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Lab 3: Filters I

Introduction

The purpose of this lab is broken up into two sections: to measure the Root Mean Square (RMS) voltage of a function generator using LabView and to become familiar with how to construct and understand different types of filters in an RC circuit. The LabView outputs of different types of waves will be compared to theoretical values to compare how close they are to one another.

Methodology

We began by exploring the concept of root-mean-square, or RMS, voltage. This quantity is also referred to as the DC equivalent voltage, because it delivers the same amount of power in a circuit as a DC source of the same value. Given a set of voltage readings from an AC waveform, the RMS value can be estimated by averaging the squares of the voltages over a complete period and taking the square root. This process can be made exact by taking the limit as the time step between measurements goes to zero. For a pure sine wave, this yields the following expression:

$$V_{rms} = \sqrt{\frac{\int_0^{2\pi} V_{max}^2 sin^2(t)dt}{2\pi}} = \frac{V_{max}}{\sqrt{2}}$$

Following a similar analysis for a triangle wave as for the sine wave, we find the RMS voltage to be

$$V_{rms} = \frac{V_{max}}{\sqrt{3}}$$

The RMS voltage of a square wave is a straightforward case, because the square of its voltage is constant over the entire waveform. As a result, the RMS voltage and the peak voltage are equivalent.

For the first part of the lab, our task was to construct a Labview program that would take a signal from a function generator and return its RMS voltage value. To ensure that we only sampled voltages from a single period of the 100 Hz waveform, we recorded 100 voltage readings at a rate of 10 kHz. The recorded data was then squared, averaged, and square rooted to obtain the RMS voltage. We performed this procedure for a sine wave, square wave, and triangle wave, and compared our results to the theoretical values.

The second part of the lab involved constructing a high-pass filter circuit. As shown in Figure 1 below, our circuit consisted of a resistor and capacitor in series, with the output voltage being measured across the resistor. Its operation takes advantage of the fact that a capacitor's impedance is frequency-dependent, as given by the following equation:

$$|Z_C| = \frac{1}{\omega C}$$

When the frequency is low, the capacitor impedance is high, causing very little voltage drop across the resistor. In contrast, when the frequency is high, the capacitor impedance is low, and most of the voltage appears across the resistor. In this way, the high-frequency components of signal can be isolated from the low-frequency parts.

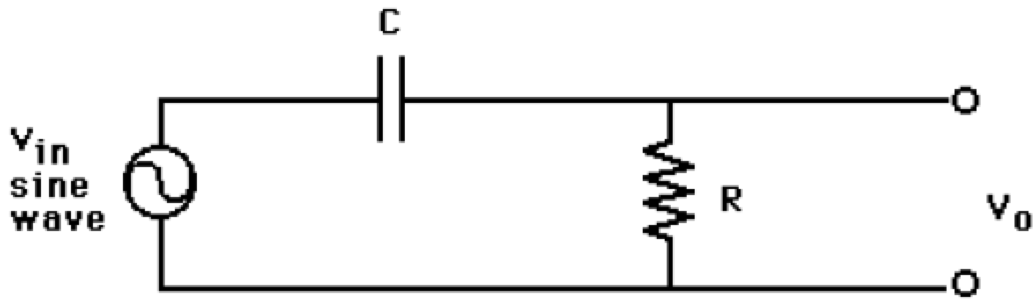

Figure 1. High pass filter circuit schematic for an RC circuit (Gannett, Lab handout, 2019).

The circuit's response to a given frequency is characterized by the gain, which is the ratio of the output voltage to the input voltage, or in decibels, 20 times the log of this ratio:

$$Gain = \frac{V_{out}}{V_{in}} = 20 \log \frac{V_{out}}{V_{in}} \ dB$$

An important gain value is the 3dB point, which occurs when the gain equals -3dB, or equivalently, $1/\sqrt{2}$. This value is also referred to as the half-power point, because it represents the point at which the output power is half of the maximum.

When the following inequality is satisfied, this circuit also functions as a differentiator:

$$\omega << \frac{1}{RC}$$

This means that the output voltage is proportional to the derivative of the input voltage. To test this, we set our function generator to output a square wave with frequency 100 Hz. As shown in the graphs on the attached pages, the output voltage spiked each time the square wave flipped polarity, while it remained at zero for the rest of the waveform. For an ideal square wave, the derivative is undefined when the polarity flips, because the change occurs instantaneously. In reality, this change occurs over a small fraction of a second, so the derivative is large but finite.

Results

V_{RMS} values for sin, square, and triangle wave functions through LabView and theoretical calculations are shown in Table 1. Appendix A contains the theoretical approach for calculating the V_{RMS}.

Table 1. V_{RMS} theoretical and LabView results for different wave functions.

Wave Type	LabView VRMS [V]	Theoretical VRMS [V]	Percent Difference [%]
Sin	0.7066	0.7071	0.071
Square	0.9979	1	0.210
Triangle	0.57725	0.5774	0.026

The desired 3 dB point given in the lab was 2kHz. Appendix B section 2.1 contains the process carried out to find the value of the capacitor to have the 3 dB point at 2 kHz. The value of the capacitor to achieve this is:

$$\mathbf{C = 7.9577x10^{-8}\ F}$$

Figure 2 displays how the gain changes as the frequency changes. Data points were obtained by recording the output voltage across different frequencies to then calculate the gain across each point with the fixed value of V_{in}. Figure 2 also displays the 3 dB point in a different marker and color.

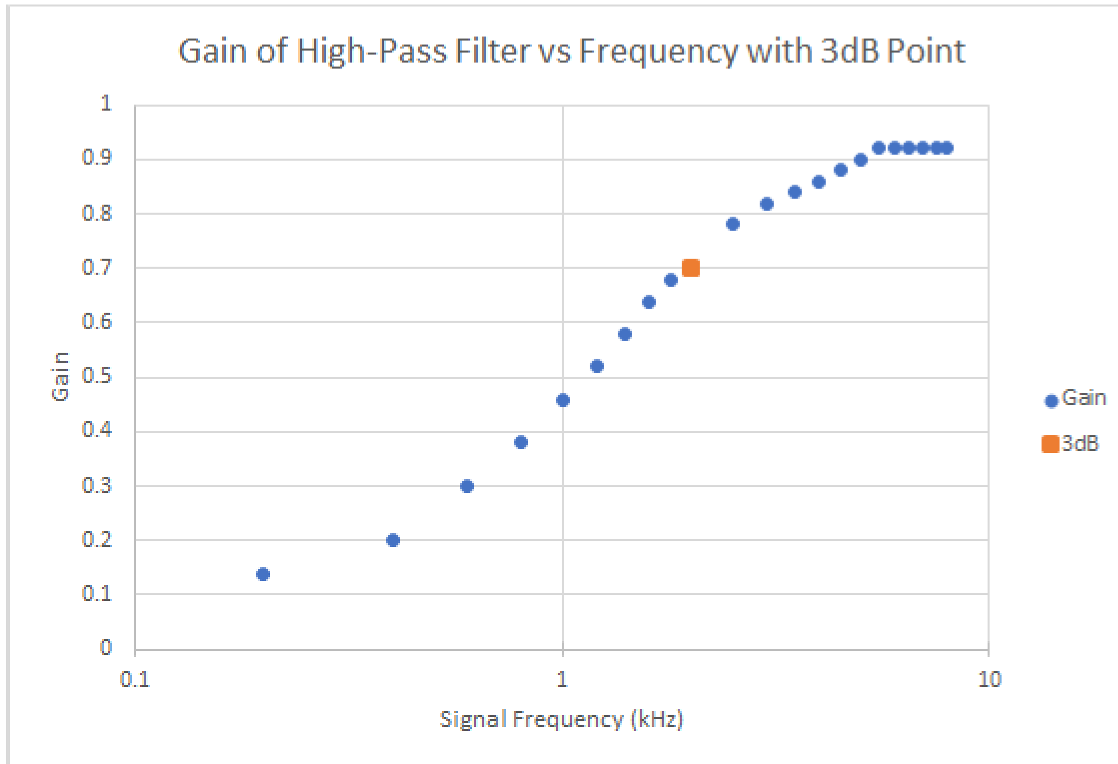

Figure 2. Relationship between voltage gain and signal frequency.

To meet the condition of the differentiator inequality, the frequency resulted as, with work shown in Appendix B section 2.2:

f = 100 Hz

Discussion

The percent difference between LabView and theoretical V_{RMS} values of sin square and triangle waves functions resulted as: 0.071, 0.210, and 0.026 respectively. This result indicates that the theoretical approach is a very close approximation of the actual value.

The 3 dB point occurs at around 0.7, which is expected because using the value of Vin of 1V in the gain equation results with a Vout of 0.707 V. Therefore, the gain would be 0.707 V, which agrees with the plotted value.

Conclusion

In this lab we explored the concept of RMS voltage and learned to construct a low-pass filter circuit. We were able to predict from theory the RMS voltages of three different waveforms with less than 1% error. In addition, using the concept of complex impedances, we designed our low-pass filter circuit such that the 3dB point occurred at 2 kHz, and used Labview to plot its gain versus frequency curve.