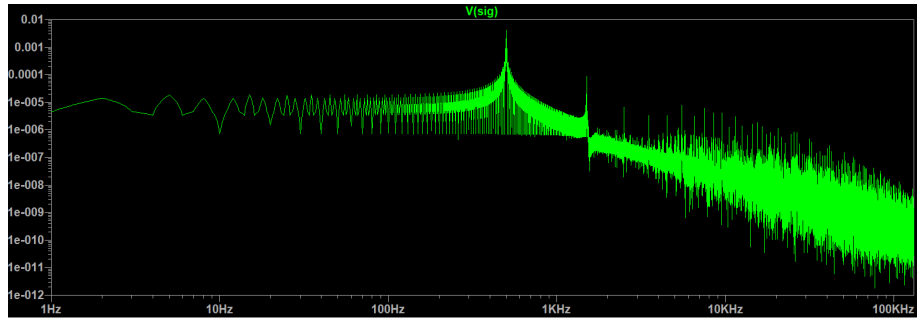


Figure 1: Fast Fourier transform of input signal with vertical axis showing log scale.

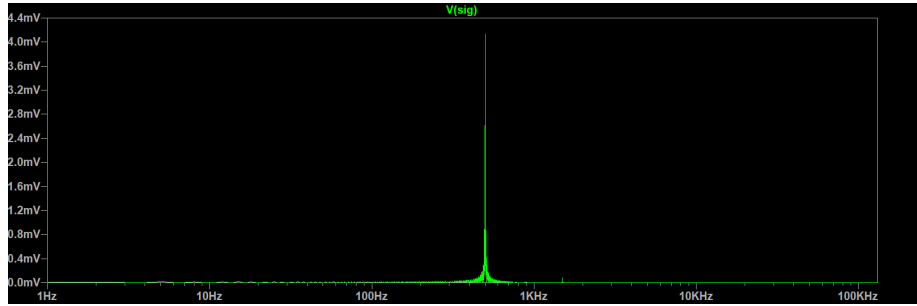


Figure 2: Fast Fourier transform of input signal with vertical axis showing linear scale.

Figure 3 depicts the effect of the  $f_{3dB}$  value on the settling time, which describes how long the output takes to lock on to the input signal. The settling time constants for the  $f_{3dB}$  settings shown in the figure, given by  $\tau = \frac{1}{f_{3dB}}$ , are shown in Table 1. Narrower  $f_{3dB}$  ranges require longer settling times.

$f_{3dB}$ (Hz)	$\tau$ (ms)
10	100
20	50
100	10

Table 1: Theoretical settling time constants for the  $f_{3dB}$  values plotted in Figure 3.

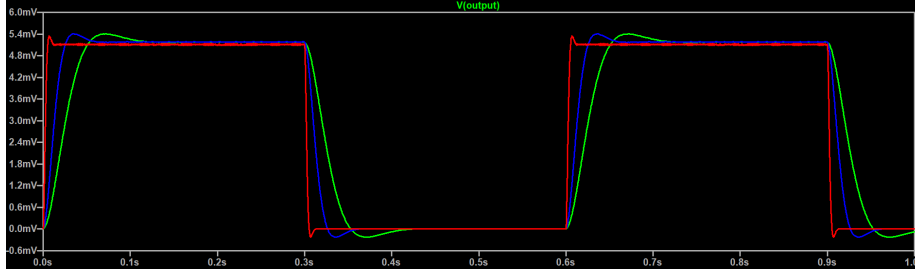


Figure 3: Output curves showing settling time for  $f_{3dB}$  values of 10Hz (green), 20Hz (blue), and 100Hz (red).

## 4 Behavior with noise sources

Figure 4 shows the signal with 60Hz, 120Hz, and  $1/f$  noise sources turned on. The true signal at 500Hz is effectively masked, as its amplitude is roughly two orders of magnitude smaller than the noise.

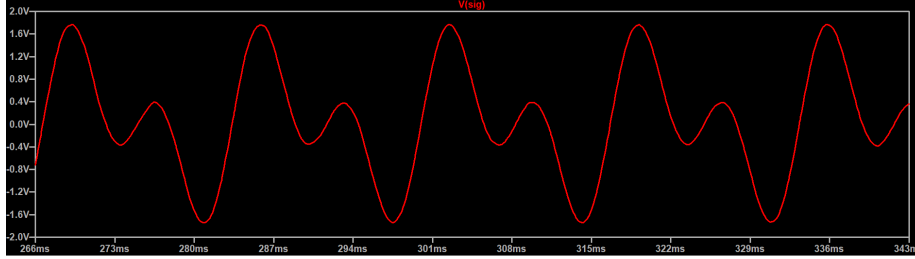


Figure 4: Signal voltage with 60Hz, 120Hz, and  $1/f$  noise sources turned on.

Figure 5 shows the Fourier transform of the noisy signal from Figure 4 plotted on a log scale. Although the peak at 500Hz is still visible, there are also two peaks at 60Hz and 120Hz with amplitudes roughly two orders of magnitude greater than the signal response. Plotted on a linear scale as shown in Figure 6, the signal response is not visible due to the axis being scaled to accommodate the noise amplitudes.

Figures 7, 8, and 9 below show the output signal from the lock-in amplifier with a 3dB width of 100Hz, 20Hz, and 1Hz, respectively. Figures 10 and 11

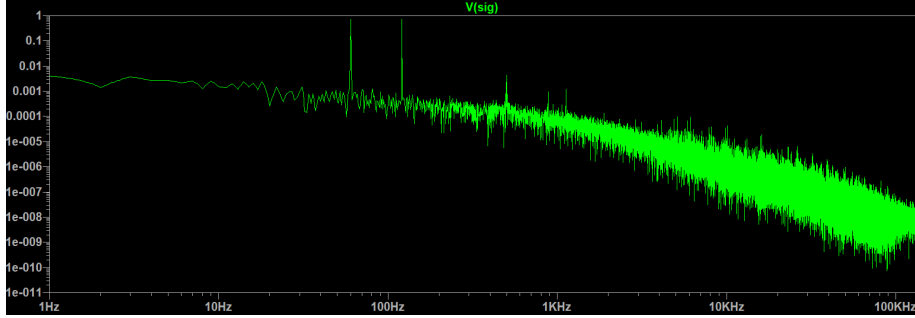


Figure 5: Fast Fourier transform of input signal with noise sources, plotted on a log scale.

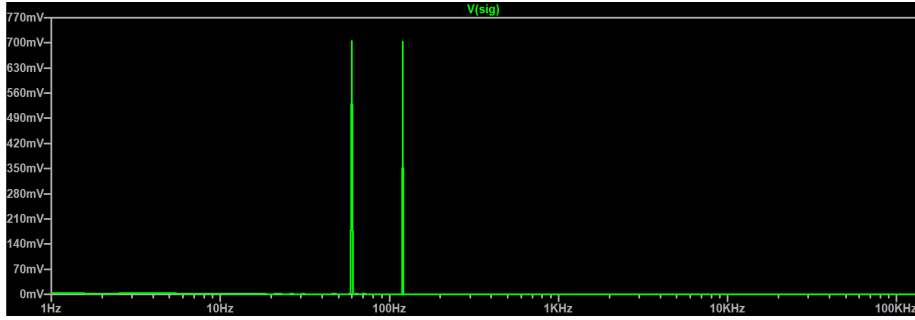


Figure 6: Fast Fourier transform of input signal with noise sources, plotted on a linear scale.

show the Fourier transforms of the 100Hz and 20Hz outputs.

The Fourier transforms show four spikes centered around the signal frequency of 500Hz and offset by  $\pm 60\text{Hz}$  and  $\pm 120\text{Hz}$ . These new frequencies result from the interference between the reference frequency and the 60Hz and 120Hz noise sources. From basic trigonometry, when the two sine waves of different frequencies are multiplied, they produce a sum of two waves: one with a frequency  $f_{ref} + f_{noise}$ , and one with a frequency of  $f_{ref} - f_{noise}$ .

Accordingly, the response to the desired signal shows up as a DC voltage (0Hz), as the reference and signal frequencies are identical. As the 3dB range is narrowed, the response from the noise sources is lessened and the output approaches a constant DC voltage at the target signal's amplitude. However, increased precision requires a longer settling time for the output, and the distortion caused by this at the 1Hz-level is apparent Figure 9.

With a 3dB width of 100Hz, the signal—the simulated effect of the experimenter periodically blocking the light source—is almost entirely obscured by noise (Figure 7). In contrast, with a 3dB width of 20Hz, there is a clear drop in the DC voltage as the “hand” interrupts the 500Hz signal (Figure 8). At a 3dB width of 1Hz, the noise is almost entirely absent, but the increased settling

time severely distorts the square wave signal (Figure 9).

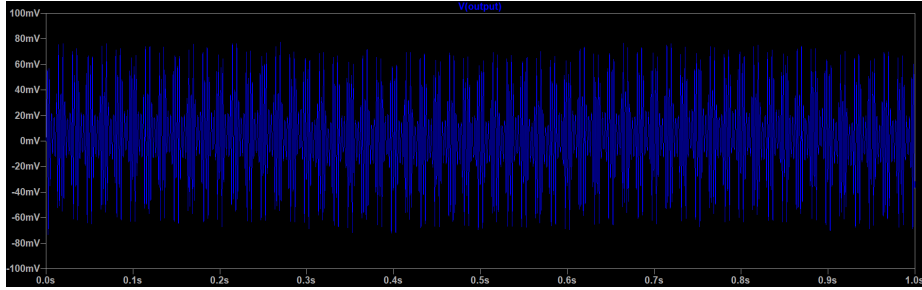


Figure 7: Output signal at 3dB width of 100Hz.

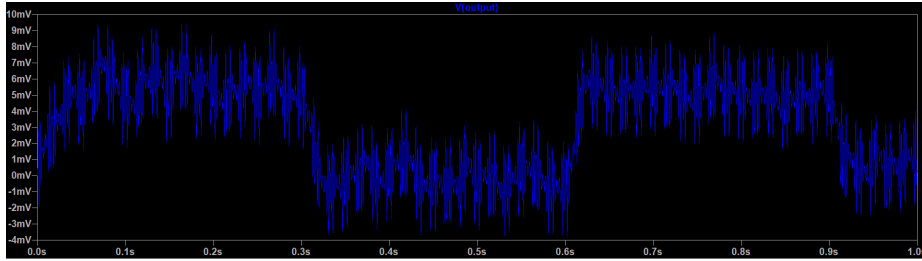


Figure 8: Output signal at 3dB width of 20Hz.

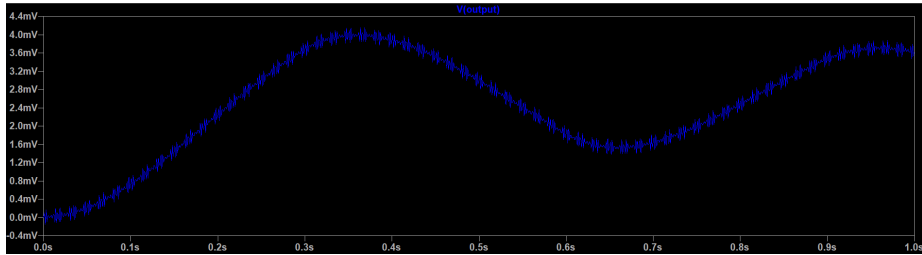


Figure 9: Output signal at 3dB width of 1Hz.

## 5 Conclusion

The results of this lab demonstrate how lock-in amplifiers allow for the measurement of weak signals in the presence of strong noise. With a signal amplitude of only 10mV and noise of up to 3V, the simulated amplifier was able to reliably detect the effect of a simulated “hand” blocking the input source. Reducing the 3dB width allows for more precise isolation of the signal from noise, but at the

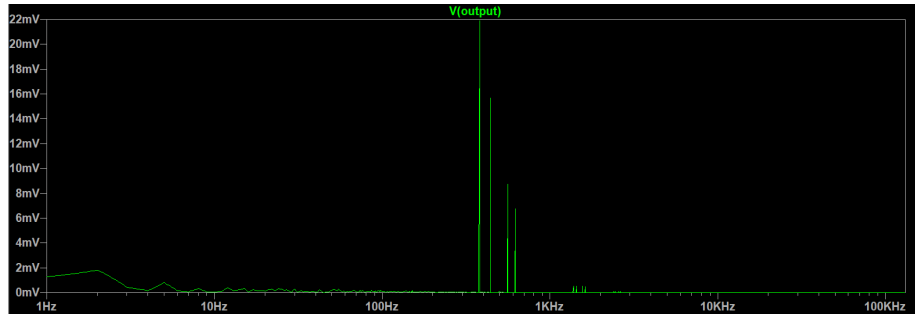


Figure 10: Fourier transform of output signal at 3dB width of 100Hz.

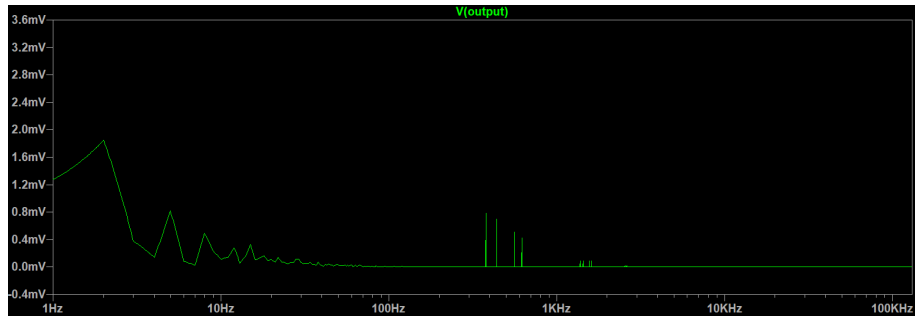


Figure 11: Fourier transform of output signal at 3dB width of 20Hz.

cost of an increased settling time, which in the extreme case can severely distort the output.