# Investigating the properties of a coaxial cable

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## 1 Abstract

Coaxial cables are ubiquitous in the modern world, with uses ranging from cable television to oscilloscopes. Two parameters of interest are $Z_0$, the characteristic impedance, and $v$, the signal propagation velocity. This report describes procedures for determining these parameters and applies them to a simulated coaxial cable in LTspice. Finally, a method is proposed for determining the propagation velocity without the need for direct time measurements.

## 2 Introduction & Theory

A coaxial cable is a material designed to transmit radio-frequency signals while reducing external noise and signal leakage. This makes it ideal both for transmitting weak signals and for preventing strong signals from interfering with nearby electronics. It consists of a central wire surrounded by a conductive shield with a layer of dielectric material in between. Each cable is designed with a characteristic impedance, $Z_0$, that depends on the cable geometry and the nature of the dielectric. In addition, the propagation velocity of the signal, $v$, depends on the dielectric constant.

The characteristic impedance in ohms and the propagation velocity as a fraction of $c$ for a particular cable can be calculated using the following equations:

$$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln(\frac{D_e}{d_e}) \tag{1}$$

$$v = \frac{1}{\sqrt{\epsilon_r}} \tag{2}$$

where $\epsilon_r$ is the relative dielectric constant of the insulator, and $D_e$ and $d_e$ are the effective diameters of the outer and inner conductors, respectively [1]. Using the values $\epsilon_r = 2.26$, $D_e = 3.63mm$, and $d_e = 1.00mm$ provided for this lab, the cable is found to have the following characteristic values:

$$Z_L = 48.4\ \Omega \tag{3}$$

$$v = 0.665\, c \tag{4}$$

When a signal is sent down the cable, it will be partially reflected at the end, with the reflection coefficient dependent on the impedance of the load. For minimum reflection and maximum power transfer, the load impedance should match the characteristic impedance of the cable. Impedances above or below this value will return a larger reflection, with those below creating an inverted reflection. In the extreme case of an open circuit—that is, a load impedance of $\infty\,\Omega$—the signal will ideally be completely reflected. In reality, there is some degree of loss during propagation through the cable, resulting in a slightly lower reflected voltage.

The reflection coefficient is given by the following equation:

$$\Gamma = \frac{V_2}{V_1} = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{5}$$

where $Z_L$ is the load impedance, $V_1$ is the outgoing signal voltage, and $V_2$ is the reflected voltage. Using this relationship, the characteristic impedance of a particular cable can be determined experimentally by the equation

$$Z_0 = \frac{V_1 - V_2}{V_1 + V_2} Z_L \tag{6}$$

The final quantity of interest in this lab is the standing wave ratio (SWR), which provides a measure of the impedance matching between the load and the cable. When the impedances are not matched properly, the reflected signal is stronger, increasing the value of the SWR. When the impedances are matched, the SWR will ideally be 1. Its value is calculated from the signal and reflection voltages by

$$SWR = \frac{|V_1| + |V_2|}{|V_1| - |V_2|} \tag{7}$$

# 3 Measurement of cable parameters

To measure the characteristic impedance and propagation velocity of a cable, the circuit in Figure 1 was simulated in LTspice. It consists of a voltage source attached to a simulated coaxial cable with a load impedance $Z_L$ across the end, as shown below in Figure 1. The voltage supply was initially programmed to send out a single 60ms square pulse. Reflections were measured for a selection of load impedances, the results of which are shown in Table 1.

Using $\Delta t$, the time from the emission of the pulse to the detection of the reflection, and the cable length of 24.15m provided in the lab, the propagation velocity was calculated to be

$$v = 2.00 \times 10^8 m/s = 0.667c \tag{8}$$

This is in agreement with the theoretical value calculated in the Introduction Theory section, as well as typical propagation velocities of around 0.66c given in coaxial cable data sheets [2].

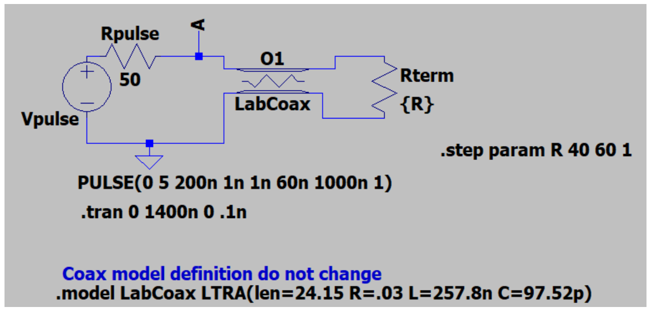



Figure 1: Schematic of simulated coaxial cable circuit.

| $Z_L(\Omega)$ | $\Delta t(ns)$ | $V_1(V)$ | $V_2(V)$ | $SWR$ | $Z_0(\Omega)$ |
|---|---|---|---|---|---|
| $10^{-3}$ | 242.0 | 2.535 | -2.460 | 66.6 | 0.066 |
| 50 | 242.0 | 2.535 | -0.030 | 1.024 | 51.20 |
| 100 | 242.0 | 2.535 | 0.795 | 1.914 | 52.25 |
| $10^8$ | 242.0 | 2.535 | 2.469 | 75.818 | $1.32 \times 10^6$ |

Table 1: Initial measurements of signal reflection through simulated coaxial cable.

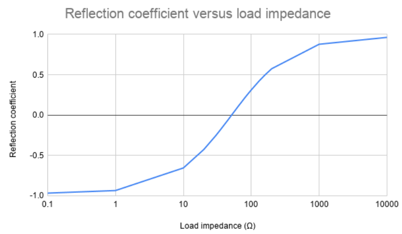
One application of knowing the propagation velocity is identifying a break in the cable. If the wire is severed at some point along its length, it will generate a reflection that returns at a different time than for an intact cable. Using the propagation velocity and the interval $\Delta t$ between emission and detection of the reflected pulse, the location of the break can be found by

$$x_{break} = \frac{1}{2} v \Delta t \tag{9}$$

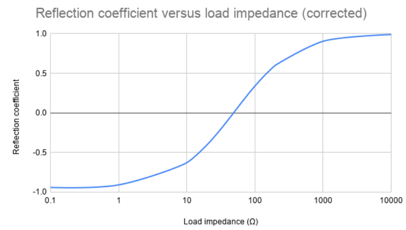
If this calculation returns a value greater than the length of the cable, it may indicate that the break is located in the shield rather than the central wire.

In an attempt to improve the calculated impedance value, the reflected voltages can be adjusted to account for loss during propagation. When the load resistance is replaced by an open circuit (approximated by a $10^8$ $\Omega$ resistor), the reflection will ideally have the same amplitude as the outgoing signal; in reality, the return voltage is slightly lower. Measuring the difference between the outgoing and incoming signals gives a value of $0.066V$, which—under the assumption that this loss is constant over all load impedances—is added to the other return voltage values as a correction. Plots of the reflection coefficient, standing wave ratio, and calculated characteristic impedance, both uncorrected and corrected, are shown below in Figures 2–4.

While the calculated impedance increases slightly with greater load impedance, the values are roughly $Z_0 = 50 \sim 52$ $\Omega$ for the uncorrected data and $Z_0 = 48 \sim 50$ $\Omega$ for the corrected data. As specific examples, at $Z_L = 50$ $\Omega$, the uncorrected and corrected values are $51.2$ $\Omega$ and $48.6$ $\Omega$, respectively. As

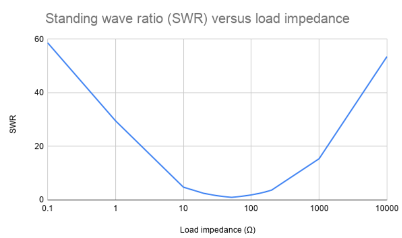



(a) Uncorrected

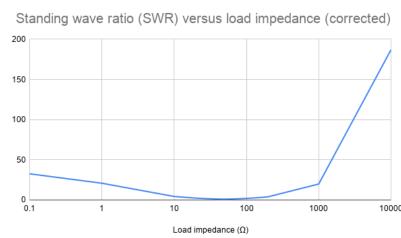



(b) Corrected

Figure 2: Reflection coefficient $\Gamma$ as function of load impedance. Negative values indicate an inverted reflection, and the inflection point at $\Gamma = 0$ shows the point of maximum power transfer, where $Z_L$ is equal to the characteristic impedance.

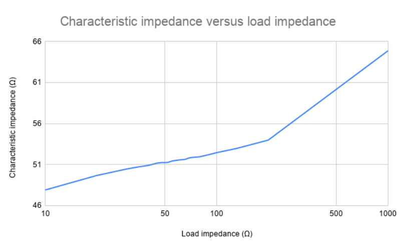



(a) Uncorrected

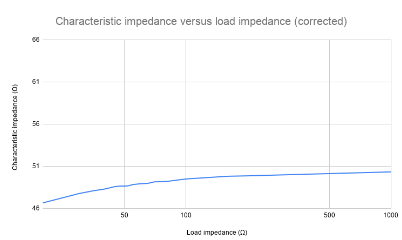



(b) Corrected

Figure 3: Standing wave ratio as function of load impedance. The bottom of the trough signifies the point of maximum power transfer.




(a) Uncorrected




(b) Corrected

Figure 4: Calculated value for characteristic impedance of the cable over a range of load impedances.

shown in the impedance plots of Figure 4, the corrected data produces a more consistent calculated value for $Z_L$.

Two common impedance ratings for coaxial cables are 50 Ω and 52 Ω. Both of these possibilities fall within the calculated ranges, so to produce a best guess, the corrected value is favored due to its greater consistency across a range of load impedances. Therefore, the measurements suggest that the cable's impedance rating is most likely 50 Ω. This conclusion is supported by the theoretical impedance of 48.4 Ω calculated previously.

# 4 Determining propagation velocity from harmonic modes

The propagation velocity can be determined using the principles of wave harmonics, with no need to measure the elapsed time directly. If an AC signal is applied to the end of the cable, with the other end of the cable shorted, the signal will reflect off the end and interfere with the input waveform. If the frequency matches one of the cable's harmonics, then the signal and inverted reflection will interfere destructively. By sweeping through the range of possible frequencies and looking for those with low-amplitude signals, the harmonics can be singled out.

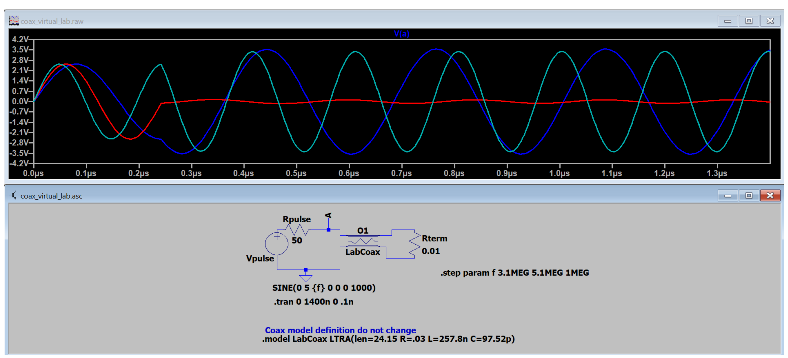



Figure 5: Example of a search for harmonics over three values. The flat red line indicates that there is a harmonic present near 4.1 MHz.

To make the amplitudes easier to distinguish, the integral of the voltage squared can be plotted as well. Ideally, the value of the integral at some fixed time would be plotted as a function of frequency, but due to lack of familiarity with LTspice, the integral was only plotted against time. Harmonics were identified by counting the curves as they were drawn by the simulation and using the step size to find the corresponding frequency value.

Although this procedure can yield harmonics, they may not necessarily include the fundamental mode, which is required to find the propagation velocity. To identify the fundamental mode, a new set of frequencies is generated by di-

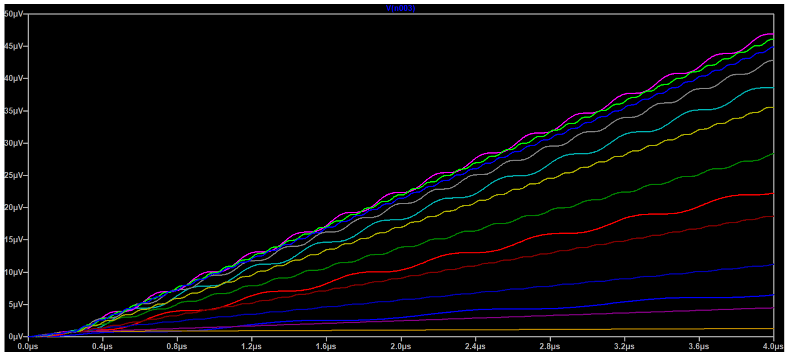


Figure 6: Plot of integral of voltage squared over frequencies 0.5MHz to 6.5MHz in steps of 0.5MHz. The line with the shallowest slope indicates a harmonic at around 4MHz.

viding the lowest frequency harmonic by each of the natural numbers until the frequency falls below a chosen lower search threshold. Each of these new frequencies is tested in the circuit, and the lowest-frequency harmonic is assumed to be the fundamental mode. The estimate is then refined by sweeping through a range of frequencies close to this value and selecting the one with the smallest amplitude.

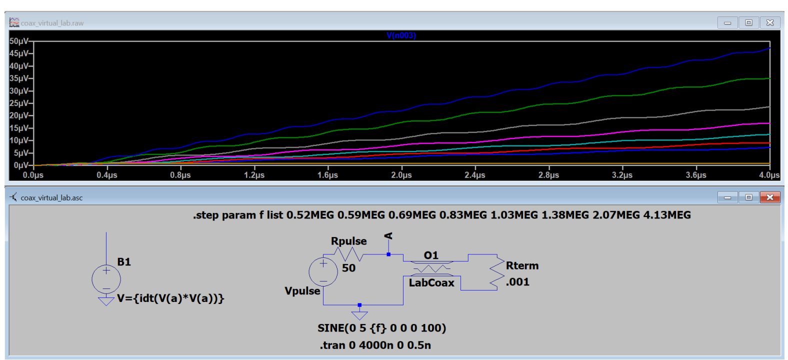



Figure 7: Depiction of process for identifying the fundamental mode. Several frequencies were tested to determine whether the harmonic in question was mode n=1 through n=8 of a harmonic series. The results indicated that the harmonic was the n=1 fundamental mode.

Based on a general familiarity with physics and electronics, a reasonable lower bound for the propagation velocity is $v_{low} = 0.1c$, which, for a cable of length 24.15m, corresponds to a frequency of $f_{low} \approx 0.62MHz$. On the other hand, the speed cannot surpass the speed of light, so the upper bound is $v_{high} = c$, with a frequency of $f_{high} \approx 6.2MHz$. If a harmonic is not found within this range, the search should be moved to a lower frequency regime.

Following this procedure for the simulated cable gives a fundamental mode of $f_0 \approx 4.13MHz$. Using the relationship $v = \lambda f_0$, where the wavelength $\lambda = 48.30m$ is twice the length of the cable, the propagation velocity is found to be

$$v = 1.99 \times 10^8 m/s = 0.66c \tag{10}$$

in agreement with the value measured previously.

# 5 Conclusion

Based on the theoretical values calculated from the cable dimensions and the dielectric constant, the methods described in this report are effective at determining the characteristic impedance to within approximately 2% and the propagation velocity to well under 1%. However, for greater precision in the characteristic impedance measurement, two issues should be addressed. First, the correction applied to the measured $V_2$ values rests on the assumption that the voltage loss in the cable was constant over all values of $Z_L$. While it successfully reduced the spread of calculated $Z_0$ values and shifted them closer to the theoretical impedance, it warrants a more rigorous justification. Second, even with the correction applied, the calculated $Z_0$ values varied by about 2.1 $\Omega$ over the range $Z_L$ = 30–200 $\Omega$. The effect of the load impedance should be investigated further to determine the most accurate way to estimate $Z_0$.

# References


[1] RFS, "Coaxial Transmission Lines: Technical Information," pp. 639-643. `http://www2.rfsworld.com/RFS_Edition4/pdfs/TechInfo_Edition4_639-672.pdf`

[2] Pasternack, "Flexible RG58 Coax Cable Single Shielded with Black PVC (NC) Jacket," RG58C/U datasheet, 2017.