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Modeling of probe-and-drogue part of an in-flight refueling system

Jie Yan

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Modeling of Probe-and-Drogue Part of an In-flight Refueling System

By
Jie Yan

**A Thesis Submitted in
Partial Fulfillment of the
Requirement for the**

**Master of Science
In
Mechanical Engineering**

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I present this work to my family- my father, my mother, my sister and her newborn daughter.

Thanks a lot for Dr. Agamemnon L. Crassidis, without his help, I can't finish this work.

1. **ABSTRACT**

In this work, a finite element model of an in-flight aircraft refueling system is developed. The model is a first attempt to describe the dynamics of a probe-and-drogue refueling system commonly used by Navy aircraft. The purpose of this work is to develop a model of the drogue system to be able to study control and sensor system requirements for an autonomous refueling system. This work is intended as a first introduction to an automated in-flight refueling system with special concentration on the modeling of a probe-and-drogue in-flight refueling system for aircrafts. An understanding of what is an in-flight refueling system and its significance and influence to the advancement of a aircraft is first presented. The main character of the system, such as mass, stiffness, damping effect is found from visual observation of an actual drogue system. While changing these parameters, the responses with different outside forces added on the system, such as impulse response, step response can be controlled. If the modeling response is stable, it is possible to simulate the simplified real outside force, such as exponential increases force used to simulate wind interaction effects in the model predicting the overall system response.

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Abbreviation and nomenclature

RAF:	Royal Air Force, UK, Farnborough
FRL:	Flight Refueling Limited
RCAF:	Royal Canadian Air Force
BSAA:	British South American Airways
FR Inc.:	Flight Refueling, Incorporated- FRL American subsidiary
SAC:	Strategic Air Command
TAC:	Tactical Air Command
NATO:	North Atlantic Treaty Organization
gpm:	gallon per minute
nm:	nautical mile, 1nm = 6,080 feet
knot:	nm per hour
Ke:	stiffness matrices of the element
Me:	mass matrix of the element
M:	mass matrix of the mathematical modal
K:	stiff matrix of the mathematical modal
C:	damping matrix of the mathematical modal
E:	Young's Modulus
i – Row, j – column	

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Chapter 1 Introduction of in-flight refueling system

1.1 What is in-flight refueling system

Fuel is required in all aircraft. It's a heavy energy source that consumes space. Fighters like the F-16 trade fuel capacity for performance and payload. For short-range air defense, the tradeoff is optimal. However, to fly long distances, designers resort to auxiliary fuel tanks in various configurations. With rare exceptions, pilots would choose not to carry auxiliary fuel tanks in combat, whether for air-to-air or for tactical missions. The auxiliary tanks increase fuel consumption, reduce ordinance payloads and tend to reduce top speed of the aircraft. Dropping the tanks before combat solves these problems, but the practice is wasteful and involves other compromises. Even with extra tanks, a fighter can't cross vast oceans by the most direct route without running dry. The only practical answer is to pick up extra fuel along the way.

Day or night, good weather or bad, in-flight refueling keeps military aircraft in the air, extending their endurance, range, and payload and vastly increasing their effectiveness. When duty calls at the far reaches of the globe, in-flight refueling allows a rapid response. Without aerial refueling, the face of modern combat would be changed. Most strategic and tactical air missions could never be undertaken.

1.2 History and achievements of in-flight refueling system

The idea of in-flight refueling aircraft appeared at the beginning of the 20th century, pilots achieved the first real in-flight refueling through a dangling-hose method. Several months later, another attempt was made. Where a border-to-border nonstop flight of 1,280 miles demonstrated how an airplane with a normal range of 275 miles could have its range quadrupled.

What became the much-publicized *Question Mark* operation went forward with a Fokker C-2A tri-motor, a high-wing monoplane of 10,935 pounds, modified into the receiver. Two 150-gallon tanks installed in its cabin supplemented its two 96-gallon wing tanks. After fuel was received into the cabin tanks it had to be pumped by hand to the wing tanks, from where it gravitated to the engines. In addition, there was a 45-gallon reserve tank for engine oil. A hatch was cut in the plane's roof to receive the refueling hose and other materials. On each side of its fuselage, the Fokker was painted with a large question mark intended to provoke wonder at how long the airplane could remain airborne. Two Douglas C-1 single-engine transports, 6,445-pound biplanes, were transformed into tankers by installing two 150-gallon tanks for offloading and a refueling hose that passed through a hatch cut in the floor. In the course of the operation on 1929, the tankers made forty-three takeoffs and landings. Crews of tankers flew forty-three sorties, twelve of them at night. Altogether, they delivered 5,660 gallons of fuel (33,960 pounds), 245 gallons of engine oil (1,838 pounds delivered in forty-nine five-gallon cans), and storage batteries, spare parts, tools, food, clothing, mail, and congratulatory telegrams. The *Question Mark* operation was predicated on its potential military utility. This achievement prompted many pilots to attempt in-flight refueling to establish long during flying record. In July 1930, the record was 647.5 hours, nearly 27 days in the air.

But although in-flight refueling promised to compensate for the airplane's many inadequacies, the modern airplane had come upon the scene and it changed everything.

In fact, until 1930s, the airplane was often unreliable and its commercial payload was helplessly uneconomical. A typical cruising speed for multiengine airplanes of the 1920s was only a trifle better than ninety miles per hour, and over a distance of 1,000 miles, most of this speed was lost at fuel stops. But within ten years, aero engine power quadrupled, which brought more powerful and long-range airplanes, such as Douglas DC-1 and Martin B-10 bomber. So with the elements of the modern airplane making it possible to build increasing range into an airplane, people felt no need for the complication of in-flight refueling.

During the World War II, aircraft with large internal fuel capacity alleviated the need for aerial refueling. At that time, in-flight refueling systems had severe drawbacks. However, changes took place after the Japanese attack on Pearl Harbor brought the United States into World War II, many Americans desperately wanted to bomb Japan. The most forward U.S. base for such an action was Wake Island, 1,983 miles from Tokyo, but the Japanese preempted its use on December 22, 1941, when they overwhelmed its small garrison of U.S. Marines. In Washington, Imperial Airways' use of in-flight refueling was recalled. The U.S. effort to develop an in-flight refueling capability went forward, although slowly.

It was Sir Alan Cobham and Flight Refueling Limited (FRL) the company he founded in 1934 brought great changes to the in-flight refueling. With World War II ended, FRL's services were more accepted than before. Six surplus Lancaster bombers were obtained and transformed into tankers and receivers. Each Lancaster tanker would deliver 2,830 U.S. gallons (16,980 pounds). During the winter of 1946–47, an intensive series of demonstration flights was flown in association with British South American Airways (BSAA). These were all-weather, day-and-night operations, and involved distant interceptions that used radar and transponders. Unlike the prewar refueling that were daylight visual flight rules operations in which tanker and receiver were rarely out of one another's sight. The object was to simulate mid-ocean rendezvous. FRL used its Lancaster tankers; BSAA used Lancaster receivers. Forty-three successful operations

were flown, twenty-six by day and seventeen at night. FRL took pains to have BSAA rotate the pilots of the receivers so the greatest number would learn that there was nothing extraordinary about the operation.

On Jun 26, 1946, the War Department's Army-Navy Aeronautical Bd agreed unanimously that the nm and the knot be adopted as standard units of distance and speed. With aviation then "going global" this made sense; it brought aviation into correspondence with geodesic measurements firmly in place since the eighteenth century. An nm is the length of one minute of the arc of a meridian at the Equator, that is, a nominal 6,080 feet. A knot is simply a rate of speed: one nm per hour. The distance from Goose Bay, Labrador, the northeastern most air base in North America, to Moscow is 3,106 nm; the distance from New York City to Moscow is 4,037 nm; and the distance from Chicago to Moscow is 4,303 nm.

As relations between the United States and the Soviet Union deteriorated after World War II, U.S. Army Air Forces' leaders started measuring distances between North America and such points in the USSR as Magnitogorsk, Novosibirsk, Omsk, and Sverdlosk. They found them to be more than a few nautical miles (nm) too far to fly. A means of range extension became urgent.

In 1948, Air Force contacted with FRL, getting two sets of FRL's in-flight refueling hardware, manufacturing rights to FRL's system and a contract with FRL to produce an additional forty refueling sets.

In 1940s, the design of some very big airplanes appeared, such as Boeing B-52 and Convair B-36, B-52 was a six-engine turboprop, with a weight as high as 490,000 pounds and a great range 10,860 nm, nearly half the globe's circumference at the equator, while B-36 had a weight of 328,000 pounds a range of 6,950 nm. People are convinced that these kinds of airplanes are too big, too slow, too vulnerable and quite obsolete. The only thing recommended was their great range, but it only achieved under the sarcastic idea of

“carrying fuel to consume fuel”. But if applied in-flight refueling system, the range reduction would be accepted, and everything else started to fall into place. The airplane was reduced to something reasonable around 300,000 pounds.

The bombardment committee emphatically concluded that the development of in-flight refueling should be the Air Force’s top priority, not only for the B-52 of the distant future, but also for existing B-29s and the new B-50s then entering the inventory.

Besides FRL, the other contributor of in-flight refueling system is Boeing. While FRL’s “looped hose” system dominated the in-flight refueling and its “probe and drogue” system was under developing, Boeing invented its “Boeing boom”, another widely used refueling system. Then in 1958 more and more B-29s were ordered to be modified into boom tankers, and became KB-29Ps. All of the new B-50 was to be boom receivers.

Another significance of 1948, twenty-five years after the world’s first aerial refueling, was the creation of the world’s first in-flight refueling unit, the 43d Air Refueling Squadron at Davis-Monthan AFB, Arizona, and the 509th at Walker AFB, Roswell, New Mexico. At the same time, FRL created an American subsidiary—Flight Refueling, Incorporated, known as FRInc.

On March, 1949, thanks to four in-flight refueling using FRL’s looped-hose system on a KB-29M, the first nonstop flight around the world was made by a B-50 named “lucky lady II” after an unremarkable ninety-four hours and one minute flight.

Promoted by world flight, at the end of 1949, SAC had six refueling squadrons and by the end of 1950, there were twelve squadrons with KB-29Ms.

In the fall of 1950, the first in-flight refueling of a jet bomber took place between a KB-29P and a North American RB-45C assigned to the 91st Strategic Reconnaissance Wing. Over the next eighteen months, the 91st developed jet bomber refueling equipment,

techniques and procedures, including the first night refueling and instrument weather refueling. By 1951, SAC had twenty squadrons with a mix of KB-29Ms and KB-29Ps, but also two with new Boeing KC-97s. The next year it had twenty refueling squadrons with 318 tankers. In 1952, SAC planners for the first time incorporated dependence on in-flight refueling into their war plans, and by 1953, SAC had almost thirty squadrons with 502 tankers, most of which were new KC-97s. At the end of 1954, SAC's refueling fleet had grown to thirty-two squadrons with 683 tankers, with an average of twenty-one airplanes per squadron.

In 1950s, while cold war got hot, refueling became vital. In 1951, during Korea war, the world's first combat mission using in-flight refueling was executed. This refueling of tip tanks to achieve range extension grew beyond occasional operations with probe-tanked F-80s and RF-80s into Project HIGH TIDE, in which the three squadrons of the 136th Fighter-Bomber Wing were equipped with probe tanks. SAC released ten KB-29Ms for this operation. HIGH TIDE's objective was an operational test of large tactical units, using in-flight refueling. The project had three phases: training the three squadrons in a series of small exercises, deploying them in combat air patrol missions over northern Japan, and deploying them in combat against targets in North Korea.

The U.S. Air Force concluded from HIGH TIDE that, although the refueling of tip tanks was a successful ad hoc operation, it was only an emergency substitute for a receiver with a single-point refueling system. Otherwise, there was no question about in-flight refueling being of value to the Tactical Air Command (TAC) and to theater air forces.

From 1948, with the assistance of in-flight refueling, flight crossing pacific became possible. After 1952, FOX PETER movements with in-flight refueling across the Pacific Ocean became routine for short-legged fighter planes.

With the development of in-flight refueling, more demands were submitted to make refueling more efficient and secure. Although the new B-47 bomber was a good sight, it also created problems.

The aircraft was not wholly compatible with SAC's slower and altitude-limited piston-engine tankers. Fully loaded at its 175,000-pound takeoff weight, a KC-97G was hard put to reach an altitude of 20,000 feet. But a B-47's cruising altitude was 35,000 feet. For its refueling, a B-47 had to descend to the KC-97's altitude and start refueling around 18,000 feet. While the tanker got lighter, the B-47 became heavier, requiring more speed to stay in the air than a KC-97's engine could match. The result was the "toboggan" maneuver in which both airplanes entered a shallow dive, a risky descent for two very large airplanes joined by a refueling boom. Two airplanes each weighing more than 150,000 pounds joined by forty-plus feet of ostensibly rigid tubing do not constitute a flying machine. Refueling could end as low as 12,000 feet, after which the B-47 had to climb back to its cruising altitude—and consume as much as *50 percent* of the fuel it had just taken on board. Clearly, the only remedy was a turbojet tanker that could deliver its offloads at altitudes that turbojet receivers found congenial. On June 1953, KC-97s fulfilled the mission to refuel B-47s in the air.

Then with the debut of B-52, new problem appeared. A KC-97 could serve two B-47s, giving each a minimum of 26,500 pounds of fuel (22.6 percent of a B-47's full load), but a B-52B's tank required 243,000 pounds to fill it, and a KC-97's total offload (53,000 lbs), was only 21 percent of this. In other words, to achieve approximately the same delivery given to two B-47s, a minimum of one KC-97 was required for each B-52B. However, a B-52's fuel consumption was greater than a B-47's, and two KC-97s were necessary to serve one of the eight-engine giants. Additionally, there remained the incompatibility between turbojet and piston-engine equipment. This not only required basing the slow tankers about 1,000 nm ahead of their receivers and establishing a rendezvous system, but also meant the B-52 had to descend for its fuel and then expend fuel climbing back to cruising altitude. Data from the operation show that it took two

KC-97s to guarantee one B-52B a minimum 26 percent refueling. A tanker larger than a KC-97, one with turbine engines that could cruise at the receiver's speed and altitude was clearly needed.

The solution was Boeing's KC-135, one the most widely used tanker now. A KC-135A could lift 31,200 gallons of JP4 (202,800 pounds of aviation jet fuel)—16,848 gallons in wing tanks, 12,178 gallons in fuselage tanks below the cargo deck, and 2,174 gallons in a tank on the tail cone's upper deck. In practice, weight limitations dictated about 5 percent less than that. The fuel in its below-deck tanks alone was enough to refuel two B-47Es to 33 percent and one B-52B to 32.5 percent.

A KC-135's below-deck fuel tanks, upper-deck and tail cone tanks, plus its center-wing tank, allowed it to replenish 57 percent of one B-52B's fuel, or 29 percent of two B-52Bs' fuel. The tanker did that at the bomber's operating altitude and comfortably within its speed envelope. A new refueling boom and pumping system delivered JP4 to receivers at a rate of 900 gpm.

At the end of 1961, the number of SAC tankers peaked at 1,095: 651 KC-97s and 444 KC-135s. This was an "air force" unto itself. In addition, the Tactical Air Command operated about 130 Boeing KB-50 tankers. The grand total was approximately 1,225 large, multiengine airplanes, all devoted to aerial refueling. The following year, at the time of the Cuban Missile Crisis, SAC's tankers included 503 KC-97s and 515 KC-135s, 1,018 airplanes—fewer total aircraft, but more jet KC-135s.

In 1949, North American the AJ-1s became the Navy's first in-flight tankers. Later AJ-1s were displaced by improved AJ-2s. After 1956, the AJ-2s were displaced by the Douglas A3D, an airplane weighing 82,000 pounds and with a combat radius of 900 nm. The best aerial tanker the Navy ever had was a modification of the Douglas A3D attack plane that could lift an offload of 3,350 gallons, or 21,775 pounds.

In 1953, to relieve the Navy's carrier space problem, Douglas developed the D-704 self-contained "buddy" refueling unit—an external store that held 300 gallons of fuel, a hose-reel unit, its own pumping system, and was self-powered by a generator turned by ram air. At first glance, the unit could be mistaken for an external fuel tank, but was distinguished by the propeller in its nose that turned its generator. The D-704 became the "granddaddy" of all buddy stores.

Operation HIGH TIDE in Korea proved the overture to the Tactical Air Command's adoption of aerial refueling. TAC worked out a system for F-101Cs to operate in pairs. One airplane carried a "dial-a-yield" nuclear weapon (ten to seventy kilotons), and the other carried a buddy-pack to refuel the bomber, whose normal radius of 690 nm could be extended to 900 nm. TAC submitted the idea of "every fighter a tanker; every fighter a bomber".

While Air Force focused on Boeing B-47, the British developed three bombers of similar weight and performance capabilities, each powered by four turbojets, the so-called "V" bombers—the Vickers Valiant, a 140,000-pound airplane of rather conservative design; the delta-wing Avro Vulcan, weighing 220,000 pounds; and the Handley Page Victor, 216,000 pounds. All had performances similar to the B-47. Later, Valiant and Victor were modified to tanker respectively. The first Victor tankers had two-point refueling, with an FRL hose-and-drogue unit beneath each wing. After 1965, later Victors had an additional unit installed beneath their fuselages, thus making them three-point tankers. Initially created as bombers, the Victor tankers already had probes for receiving; once converted; they could both give and receive. This double-duty outfitting permitted relay refueling in which two or more tankers could accompany a receiver to give it maximum range, with the tankers topping off one another. Some years later, the Vulcans, too, were converted from bombers to tankers, and they also had that double-ended capability. Much later in 1981, the U.S. Air Force first exploited the versatility of the tanker-receiver by designing that feature into the Douglas KC-10.

In 1960s, during war in Southeast Asia, in-flight refueling tanker, especially KC-135 did great achievements. By rule of thumb since 1915, the internal fuel of any fighter plane at any point in time is something less than 25 percent of its nominal maximum takeoff weight. A requirement for more fuel involves strapping on external fuel tanks. If that is not enough, in-flight refueling is necessary.

In a saga of aerial refueling, this KC-135 replenished two Navy KA-3 tankers, two Navy F-8s, and two F-4s returning from the strike, in addition to its F-104s. While it was pumping fuel to one of the KA-3s, the F-8 fighters came on the scene, desperate for fuel. The KA-3 reeled out its hose for them. For a few minutes, a K-135 was refueling a KA-3, which at the same time was refueling F-8s—a tri-level refueling. Without the service from this obliging KC-135, the Navy airplanes probably would not have reached their carrier.

And it was in this war, a KC-135 crew received proper credit: the Mackay Trophy, an award dating back to 1912 and given for the most extraordinary aerial flight of the year; this was the first time a tanker crew had been so honored.

Another extraordinary work of in-flight refueling tanker was saving leaking fighters. KC-135s occasionally flew leaking fighter planes back to their bases attached to their refueling booms, meanwhile pumping enough fuel to keep the fighter's engine barely running. The KA-3s did the same for Navy fighters and attacked planes, leading them at the ends of their refueling hoses.

In the course of the war, KC-135 tankers flew 194,687 sorties, averaging 21,631 sorties per year. They executed 813,378 in-flight refueling, an average of 90,375 per year, 7,531 per month, 251 per day. Total flying time was 911,364 hours, an average of 10,126 hours annually, 8,438 hours each month. In total, they delivered almost 1.4 billion gallons of fuel weighing almost 9 billion pounds.

At the same time, the application of in-flight refueling to helicopter enhanced its rescuing capability greatly. From its inception, the helicopter was a severely range-limited aircraft. The Sikorsky R-4, the first operational helicopter of the U.S. military, was a charming little machine of 2,020 pounds that had an operating radius of sixty statute miles. In any vehicle that lifts its own weight directly off the earth, weight is worse than critical—it is *everything*. Fuel is range, but fuel quickly adds up to lots of weight.

Nonetheless, military aviators, from the very beginning of rotary wing aviation, viewed the helicopter as an instrument to rescue people from predicaments with difficult access. From the first covert U.S. intervention in the affairs of South Vietnam and Laos in 1962, downed airmen needed rescue. This meant helicopters, and the effort quickly grew to more than just rescue. It meant dashing in to snatch these airmen out from under the guns of the enemy, being shot at, and shooting back. These circumstances generated requirements for range extension and an expanded loiter time. Helicopter advocates faced the same problem as their Aircraft and Weapons Board counterparts in 1947: either build a ridiculously large and hopelessly conspicuous flying machine, or go to in-flight refueling.

On December 1965, a CH-3 helicopter and a KC-130 tanker made success in the first experiment. In that moment, not only had air rescue operations been revolutionized, but also had the general scope of helicopter operations.

In the course of the war, the helicopters and crews of the Air Rescue Service picked up 3,383. It is doubtful that score would have been what it was without in-flight refueling extending the range and expanding the endurance, which was necessary for so many successful executions of the mission. The U.S. Navy, Marines, and Coast Guard quickly adopted in-flight refueling for helicopters built for rescue work and special missions.

Ever since then, in-flight refueling nearly appeared in every major war involved USA. When United States could not depend on its allies, couldn't use bases in those countries,

it needed an airlift capability independent of those bases, one it could deploy unilaterally. In-flight refueling was the solution. All the while, calls for in-flight refueling services were increasing. Since then KC-10 appeared, a tanker even more capable than KC-135, proving US's refueling capability.

The KC-10 is a 590,000-pound aircraft with a total capacity of 365,000 pounds of fuel (56,153 gallons), 61 percent of its takeoff weight, theoretically, all deliverable. A wholly new boom of McDonnell-Douglas design and its pumping system can deliver more than 1,000 gallons per minute. The KC-10A is air refuelable, making possible range extension by relay, refueling by a series of tankers. Additionally, some KC-10s have been equipped with wing-tip refueling pods with FRL hose-reel systems, thereby making the KC-10A the Air Force's first two-system tanker (boom *and* hose), and its first three-point tanker since the KB-50s were retired in 1965.

In 1991, during forty-three-day operation of DESERT STORM, one of the most interesting things was on any given day 18 percent of the airplanes in the air—almost one-fifth of the force—was tankers. The tankers were “first in and last out”.

The only regret of in-flight refueling, despite Sir Alan Cobham's vision of using it for commercial aircraft, despite all the technological advances, commercial aircraft designers never adopted aerial refueling. They preferred to build aircraft with large internal fuel tanks because this was cheaper than operating a dedicated fleet of aircraft that simply served as flying gas tanks. In-flight refueling now exclusively a military operation,

1.3 Different types of in-flight refueling system

1) Dangle – and – grab system, the first effort of in-flight refueling system,

On June 27, 1923, at an altitude of about 500 feet above Rockwell Field on San Diego's North Island, a hose linked two U.S.Army Air Service airplanes, and one airplane refueled the other. While only seventy-five gallons of gasoline were transferred, the event is memorable because it was a first. Airplanes were de Havilland DH-4Bs, single-engine biplanes of 4,600 pounds. 1st Lt. Virgil Hine piloted the tanker; 1st Lt. Frank W. Seifert occupied the rear cockpit and handled the fueling hose. Capt. Lowell H. Smith flew the receiver while 1st Lt. John Paul Richter handled the refueling from the rear cockpit. The refueling system consisted of a fifty-foot length of rubber hose, trailed from the tanker, with a manually operated quick-closing valve at each end. The process is best described in terms of "you dangle it; I'll grab it."

Ever since then, several attempts were made to demonstrate the practical application for in-flight refueling, and people began to realize how an airplane could have its range quadrupled. However, after the first fatal accident happened on November 18, 1923, when an airplane was wrecked and a pilot killed while trying to demonstrate in-flight refueling during an air show, the experiments were dismissed as stunts.

People had realized that dangle and grab technique was primitive, clumsy and dangerous. It was necessary to develop a technique for the fueling hookup that did not demand unusual flying skill.

2) Looped hose system

The situation lasted till L. R. Atcherly worked out his own in-flight refueling system – the looped hose system. During in-flight refueling there is a cruising airplane and a maneuvering airplane. Ordinarily, the tanker cruised while the receiver maneuvered to grab the hose. Atcherly reversed that order of work and put almost the whole burden of the operation on the tanker, whose crew would inevitably have more experience with refueling than would the crews of occasional receivers. The Atcherly System had both the

tanker and the receiver trailing cables with grapnels at their ends. While trailing its cable, the receiver flew a straight course, and the tanker crossed its track from behind, trailing its cable across the receiver's cable until the two grapnels connected. With the two airplanes now joined by their cables and flying side-by-side, a winch aboard the receiver pulled in its cable and along with it the tanker's cable. The refueling hose was attached to the other end of the tanker's cable and winched into the receiver, where it was made fast to a fueling connection. With the two aircraft joined by a huge bight of hose some 300 feet long, the tanker climbed to a position slightly higher than the receiver to put a gravity head on the offload, valves were opened, and refueling began.

When refueling was finished, the receiver disconnected the hose and the tanker reeled it in, but the two airplanes remained joined by the cables of the original connection. The tanker then turned away, breaking a weak link in the cable connection.

Sir Alan Cobham brought great changes to the in-flight refueling system. In spite of the failure happened before, Cobham was convinced that in-flight refueling had a practical future. Well aware that the dangle-and-grab fueling system had no future, by mid-1938 Cobham and the company he founded Flight Refueling Limited (FRL) had a workable system, which came to be known as the "looped hose." Although it was quite similar to what Atcherly had worked out in the early 1930s. FRL's distinct contribution was the invention and development of the small but vital fittings and hose connections that transformed in-flight refueling from stunts and experiments to rational flight operations that could be performed routinely.

And later, the idea of single-point refueling was introduced. Before 1948, airplanes were refueled much like automobiles, except that big airplanes had more than one fuel tank. At an airfield, a fuel truck's hose was moved from gas tank to gas tank, the filler caps usually located on a wing's upper surface and the fuel flowed by gravity. It was slow and awkward, but that was the way it had always been done. In 1948, Air Force introduced a necessary change in in-flight refueling, "single-point refueling". Hereafter, fueling would

be at a single point on the airplane into an integrated and well-vented fuel system, and accomplished under pressure.

3) **Boeing boom**

The cumbersome looped-hose system clearly had its limitations, and the Air Force had asked Boeing to investigate alternatives. The result was the "Boeing boom". In 1948, the Boeing Company began testing the "Boeing boom" system, consisting of a large-diameter pipe connected to the rear of a B-29 and fitted with small wings at the end. An operator, sitting in the tail of the tanker aircraft, controlled the boom to "fly" to a receptacle in the receiver. The boom was lowered and "flown" to a connector on the receiver aircraft. This allowed fuel transfers to take place at higher speeds and, more importantly, allowed more than six times as much fuel to flow per minute. FRL's looped-hose system had an inside diameter of only 2.5 inches and did well to deliver 110 gallons per minute (gpm). As most mathematicians and *all* plumbers know, if the diameter of pipe is doubled, its capacity is quadrupled. With an inside diameter of four inches and a powerful pumping system, the early-model Boeing boom delivered 700 gallon per minute.

Another important development was the "single-point refueling system" on receiver aircraft, which allowed all of an airplane's several fuel tanks to be refilled from a single spot instead of from multiple nozzles around the airplane.

4) **Probe – and – drogue system**

As early as 1939, FRL's looped hose provided a workable in-flight refueling system for large, multiengine airplanes, but its adaptation to fighter planes in which there was no crew to connect the hose was clearly impossible. Locked into the idea of doing something with a hose, FRL soon hit upon what became known as the probe-and-drogue system, whereby a small plane was equipped with a probe that could be plugged into a drogue at the end of a refueling hose trailing behind a tanker. In this system, the tanker un-reels a refueling hose from the rear or from wing pods. At the end of the hose is a funnel –

shaped drogue, a basket that resembles an oversized shuttlecock. The drogue stabilizes the trailing hose and provides a lead into the connector at the end of the hose. The receiver aircraft is fitted with a probe that extends out from the side of the fuselage or from the leading edge of the wing. The pilot guides the probe into the drogue to make the connection.

On April 1949, the first test was given. Air Force flew four B-29s and a pair of F-84Es to England for FRL's modification. Two B-29s were given probes that jutted out conspicuously from the upper curve of the nose of their fuselages, making them receivers; these aircraft became known as "unicorns." A third B-29 was modified with a single hose-reel unit in its fuselage and with hose-reel units in pods, one on each wing tip, enabling it to refuel three fighters in a single contact. It was designated the YKB-29T; its three-hose capability led to its being called the "Triple Nipple." The two F-84Es were fitted with single-point fueling systems.

Later, the idea of using the new probe- and -drogue system to refuel only the external drop tanks of the fighters was introduced. A receiver probe and its valve were welded on the inside forward curvature of an external drop tank. Instead of filling the internal tanks, the receiver pilot simply filled his wing tanks. For straight-wing F-80s and F-84Es, which had wing-tip tanks, subsequent operations were.

The capacities of the external tanks varied from 160 to 260 gallons (1,040–1,690 lbs), and the movement generated by a half-ton of fuel suddenly placed at the wing tip could make an airplane uncontrollable. To avoid making the airplane unstable in the roll axis, refueling the wing-tip tanks of an F-80 or an F-84E involved three fueling contacts. The receiver pilot filled his left tank half full, disconnected from the drogue, and connected with his right tank. When it overflowed, he disconnected that tank and reconnected his half-full left tank, filling it to an overflow. With both wing tanks full, he flew away to execute his mission.

5) Boom –drogue – adapter refueling system

Many hailed probe-and-drogue refueling as the system of the future. It was simpler, cheaper, and it weighed less than a Boeing boom. It imposed less aerodynamic drag on the tanker and did not require a skilled operator. But there were also some problems. Probe-and-drogue involved a lot of rubber, a material that could become unreliable in the –60°F temperatures above 30,000 feet. Furthermore it seemed a bit too much to expect a tired pilot of the sluggish mass of a 200,000-pound airplane to chase through 180 degrees of his vision ahead to put a probe into the small, dancing target of a drogue's refueling basket. Finally, the optimum transfer capacity of a probe-and-drogue system was only 250 gallons per minute, compared with the Boeing boom's 700 gpm. On the other hand, the receptacle for boom refueling could be placed anywhere on the upper surface of the receiver. It could be on the fuselage behind the canopy or on the leading edge of the left wing. The best position was eventually determined to be outside the pilot's vision, allowing the pilot to concentrate on the receiver's position relative to the tanker.

As a result, pilots of small maneuverable airplanes liked probe-and- drogue; those who flew big airplanes preferred the boom.

In 1959, the boom's incompatibility with probe-equipped airplanes was resolved with a boom-drogue-adapter—a flexible “tassel” fitted to the end of the boom with a basket at its end to receive a probe. However, when a boom tanker had an adapter attached, it could not serve receivers fitted with a boom receptacle.

In 1962, new modifications appeared. Use KB-50 as an example, in addition to hose-reel unit in the tanker's tail there was on in a pod at each wing tip. Each hose-reel unit deployed seventy-five feet of hose. While pumping to three receivers at the same time, the system delivered about 285 gallons per minute.

Chapter 2 Theory of finite element

2.1 Introduction of finite element

The finite element method is a numerical analysis technique for obtaining approximate solutions to a wide variety of engineering problems. Although originally developed to study stresses in complex airframe structures, it has since been extended and applied to the broad field of continuum mechanics. Because of its diversity and flexibility as an analysis tool, it is receiving much attention in engineering schools and in industry.

In reality, either the geometry or some other feature of the problem is irregular or “arbitrary”. Analytical solutions to problems seldom exist. One possible solution is to make simplifying assumptions- to ignore the difficulties and reduce the problem to one that can be handled. Sometimes this procedure works; but more often than not, it leads to serious inaccuracies or erroneous results. Now, as computers are widely available, a more viable alternative is to retain the complexities of the problem and find an approximate numerical solution.

One of the most widely used numerical analysis methods is finite element method. The finite element method envisions the solution regions as built up of many small interconnected sub-regions or elements. A finite element model of a problem gives a piecewise approximation to the governing equations. The basic premise of the finite element method is that a solution region can be analytically modeled or approximated by replacing it with an assemblage of discrete elements. Since these elements can be put together in a variety of ways, they can be used to represent exceedingly complex shapes.

2.2 Theory of finite element method

In a continuum problem of any dimension the field variable (whether it is pressure, temperature, displacement, stress or some other quantity) possesses infinitely many values because it is a function of each generic point in the body or solution region. Consequently, the problem is one with an infinite number of unknowns. The finite element discretization procedure reduce the problem to one of a finite number of unknowns by dividing the solution region into elements and by expressing the unknown field variable in terms of assumed approximating function with each element. The approximating functions (interpolation functions) are defined in terms of the values of the field variables at specified points called nodes. Nodes usually lay on the element boundaries where adjacent elements are connected. In addition to boundary nodes, an element may also have a few interior nodes. The nodal values of the field variable and the interpolation functions for the elements completely define the behavior of the field variable within the elements. For the finite element representation of a problem the nodal values of the field variable become the unknowns. Once these unknowns are found, the interpolation functions define the field variable throughout the assemblage of elements.

Clearly, the nature of the solution and the degree of approximation depend not only on the size and number of the elements used but also on the interpolation functions selected. However, the functions can't be chose, because certain compatibility conditions should be satisfied. Often functions are chosen so that the field variable or its derivatives are continuous across adjoining element boundaries.

Another very important feature of finite element method is the ability to formulate solutions for individual elements before putting them together to represent the entire problem. This means, we could reduce a complex problem by considering a series of greatly simplified problems. For example, if we are treating a problem in stress analysis, we find the force-displacement or stiffness characteristics of each individual element first, and then assemble the elements to find the stiffness of the whole structure, formulated in the chapters to follow.

Although using the finite element method, we could approach the formulation of properties of individual elements in different ways; the solution of a continuum problem by the finite element method always follows an orderly step-by-step process as blow:

1. Discretize the continuum

This is the first step to divide the continuum or solution region into elements. During this process, a variety of element shapes may be used, and different element shapes may be employed in the same solution region.

2. Select interpolation functions

In this step, we should assign nodes to each element and then choose the interpolation function to represent the variation of the field variable over the element. The field variable may be a scalar, a vector, or a higher-order tensor. Often, polynomials are selected as interpolation functions for the field variable because they are easy to integrate and differentiate. The degree of the polynomial chosen depends on the number of nodes assigned to the elements, the nature and number of unknowns at each node, and certain continuity requirements imposed at the nodes and along the element boundaries. The magnitude of the field variable as well as the magnitude of its derivatives may be the unknowns at the nodes.

3. Find the element properties

Once the finite element model has been established, which means once the elements and their interpolation functions have been selected, we are ready to determine the matrix equations expressing the properties of the individual elements.

4. Assembly the element properties to obtain the system equations

To find the properties of the overall system modeled by the network of elements we must “assemble” all the element properties. In other word, we combine the matrix equations expressing the behavior of the elements and form the matrix equations expressing the behavior of the entire system. The matrix equations for the system have the same form as

the equations for an individual element except they contain many more terms because they include all nodes.

The basis for the assembly procedure stems from the fact that at a node, where elements are interconnected, the value of the field variable is the same for each element sharing that node. A unique feature of the finite element method is that assembly of the individual element equations generates the system equations.

5. Impose the boundary conditions

Before the system equations are ready for solution they must be modified to account for the boundary conditions of the problem. At this stage we impose known nodal values of the dependent variables or nodal loads.

6. Solve the system equations

The assembly process gives a set of simultaneous equations that we solve to obtain the unknown nodal values of the problem. If the problem describes steady or equilibrium behavior then we must solve a set of linear or nonlinear algebraic equations. If the problem is unsteady, the nodal unknowns are a function of time, and we must solve a set of linear or nonlinear ordinary differential equations.

2.3 The application of finite element method to the case

In the case of the in-flight refueling system, the system consists of a long flexible steel hose with an attached drogue at the end. The hose is more than fifty feet long and the outside diameter is approximately 2.5 – 3 inches. Comparing the length of the hose to the diameter, the diameter is small so that we simulate the probe-and-drogue system as a flexible beam and create the mathematical model as a beam system. According to the first step of finite element method, we could discretize the continuum beam system into elements and analyze the beam element first.

Describe beam element as below:

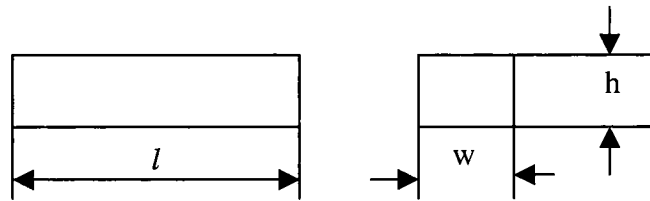


Figure 2.1 Beam elements

- l : length of beam element
- w : width of beam element
- h : height of beam element
- E : Young's modulus
- I : moment of inertia, $I = \frac{1}{12} wh^3$

First, the parameters of a beam element, stiffness matrices and mass matrices are developed. Generally, the relative axial displacements will be small compared to the lateral displacements of the beam element and can be assumed to be zero.

The stiffness for a beam element is expressed as shown below:

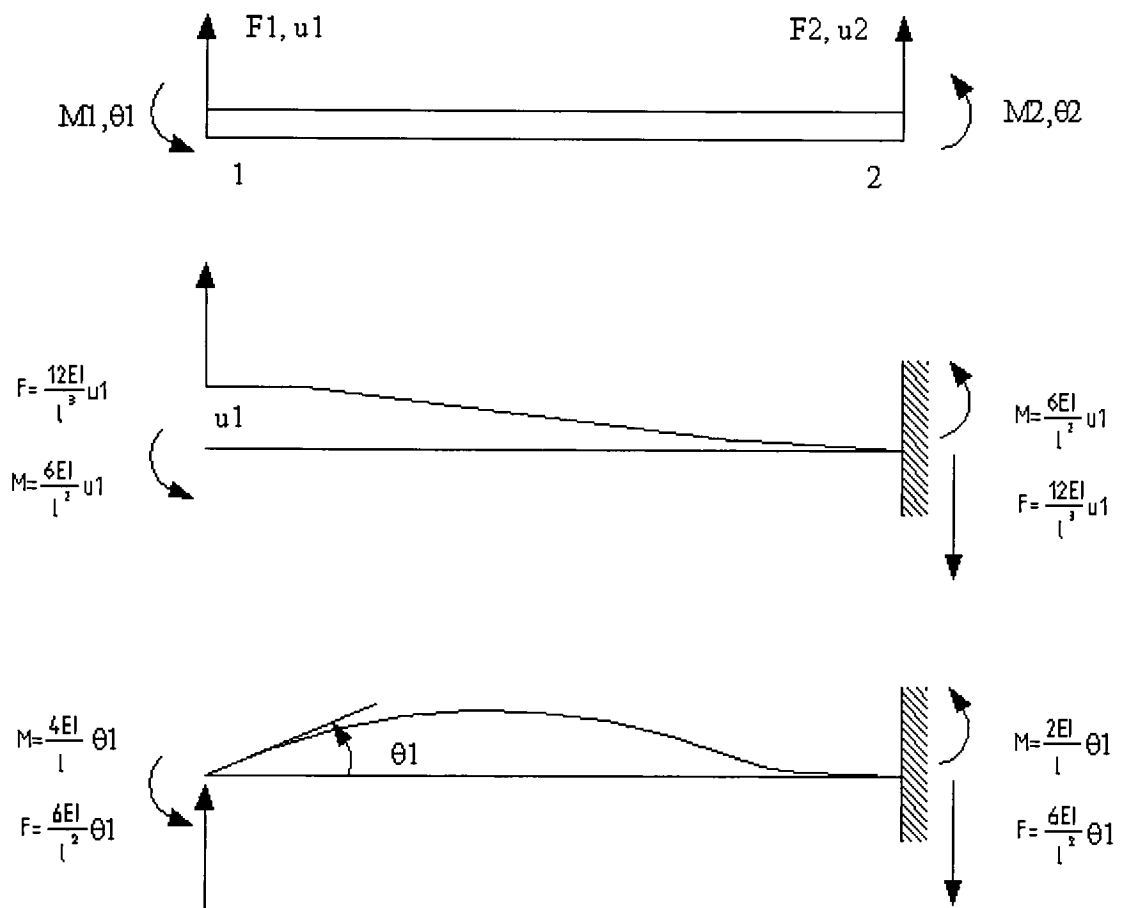


Figure 2.2 Stiffness for beam element

The local coordinates for the beam element are the lateral displacements and rotation at the two ends. Each end will have a lateral displacement and a rotation, resulting in four coordinates, u_1, θ_1 for the first end and u_2, θ_2 for the second end. The displacement can be considered to be the superposition of the four shapes, as $\phi_1(x)$, $\phi_2(x)$, $\phi_3(x)$ and $\phi_4(x)$.

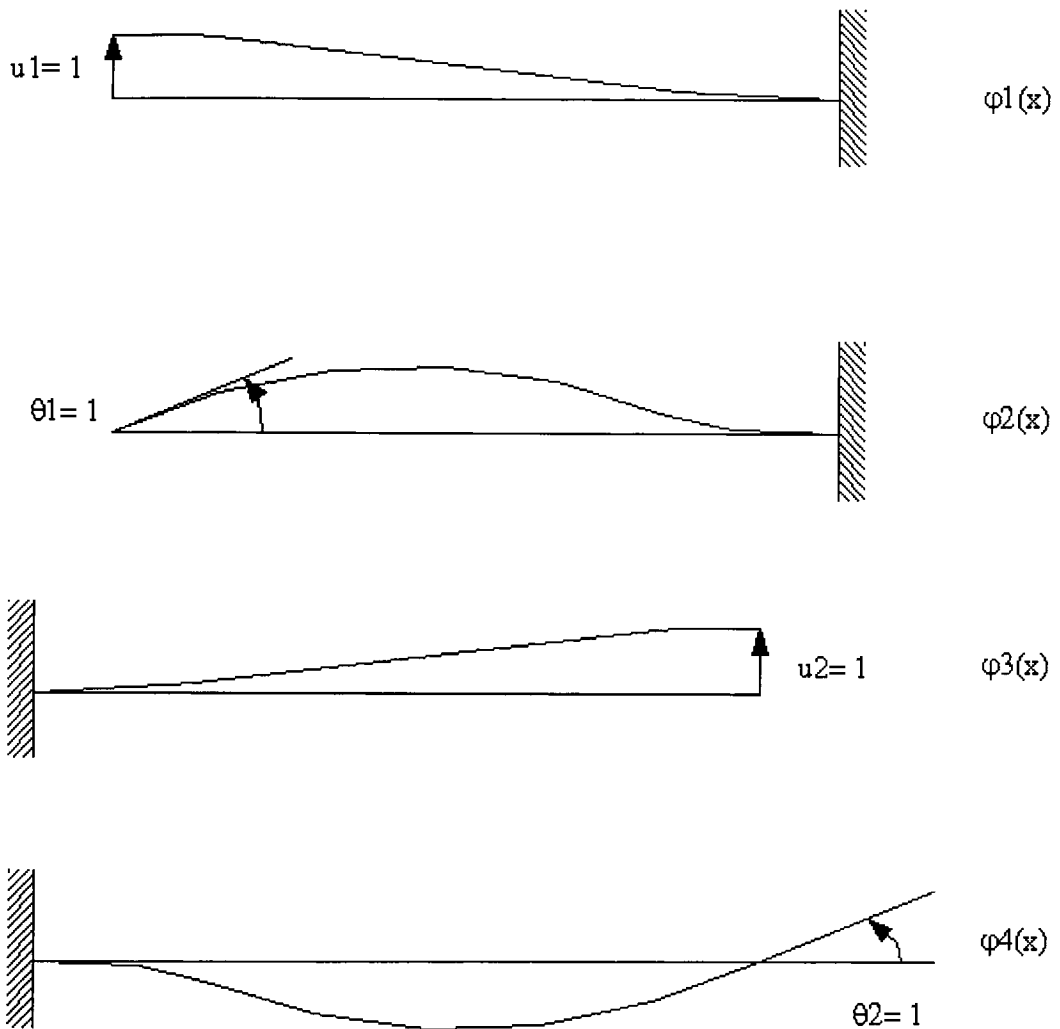


Figure 2.3 Positive sense of beam displacement

The 4 X 4 stiffness matrix of the element is given by:

$$K^e = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

For the development of the general equation of the beam, which is a cubic polynomial, the deflection is expressed in the form

$$u(x) = P_1 + P_2 \xi + P_3 \xi^2 + P_4 \xi^3$$

Where

$$\xi = \frac{x}{l} \text{ and } P_i = \text{constants}$$

Differentiating yields the slope equation

$$l\theta(x) = P_2 + 2P_3 \xi + 3P_4 \xi^2$$

The boundary conditions are

$\xi = 0$ at $x = 0$ and $\xi = 1$ at $x = l$; the boundary equations can be expressed by the following matrix equation:

$$\begin{bmatrix} u_1 \\ l\theta_1 \\ u_2 \\ l\theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

If we express parameters P_i by displacements, the equation becomes:

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & -2 & 3 & -1 \\ 2 & 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ l\theta_1 \\ u_2 \\ l\theta_2 \end{bmatrix}$$

Set $u_1(x) = 1$, $\theta_1(x) = 0$, $u_2(x) = 0$, $\theta_2(x) = 0$, we obtain a group of constants P_i to define shape function of $\varphi_1(x)$;

$$P_1 = 1, P_2 = 0, P_3 = -3, P_4 = 2;$$

Then set $u_1(x) = 0$, $\theta_1(x) = 1$, $u_2(x) = 0$, $\theta_2(x) = 0$ to define $\varphi_2(x)$, get

$$P_1 = 0, P_2 = l, P_3 = -2l, P_4 = l;$$

Then set $u_1(x) = 0$, $\theta_1(x) = 0$, $u_2(x) = 1$, $\theta_2(x) = 0$ to define $\varphi_3(x)$, get

$$P_1 = 0, P_2 = 0, P_3 = 3, P_4 = -2;$$

Then set $u_1(x) = 0$, $\theta_1(x) = 0$, $u_2(x) = 0$, $\theta_2(x) = 1$ to define $\varphi_4(x)$, get

$$P_1 = 0, P_2 = 0, P_3 = -l, P_4 = l;$$

Therefore, the following shape functions are derived:

$$\varphi_1(x) = 1 - 3\xi^2 + 2\xi^3$$

$$\varphi_2(x) = l\xi - 2l\xi^2 + l\xi^3$$

$$\varphi_3(x) = 3\xi^2 - 2\xi^3$$

$$\varphi_4(x) = -l\xi^2 + l\xi^3$$

Considering the displacement in general to be the superposition of the four shape functions, i.e.,

$$\begin{aligned} y(x) &= \varphi_1 u_1 + \varphi_2 \theta_1 + \varphi_3 u_2 + \varphi_4 \theta_2 \\ &= \varphi_1 q_1 + \varphi_2 q_2 + \varphi_3 q_3 + \varphi_4 q_4 \end{aligned}$$

The kinetic energy

$$\begin{aligned} T &= \frac{1}{2} \int \dot{y}^2 m dx = \frac{1}{2} \sum_i \sum_j \dot{q}_i \dot{q}_j \int \varphi_i \varphi_j m dx \\ &= \frac{1}{2} \sum_i \sum_j m_{ij} \dot{q}_i \dot{q}_j \end{aligned}$$

in which the generalized mass m_{ij} forms the elements of mass matrix. The m_{ij} is defined as

$$m_{ij} = \int_0^l \varphi_i \varphi_j m dx$$

Thus, the mass matrix of the element is a 4 X 4 matrix

$$M^e = \frac{ml}{420} \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}$$

Chapter 3 Theory of mathematical model

3.1 Theory of mathematical model

The goal of modeling a probe-and-drogue in-flight refueling system is to analyze the motion of the probe-and-drogue part of the system, and understand the movements, then develop a system for autonomous in-flight refueling.

As discussed in the chapter 2, comparing the diameter of the pipe to the length, the diameter is small, so the probe-and-drogue part of the refueling system can be treated as a long and thin flexible steel cantilever beam. The end of the beam is rigidly connected to the tanker, and the pipe will act like a beam with moments and lateral forces acting at the joints. The relative axial displacements will be very small compared to the lateral displacements, which could be assumed to be zero.

The figure below displays the simplified model of the probe-and-drogue system.



Figure 3.1 simplification of probe and drogue

L: the length of the beam

W: width of the beam

H: height of the beam

I: area moment of inertia = $\frac{1}{12} WH^3$

In the finite element method, the continuum beam system is discretized into several elements to analyze the beam element first. The assembly of the system matrix is found by superimposing the preceding element matrices. For example, consider a 2-element system shown below:

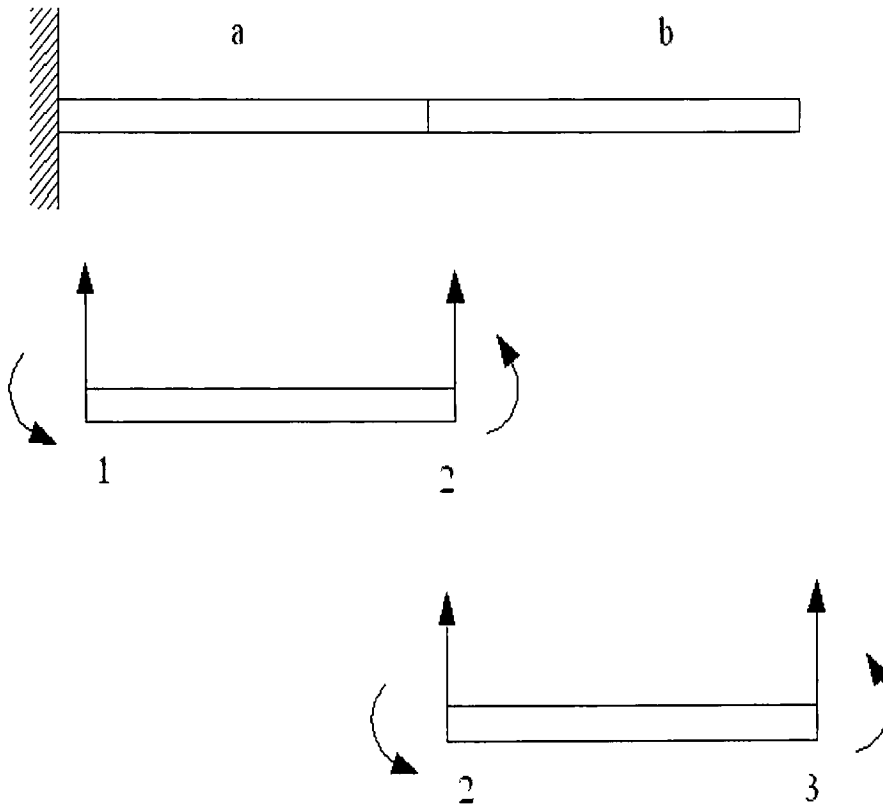


Figure 3.2 Uniform two-beam elements

Element a and element b are identical elements except for the displacement vector.

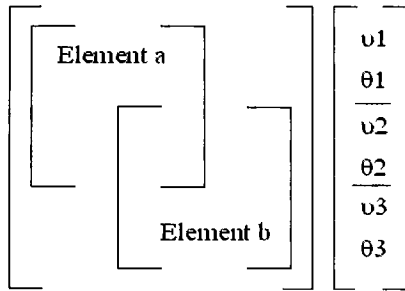
Element a:

$$\begin{Bmatrix} u1 \\ \theta1 \\ u2 \\ \theta2 \end{Bmatrix}$$

Element b:

$$\begin{Bmatrix} u2 \\ \theta2 \\ u3 \\ \theta3 \end{Bmatrix}$$

We superimpose the preceding matrix of “a” with matrix “b” to obtain the following 6 X 6 system matrix:



For example, if there are two beam elements, then the stiffness and mass matrices of the system is shown as below:

Stiffness matrix:

$$K = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l & & \\ 6l & 4l^2 & -6l & 2l^2 & & \\ -12 & -6l & 12+12 & -6l+6l & -12 & 6l \\ 6l & 2l^2 & -6l+6l & 4l^2+4l^2 & -6l & 2l^2 \\ & & -12 & -6l & 12 & -6l \\ & & 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

$$= \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l & 0 & 0 \\ 6l & 4l^2 & -6l & 2l^2 & 0 & 0 \\ -12 & -6l & 24 & 0 & -12 & 6l \\ 6l & 2l^2 & 0 & 8l^2 & -6l & 2l^2 \\ 0 & 0 & -12 & -6l & 12 & -6l \\ 0 & 0 & 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

Mass matrix:

$$M = \frac{ml}{420} \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156+156 & -22l+22l & 54 & -13l \\ -13l & -3l^2 & -22l+22l & 4l^2+4l^2 & 13l & -3l^2 \\ & 54 & 13l & 156 & -22l \\ & -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}$$

$$= \frac{ml}{420} \begin{bmatrix} 156 & 22l & 54 & -13l & 0 & 0 \\ 22l & 4l^2 & 13l & -3l^2 & 0 & 0 \\ 54 & 13l & 312 & 0 & 54 & -13l \\ -13l & -3l^2 & 0 & 8l^2 & 13l & -3l^2 \\ 0 & 0 & 54 & 13l & 156 & -22l \\ 0 & 0 & -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}$$

If there are more than two elements, an extension of the matrix in the same manner is made. The overall stiffness and mass matrices of the system can then be developed.

3.2 The equation of motion

A second-order equation of a system with viscous damping and arbitrary excitation F can be presented in matrix form as:

$$[M]\ddot{X} + [C]\dot{X} + [K]X = [F]$$

where

- [M]: mass matrix of the system
- [K]: stiffness matrix of the system
- [C]: damping matrix of the system
- [F]: excitation force

$[M]$ and $[K]$ are determined by the character of the system.

3.2.1 Free vibration of the un-damped system

We first discuss free vibration of the un-damped system, the equation of motion expressed above becomes:

$$[M]\ddot{X} + [K]X = 0$$

If we pre-multiply the equation by M^{-1} , we get the following terms:

$$M^{-1}M = I \text{ (unit matrix)}$$

$$M^{-1}K = A \text{ (a system matrix)}$$

And

$$I\ddot{X} + AX = 0$$

The system matrix A defines the dynamic properties of the system. Assuming harmonic motion

$$\ddot{X} = -\lambda X \text{ and } \lambda = \omega^2, \text{ we get } [A - \lambda I][X] = 0,$$

Then the characteristic equation of the system is the determinant equated to zero, or

$$[M^{-1}K - \lambda I] = [A - \lambda I] = 0$$

the roots λ_i are eigenvalues, and the natural frequencies of the system are determined by the relationship

$$\lambda_i = \omega_i^2$$

If substitute λ into the characteristic equation, we obtain the corresponding eigenvector X , and it is possible to make X orthogonal.

3.2.2 Modal matrix P

In the discussion above, the eigenvalues and eigenvectors of the system were developed. For each eigenvalue, a corresponding eigenvector exists. The modal matrix for an n degree-of-freedom system may appear as

$$P = [X_1 \ X_2 \dots \ X_n]$$

In which, each X_n is a column orthogonal vector and corresponding to a system mode.

The transpose of P is

$$P' = [X_1 \ X_2 \dots \ X_n]'$$

If we form the product of $P'MP$ and $P'KP$, diagonal matrices as below are developed:

$$\begin{aligned} P'MP &= [X_1 \ X_2 \dots \ X_n]' [M] [X_1 \ X_2 \dots \ X_n] \\ &= \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \end{aligned}$$

in which, the off-diagonal terms are zero because of the orthogonal property, and the diagonal terms are the generalized mass M_i .

Similarly, for stiffness matrix K ,

$$\begin{aligned} P'KP &= [X_1 \ X_2 \dots \ X_n]' [K] [X_1 \ X_2 \dots \ X_n] \\ &= \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \end{aligned}$$

in which, the diagonal terms are the generalized stiffness K_i .

If we divide each column of P by the square root of generalized mass M_i , we obtain the weighted modal matrix \bar{P} . The diagonal matrix developed from the weighted modal matrix results in the unit matrix

$$\bar{P}' M \bar{P} = I$$

Since $M_i^{-1} K_i = \lambda_i$, the stiffness matrix treated by the weighted modal matrix becomes a diagonal matrix of the eigenvalues

$$\bar{P}' K \bar{P} = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \lambda_n \end{bmatrix}$$

3.2.3 Motion of n degree-of-freedom system with damping

After development of the eigenvalues, eigenvector and modal matrix, we consider the equation of motion of an n degree-of-freedom system with viscous damping and arbitrary excitation F is presented by the form:

$$[M]\ddot{X} + [C]\dot{X} + [K]X = [F]$$

that is generally a set of n coupled equations.

As we have found the solution of the homogeneous un-damped equation

$$[M]\ddot{X} + [K]X = 0$$

which leads to the eigenvalues and eigenvectors that describe the normal modes of the system and the modal matrix P or \bar{P} . If we set $X = \bar{P}Y$, and pre-multiply by \bar{P}' , the equation above becomes:

$$\bar{P}'[M]\bar{P}\ddot{Y} + \bar{P}'[C]\bar{P}\dot{Y} + \bar{P}'[K]\bar{P}Y = \bar{P}'[F]$$

in which, $\bar{P}'M\bar{P}$ and $\bar{P}'K\bar{P}$ are diagonal matrices. Although in general, $\bar{P}'C\bar{P}$ is not diagonal, if C is proportional to M or K , it is evident that $\bar{P}'C\bar{P}$ becomes diagonal. Then the system has proportional damping. The equation of system motion becomes:

$$\ddot{y}_i + 2\xi_i\omega_i\dot{y}_i + \omega_i^2 y_i = f_i$$

we set the proportional damping in the form shown below:

$$[C] = \alpha[M] + \beta[K]$$

where α and β are constants. When different values for parameters α and β are chosen, a new damping matrix $[C]$ is developed.

Applying the weighted modal matrix \bar{P} to $[C]$ results in

$$\bar{P}'C\bar{P} = \alpha\bar{P}'M\bar{P} + \beta\bar{P}'K\bar{P} = \alpha I + \beta\bar{P}'K\bar{P}$$

$$= \alpha I + \beta \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \lambda_n \end{bmatrix}$$

$$= \alpha I + \beta \begin{bmatrix} \omega_1^2 & & 0 \\ & \omega_2^2 & \\ 0 & & \omega_n^2 \end{bmatrix}$$

So that

$$\ddot{y}_i + (\alpha + \beta \omega_i^2) \dot{y}_i + \omega_i^2 y_i = f_i$$

comparing to

$$\ddot{y}_i + 2\xi_i \omega_i \dot{y}_i + \omega_i^2 y_i = f_i$$

it is clear that

$$2\xi_i \omega_i = \alpha + \beta \omega_i^2 \text{ or } \beta \omega_i^2 - 2\xi_i \omega_i + \alpha = 0$$

3.2.4 State space version

Converting the equation

$$[M]\ddot{X} + [C]\dot{X} + [K]X = [F]$$

to a state space model, i.e.,

$$\dot{X} = AX + BU$$

$$Y = DX + EU$$

where X is state variable, and U is input, if the system is an n th order dynamic system with m inputs, then X will be an n th order vector, U will be an m th vector, A will be a $(n \times n)$ matrix, B will be an $(n \times m)$ matrix. If p output variables are to be monitored, Y will be a p th order vector, D will be $(p \times n)$, and E will be $(p \times m)$.

Set

$$X_1 = X$$

$$X_2 = \dot{X}_1 = \dot{X}$$

$$\begin{aligned}\dot{X}_2 = \ddot{X}_1 = \ddot{X} &= [M]^{-1}[F] - [M]^{-1}[C]\dot{X} - [M]^{-1}[K]X \\ &= [M]^{-1}[F] - [M]^{-1}[K]X_1 - [M]^{-1}[C]X_2\end{aligned}$$

Then

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ [M]^{-1}[F] \end{bmatrix}$$

If we put force F into matrix [B], and set [U] as a unit input, then the equation becomes

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ [M]^{-1}[F] \end{bmatrix} [U]$$

$$[Y] = [D] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + [E][U]$$

If we set [E] matrix to zero and set [D] matrix as [1 0], then

$$[Y] = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = X_1$$

If we set [D] matrix as [0 1], then

$$[Y] = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = X_2$$

Analysis above discussed situations where X_1, X_2 are single variables. If both X_1, X_2 are vectors, with n elements, then the state space equation can be modified to:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} \text{zeros} & I \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ [M]^{-1}[F] \end{bmatrix} [U]$$

In which, *zeros* is an $n \times n$ matrix with all elements to be zero and I is an $n \times n$ identity matrix with all elements on the diagonal to be 1.

Chapter 4 Creation of mathematic model of the in-flight refueling system

4.1 The Forcing function

For the mathematical model of probe-and-drogue part, first consider the free vibration condition (i.e., there is no any outside force added on the system). The only existing force is the gravity. The longer the pipe, the larger the gravity force, although the gravity acceleration is the constant, to generalize the situation, a linearly increasing force on the system is assumed, shown below:

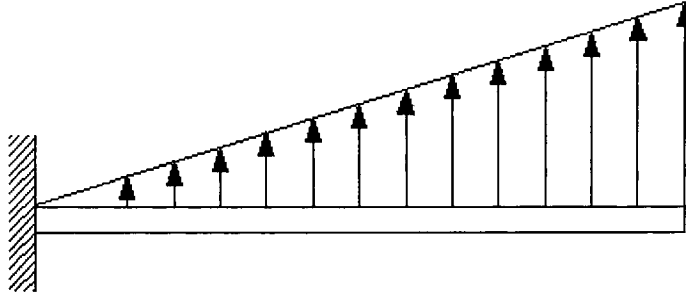


Figure 4.1 Model of free vibration

For the equation $[M]\ddot{X} + [C]\dot{X} + [K]X = [F]$, a known stiffness and mass matrices of the system can be derived, and by defining parameter α and β , the damping matrix is also derived, leaving only the force function $[F]$. According to theory of virtual work of applied force, the shape function displacement is expressed as follows:

$$y(x) = \varphi_1 u_1 + \varphi_2 \theta_1 + \varphi_3 u_2 + \varphi_4 \theta_2$$

The virtual work of the applied force gravity on the beam is:

$$\begin{aligned} \delta W &= \int_0^l p(x) \delta y(x) dx \\ &= \delta v_1 \int_0^l p(x) \varphi_1(x) dx + \delta \theta_1 \int_0^l p(x) \varphi_2(x) dx + \delta v_2 \int_0^l p(x) \varphi_3(x) dx + \delta \theta_2 \int_0^l p(x) \varphi_4(x) dx \end{aligned}$$

where $\varphi_{1-4}(x)$ are shape functions for a simple element, shown below:

$$\begin{aligned} \varphi_1(x) &= 1 - 3\xi^2 + 2\xi^3 \\ \varphi_2(x) &= l\xi - 2l\xi^2 + l\xi^3 \end{aligned}$$

$$\varphi_3(x) = 3\xi^2 - 2\xi^3$$

$$\varphi_4(x) = -l\xi^2 + l\xi^3$$

$$\text{with } \xi = \frac{x}{l}$$

If the same procedure is applied to the end forces, F_1 , M_1 , F_2 and M_2 , the total virtual work is represented by

$$\delta W = F_1 \delta u_1 + M_1 \delta \theta_1 + F_2 \delta u_2 + M_2 \delta \theta_2$$

Equating the virtual work above, we could obtain the following relationship

$$F_1 = \int_0^l p(x) \varphi_1(x) dx$$

$$M_1 = \int_0^l p(x) \varphi_2(x) dx$$

$$F_2 = \int_0^l p(x) \varphi_3(x) dx$$

$$M_2 = \int_0^l p(x) \varphi_4(x) dx$$

where l is the length of element and $p(x)$ is obtained by examining Figure 4.2,

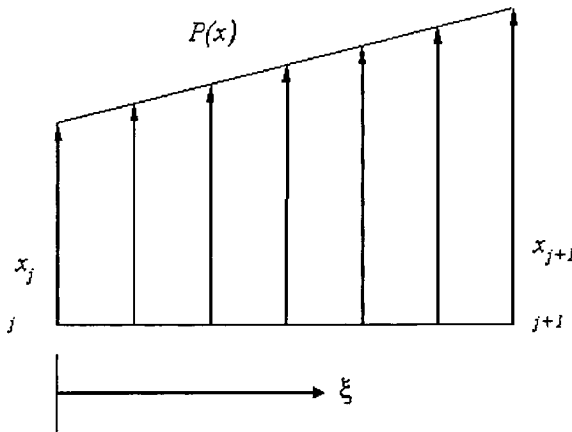


Figure 4.2 Forcing function for a single beam element

$$p(x) = (x_{j+1} - x_j)\xi + x_j$$

The force for the j th node can be given by

$$F_j = \int_0^l p(x) \varphi_1(x) dx = l \int_0^1 [(x_{j+1} - x_j)\xi + x_j](1 - 3\xi^2 + 2\xi^3) d\xi$$

$$F_j = l \left(\frac{1}{6} x_{j+1} + \frac{1}{3} x_j \right)$$

$$M_j = \int_0^l p(x) \varphi_2(x) dx = l \int_0^1 [(x_{j+1} - x_j) \xi + x_j] (\xi - 2\xi^2 + \xi^3) d\xi$$

$$M_j = l^2 \left(\frac{1}{30} x_{j+1} + \frac{1}{20} x_j \right)$$

The force for the (j+1)th node can be given by

$$F_{j+1} = \int_0^l p(x) \varphi_1(x) dx = l \int_0^1 [(x_{j+1} - x_j) \xi + x_j] (3\xi^2 - 2\xi^3) d\xi$$

$$F_{j+1} = l \left(\frac{7}{20} x_{j+1} + \frac{3}{20} x_j \right)$$

$$M_{j+1} = \int_0^l p(x) \varphi_2(x) dx = l \int_0^1 [(x_{j+1} - x_j) \xi + x_j] (\xi^3 - \xi^2) d\xi$$

$$M_{j+1} = l^2 \left(-\frac{1}{20} x_{j+1} - \frac{1}{30} x_j \right)$$

The system forcing function F_i for the i th link can be assembled by superimposing the proceeding expressions for the general elements. For n nodes (i.e. 1, 2, 3, ..., n_{i+1}), the following are used to obtain the equivalent nodal force, where we assume the beam is divided into n elements with equal length.

$$\bigcap_{i=1}^{n+1} = \begin{cases} F(i) = l \left(\frac{7}{20} x_{(i)j+1} + \frac{3}{20} x_{(i)j} \right) + l \left(\frac{3}{20} x_{(i+1)j+1} + \frac{7}{20} x_{(i+1)j} \right) \\ M(i) = l^2 \left(-\frac{1}{20} x_{(i)j+1} - \frac{1}{30} x_{(i)j} \right) + l^2 \left(\frac{1}{30} x_{(i+1)j+1} + \frac{1}{20} x_{(i+1)j} \right) \end{cases}$$

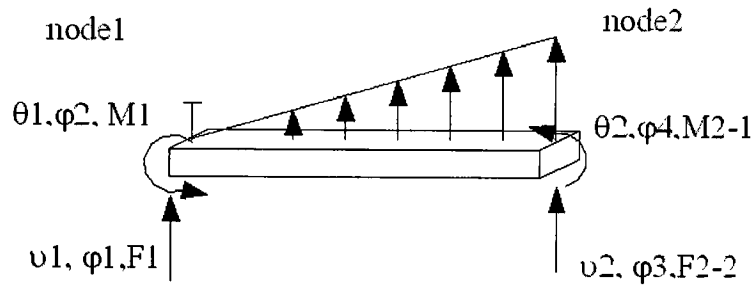
$$F(n+1) = l \left(\frac{7}{20} x_{(i)j+1} + \frac{3}{20} x_{(i)j} \right)$$

$$M(n+1) = l^2 \left(-\frac{1}{20} x_{(i)j+1} - \frac{1}{30} x_{(i)j} \right)$$

4.2 Stiffness and forcing function matrices

The beam is divided into several elements. For every beam element, gravity is the only force applied on it.

For the first element



and for the second element

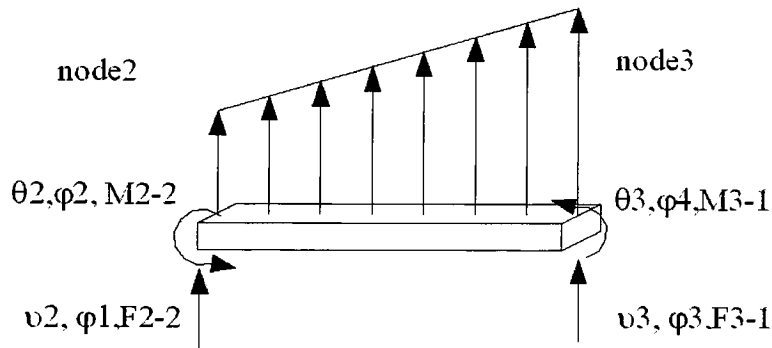


Figure 4.3 Beam elements for free vibration model

The ending point of the previous element is the starting point of the next element, so for every node, the generalized force is consisted of two parts, which are derived from different shape functions and forces and moment on the element. Denote

- P1: applied force on previous element,
- P2: applied force on following element

ψ_1 : shape function of starting point of the element

ψ_2 : shape function of ending point of the element

See table below:

Previous element		Following element
P1, ψ_1	P1, ψ_2	
	P2, ψ_1	P2, ψ_2

Table 4.1 Uniform two beam elements

After the generalized force for each element is found, we superimpose the previous elemental force to the next one.

If we divide the beam to n elements with equal length l , there are $n+1$ nodes on the beam, For the first node, the generalized force is shown below:

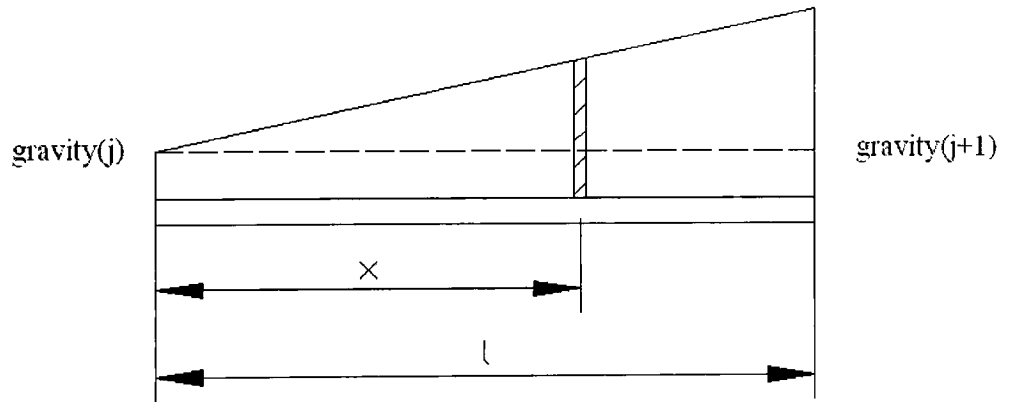


Figure 4.4 Gravity on the beam

Set $\xi = \frac{x}{l}$, integrate from the starting point to the ending point, at every point, denote

gravity as

$$\text{Gravity}_i = (\text{gravity}(j+1) - \text{gravity}(j)) * \xi + \text{gravity}(j)$$

$$F = \int_0^l p(x) \varphi(i)(x) dx = \int_0^l \text{Gravity}_i * \varphi(i)(\xi) * l d\xi = l \int_0^l \text{Gravity}_i * \varphi(i)(\xi) d\xi$$

If we divide the beam into n elements, followed by a separate element for the attached drogue, there should be $n+1$ elements total and $n+2$ nodes. If we consider two degrees-of-freedom at every node, then the size of the relative matrices is:

Stiffness matrix	Mass matrix	Damping matrix	Generalized force
$2*(n+2) \times 2*(n+2)$			$2*(n+2) \times 1$
If $n = 3$, 10×10			10×1

Table 4.2 Matrices of free vibration

As for the first point of the pipe, we assume that if it is clamped to the tanker, so that the displacement and rotation at this point equals zero. Thus we need only consider the left $n+1$ node. When there is only one input, for state space expression becomes:

A	B	D	E
$2*2*(n+1) \times 2*2*(n+1)$	$2*2*(n+1) \times 1$	$1 \times 2*2*(n+1)$	1×1
If $n=3$, 16×16	16×1	1×16	1×1

Table 4.3

After understanding the application of finite element method on our system, a simulation is performed of the system.

In order to simplify the model, we treat the probe-and-drogue system as a steel beam with a rectangle cross section. Since the length and the relative displacement in the axial direction is small we assume it to be zero. We set two degrees-of-freedom on every node, the horizontal and vertical displacement and rotation. First, we divide the beam to three elements of equal length. As the drogue is attached to the free end of pipe, there are four elements total with five nodes. The drogue element has a different density and dimension, consequently different stiffness, mass and damping matrices. Gravity is the only force applied on the beam. In order to make the analysis more general, we assume the gravity is distributed linear increasing as shown below:

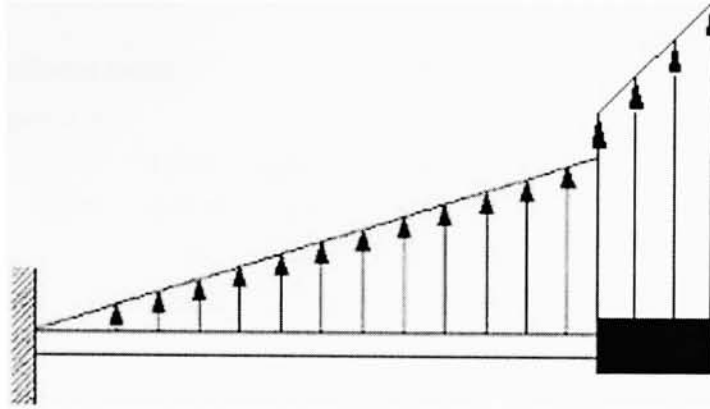


Figure 4.5 Gravity on beam and drogue

The boundary condition for the displacement and rotation at the fixed clamped end are zero. When we convert the motion equation

$$[M]\ddot{X} + [C]\dot{X} + [K]X = [F]$$

to state space function

$$\begin{aligned}\dot{X} &= AX + BU \\ Y &= DX + EU\end{aligned}$$

We can solve it by *simulation* software.

Now let's derive the stiffness, mass matrices and forcing function for the system model. As mentioned before, the displacement in axial direction is ignored, so there are 2 degrees of freedom at each node. The beam is divided to 3 elements with equal length and the drogue has different density and dimension.

See program 1 in Appendix I for the calculation. The results are shown below:

The element stiffness matrix:

$$K^e = 1.0e+005 * \begin{bmatrix} 1.0417 & 0.5208 & -1.0417 & 0.5208 \\ 0.5208 & 0.3472 & -0.5208 & 0.1736 \\ -1.0417 & -0.5208 & 1.0417 & -0.5208 \\ 0.5208 & 0.1736 & -0.5208 & 0.3472 \end{bmatrix};$$

The drogue stiffness matrix

$K_{\text{drogue}} = 1.0\text{e}+004 * [$

1.3021	0.6510	-1.3021	0.6510
0.6510	0.4340	-0.6510	0.2170
-1.3021	-0.6510	1.3021	-0.6510
0.6510	0.2170	-0.6510	0.4340

$];$

The stiffness matrix of the model:

$K = 1.0\text{e}+005 * [$

1.0417	0.5208	-1.0417	0.5208	0	0	0	0	0	0
0.5208	0.3472	-0.5208	0.1736	0	0	0	0	0	0
-1.0417	-0.5208	2.0833	0	-1.0417	0.5208	0	0	0	0
0.5208	0.1736	0	0.6944	-0.5208	0.1736	0	0	0	0
0	0	-1.0417	-0.5208	2.0833	0	-1.0417	0.5208	0	0
0	0	0.5208	0.1736	0	0.6944	-0.5208	0.1736	0	0
0	0	0	0	-1.0417	-0.5208	1.1719	-0.4557	-0.1302	0.0651
0	0	0	0	0.5208	0.1736	-0.4557	0.3906	-0.0651	0.0217
0	0	0	0	0	0	-0.1302	-0.0651	0.1302	-0.0651
0	0	0	0	0	0	0.0651	0.0217	-0.0651	0.0434

$];$

The mass matrix of the element:

$m^e = [$

0.0202	0.0029	0.0070	-0.0017
0.0029	0.0005	0.0017	-0.0004
0.0070	0.0017	0.0202	-0.0029
-0.0017	-0.0004	-0.0029	0.0005

$];$

The mass matrix of the drogue:

$$M_{\text{drogue}} = \begin{bmatrix} 0.0025 & 0.0004 & 0.0009 & -0.0002 \\ 0.0004 & 0.0001 & 0.0002 & -0.0000 \\ 0.0009 & 0.0002 & 0.0025 & -0.0004 \\ -0.0002 & -0.0000 & -0.0004 & 0.0001 \end{bmatrix};$$

The mass matrix of the model:

$$M = \begin{bmatrix} 0.0202 & 0.0029 & 0.007 & -0.0017 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0029 & 0.0005 & 0.0017 & -0.0004 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.007 & 0.0017 & 0.0405 & 0 & 0.007 & -0.0017 & 0 & 0 & 0 & 0 \\ -0.0017 & -0.0004 & 0 & 0.001 & 0.0017 & -0.0004 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.007 & 0.0017 & 0.0405 & 0 & 0.007 & -0.0017 & 0 & 0 \\ 0 & 0 & -0.0017 & -0.0004 & 0 & 0.001 & 0.0017 & -0.0004 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.007 & 0.0017 & 0.0228 & -0.0025 & 0.0009 & -0.0002 \\ 0 & 0 & 0 & 0 & -0.0017 & -0.0004 & -0.0025 & 0.0006 & 0.0002 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0009 & 0.0002 & 0.0025 & -0.0004 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.0002 & 0 & -0.0004 & 0.0001 \end{bmatrix};$$

The generalized force vector:

$$FM = \begin{bmatrix} -0.2631 \\ -0.0585 \\ -1.7537 \\ -0.1169 \\ -3.5074 \\ -0.1169 \\ -2.4004 \\ 0.3727 \\ -0.0767 \\ 0.0110 \end{bmatrix}$$

There are 4 elements and 5 for the system model and we assume the displacement and rotation at the fixed end are zero. There are two degrees of freedom at every node. The dimension of A and B matrices are 16X16 and D matrix is 1X16.

Result of free vibration:

The damping matrix [C] is assumed to be Rayleigh damping (reference “Theory of Vibration with Application”), i.e.:

$$[C] = \alpha[M] + \beta[K]; \text{ Set } \alpha=1, \beta = 0.001$$

The damping matrix of the element:

C= [

104.187	52.0862	-104.16	52.0816	0	0	0	0	0	0	0
52.0862	34.7227	-52.0816	17.3607	0	0	0	0	0	0	0
-104.16	-52.0816	208.374	0	-104.16	52.0816	0	0	0	0	0
52.0816	17.3607	0	69.4455	-52.0816	17.3607	0	0	0	0	0
0	0	-104.16	-52.0816	208.373	0	-104.16	52.0816	0	0	0
0	0	52.0816	17.3607	0	69.4455	-52.0816	17.3607	0	0	0
0	0	0	0	-104.16	-52.0816	117.210	-45.5754	-13.02	6.5102	
0	0	0	0	52.0816	17.3607	-45.5754	39.0631	-6.5102	2.1701	
0	0	0	0	0	0	-13.02	-6.5102	13.0234	-6.5108	
0	0	0	0	0	0	6.5102	2.1701	-6.5108	4.3403	

];

The displacement at the free end of the drogue transient response for a unit force impulse input at the drogue element with the previously defined parameters is shown by Figure 4.6

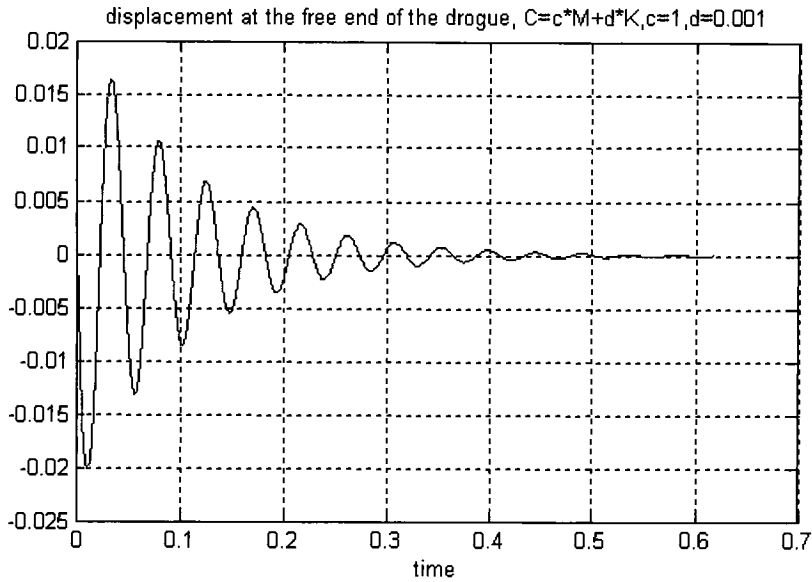


Figure 4.6 Displacement at the free end of the drogue, $c=1$, $d=0.001$

The velocity response is shown by Figure 4.7

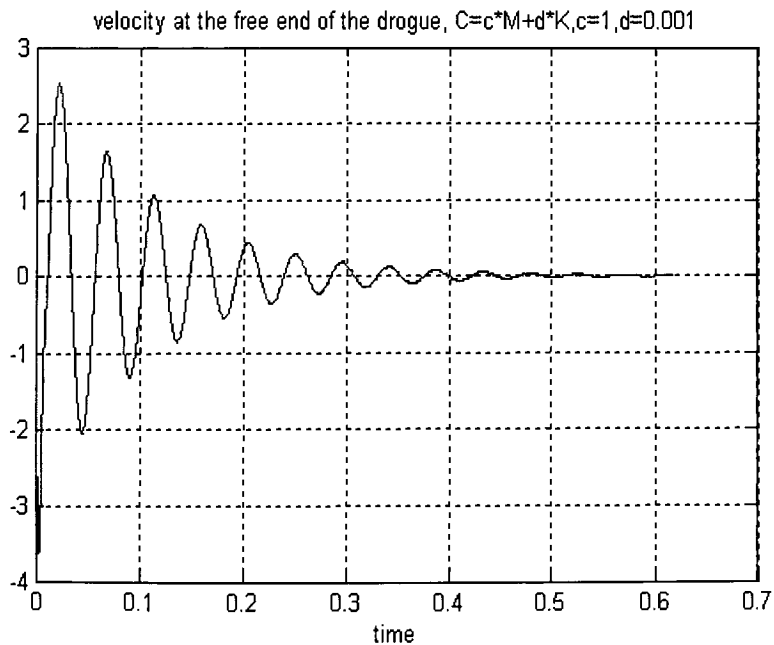


Figure 4.7 Velocity at the free end of the drogue, $c=1$, $d=0.001$

When different parameters α and β in equation $[C] = \alpha[M] + \beta[K]$ are chosen, a different damping matrix is found. For a new damping criterion, the solution of two variable mentioned above are showed as below:

Set $\alpha=1$, $\beta = 0.002$

$C=[$

208.35	104.17	-208.33	104.17	0.00	0.00	0.00	0.00	0.00	0.00
104.17	69.45	-104.17	34.72	0.00	0.00	0.00	0.00	0.00	0.00
-208.33	-104.17	416.71	0.00	-208.33	104.17	0.00	0.00	0.00	0.00
104.17	34.72	0.00	138.89	-104.17	34.72	0.00	0.00	0.00	0.00
0.00	0.00	-208.33	-104.17	416.71	0.00	-208.33	104.17	0.00	0.00
0.00	0.00	104.17	34.72	0.00	138.89	-104.17	34.72	0.00	0.00
0.00	0.00	0.00	0.00	-208.33	-104.17	234.40	-91.15	-26.04	13.02
0.00	0.00	0.00	0.00	104.17	34.72	-91.15	78.13	-13.02	4.34
0.00	0.00	0.00	0.00	0.00	0.00	-26.04	-13.02	26.04	-13.02
0.00	0.00	0.00	0.00	0.00	0.00	13.02	4.34	-13.02	8.68

];

The displacement at the free end of the drogue transient response for a unit force impulse input at the drogue element with the previously defined parameters is shown by Figure 4.8

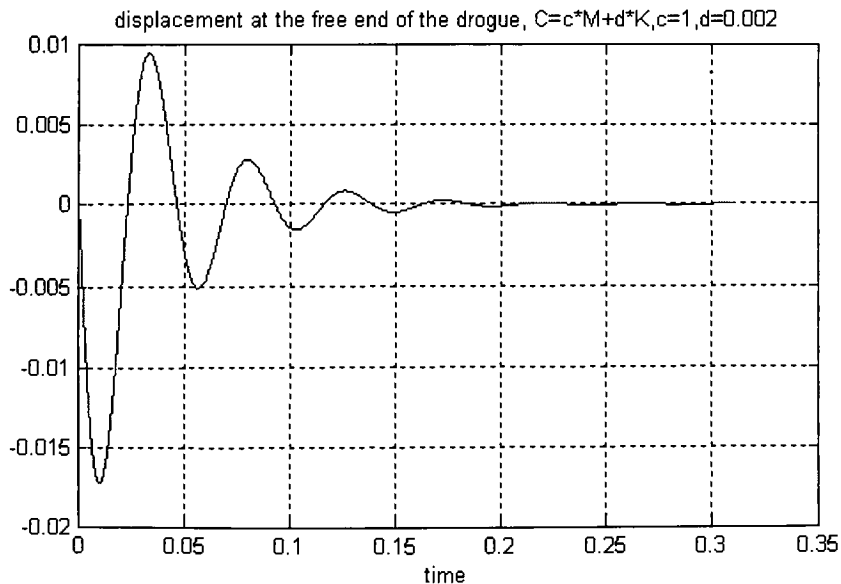


Figure 4.8 Displacement at the free end of the drogue, $c=1$, $d=0.002$

The velocity response is shown by Figure 4.9

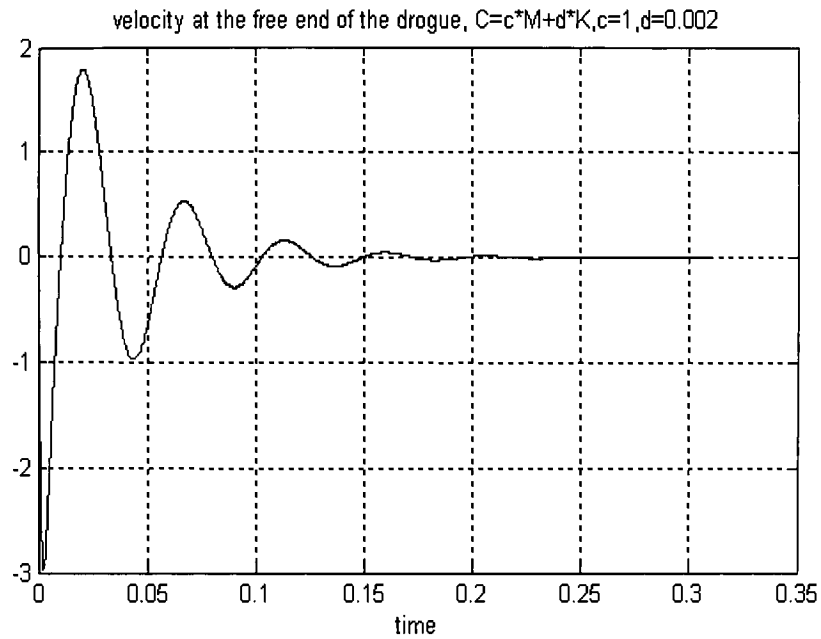


Figure 4.9 Velocity at the free end of the drogue, $c=1, d=0.002$

Set $\alpha=1, \beta = 0.005$;

$C= 1.0e+003 * [$

0.5209	0.2604	-0.5208	0.2604	0	0	0	0	0	0
0.2604	0.1736	-0.2604	0.0868	0	0	0	0	0	0
-0.5208	-0.2604	1.0417	0	-0.5208	0.2604	0	0	0	0
0.2604	0.0868	0	0.3472	-0.2604	0.0868	0	0	0	0
0	0	-0.5208	-0.2604	1.0417	0	-0.5208	0.2604	0	0
0	0	0.2604	0.0868	0	0.3472	-0.2604	0.0868	0	0
0	0	0	0	-0.5208	-0.2604	0.586	-0.2279	-0.0651	0.0326
0	0	0	0	0.2604	0.0868	-0.2279	0.1953	-0.0326	0.0109
0	0	0	0	0	0	-0.0651	-0.0326	0.0651	-0.0326
0	0	0	0	0	0	0.0326	0.0109	-0.0326	0.0217

];

The displacement at the free end of the drogue transient response for a unit force impulse input at the drogue element with the previously defined parameters is shown by Figure 4.10

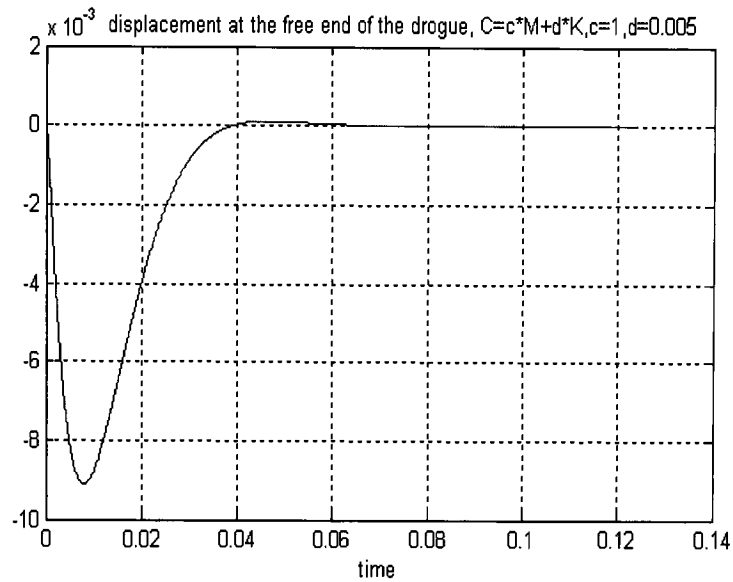


Figure 4.10 Displacement at the free end of the drogue, $c=1$, $d=0.005$

The velocity response is shown by Figure 4.11

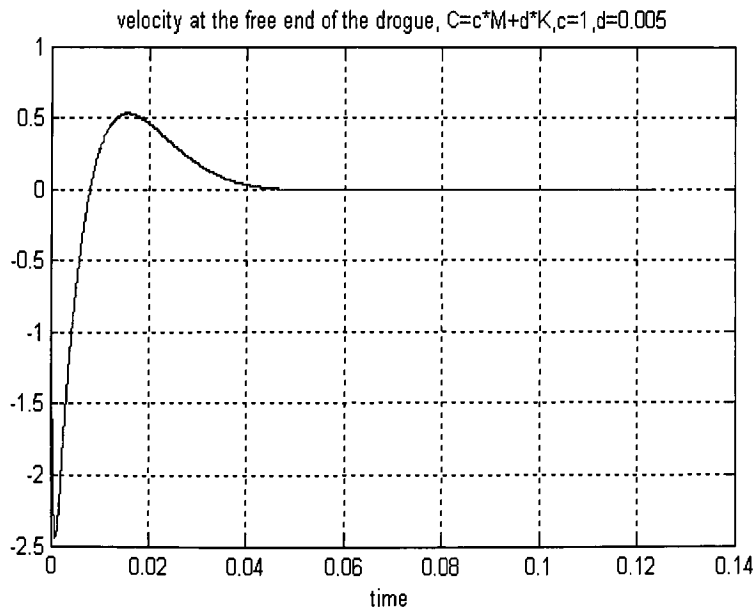


Figure 4.11 Velocity at the free end of the drogue, $c=1$, $d=0.005$

The result under the influence of the gravity

However, the results indicated above do not include gravity. The gravity is the force applied on the system, so under this circumstance, we should re-analyze the motion of the system with gravity effects.

When $\alpha=1$, $\beta = 0.001$ and including gravity. The displacement at the free end of the drogue transient response for a unit force impulse input at the drogue element with the previously defined parameters is shown by Figure 4.12

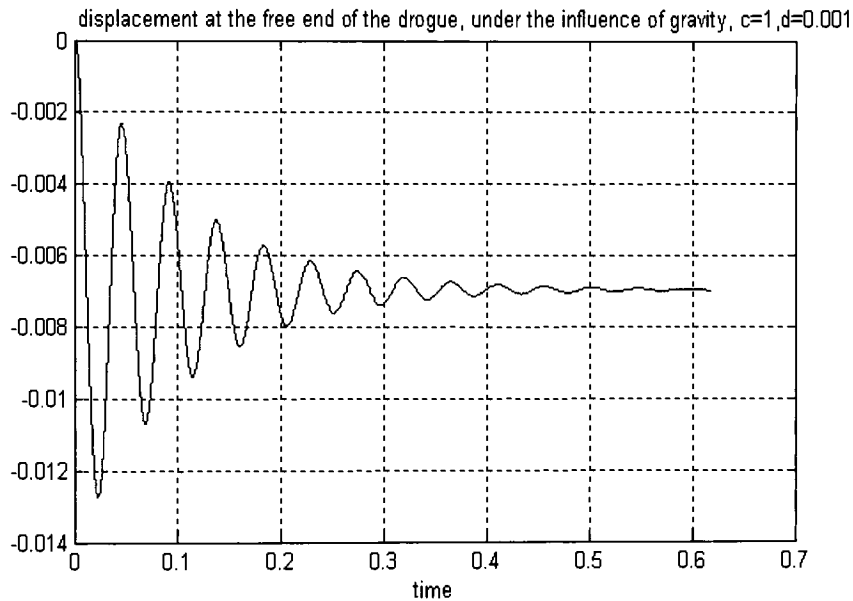


Figure 4.12 Displacement at the free end of the drogue with gravity, $c=1$, $d=0.001$

The velocity response is shown by Figure 4.13

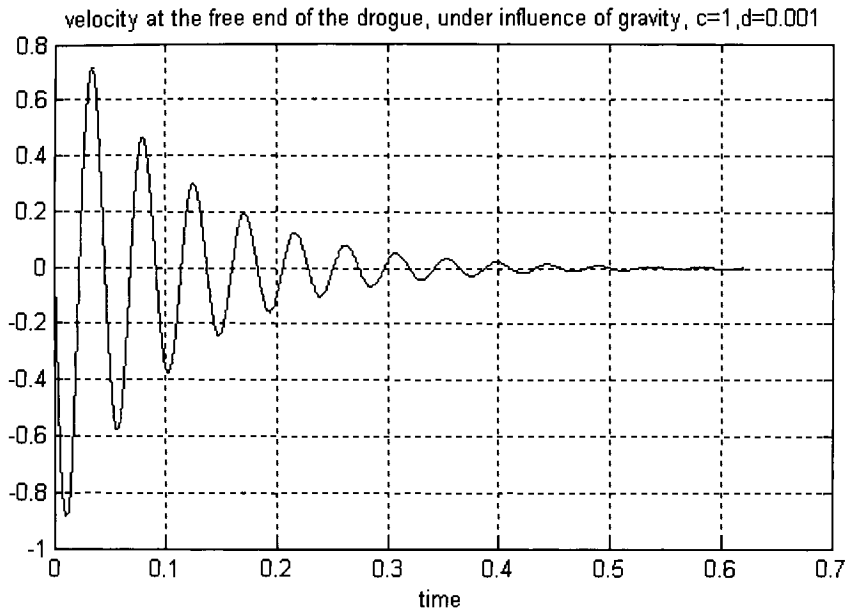


Figure 4.13 velocity at the free end of the drogue with gravity, $c=1$, $d=0.001$

When $\alpha=1$, $\beta = 0.002$ and including gravity. The displacement at the free end of the drogue transient response for a unit force impulse input at the drogue element with the previously defined parameters is shown by Figure 4.14

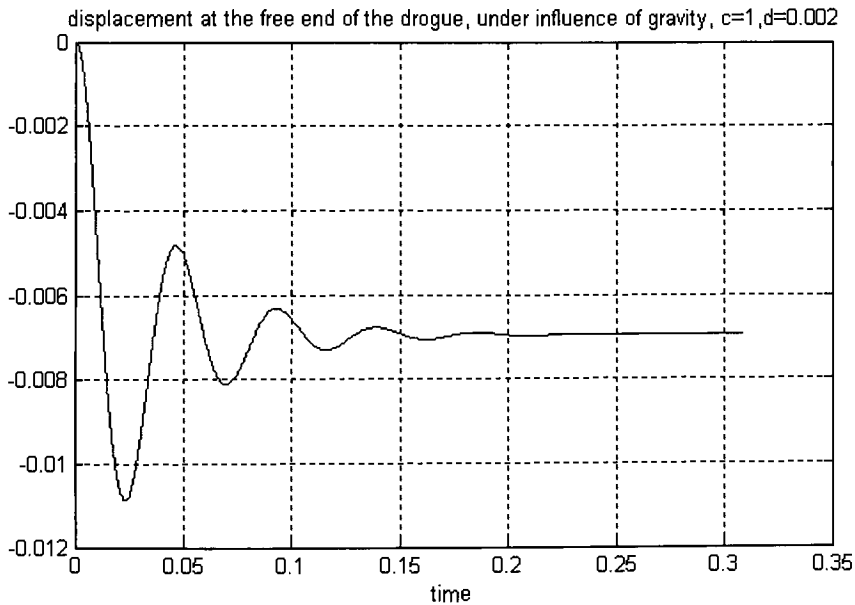


Figure 4.14 Displacement at the free end of the drogue with gravity, $c=1$, $d=0.002$

The velocity response is shown by Figure 4.15

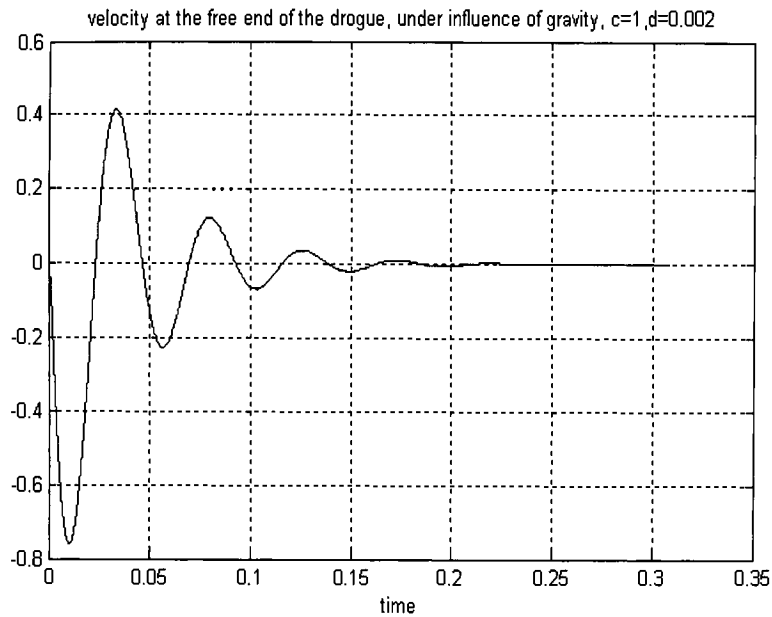


Figure 4.15 Velocity at the free end of the drogue with gravity, $c=1$, $d=0.002$

When $\alpha=1$, $\beta = 0.005$ and including gravity. The displacement at the free end of the drogue transient response for a unit force impulse input at the drogue element with the previously defined parameters is shown by Figure 4.16

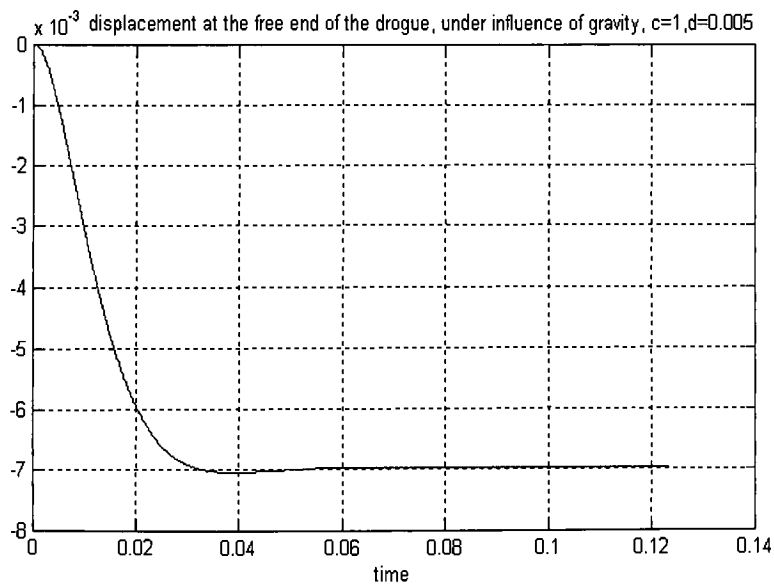


Figure 4.16 Displacement at the free end of the drogue with gravity, $c=1$, $d=0.005$

The velocity response is shown by Figure 4.17

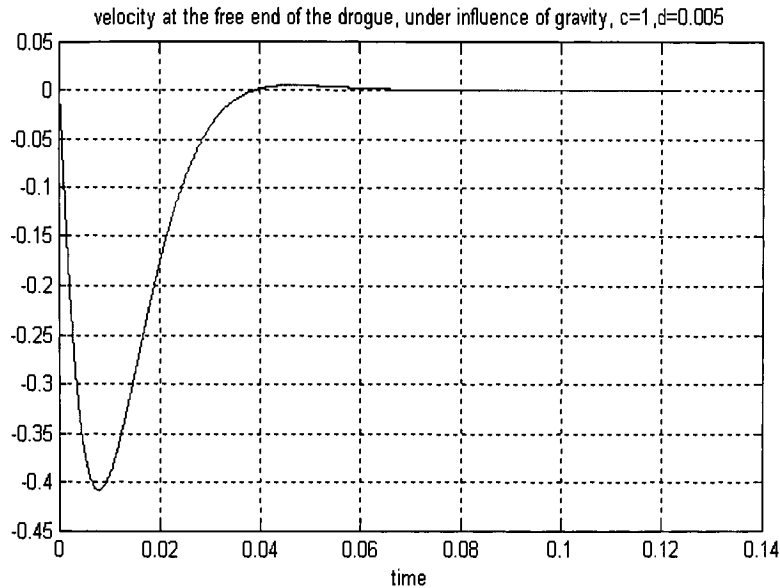


Figure 4.17 Velocity at the free end of the drogue with gravity, $c=1$, $d=0.005$

Stiffness and mass matrices of 4 degree-of-freedom in every node

In the discussion above, we assume there are two degrees of freedom for every node of the beam element. One is the lateral displacement and the other is lateral rotation. But in the real discussion of our case, there is not only displacement and rotation in lateral direction, but also displacement and rotation in vertical direction. So we need to extend the degree-of-freedom of every node to 4.

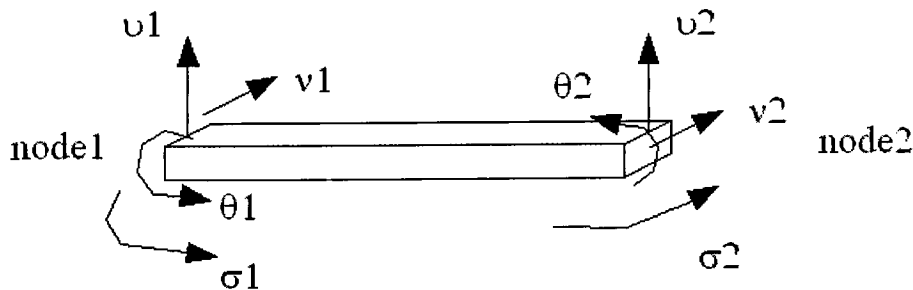


Figure 4.18 Free vibration model with 4 degree-of-freedom

In up and down direction and lateral direction, the stiffness and mass matrices follow the same theory except different moment of inertia.

In up and down direction, the moment of inertia

$$I_u = \frac{1}{12} wh^3$$

w : The width of beam element

h : The height of beam element

In lateral direction, the moment of inertia

$$I_l = \frac{1}{12} hw^3$$

So the modified stiffness and mass matrices of beam element is 8 X 8, as below:

$$K^e = \frac{E}{l^3} \begin{bmatrix} 12I_u & 6Il_u & 0 & 0 & -12I_u & 6Il_u & 0 & 0 \\ 6Il_u & 4l^2 I_u & 0 & 0 & -6Il_u & 2l^2 I_u & 0 & 0 \\ 0 & 0 & 12I_l & 6Il_l & 0 & 0 & -12I_l & 6Il_l \\ 0 & 0 & 6Il_l & 4l^2 I_l & 0 & 0 & -6Il_l & 2l^2 I_l \\ -12I_u & -6Il_u & 0 & 0 & 12I_u & -6Il_u & 0 & 0 \\ 6Il_u & 2l^2 I_u & 0 & 0 & -6Il_u & 4l^2 I_u & 0 & 0 \\ 0 & 0 & 12I_l & -6Il_l & 0 & 0 & 12I_l & -6Il_l \\ 0 & 0 & 6Il_l & 2l^2 I_l & 0 & 0 & -6Il_l & 4l^2 I_l \end{bmatrix}$$

$$M^e = \frac{ml}{420} \begin{bmatrix} 156 & 22l & 0 & 0 & 54 & -13l & 0 & 0 \\ 22l & 4l^2 & 0 & 0 & 13l & -3l^2 & 0 & 0 \\ 0 & 0 & 156 & 22l & 0 & 0 & 54 & -13l \\ 0 & 0 & 22l & 4l^2 & 0 & 0 & 13l & -3l^2 \\ 54 & 13l & 0 & 0 & 156 & -22l & 0 & 0 \\ -13l & -3l^2 & 0 & 0 & -22l & 4l^2 & 0 & 0 \\ 0 & 0 & 54 & 13l & 0 & 0 & 156 & -22l \\ 0 & 0 & -13l & -3l^2 & 0 & 0 & -22l & 4l^2 \end{bmatrix}$$

Then the size of relative matrices of the system should be modified as below:

Stiffness matrix	Mass matrix	Damping matrix	Generalized force
$4*(n+2) \times 4*(n+2)$			$4*(n+2) \times 1$
If $n = 3$, 20×20			20×1

Table 4.4

A	B	D	E
$2*4*(n+1) \times 2*4*(n+1)$	$2*4*(n+1) \times 1$	$1 \times 2*4*(n+1)$	1×1
If $n=3$, 32×32	32×1	1×32	1×1

Table 4.5

Now considering shape functions of variables denoting two new degree-of-freedom, displacement and rotation in lateral direction, they are exactly the same as the shape functions of displacement and rotation in up-and-down direction, which means:

Displacement shape function for the starting point:

$$\varphi_1(x) = 1 - 3\xi^2 + 2\xi^3$$

Rotation shape function for the starting point:

$$\varphi_2(x) = l\xi - 2l\xi^2 + l\xi^3$$

Displacement shape function for the ending point:

$$\varphi_3(x) = 3\xi^2 - 2\xi^3$$

Rotation shape function for the starting point:

$$\varphi_4(x) = -l\xi^2 + l\xi^3$$

The generalized force of the system can now be determined and the motion equation can be solved as before.

Apply real force on the beam besides gravity

From the analysis above, we have shown the solution of the free vibration condition of the system. But in the real situation, the applied force on the probe-and-drogue is not only gravity; there is also the influence of other factors, such as wind and the reaction between tanker and aircraft receiving fuel. The wind interaction disturbance effect between two flying objects is highly nonlinear and difficult to model correctly. These effects are highly problem dependent and thus vary from aircraft to aircraft and types of refueling system. The attempt here is to simplify these effects so that a reasonably model can be developed for analysis. The proposed model is a simple exponential model that is time and relative distance dependent between the two approaching aircrafts. The exponential dynamics are input to the probe-drogue refueling system through the external disturbance input model. The magnitude of the interaction effects can be varied as a function of time and relative distance between the tow approaching objects. The resulting interaction effects to the receiving aircraft are summed with the body positions degree-of-freedom dynamical model of the receiving aircraft. Although the supply aircraft will also feel this interaction effect, this effect is assumed to be small with the assumption that the supply aircraft is much heavier compared to the receiving aircraft. However, provisions are provided in the two aircraft simulation to include this effect to the supply aircraft.

So let's simulate the real applied forces on the pipe and drogue. Just to simplify the simulation, we begin from the simplest one, the exponential force. Express it in mathematical equation:

Set t as time, and y is the exponential function of time

$$Y = 1 - e^{-t}$$

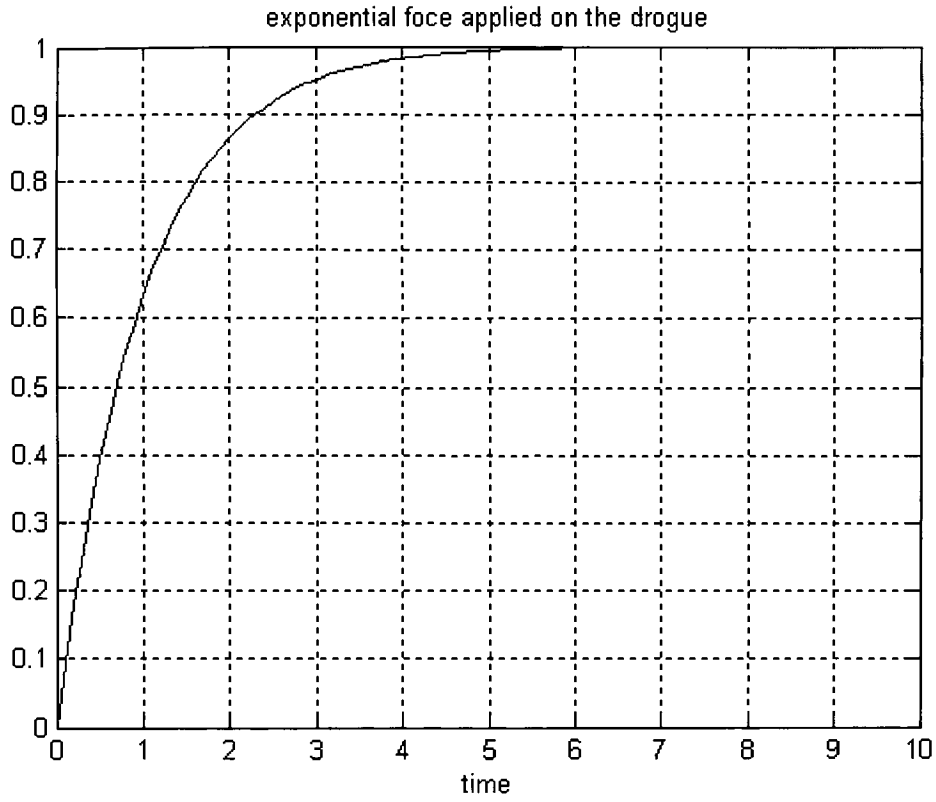


Figure 4.19 Exponential force

Considering the application of both gravity and wind interaction external force on the pipe, now there are two inputs on the system, therefore for the general modeling equation

$$[M]\ddot{X} + [C]\dot{X} + [K]X = [F]$$

$[F]$ is a 2×1 vector, which defines inputs to the system and the state space model becomes:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} \text{zeros} & I \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + [B] \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

where $[B]$ is a 2×2 matrix:

$$[B] = \begin{bmatrix} 0 & 0 \\ [M]^{-1}[F_1] & [M]^{-1}[F_2] \end{bmatrix}$$

and

$$[Y] = [D] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + [E] \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

[D] is a 1X2 matrix and [E] is a 2X2 matrix.

If the pipe is divided into we divide the pipe to n beam elements, plus one drogue element at the end, there are n+1 total elements, and there are m forces (m inputs) applied. One is gravity along the whole length of the system; the other is the exponential force applied at the end of the system, the drogue element.

As there is no change in the number of variables discussed, there is no change in stiffness, mass and damping matrix. The matrix of generalized force becomes a matrix with m columns.

Stiffness matrix	Mass matrix	Damping matrix	Generalized force
$4*(n+2) \times 4*(n+2)$			$4*(n+2) \times m$
If n = 3, m=2, 20 X 20			20X2

Table 4.6

Relative changes take place in the matrices [B] and [E].

A	B	D	E
$2*4*(n+1) \times 2*4*(n+1)$	$2*4*(n+1) \times m$	$1 \times 2*4*(n+1)$	$1 \times m$
If n=3, m=2, 32X32	32X2	1X32	1 X 2

Table 4.7

Consider the case where the exponential external wind interaction force is applied on the drogue. There are 4 degrees of freedom for each node with 2 applied forces. One is linear increasing gravity, the other is exponential force added on the drogue element with an angle of $\pi/4$ to the Z-axis. Later, more inputs on the system will be added to simulate a more realistic case. Figure 4.20 displays the applied forces.

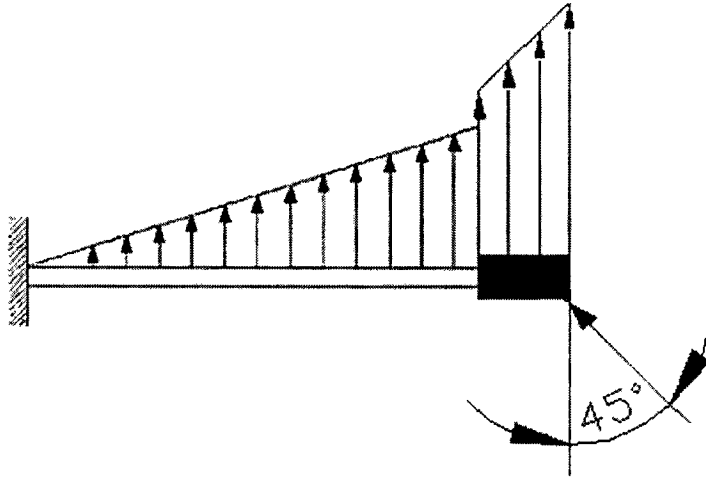


Figure 4.20 Applied forces on the system model

See program 2 in the Appendix II for calculation, and results are shown below:

The element stiffness matrix including 4 degree-of-freedom is:

$K^e = 1.0e+005 * [$

1.0417	0.5208	0	0	-1.0417	0.5208	0	0
0.5208	0.3472	0	0	-0.5208	0.1736	0	0
0	0	2.3438	1.1719	0	0	-2.3438	1.1719
0	0	1.1719	0.7813	0	0	-1.1719	0.3906
-1.0417	-0.5208	0	0	1.0417	-0.5208	0	0
0.5208	0.1736	0	0	-0.5208	0.3472	0	0
0	0	-2.3438	-1.1719	0	0	2.3438	-1.1719
0	0	1.1719	0.3906	0	0	-1.1719	0.7813

];

The stiffness matrix of the drogue is:

$K_{drogue} = 1.0e+004 * [$

1.3021	0.651	0	0	-1.3021	0.651	0	0
0.651	0.434	0	0	-0.651	0.217	0	0
0	0	2.9297	1.4648	0	0	-2.9297	1.4648
0	0	1.4648	0.9766	0	0	-1.4648	0.4883
-1.3021	-0.651	0	0	1.3021	-0.651	0	0
0.651	0.217	0	0	-0.651	0.434	0	0
0	0	-2.9297	-1.4648	0	0	2.9297	-1.4648
0	0	1.4648	0.4883	0	0	-1.4648	0.9766

];

The stiffness matrix of the model can be derived by superimposing the preceeding matrices, which is a 20X20 matrix.

The element mass matrix is:

$M^e = [$

0.0202	0.0029	0	0.0070	-0.0017	0	0
0.0029	0.0005	0	0	0.0017	-0.0004	0
0	0	0.0202	0.0029	0	0	0.007
0	0	0.0029	0.0005	0	0	0.0017
0.007	0.0017	0	0	0.0202	-0.0029	0
-0.0017	-0.0004	0	0	-0.0029	0.0005	0
0	0	0.007	0.0017	0	0	0.0202
0	0	-0.0017	-0.0004	0	0	-0.0029

];

The mass matrix of the drogue is:

Mdrogue = [

$$\begin{bmatrix} 0.0025 & 0.0004 & 0 & 0 & 0.0009 & -0.0002 & 0 & 0 \\ 0.0004 & 0.0001 & 0 & 0 & 0.0002 & 0 & 0 & 0 \\ 0 & 0 & 0.0025 & 0.0004 & 0 & 0 & 0.0009 & -0.0002 \\ 0 & 0 & 0.0004 & 0.0001 & 0 & 0 & 0.0002 & 0 \\ 0.0009 & 0.0002 & 0 & 0 & 0.0025 & -0.0004 & 0 & 0 \\ -0.0002 & 0 & 0 & 0 & -0.0004 & 0.0001 & 0 & 0 \\ 0 & 0 & 0.0009 & 0.0002 & 0 & 0 & 0.0025 & -0.0004 \\ 0 & 0 & -0.0002 & 0 & 0 & 0 & -0.0004 & 0.0001 \end{bmatrix};$$

The mass matrix of the model can be derived by superimposing the preceeding matrices, which is a 20X20 matrix.

The generalized force vector is:

$$F = \begin{bmatrix} -0.26 \\ -0.06 \\ 0 \\ 0 \\ -1.75 \\ -0.12 \\ 0 \\ 0 \\ 3.51 \\ -0.12 \\ 0 \\ 0 \\ 2.40 \\ 0.37 \\ 0 \\ 0 \\ 0 \\ -0.08 \\ 0.01 \\ 0 \\ 0 \end{bmatrix}$$

The exponential force applied on the drogue is a function of time. The values of exponential force at the first 10 iteration is:

ExpForce =

0.00	0	0	0	0	0	0	0	0	0
0.00	0	0	0	0	0	0	0	0	0
0.00	0	0	0	0	0	0	0	0	0
0.00	0	0	0	0	0	0	0	0	0
0.00	0	0	0	0	0	0	0	0	0
0.00	0	0	0	0	0	0	0	0	0
0.00	0	0	0	0	0	0	0	0	0
0.00	0	0	0	0	0	0	0	0	0
0.00	0	0	0	0	0	0	0	0	0
0.00	0	0	0	0	0	0	0	0	0
0.00	0	0	0	0	0	0	0	0	0
0.00	0	0	0	0	0	0	0	0	0
0.03	0.0641	0.0916	0.1166	0.1391	0.1595	0.178	0.1947	0.2098	0.2235
0.01	0.0107	0.0153	0.0194	0.0232	0.0266	0.0297	0.0324	0.035	0.0372
0.00	0	0	0	0	0	0	0	0	0
0.00	0	0	0	0	0	0	0	0	0
0.03	0.0641	0.0916	0.1166	0.1391	0.1595	0.178	0.1947	0.2098	0.2235
-0.01	-0.0107	-0.0153	-0.0194	-0.0232	-0.0266	-0.0297	-0.0324	-0.035	-0.0372
0.00	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

];

$Y = 1 - e^{-t}$, When $t = 1$, $[C] = \alpha[M] + \beta[K]$;

Set $\alpha=1$, $\beta = 0.001$ and with gravity and exponential force included. The displacement at the free end of the drogue transient response for a unit force impulse input at the drogue element with the previously defined parameters is shown by Figure 4.21

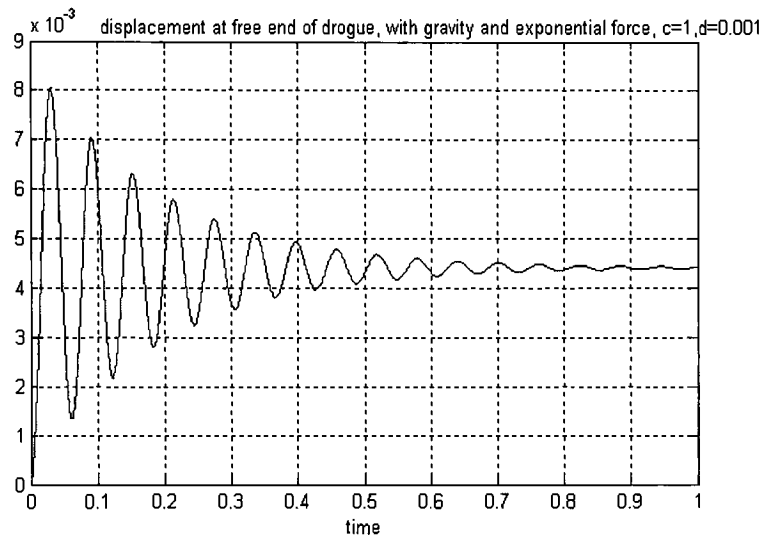


Figure 4.21 Displacement at the free end of the drogue with outside forces, $d=0.001$

The velocity response is shown by Figure 4.22

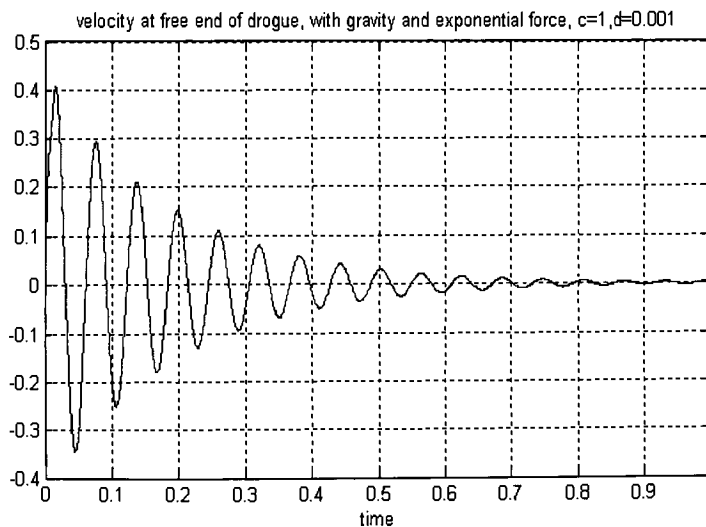


Figure 4.22 Velocity at the free end of the drogue with outside forces, $d=0.001$

As before, modify the parameters α and β to change the damping matrix $[C]$, to analyze influence of damping to the vibration of the system:

Set $\alpha=1$, $\beta = 0.002$. The displacement at the free end of the drogue transient response for a unit force impulse input at the drogue element with the previously defined parameters is shown by Figure 4.23

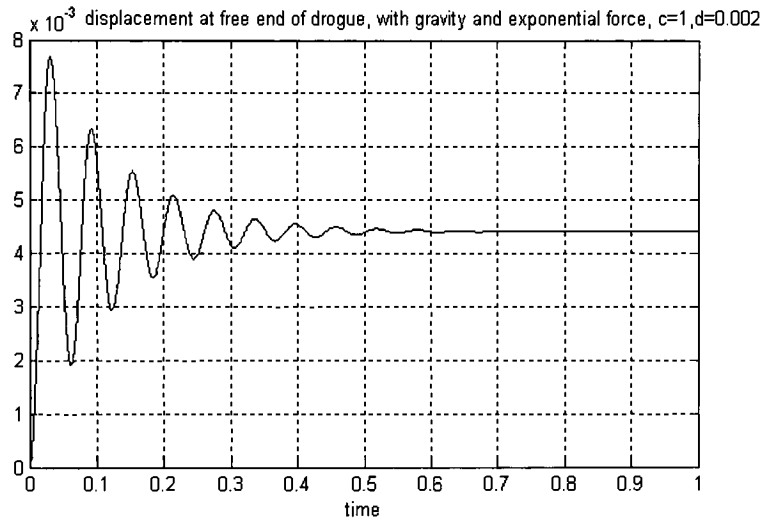


Figure 4.23 Displacement at the free end of the drogue with outside forces, $d=0.002$

The velocity response is shown by Figure 4.24

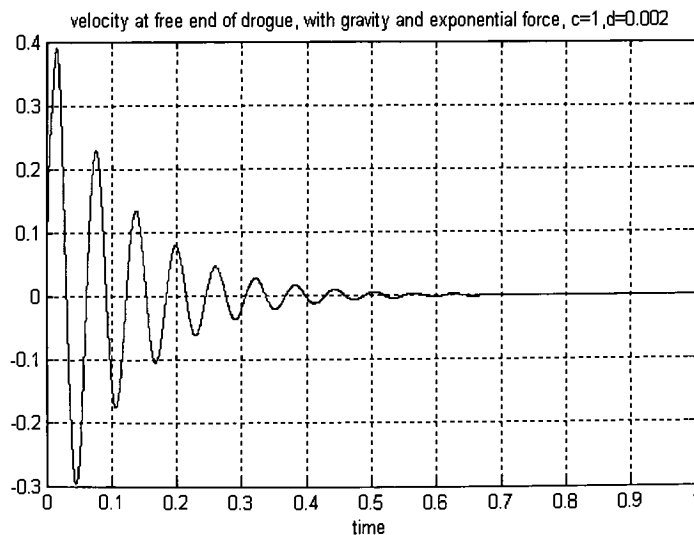


Figure 4.24 Velocity at the free end of the drogue with outside forces, $d=0.002$

Set $\alpha=1$, $\beta = 0.005$, The displacement at the free end of the drogue transient response for a unit force impulse input at the drogue element with the previously defined parameters is shown by Figure 4.25

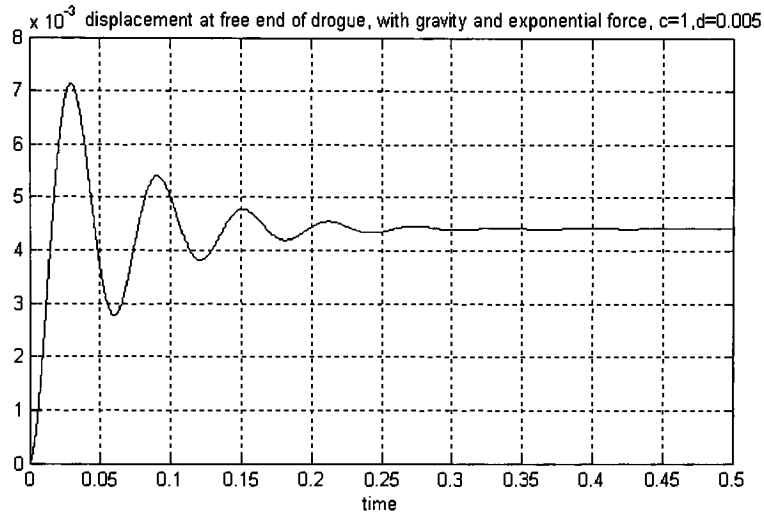


Figure 4.25 Displacement at the free end of the drogue with outside forces, $d=0.005$

The velocity response is shown by Figure 4.26

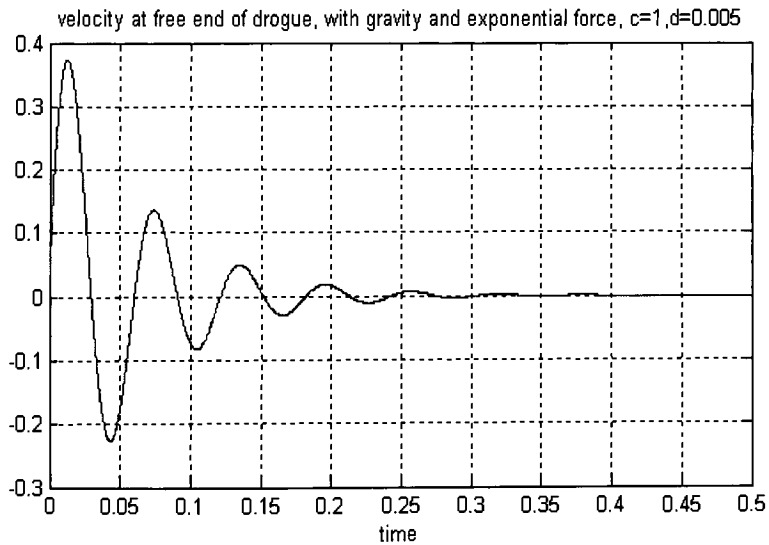


Figure 4.26 Velocity at the free end of the drogue with outside forces, $d=0.005$

Simulation in the real condition

In real situation, the flexible steel hose connected to the tanker is usually 50 – 90 feet long. As we have simplified the hose to a cantilever beam, so the cross-area section of the beam is assumed to be rectangular. However, it is more accurate to define the pipe as a hollow cylinder rather than a rectangular beam, so that the cross section of the pipe is hollow. With the inner and outer diameters known, the moment-of-inertia can be found by:

$$I = \frac{1}{4}\pi(R_o^4 - R_i^4)$$

R_i is inner diameter and R_o is outer diameter.

The end of the hose is a funnel –shaped drogue, a basket that resembles an oversized shuttlecock (the drogue stabilizes the trailing hose and provides a lead into the connector at the end of the hose). To simplification, we assume the drogue to be a thin-walled hollow cylinder. The moment of inertia can therefore be found.

As the exponential force is the function of time, the responded forces should also be the function of time.

The parameters of the system model are defined as following:

Pipe length:	50 feet
Inside diameter:	1.5 inches
Outside diameter:	2.5 inches
Drogue length:	2 foot
Outside diameter:	3.5 inches
Thickness of the drogue:	1/8 inchs
Young's Modulus:	4.18e009
Length density of the pipe:	10.7lb/ft
Density of drogue:	4.5lb/ft

The probe-and-drogue system is divided to 9 beam elements. Pls. See Appendix III for program 3 for calculation and see results below:

The stiffness matrix of the element:

$K^e = 1.0e+005 * [$

0.1654	0.5168	0	0	-0.1654	0.5168	0	0
0.5168	2.1532	0	0	-0.5168	1.0766	0	0
0	0	0.1654	0.5168	0	0	-0.1654	0.5168
0	0	0.5168	2.1532	0	0	-0.5168	1.0766
-0.1654	-0.5168	0	0	0.1654	-0.5168	0	0
0.5168	1.0766	0	0	-0.5168	2.1532	0	0
0	0	-0.1654	-0.5168	0	0	0.1654	-0.5168
0	0	0.5168	1.0766	0	0	-0.5168	2.1532

];

The stiffness matrix of the drogue:

$K_{drogue} = [$

0.7142	1.4285	0	0	-0.7142	1.4285	0	0
1.4285	3.8092	0	0	-1.4285	1.9046	0	0
0	0	0.7142	1.4285	0	0	-0.7142	1.4285
0	0	1.4285	3.8092	0	0	-1.4285	1.9046
-0.7142	-1.4285	0	0	0.7142	-1.4285	0	0
1.4285	1.9046	0	0	-1.4285	3.8092	0	0
0	0	-0.7142	-1.4285	0	0	0.7142	-1.4285
0	0	1.4285	1.9046	0	0	-1.4285	3.8092

];

The mass matrix of the element:

$M^e = [$

24.85	21.91	0.00	0.00	8.60	-12.95	0.00	0.00
21.91	24.89	0.00	0.00	12.95	-18.67	0.00	0.00
0.00	0.00	24.85	21.91	0.00	0.00	8.60	-12.95
0.00	0.00	21.91	24.89	0.00	0.00	12.95	-18.67
8.60	12.95	0.00	0.00	24.85	-21.91	0.00	0.00
-12.95	-18.67	0.00	0.00	-21.91	24.89	0.00	0.00
0.00	0.00	8.60	12.95	0.00	0.00	24.85	-21.91
0.00	0.00	-12.95	-18.67	0.00	0.00	-21.91	24.89

];

The mass matrix of the drogue:

$M_{drogue} = [$

0.00	6.71	3.79	0.00	0.00	2.32	-2.24	0.00
0.00	3.79	2.75	0.00	0.00	2.24	-2.06	0.00
-2.24	0.00	0.00	6.71	3.79	0.00	0.00	2.32
-2.06	0.00	0.00	3.79	2.75	0.00	0.00	2.24
0.00	2.32	2.24	0.00	0.00	6.71	-3.79	0.00
0.00	-2.24	-2.06	0.00	0.00	-3.79	2.75	0.00
-3.79	0.00	0.00	2.32	3.50	0.00	0.00	6.71
2.75	0.00	0.00	-3.50	-5.04	0.00	0.00	-3.79

];

Chapter 5 Discussion and conclusion

As shown in Chapter 4, the stiffness, mass and damping matrix were derived for a candidate drogue in-flight refueling model. The dimensions of matrices are 40 by 40, which are too large to display here. However, it is easier to display the natural frequency of the system. As mentioned before, since the first node of the system is fixed on the tanker, we assume the movement in any direction equals zero. The eigenvalues of the model system are shown below. Consequently, the natural frequency of the can be found.

By solving the motion equation model $[M]\ddot{X} + [C]\dot{X} + [K]X = [F]$

We obtain the system's eigenvalue and natural frequency. In the vertical direction, for the vertical displacement:

$$\lambda = \begin{bmatrix} 741181.63 \\ 56567.52 \\ 32156.44 \\ 9306.73 \\ 3705.44 \\ 1286.50 \\ 363.42 \\ 60.53 \\ 2.08 \end{bmatrix} \text{ rad / sec} \quad \omega = \begin{bmatrix} 860.92 \\ 237.84 \\ 179.32 \\ 96.47 \\ 60.87 \\ 35.87 \\ 19.06 \\ 7.78 \\ 1.44 \end{bmatrix} \text{ rad / sec}$$

In the vertical direction, for the vertical rotation:

$$\lambda = \begin{bmatrix} 7586698.92 \\ 32568.31 \\ 21807.51 \\ 9263.51 \\ 3695.05 \\ 1283.63 \\ 362.89 \\ 60.49 \\ 2.08 \end{bmatrix} \text{ rad / sec} \quad \omega = \begin{bmatrix} 2754.40 \\ 180.47 \\ 147.67 \\ 96.25 \\ 60.79 \\ 35.83 \\ 19.05 \\ 7.78 \\ 1.44 \end{bmatrix} \text{ rad / sec}$$

In the lateral direction, for the lateral displacement:

$$\lambda = \begin{bmatrix} 63865.67 \\ 44236.76 \\ 14425.67 \\ 5919.80 \\ 2176.43 \\ 713.29 \\ 163.54 \\ 16.01 \\ 0.05 \end{bmatrix} \text{ rad / sec} \quad \omega = \begin{bmatrix} 252.7166 \\ 210.3254 \\ 120.1069 \\ 76.9402 \\ 46.6522 \\ 26.7076 \\ 12.7881 \\ 4.0018 \\ 0.2324 \end{bmatrix} \text{ rad / sec}$$

In the lateral direction, for the lateral rotation:

$$\lambda = \begin{bmatrix} 44856.78 \\ 22004.87 \\ 14334.68 \\ 5898.53 \\ 2167.07 \\ 711.97 \\ 163.37 \\ 16.01 \\ 0.05 \end{bmatrix} \text{ rad / sec} \quad \omega = \begin{bmatrix} 211.7942 \\ 148.3404 \\ 119.7275 \\ 76.8019 \\ 46.5518 \\ 26.6828 \\ 12.7816 \\ 4.0012 \\ 0.2324 \end{bmatrix} \text{ rad / sec}$$

From data above, we see the natural frequency of the system distributes from high frequency to lower ones. Since the system damping was assumed to be proportional, i.e.

$$[C] = \alpha[M] + \beta[K]$$

We can equate the following equations:

$$2\xi_i\omega_i = \alpha + \beta\omega_i^2 \text{ or } \beta\omega_i^2 - 2\xi_i\omega_i + \alpha = 0$$

In our real object simulation, since the stiffness matrix is significantly larger than mass matrix, parameter β will be sensitive in influencing the value of $[C]$ or the damping ratio. Equation can be reunited as follows:

$$\xi_i = \frac{\alpha + \beta \omega_i^2}{2\omega_i}$$

Which indicates, when any one of α and β increases, ξ will increase. As a result, the damped frequency of the system defined by equation

$$\omega_d = \omega_n \sqrt{1 - \xi^2} .$$

Decreases.

▪ Influence of damping

To analyze the influence of α and β on the system, compare the result of different values of α and β . Set $\alpha = 0.2$, and $\beta = 0.0005$ and $\alpha = 1$, and $\beta = 0.001$ separately.

The overall system damping is shown below for the two different cases.

In the vertical direction, for the vertical displacement:

$$\alpha = 0.2, \beta = 0.0005$$

$$\xi = \begin{bmatrix} 0.2153 \\ 0.0599 \\ 0.0454 \\ 0.0252 \\ 0.0169 \\ 0.0118 \\ 0.0100 \\ 0.0148 \\ 0.0697 \end{bmatrix}$$

$$\alpha = 1, \beta = 0.001$$

$$\xi = \begin{bmatrix} 0.4310 \\ 0.1210 \\ 0.0924 \\ 0.0534 \\ 0.0387 \\ 0.0319 \\ 0.0358 \\ 0.0682 \\ 0.3475 \end{bmatrix}$$

In the vertical direction, for the vertical rotation:

$$\alpha = 0.2, \beta = 0.0005$$

$$\xi = \begin{bmatrix} 0.6886 \\ 0.0457 \\ 0.0376 \\ 0.0251 \\ 0.0168 \\ 0.0117 \\ 0.0100 \\ 0.0148 \\ 0.0697 \end{bmatrix}$$

$$\alpha = 1, \beta = 0.001$$

$$\xi = \begin{bmatrix} 1.3774 \\ 0.0930 \\ 0.0772 \\ 0.0533 \\ 0.0386 \\ 0.0319 \\ 0.0358 \\ 0.0682 \\ 0.3476 \end{bmatrix}$$

In the lateral direction, for the lateral displacement:

$$\begin{array}{cc} \alpha = 0.2, \beta = 0.0005 & \alpha = 1, \beta = 0.001 \\ \xi = \begin{bmatrix} 0.0636 \\ 0.0531 \\ 0.0309 \\ 0.0205 \\ 0.0138 \\ 0.0104 \\ 0.0110 \\ 0.0260 \\ 0.4304 \end{bmatrix} & \xi = \begin{bmatrix} 0.1283 \\ 0.1075 \\ 0.0642 \\ 0.0450 \\ 0.0340 \\ 0.0321 \\ 0.0455 \\ 0.1269 \\ 2.1520 \end{bmatrix} \end{array}$$

In the lateral direction, for the lateral rotation:

$$\begin{array}{cc} \alpha = 0.2, \beta = 0.0005 & \alpha = 1, \beta = 0.001 \\ \xi = \begin{bmatrix} 0.0534 \\ 0.0378 \\ 0.0308 \\ 0.0205 \\ 0.0138 \\ 0.0104 \\ 0.0110 \\ 0.0260 \\ 0.4304 \end{bmatrix} & \xi = \begin{bmatrix} 0.1083 \\ 0.0775 \\ 0.0640 \\ 0.0449 \\ 0.0340 \\ 0.0321 \\ 0.0455 \\ 0.1270 \\ 2.1520 \end{bmatrix} \end{array}$$

The results indicate that when α or β increases, the damping increases, and the magnitude of system motion decreases as well. See analysis results below:

Set $\alpha = 0.2$, and $\beta = 0.0005$, including the effect of the exponential force.

The displacement at the vertical free end of the drogue with the previously defined parameters is shown by Figure 5.1,

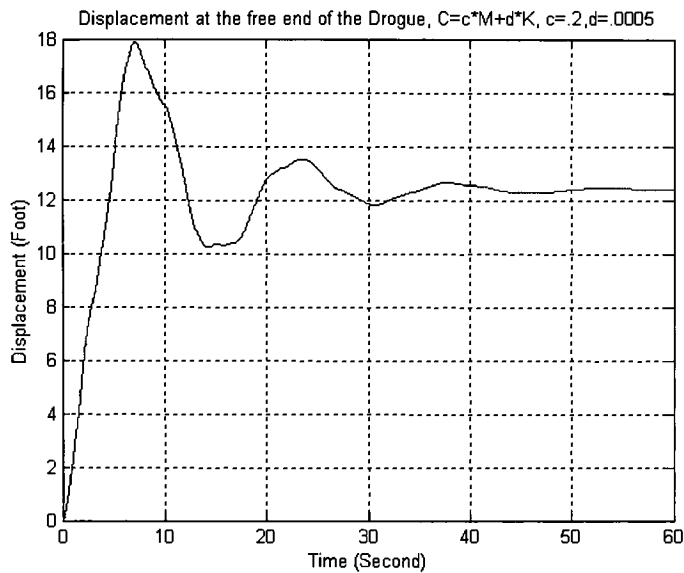


Figure 5.1 Displacement at the freed end of the drogue, $c = 0.2$ $d = 0.0005$

Set $\alpha = 0.5$, and $\beta = 0.001$, including the effect of the exponential force.

The displacement at the free end of the with the previously defined parameters is shown by Figure 5.2,

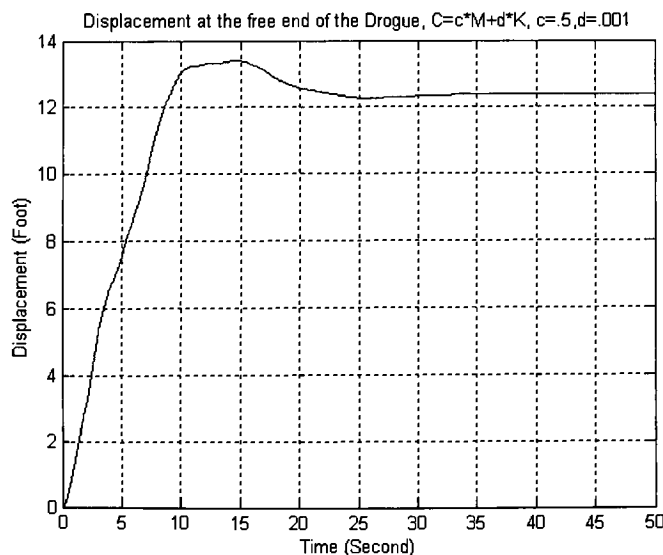


Figure 5.2 Displacement at the freed end of the drogue, $c = 0.5$ $d = 0.001$

Set $\alpha = 1$, and $\beta = 0.001$, including the effect of the exponential force.

The displacement at the free end of the with the previously defined parameters is shown by Figure 5.3,

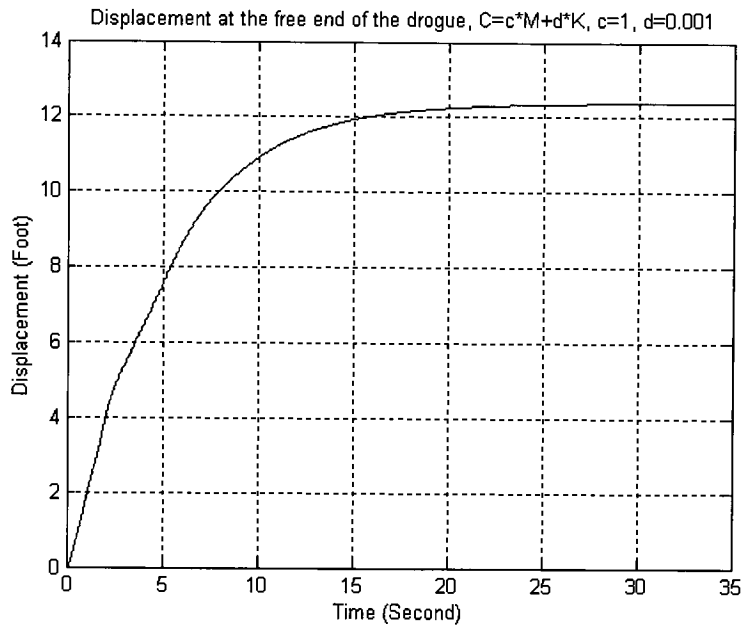


Figure 5.3 Displacement at the freed end of the drogue, $c=1$ $d=0.001$

From figures above, as α or β increases, the overall system damping increases as expected. Since the stiffness matrix is significantly larger in the magnitude than the mass matrix, the system is more sensitive to changes in β than to changes in α .

The probe-and-drogue system is divided into 9 beam elements, so there are 10 nodes in the model. Since the 1st node is rigidly connected to the tanker, the displacement at the 1st node is assumed to be zero. In the analysis above, we have discussed the displacement at the free end of the drogue, which is the 10th node.

The displacement at each node with the previously defined parameters is shown by Figure 5.4,

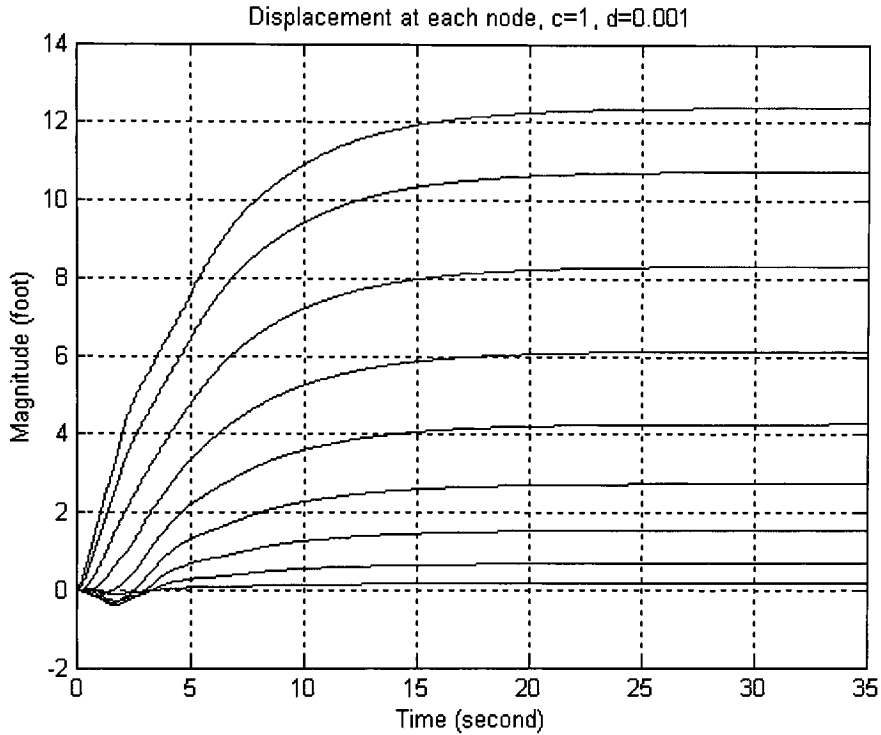


Figure 5.4 Displacements at each node, c= 1 d=0.001

(See Appendix IV for details of the vertical displacements for all nodes). From plot above and plots in Appendix IV, we see that the further the distance to the tanker, the larger the vertical displacement.

To analyze the damping's influence on the settling time, apply the Laplace transformation to the following equation:

$$\ddot{y}_i + 2\xi\omega \dot{y}_i + \omega^2 y_i = f_i$$

where f_i is the generalized force of the system. The transfer function for the above equation is:

$$H(S) = \frac{F(S)}{S^2 + 2\xi\omega S + \omega^2}$$

Focus on the movement of the free end of the drogue in the vertical direction. The generalized force at the free end of the drogue in the vertical direction is -25.2 . The system's natural frequency of the vertical movement is

$$\omega = \begin{bmatrix} 860.92 \\ 237.84 \\ 179.32 \\ 96.47 \\ 60.87 \\ 35.87 \\ 19.06 \\ 7.78 \\ 1.44 \end{bmatrix} \text{ rad / sec}$$

the damped natural frequency is defined by:

$$\omega_d = \omega_n \sqrt{1 - \xi^2}.$$

Consider the highest natural frequency of the system 860.92 rad/sec, the corresponding damping characteristics are:

$$\alpha = 0.2, \beta = 0.0005$$

$$\xi = 0.2153$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$= 860.92 * \sqrt{1 - 0.2153^2}$$

$$= 840.7 \text{ rad / sec}$$

$$\alpha = 1, \beta = 0.001$$

$$\xi = 0.4310$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$= 860.92 * \sqrt{1 - 0.4310^2}$$

$$= 777 \text{ rad / sec}$$

With $\alpha = 0.2$ and $\beta = 0.0005$, the motion equation becomes:

$$\ddot{y}_i + 2 * 0.2158 * 860.9 \dot{y}_i + 860.9^2 y_i = -25.2$$

The transfer function for the above equation is:

$$H(s) = \frac{-25.2}{s^2 + 371s + 741200}$$

With $\alpha = 1$ and $\beta = 0.001$

$$\ddot{y}_i + 2 * 0.4307 * 860.9 \dot{y}_i + 860.9^2 y_i = -25.2$$

The transfer function for the above equation is:

$$H(s) = \frac{-25.2}{s^2 + 742.2s + 741200}$$

The impulse response of the system:

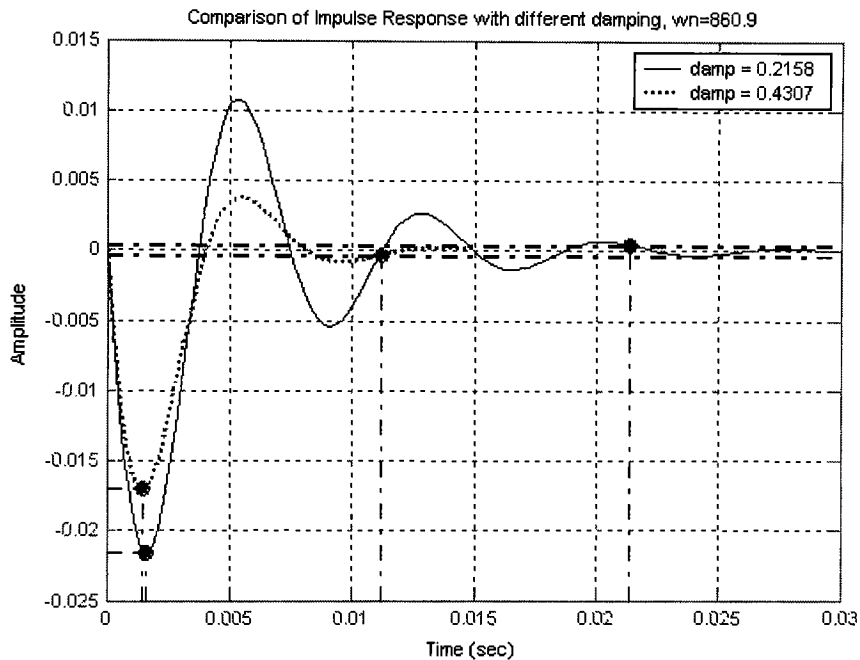


Figure 5.5 Comparison of Impulse Response at high frequency domain

Consider the lowest natural frequency of the system 1.44rad/sec, the corresponding damping characteristics are:

$$\alpha = 0.2, \beta = 0.0005$$

$$\xi = 0.07$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$= 1.44 * \sqrt{1 - 0.07^2}$$

$$= 1.4 \text{ rad/sec}$$

$$\alpha = 1, \beta = 0.001$$

$$\xi = 0.3476$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$= 1.44 * \sqrt{1 - 0.4310^2}$$

$$= 1.35 \text{ rad/sec}$$

With $\alpha = 0.2$ and $\beta = 0.0005$, the motion equation becomes:

$$\ddot{y}_i + 2 * 0.07 * 1.44 \dot{y}_i + 1.44^2 y_i = -25.2$$

The transfer function for the above equation is:

$$H(s) = \frac{-25.2}{s^2 + 0.2s + 2.1}$$

With $\alpha = 1$ and $\beta = 0.001$, the motion equation becomes:

$$\ddot{y}_i + 2 * 0.3476 * 1.44 \dot{y}_i + 1.44^2 y_i = -25.2$$

The transfer function for the above equation is:

$$H(s) = \frac{-25.2}{s^2 + s + 2.1}$$

The impulse response of the system is shown in Figure 5.6,

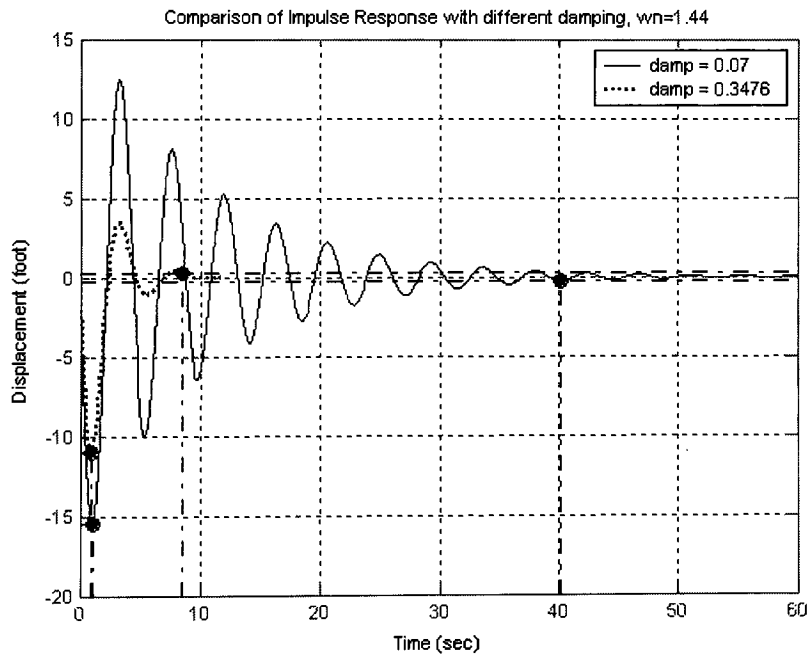


Figure 5.6 Comparison of Impulse Response at low frequency domain

In summary, as the damping increases, both the displacement peak and settling time decreases. Therefore, the system tends to settle down quickly.

▪ Influence of characteristic values

To analyze the influence of the characteristic values (i.e. the mass and stiffness of the system), we change the mass density of the system. Before, structural steel was used as the material, while keeping the same dimensions of the pipe and the drogue, aviation Buna-N tube, which is much lighter than structural steel, is used. The length density of aviation Buna-N tube is 5.33lb/ft. The stiffness and mass matrix of the beam element are as below:

The stiffness matrix of the element:

$$K^e = 1.0e+005 * [$$

0.1654	0.5168	0	0	-0.1654	0.5168	0	0
0.5168	2.1532	0	0	-0.5168	1.0766	0	0
0	0	0.1654	0.5168	0	0	-0.1654	0.5168
0	0	0.5168	2.1532	0	0	-0.5168	1.0766
-0.1654	-0.5168	0	0	0.1654	-0.5168	0	0
0.5168	1.0766	0	0	-0.5168	2.1532	0	0
0	0	-0.1654	-0.5168	0	0	0.1654	-0.5168
0	0	0.5168	1.0766	0	0	-0.5168	2.1532

];

The stiffness matrix of the drogue:

$$K_{\text{drogue}} = [$$

0.7142	1.4285	0	0	-0.7142	1.4285	0	0
1.4285	3.8092	0	0	-1.4285	1.9046	0	0
0	0	0.7142	1.4285	0	0	-0.7142	1.4285
0	0	1.4285	3.8092	0	0	-1.4285	1.9046
-0.7142	-1.4285	0	0	0.7142	-1.4285	0	0
1.4285	1.9046	0	0	-1.4285	3.8092	0	0
0	0	-0.7142	-1.4285	0	0	0.7142	-1.4285
0	0	1.4285	1.9046	0	0	-1.4285	3.8092

];

The element mass matrix is:

$$M^e = \begin{bmatrix} 12.148 & 10.71 & 0 & 0 & 4.2051 & -6.327 & 0 & 0 \\ 10.707 & 12.17 & 0 & 0 & 6.3271 & -9.126 & 0 & 0 \\ 0 & 0 & 12.148 & 10.707 & 0 & 0 & 4.2051 & -6.327 \\ 0 & 0 & 10.707 & 12.168 & 0 & 0 & 6.3271 & -9.126 \\ 4.2051 & 6.33 & 0 & 0 & 12.148 & -10.71 & 0 & 0 \\ -6.327 & -9.13 & 0 & 0 & -10.71 & 12.168 & 0 & 0 \\ 0 & 0 & 4.2051 & 6.3271 & 0 & 0 & 12.148 & -10.71 \\ 0 & 0 & -6.327 & -9.126 & 0 & 0 & -10.71 & 12.168 \end{bmatrix};$$

The mass matrix of the drogue is

$$M_{\text{drogue}} = \begin{bmatrix} 3.28 & 1.85 & 0 & 0 & 1.135 & 1.0933 & 0 & 0 \\ 1.8502 & 1.35 & 0 & 0 & 1.0933 & -1.009 & 0 & 0 \\ 0 & 0 & 3.28 & 1.8502 & 0 & 0 & 1.1354 & -1.093 \\ 0 & 0 & 1.8502 & 1.3456 & 0 & 0 & 1.0933 & -1.009 \\ 1.1354 & 1.09 & 0 & 0 & 3.28 & -1.85 & 0 & 0 \\ -1.093 & -1.01 & 0 & 0 & -1.85 & 1.3456 & 0 & 0 \\ 0 & 0 & 1.1354 & 1.7083 & 0 & 0 & 3.28 & -1.85 \\ 0 & 0 & -1.708 & -2.464 & 0 & 0 & -1.85 & 1.3456 \end{bmatrix};$$

The system's eigenvalue and natural frequency can be determined. In the vertical direction, for the vertical displacement:

$$\lambda = \begin{bmatrix} 1482363.26 \\ 113135.04 \\ 64312.88 \\ 18613.46 \\ 7410.87 \\ 2573.01 \\ 726.83 \\ 121.07 \\ 4.16 \end{bmatrix} \text{ rad / sec} \quad \omega = \begin{bmatrix} 1217.52 \\ 336.36 \\ 253.60 \\ 136.43 \\ 86.09 \\ 50.72 \\ 26.96 \\ 11.00 \\ 2.04 \end{bmatrix} \text{ rad / sec}$$

In the vertical direction, for the vertical rotation

$$\lambda = \begin{bmatrix} 15173397.83 \\ 65136.63 \\ 43615.02 \\ 18527.01 \\ 7390.09 \\ 2567.25 \\ 725.79 \\ 120.99 \\ 4.16 \end{bmatrix} \text{ rad / sec} \quad \omega = \begin{bmatrix} 3895.30 \\ 255.22 \\ 208.84 \\ 136.11 \\ 85.97 \\ 50.67 \\ 26.94 \\ 11.00 \\ 2.04 \end{bmatrix} \text{ rad / sec}$$

In the lateral direction, for the lateral displacement

$$\lambda = \begin{bmatrix} 127731.35 \\ 88473.51 \\ 28851.35 \\ 11839.60 \\ 4352.85 \\ 1426.59 \\ 327.07 \\ 32.03 \\ 0.11 \end{bmatrix} \text{ rad / sec} \quad \omega = \begin{bmatrix} 357.40 \\ 297.44 \\ 169.86 \\ 108.81 \\ 65.98 \\ 37.77 \\ 18.09 \\ 5.66 \\ 0.33 \end{bmatrix} \text{ rad / sec}$$

In the lateral direction, for the lateral rotation

$$\lambda = \begin{bmatrix} 89713.57 \\ 44009.75 \\ 28669.36 \\ 11797.06 \\ 4334.14 \\ 1423.95 \\ 326.74 \\ 32.02 \\ 0.11 \end{bmatrix} \text{ rad / sec} \quad \omega = \begin{bmatrix} 299.52 \\ 209.79 \\ 169.32 \\ 108.61 \\ 65.83 \\ 37.74 \\ 18.08 \\ 5.66 \\ 0.33 \end{bmatrix} \text{ rad / sec}$$

From the presented data above, we observe that as the magnitude of the mass matrix decreases the natural frequency will increase.

Compare the system response with different mass matrices.

Set $\alpha=1$, $\beta = 0.001$, including the effect of the exponential force.

The displacement at the free end of the drogue with the previously defined parameters is shown by Figure 5.7,

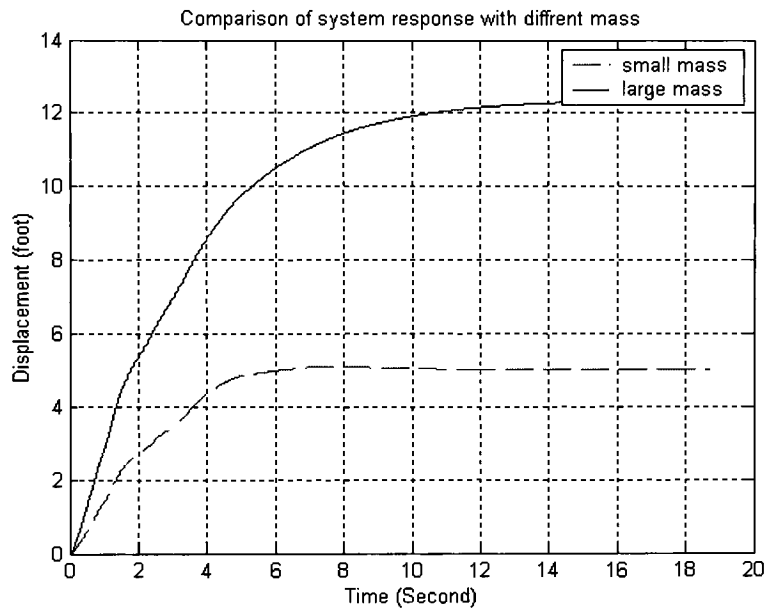


Figure 5.7 Comparison of system response with different Mass

Set $\alpha=0.2$, $\beta = 0.0005$, including the effect the exponential force.

The displacement at the free end of the drogue with the previously defined parameters is shown by Figure 5.8

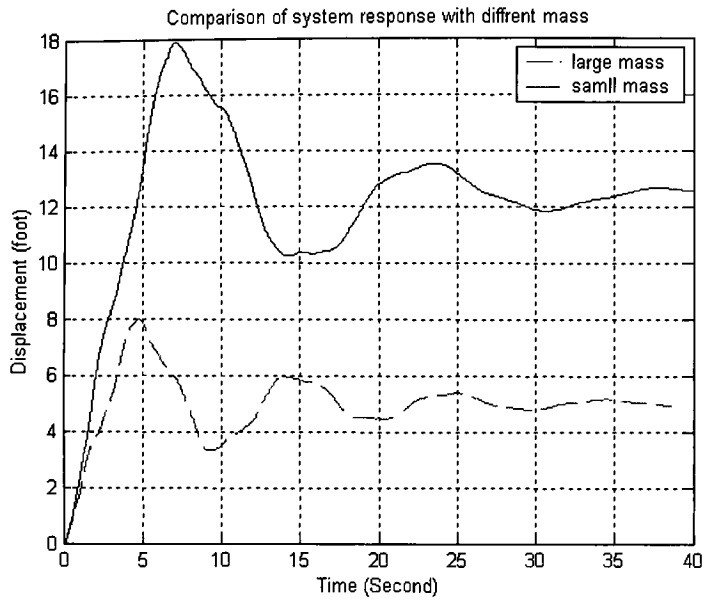


Figure 5.8 Comparison of system response with different Mass

In summary, as shown before, as the system's mass decreases, a decrease in the tip displacement is observed. Based on visual observation of actual in-flight refueling, (reference VISTA/JSF Video), the system response shown by the "small mass" in Figure 5.7 accurately describes the motion of the drogue system.

Summarize the results shown previously, we conclude:

1. Assume the system damping to be proportional, i.e.

$$[C] = \alpha[M] + \beta[K],$$

and

$$\xi_i = \frac{\alpha + \beta\omega_i^2}{2\omega_i}$$

increases in α or β result in increases in ξ and decreases in the damped frequency of the system $\omega_d = \omega_n\sqrt{1 - \xi^2}$.

2. Increases in damping ratio result in a decrease in the magnitude of system motion, and therefore the decreases in the system settling time.
3. Decrease in the system's overall mass results in a decrease in the magnitude of the system motion.
4. Model parameters were varied to match observation results obtained from video of an actual in-flight probe-and-drogue refueling.

Parameters described above may be further tuned to more accurately describe the dynamical behavior of the drogue refueling system.

Suggestion for the further work:

1. Instrument an actual drogue system to compare the developed model response with measured data.
2. Develop a control law for controlling drogue position as a precursor for an automatic in-flight refueling system.
3. Analyze required sensors for control of the drogue system.
4. The transverse effect was not discussed in this work. To include transverse effect, the displacement in the axial direction should be considered. So the assumption of zero displacement in axial direction is no longer needed and there are five degree-of-free at each node of the beam system – one more degree-of –freedom is added for displacement in axial direction.

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Chapter 7 APPENDICES:

APPENDIX I

Program1

```
% not consider axial force of the beam, there are two degree-
of-freedom
% for every node, the displacement and rotation in up and down
direction
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
close all, clear all, clc;
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
% symbols of mental pipe features
```

```
%length of pipe
Lpipe = 3.0;
```

```
%young's modulus of pipe
Epipe = 1.44e09;
```

```
%density of pipe
DensityPipe = 5.233;
```

```
%thickness of pipe
Hpipe = 16 * (1/16)/12;
```

```
%width of pipe
Wpipe = .125;
```

```
% cross area of pipe
Apipe = Hpipe * Wpipe;
```

```
% unit mass of pipe
Mpipe = DensityPipe * (Hpipe * Wpipe);
```

```
% area moment of inertia in up and down direction
Ipipe = (1/12) * (Wpipe * Hpipe * Hpipe * Hpipe);
```

```
% divide pipe to 3 element
n = 3;
l = Lpipe/n;
```

```
% get shape functions of beam,
syms theta;
```

```

% shape function of up and down movement of starting point
N1=1-3*theta^2+2*theta^3;

% shape function of up and down rotation of starting point
N2=1*theta-2*1*theta^2+1*theta^3;

% shape function of up and down movement of end point
N3=3*theta^2-2*theta^3;

% shape function of up and down rotation of end point
N4=-1*theta^2+1*theta^3;

%length of drogue
ld = 1;

%young's modulus of drogue
Edrogue = Epipe * 2;

%density of drogue
DensityDrogue = 1/2 * DensityPipe;

%thickness of drogue
Hdrogue = 1/2 * Hpipe;

%width of drogue
Wdrogue = 1/2 * Wpipe;

% cross area of drogue
Adrogue = Hdrogue * Wdrogue;

% unit mass of drogue
Mdrogue = DensityDrogue * (Hdrogue * Wdrogue);

% area moment of inertia on up and down direction
Idrogue = (1/12) * (Wdrogue * Hdrogue * Hdrogue * Hdrogue);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%

% stiffness matrix of one element, treat a as coefficient

a = Epipe * Ipipe /l^3;
kkA = Epipe*Apipe/l;

kk= a*[
    12      6*1      -12      6*1;
    6*1     4*1^2     -6*1     2*1^2;

    -12     -6*1      12      -6*1;
    6*1     2*1^2     -6*1     4*1^2;

```

```

];

% K is the stiffness matrix of the system
K = zeros(2*(n+1),2*(n+1));

% superimposing preceding matrices for elements to get the
stiffness matrix
for i=1:n;
q = zeros(2*(n+1),2*(n+1));
q = [zeros(2*(i-1),2*(n+1));zeros(4,2*(i-1)) kk zeros(4,2*(n-
i)); zeros(2*(n-i),2*(n+1))];
K = K + q;
end;

% stiffness matrix of drogue, treat aa as coefficient
aa = Edrogue * Idrogue / ld^3;
kkDA = Edrogue * Adrogue / ld;

kkDrogue= aa*[
    12      6*ld    -12      6*ld;
    6*ld    4*ld^2  -6*ld    2*ld^2;
    -12     -6*ld   12      -6*ld;
    6*ld    2*ld^2  -6*ld    4*ld^2;
];

% superimposing stiffness matrix of mental pipe to drogue
K = [K zeros(2*(n+1),2); zeros(2,2*(n+2))];
kkDrogue = [zeros(2*n, 2*(n+2)); zeros(4,2*n) kkDrogue];
K = K + kkDrogue;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% get the mass matrix of the beam, ignoring the axial
displacement

% mass matrix of one element, treat b as coefficient
b = Mpipe * 1/420;
N = 140;
mm= b*[
    156      22*1    54      -13*1;
    22*1     4*1^2   13*1     -3*1^2;
    54       13*1    156      -22*1;
    -13*1    -3*1^2  -22*1    4*1^2;
];

% superimposing preceding matrices for elements to get the
mass matrix
M = zeros(2*(n+1),2*(n+1));

```

```

for i = 1:n;
q = zeros(2*(n+1),2*(n+1));
q = [zeros(2*(i-1),2*(n+1));zeros(4,2*(i-1)) mm zeros(4,2*(n-
i)); zeros(2*(n-i),2*(n+1))];
M = M + q;
end;

% mass matrix of drogue, treat bb as coefficient
bb = Mdrogue * ld/420;

mmDrogue = bb*[
    156      22*ld    54      -13*ld;
    22*ld    4*ld^2  13*ld    -3*ld^2;
    54      13*ld   156      -22*ld;
    -13*ld   -3*ld^2 -22*ld   4*ld^2;
];

% superimposing mass matrix of mental pipe to drogue

M = [M zeros(2*(n+1),2); zeros(2,2*(n+2))];
mmDrogue = [zeros(2*n, 2*(n+2)); zeros(4,2*n) mmDrogue];
M = M + mmDrogue;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%

% get matrix of damping, C = c*M + d*K;
% also consider M*(ddX) + C*(dX) + K*X = 0;
% ddX/(K/M) + dX*(C/K) + X = 0;
% ddX/(wn^2) + dX*2*damping/wn + X = 0;
% C = 2*damping*sqrt(K*M);

% first set c = 1, d =0.001;
c = 1;
d = 0.001;
C= c*M + d*K;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%

% discuss the situation of triangular force applied on the
system
% if the force applied on the beam is uniform, then we could
divide it to % two identical triangle forces applied on the
left and right side of beam % separately

% if the force applied on the beam is only gravity
% g is the gravitational acceleration
% unitGravity is the gravity of unit length

```

```

g = -32.172;
unitGravity = Mpipe * g;
unitGravityDrogue = Mdrogue * g;

% denote gravity as the gravity of every element of the pipe
for j = 1:n+1;
    gravity(j)= unitGravity*(j-1)*1;
end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%get forces and moments of starting point

% get the force of up and down direction of starting point
for j = 1:n;
    Forcer1(j) = int( ((gravity(j+1)-gravity(j))*theta +
        gravity(j))*N1*1, theta, 0, 1 );
end;

% get the moment of up and down rotation of starting point
for j = 1:n;
    Momentr1(j) = int( ((gravity(j+1)-gravity(j))*theta
        +gravity(j))*N2*1, theta, 0, 1 );
end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
% get forces and moments of end point
% get the axial force of the end point

% get the force of up and down direction of end point
for j = 1:n;
    Forcer2(j) = int( ((gravity(j+1)-gravity(j))*theta +
        gravity(j))*N3*1, theta, 0, 1 );
end;

% get the moment of up and down rotation of end point
for j = 1:n;
    Momentr2(j) = int( ((gravity(j+1)-gravity(j))*theta
        +gravity(j))*N4*1, theta, 0, 1 );
end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% gravityDrogue is the gravity of drogue
for j = 1:2;

```

```

    gravityDroque(j)= unitGravityDroque * (j-1) * ld;
end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

%get forces and moments of starting point of drogue

% get the force of up and down direction of starting point of
drogue
    ForceDr1 = int( ( (gravityDroque(2) - gravityDroque(1)
        ) * theta + gravityDroque(1) ) * N1 * ld, theta, 0, 1
    );

% get the moment of up and down rotation of starting point of
drogue
    MomentDr1 = int( ( (gravityDroque(2) - gravityDroque(1)
        ) * theta + gravityDroque(1) ) * N2 * ld, theta, 0, 1 );

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% get forces and moments of end point of drogue
% get the axial force of the end point of drogue

% get the force of up and down direction of end point of
drogue
    ForceDr2 = int( ( (gravityDroque(2) - gravityDroque(1)
        ) * theta + gravityDroque(1) ) * N3 * ld, theta, 0, 1 );

% get the moment of up and down rotation of end point of
drogue
    MomentDr2 = int( ( (gravityDroque(2) - gravityDroque(1)
        ) * theta + gravityDroque(1) ) * N4 * ld, theta, 0, 1 );

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% extend force and moment vector
Forcer1 = [Forcer1 zeros(1,2)];
Momentr1 = [Momentr1 zeros(1,2)];

Forcer2 = [zeros(1,1) Forcer2 zeros(1,1)];
Momentr2 = [zeros(1,1) Momentr2 zeros(1,1)];

ForceDr = [zeros(1,3) ForceDr1 ForceDr2];
MomentDr = [zeros(1,3) MomentDr1 MomentDr2];

% get the whole response
Force = Forcer1 + Forcer2 + ForceDr;
Moment = Momentr1 + Momentr2 + MomentDr;

```

```

% get the whole Force and Moment vector
for i = 0:n+1;

    FM(2*i+1) = Force(i+1);
    FM(2*i+2) = Moment(i+1);

end;

FM = FM';

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% get the state space function of the system
%  $\dot{X}_2 = [M]^{-1}[F] - [M]^{-1}[C]\dot{X} - [M]^{-1}[K]X = [M]^{-1}[F] - [M]^{-1}[K]X_1 - [M]^{-1}[C]X_2$ 
% as the starting end of the pipe is fixed, so the displacement
% equals zero

Mr = M(1:2, 1:2);
Ms = M(3:2*(n+2), 3:2*(n+2));
Msinv = inv(Ms);

Kr = K(1:2, 1:2);
Ks = K(3:2*(n+2), 3:2*(n+2));

Cr = C(1:2, 1:2);
Cs = C(3:2*(n+2), 3:2*(n+2));

FMr = FM(1:2,1);
FMs = FM(3:2*(n+2),1);

sz = size(Ms);

A = [zeros(sz) eye(sz); -Msinv*Ks -Msinv*Cs ];
B = [zeros(sz(1),1); Msinv*FMs];

% set the first element to be 1, results the solution of first
% output
% if we set the desired element to be 1 and others to be
% zeros, we could
% get the solution of that variable

D = [1, zeros(1,2*sz(2)-1)];
E = 0;

```


APPENDIX II

Program 2

```
% didn't consider axial force of the beam

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
close all, clear all, clc;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
% symbol of mental pipe features

%length of pipe
Lpipe = 3.0;

%young's modulus of pipe
Epipe = 1.44e09;

%density of pipe
DensityPipe = 5.233;

%thickness of pipe
Hpipe = 16 * (1/16)/12;

%width of pipe
Wpipe = .125;

% cross area of pipe
Apipe = Hpipe * Wpipe;

% unit mass of pipe
Mpipe = DensityPipe * (Hpipe * Wpipe);

% area moment of inertia on up and down direction
Ipipe = (1/12) * (Wpipe * Hpipe * Hpipe * Hpipe);

% area moment of inertia on lateral direction
Ipipe2 = (1/12) * (Hpipe * Wpipe * Wpipe * Wpipe);

n = 3;
l = Lpipe/n;

% shape function of beam
syms theta;

% up and down movement of starting point
N1=1-3*theta^2+2*theta^3;
```

```

% up and down rotation of starting point
N2=1*theta-2*1*theta^2+1*theta^3;

% up and down movement of end point
N3=3*theta^2-2*theta^3;

% up and down rotation of end point
N4=-1*theta^2+1*theta^3;

ld = 1; %length of drogue
%young's modulus of drogue
Edrogue = Epipe * 2;

%density of drogue
DensityDrogue = 1/2 * DensityPipe;

%thickness of drogue
Hdrogue = 1/2 * Hpipe;

%width of drogue
Wdrogue = 1/2 * Wpipe;

% cross area of drogue
Adrogue = Hdrogue * Wdrogue;

% unit mass of drogue
Mdrogue = DensityDrogue * (Hdrogue * Wdrogue);

% area moment of inertia on up and down direction
Idrogue = (1/12) * (Wdrogue * Hdrogue * Hdrogue * Hdrogue);

% area moment of inertia on lateral direction
Idrogue2 = (1/12) * (Hdrogue * Wdrogue * Wdrogue * Wdrogue);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

%stiffness matrix of one element, treat a as coefficient

a = Epipe * Ipipe /l^3;
a2= Epipe * Ipipe2 /l^3;
kkA = Epipe*Apipe/l;

kk= [
a*12      a*6*1      0      0      -a*12      a*6*1      0
0;
a*6*1      a*4*1^2    0      0      -a*6*1      a*2*1^2    0
0;
0          0          a2*12   a2*6*1    0          0          -
a2*12      a2*6*1
0          0          a2*6*1   a2*4*1^2  0          0          -
a2*6*1      a2*2*1^2

```

```

-a*12    -a*6*1    0    0    a*12    -a*6*1    0
0;
a*6*1    a*2*1^2    0    0    -a*6*1    a*4*1^2    0
0;
0    0    -a2*12    -a2*6*1    0    0
a2*12    -a2*6*1
0    0    a2*6*1    a2*2*1^2    0    0    -
a2*6*1    a2*4*1^2
];

```

```

% divide the mental pipes to 10 elements
K = zeros(4*(n+1),4*(n+1));

```

```

%superimposing preceding matrices for elements to get the
striffness matrix
for i=1:n;
q = zeros(4*(n+1),4*(n+1));
q = [zeros(4*(i-1),4*(n+1));zeros(8,4*(i-1)) kk zeros(8,4*(n-
i)); zeros(4*(n-i),4*(n+1))];
K = K + q;
end;

```

```

%stiffness matrix of drogue, treat aa aa2 as coefficient
aa = Edrogue * Idrogue / ld^3;
aa2 = Edrogue * Idrogue2 / ld^3;
kkDA = Edrogue * Adrogue / ld;

```

```

kkDrogue = [
aa*12    aa*6*ld    0    0    -aa*12    aa*6*ld
0    0;
aa*6*ld    aa*4*ld^2    0    0    -aa*6*ld    aa*2*ld^2
0    0;
0    0    aa2*12    aa2*6*ld    0    0
-aa2*12    aa2*6*ld
0    0    aa2*6*ld    aa2*4*ld^2    0    0
-aa2*6*ld    aa2*2*ld^2

-aa*12    -aa*6*ld    0    0    aa*12    -aa*6*ld
0    0;
aa*6*ld    aa*2*ld^2    0    0    -aa*6*ld    aa*4*ld^2
0    0;
0    0    -aa2*12    -aa2*6*ld    0    0
aa2*12    -aa2*6*ld
0    0    aa2*6*ld    aa2*2*ld^2    0    0
-aa2*6*ld    aa2*4*ld^2
];

```

```

%superimposing stiffness matrix of mental pipe to drogue

```

```

K = [K zeros(4*(n+1),4); zeros(4,4*(n+2))];
kkDrogue = [zeros(4*n, 4*(n+2)); zeros(8,4*n) kkDrogue];
K = K + kkDrogue;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%

%get the mass matrix of the beam, ignoring the axial
displacement

%mass matrix of one element, treat b as coefficient
b = Mpipe * l/420;
N = 140;
mm = b*[
    156      22*1      0      0      54      -13*1      0      0;
    22*1      4*1^2      0      0      13*1      -3*1^2      0      0;
    0      0      156      22*1      0      0      54      -
13*1;
    0      0      22*1      4*1^2      0      0      13*1      -
3*1^2;
    54      13*1      0      0      156      -22*1      0      0;
    -13*1      -3*1^2      0      0      -22*1      4*1^2      0      0;
    0      0      54      13*1      0      0      156      -
22*1;
    0      0      -13*1      -3*1^2      0      0      -22*1
4*1^2;
    ];

%superimposing precding matrices for elements to get the mass
matrix
M = zeros(4*(n+1),4*(n+1));

for i = 1:n;
q = zeros(4*(n+1),4*(n+1));
q = [zeros(4*(i-1),4*(n+1));zeros(8,4*(i-1)) mm zeros(8,4*(n-
i)); zeros(4*(n-i),4*(n+1))];
M = M + q;
end;

%mass matrix of drogue, treat bb as coefficient
bb = Mdrogue * ld/420;

mmDrogue = bb*[
    156      22*ld      0      0      54      -13*ld      0
0;
    22*ld      4*ld^2      0      0      13*ld      -3*ld^2      0
0;
    0      0      156      22*ld      0      0      54
-13*ld;

```

```

0      0      22*ld    4*ld^2    0      0      13*ld
-3*ld^2;

54      13*ld    0      0      156      -22*ld    0
0;
-13*ld   -3*ld^2    0      0      -22*ld    4*ld^2    0
0;
0      0      54      13*1    0      0      156
-22*ld;
0      0      -13*1    -3*1^2    0      0      -22*ld
4*ld^2;
];

```

```
%superimposing mass matrix of mental pipe to drogue
```

```

M = [M zeros(4*(n+1),4); zeros(4,4*(n+2))];
mmDrogue = [zeros(4*n, 4*(n+2)); zeros(8,4*n) mmDrogue];
M = M + mmDrogue;

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

```

```

% get matrix of damping, C = c*M + d*K;
% also consider M*(ddX) + C*(dX) + K*X = 0;
% ddX/(K/M) + dX*(C/K) + X = 0;
% ddX/(wn^2) + dX*2*damping/wn + X = 0;
% C = 2*dampig*sqrt(K/M);

```

```

% first set c = 1, d =0.001;
c = 1;
d = 0.001;
C= c*M + d*K;

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

```

```

% discuss the situation of triangular force applied on
% if the force applied on the beam is uniform, then
% we could devide it to two identical triangle forces
% applied on the left and right side of beam separately

```

```

% if the force applied on the beam is only gravity
% g is the gravitational acceleration
% unitGravity is the gravity of unit length

```

```

g = -32.172;
unitGravity = Mpipe * g;
unitGravityDrogue = Mdrogue * g;

```

```
% gravity is the gravity of every element of the pipe
```

```

for j = 1:n+1;
    gravity(j)= unitGravity*(j-1)*1;
end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

%get forces and moments of starting point

% get the force of up and down direction of starting point
for j = 1:n;
    Forcer1(j) = int( ((gravity(j+1)-gravity(j))*theta +
gravity(j))*N1*1, theta, 0, 1 );
end;

% get the moment of up and down rotation of starting point
for j = 1:n;
    Momentr1(j) = int( ((gravity(j+1)-gravity(j))*theta +
gravity(j))*N2*1, theta, 0, 1 );
end;

% get the force of lateral direction of starting point
for j = 1:n;
    Forcelat1(j) = int( 0*N1*1, theta, 0, 1 );
end;

% get the moment of lateral rotation of starting point
for j = 1:n;
    Momentlat1(j) = int( 0*N2*1, theta, 0, 1 );
end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% get forces and moments of end point
% get the axial force of the end point

% get the force of up and down direction of end point
for j = 1:n;
    Forcer2(j) = int( ((gravity(j+1)-gravity(j))*theta +
gravity(j))*N3*1, theta, 0, 1 );
end;

% get the moment of up and down rotation of end point
for j = 1:n;
    Momentr2(j) = int( ((gravity(j+1)-gravity(j))*theta +
gravity(j))*N4*1, theta, 0, 1 );
end;

% get the force of lateral direction of end point
for j = 1:n;

```

```

    Forcelat2(j) = int( 0*N3*1, theta, 0, 1 );
end;

% get the moment of lateral rotation of end point
for j = 1:n;
    Momentlat2(j) = int( 0*N4*1, theta, 0, 1 );
end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% gravityDroque is the gravity of drogue
for j = 1:2;
    gravityDroque(j)= unitGravityDroque * (j-1) * ld;
end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

%get forces and moments of starting point of drogue

% get the force of up and down direction of starting point of
drogue
    ForceDr1 = int( ( gravityDroque(2) - gravityDroque(1)
)*theta + gravityDroque(1) )*N1*ld, theta, 0, 1 );

% get the moment of up and down rotation of starting point of
drogue
    MomentDr1 = int( ( gravityDroque(2) - gravityDroque(1)
)*theta + gravityDroque(1) )*N2*ld, theta, 0, 1 );

% get the force of lateral direction of starting point of
drogue
    ForceDlat1 = int( 0*N1*ld, theta, 0, 1 );

% get the moment of lateral rotation of starting point of
drogue
    MomentDlat1 = int( 0*N2*ld, theta, 0, 1 );

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% get forces and moments of end point of drogue
% get the axial force of the end point of drogue

% get the force of up and down direction of end point of
drogue
    ForceDr2 = int( ( gravityDroque(2) - gravityDroque(1)
)*theta + gravityDroque(1) )*N3*ld, theta, 0, 1 );

```

```

% get the moment of up and down rotation of end point of
drogue
    MomentDr2 = int( ( gravityDroque(2) - gravityDroque(1)
    ) * theta + gravityDroque(1) ) * N4 * ld, theta, 0, 1 );

% get the force of lateral direction of end point of drogue
    ForceDlat2 = int( 0 * N3 * ld, theta, 0, 1 );

% get the moment of lateral rotation of end point of drogue
    MomentDlat2 = int( 0 * N4 * ld, theta, 0, 1 );

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% get the whole force and moment response of system under the
influence of gravity
% extend force and moment vector
Forcer1 = [Forcer1 zeros(1,2)];
Momentr1 = [Momentr1 zeros(1,2)];
Forcelat1 = [Forcelat1 zeros(1,2)];
Momentlat1 = [Momentlat1 zeros(1,2)];

Forcer2 = [zeros(1,1) Forcer2 zeros(1,1)];
Momentr2 = [zeros(1,1) Momentr2 zeros(1,1)];
Forcelat2 = [zeros(1,1) Forcelat2 zeros(1,1)];
Momentlat2 = [zeros(1,1) Momentlat2 zeros(1,1)];

ForceDr = [zeros(1,3) ForceDr1 ForceDr2];
MomentDr = [zeros(1,3) MomentDr1 MomentDr2];
ForceDlat = [zeros(1,3) ForceDlat1 ForceDlat2];
MomentDlat = [zeros(1,3) MomentDlat1 MomentDlat2];

% get the whole response
Force = Forcer1 + Forcer2 + ForceDr;
Moment = Momentr1 + Momentr2 + MomentDr;
Forcelat = Forcelat1 + Forcelat2 + ForceDlat;
Momentlat = Momentlat1 + Momentlat2 + MomentDlat;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% get the whole Force and Moment vector
for i = 0:n+1;

    FM(4*i+1) = Force(i+1);
    FM(4*i+2) = Moment(i+1);
    FM(4*i+3) = Forcelat(i+1);
    FM(4*(i+1)) = Momentlat(i+1);

end;

```



```

FM = FM';

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% add an exponential force on the end of drogue
t=0:0.1:10;
account = size(t);

expInput = -1./exp(t);
expInput = 1 + expInput;

% define the angle of expnential force
angle = pi/4;

% get the exponential force division in the up and down
direction
expInputZ = expInput * cos(angle);

% get the force response under the effection of exponential
force
% as there is no force distribution in the lateral direction,
the force
% response is 0

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%

%get forces and moments of starting point of drogue

% get the force of up and down direction of starting point of
drogue
expForceDr1 = int( expInputZ*N1*ld, theta, 0, 1 );

% get the moment of up and down rotation of starting point of
drogue
expMomentDr1 = int( expInputZ*N2*ld, theta, 0, 1 );

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%

% get forces and moments of end point of drogue

% get the force of up and down direction of end point of
drogue
expForceDr2 = int( expInputZ*N3*ld, theta, 0, 1 );

% get the moment of up and down rotation of end point of
drogue
expMomentDr2 = int( expInputZ*N4*ld, theta, 0, 1 );

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%

```

```

% get the whole force and moment response of system under the
influence of % exponential force

expForceP = zeros( n+2, account(2) );
expMomentP = zeros( n+2, account(2) );

expForceDr = [zeros( 3, account(2) ); expForceDr1;
expForceDr2];
expMomentDr = [zeros( 3, account(2) ); expMomentDr1;
expMomentDr2];

% get the whole response
for i = 1:account(2);
    expForce = expForceP(i) + expForceDr;
    expMoment = expMomentP(i) + expMomentDr;
end;

expForcelat = zeros( n+2, account(2) );
expMomentlat = zeros( n+2, account(2) );

% get the whole Force and Moment vector
for j = 1 : account(2);

    for i = 0:n+1;
        expFM(4*i+1,j) = expForce(i+1,j);
        expFM(4*i+2,j) = expMoment(i+1,j);
        expFM(4*i+3,j) = expForcelat(i+1,j);
        expFM(4*(i+1),j) = expMomentlat(i+1,j);
    end;

end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%

% get the state space function of the system

% as the starting end of the pipe is fixed, so the displace
equals zero

Mr = M(1:4, 1:4);
Ms = M(5:4*(n+2), 5:4*(n+2));
Msinv = inv(Ms);

Kr = K(1:4, 1:4);
Ks = K(5:4*(n+2), 5:4*(n+2));

Cr = C(1:4, 1:4);
Cs = C(5:4*(n+2), 5:4*(n+2));

FMr = FM(1:4,1);

```

```

FMs = FM(5:4*(n+2),1);

expFMr = expFM(1:4,:);
expFMs = expFM(5:4*(n+2),:);

sz = size(Ms);
A = [ zeros(sz) eye(sz); -Msinv*Ks -Msinv*Cs ];

for i = 1: account(2);
    BB(:,i) = Msinv*expFMs(:,i);
end;

B = [ zeros(sz(1), 2); Msinv*FMs BB(:,100) ];

D = [zeros(1,2*sz(2))];
E = zeros( 2, 2 );

```

APPENDIX III

Program3

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
close all, clear all, clc;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
% features of pipe and drogue
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% length of pipe, unit ft
Lpipe = 50;

% young's modulus, unit lb/ft^2
E=4.18e09;

% density of pipe, unit lb/ft^3
Density = 0.284 * 12^3;

% inside diameter of the pipe, unit ft
DoPipe = 2.5/12;

% outside diameter of the pipe, unit ft
DiPipe = 1.5/12;

% cross area of pipe, unit ft^2
Apipe = 1/4 * pi* (DoPipe^2 - DiPipe^2);

% % unit mass of pipe, lb/ft
Mpipe = Density * Apipe;

% area moment of inertia on lateral direction
Ipipe = (1/64) * pi* ( DoPipe^4 - DiPipe^4 );

% divide pipe to pipe elements
n = 8;

% length of pipe element, unit ft
l = Lpipe/n;

% length of drogue, unit ft
ld = 4;

% the overall length of the system
L = Lpipe + ld;

%outside diameter of the drogue, unit ft
DoDrogue = 3.5/12;

```

```

%insider diameter of the drogue, unit ft
DiDrogue = 3.25/12;

% cross area of drogue
Adrogue = (1/4) * pi * (DoDrogue^2 - DiDrogue^2);

% unit mass of drogue
Mdrogue = Density * Adrogue;

% area moment of inertia of the drogue
Idrogue = (1/64) * pi * ( DoDrogue^4 - DiDrogue^4 );

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

%shape function

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% shape function of beam
syms theta;

% movement of starting point
N1=1-3*theta^2+2*theta^3;

% rotation of starting point
N2=1*theta-2*1*theta^2+1*theta^3;

% movement of end point
N3=3*theta^2-2*theta^3;

% rotation of end point
N4=-1*theta^2+1*theta^3;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

%get stiffness matrices of the system

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%stiffness matrix of one pipe element, treat a as coefficient
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

a = E * Ipipe /l^3;

kk= a*[

```

```

12      6*1      0      0      -12      6*1      0      0;
6*1     4*1^2    0      0      -6*1     2*1^2    0      0;
0       0       12     6*1     0       0       -12     6*1
0       0       6*1     4*1^2    0       0       -6*1     2*1^2

-12     -6*1     0      0      12     -6*1     0      0;
6*1     2*1^2    0      0      -6*1     4*1^2    0      0;
0       0       -12     -6*1     0       0      12     -6*1
0       0       6*1     2*1^2    0       0      -6*1     4*1^2
];

```

```

% divide the mental pipes to 10 elements
K = zeros(4*(n+1),4*(n+1));

```

```

% superimposing precding matrices for elements to get the
striffness % matrix

```

```

for i=1:n;
q = zeros(4*(n+1),4*(n+1));
q = [zeros(4*(i-1),4*(n+1));zeros(8,4*(i-1)) kk zeros(8,4*(n-
i)); zeros(4*(n-i),4*(n+1))];
K = K + q;
end;

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%stiffness matrix of drogue, treat aa as coefficient
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

```

```

aa = E * Idrogue / ld^3;

```

```

kkDrogue = aa*[
12      6*ld      0      0      -12      6*ld      0      0;
6*ld     4*ld^2    0      0      -6*ld     2*ld^2    0      0;
0       0       12     6*ld     0       0       -12
6*ld
0       0       6*ld     4*ld^2    0       0       -6*ld
2*ld^2

-12     -6*ld     0      0      12     -6*ld     0
0;
6*ld     2*ld^2    0      0      -6*ld     4*ld^2    0
0;
0       0       -12     -6*ld     0       0      12      -
6*ld
0       0       6*ld     2*ld^2    0       0      -6*ld
4*ld^2
];

```

```
%superimposing stiffness matrix of mental pipe to drogue
K = [K zeros(4*(n+1),4); zeros(4,4*(n+2))];
kkDrogue = [zeros(4*n, 4*(n+2)); zeros(8,4*n) kkDrogue];
K = K + kkDrogue;
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
```

```
%get mass matrices of the system
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
```

```
%mass matrix of one pipe element, treat b as coefficient
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
```

```
b = Mpipe * 1/420;
N = 140;
mm = b*[
    156      22*1      0      0      54      -13*1      0      0;
    22*1      4*1^2      0      0      13*1      -3*1^2      0      0;
    0      0      156      22*1      0      0      54      -
13*1;
    0      0      22*1      4*1^2      0      0      13*1      -
3*1^2;

    54      13*1      0      0      156      -22*1      0      0;
    -13*1      -3*1^2      0      0      -22*1      4*1^2      0      0;
    0      0      54      13*1      0      0      156      -
22*1;
    0      0      -13*1      -3*1^2      0      0      -22*1
4*1^2;
];
```

```
%superimposing precding matrices for elements to get the mass
matrix
```

```
M = zeros(4*(n+1),4*(n+1));
```

```
for i = 1:n;
q = zeros(4*(n+1),4*(n+1));
q = [zeros(4*(i-1),4*(n+1));zeros(8,4*(i-1)) mm zeros(8,4*(n-
i)); zeros(4*(n-i),4*(n+1))];
M = M + q;
end;
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
```

```
%mass matrix of drogue, treat bb as coefficient
```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%

bb = Mdrogue * ld/420;

mmDroque = bb*[
    156      22*ld      0      0      54      -13*ld      0
0;
    22*ld    4*ld^2    0      0      13*ld    -3*ld^2    0
0;
    0        0        156    22*ld    0        0        54
-13*ld;
    0        0        22*ld    4*ld^2    0        0        13*ld
-3*ld^2;

    54      13*ld      0      0      156      -22*ld      0
0;
    -13*ld   -3*ld^2    0      0      -22*ld    4*ld^2      0
0;
    0        0        54      13*ld    0        0        156
-22*ld;
    0        0        -13*ld   -3*ld^2    0        0        -22*ld
4*ld^2;
    ];

%superimposing mass matrix of mental pipe to drogue

M = [M zeros(4*(n+1),4); zeros(4,4*(n+2))];
mmDroque = [zeros(4*n, 4*(n+2)); zeros(8,4*n) mmDroque];
M = M + mmDroque;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%

% get matrix of damping, C = c*M + d*K;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%

% also consider M*(ddX) + C*(dX) + K*X = 0;
% ddX/(K/M) + dX*(C/K) + X = 0;
% ddX/(wn^2) + dX*2*damping/wn + X = 0;
% C = 2*damping*sqrt(K/M);

% first set c = 1, d =0.001;
c = 1;
d = 0.001;
C= c*M + d*K;

```



```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%

%discuss influence of gravity, gravity acceleration is a
constant

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%

% define gravity on the system
g = -32.172;

unitGravity = Mpipe * g;

unitGravityDrogue = Mdrogue * g;

ratio = 1/L;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
%get forces and moments of starting point
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%

% get the force of up and down direction of starting point
for j = 1:n;
    Forcer1(j) = int( unitGravity*N1*L, theta, (j-1)*ratio,
j*ratio );
end;

% get the moment of up and down rotation of starting point
for j = 1:n;
    Momentr1(j) = int( unitGravity*N2*L, theta, (j-1)*ratio,
j*ratio );
end;

% get the force of lateral direction of starting point
for j = 1:n;
    Forcelat1(j) = int( 0*N1*L, theta, (j-1)*ratio, j*ratio );
end;

% get the moment of lateral rotation of starting point
for j = 1:n;
    Momentlat1(j) = int( 0*N2*L, theta, (j-1)*ratio, j*ratio
);
end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
% get forces and moments of end point

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% get the force of up and down direction of end point
for j = 1:n;
    Forcer2(j) = int( unitGravity*N3*L, theta, (j-1)*ratio,
j*ratio );
end;

% get the moment of up and down rotation of end point
for j = 1:n;
    Momentr2(j) = int( unitGravity*N4*L, theta, (j-1)*ratio,
j*ratio );
end;

% get the force of lateral direction of end point
for j = 1:n;
    Forcelat2(j) = int( 0*N3*L, theta, (j-1)*ratio, j*ratio );
end;

% get the moment of lateral rotation of end point
for j = 1:n;
    Momentlat2(j) = int( 0*N4*L, theta, (j-1)*ratio, j*ratio
);
end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%get forces and moments of starting point of drogue
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% get the force of up and down direction of starting point of
drogue
    ForceDr1 = int( unitGravityDrogue*N1*L, theta, Lpipe/L, 1
);

% get the moment of up and down rotation of starting point of
drogue
    MomentDr1 = int( unitGravityDrogue*N2*ld, theta, Lpipe/L,
1 );

% get the force of lateral direction of starting point of
drogue
    ForceDlat1 = int( 0*N1*ld, theta, Lpipe/L, 1 );

% get the moment of lateral rotation of starting point of
drogue
    MomentDlat1 = int( 0*N2*ld, theta, Lpipe/L, 1 );

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
% get forces and moments of end point of drogue
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% get the force of up and down direction of end point of
drogue
    ForceDr2 = int( unitGravityDrogue*N3*L, theta, Lpipe/L, 1
);

% get the moment of up and down rotation of end point of
drogue
    MomentDr2 = int( unitGravityDrogue*N4*L, theta, Lpipe/L, 1
);

% get the force of lateral direction of end point of drogue
    ForceDlat2 = int( 0*N3*L, theta, Lpipe/L, 1);

% get the moment of lateral rotation of end point of drogue
    MomentDlat2 = int( 0*N4*L, theta, Lpipe/L, 1 );

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
% get the whole force and moment response of system under the
% influence % of gravity
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% extend force and moment vector
Forcer1 = [Forcer1 zeros(1,2)];
Momentr1 = [Momentr1 zeros(1,2)];
Forcelat1 = [Forcelat1 zeros(1,2)];
Momentlat1 = [Momentlat1 zeros(1,2)];

Forcer2 = [zeros(1,1) Forcer2 zeros(1,1)];
Momentr2 = [zeros(1,1) Momentr2 zeros(1,1)];
Forcelat2 = [zeros(1,1) Forcelat2 zeros(1,1)];
Momentlat2 = [zeros(1,1) Momentlat2 zeros(1,1)];

ForceDr = [zeros(1,n) ForceDr1 ForceDr2];
MomentDr = [zeros(1,n) MomentDr1 MomentDr2];
ForceDlat = [zeros(1,n) ForceDlat1 ForceDlat2];
MomentDlat = [zeros(1,n) MomentDlat1 MomentDlat2];

% get the whole response
Force = Forcer1 + Forcer2 + ForceDr;
Moment = Momentr1 + Momentr2 + MomentDr;
Forcelat = Forcelat1 + Forcelat2 + ForceDlat;
Momentlat = Momentlat1 + Momentlat2 + MomentDlat;

```

```
% get the whole Force and Moment vector
for i = 0:n+1;
```

```
    FM(4*i+1) = Force(i+1);
    FM(4*i+2) = Moment(i+1);
    FM(4*i+3) = Forcelat(i+1);
    FM(4*(i+1)) = Momentlat(i+1);
```

```
end;
```

```
FM = FM';
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
```

```
% add an exponential force on the system
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
```

```
%define exponential force
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
```

```
t=0:1:10;
```

```
account = size(t);
```

```
expInput1 = -1./exp(t);
expInput1 = 10 + 10*expInput1;
```

```
% define the angle of expnential force
angle = pi/4;
```

```
% get the exponential force division in the up and down
direction
expInputZ1 = expInput1 * cos(angle);
```

```
% get the exponential force division in lateral direction
expInputY1 = expInput1 * sin(angle);
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
```

```
% get response of exponential forces and moments of starting
point % of pipe element
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
```

```

% get the force of up and down direction of starting point
for j = 1:n;
    for i = 1:account(2);
        expForcer1(j,i) = int( expInputZ1(i) * N1*L, theta,
(j-1)*ratio, j*ratio );
    end;
end;

% get the moment of up and down rotation of starting point
for j = 1:n;
    for i = 1:account(2);
        expMomentr1(j,i) = int( expInputZ1(i) * N2*L, theta,
(j-1)*ratio, j*ratio );
    end;
end;

% get the force of lateral direction of starting point
for j = 1:n;
    for i = 1:account(2);
        expForcelat1(j,i) = int( expInputY1(i)*N1*L, theta, (j-
1)*ratio, j*ratio );
    end;
end;

% get the moment of lateral rotation of starting point
for j = 1:n;
    for i = 1:account(2);
        expMomentlat1(j,i) = int( expInputY1(i)*N2*L, theta,
(j-1)*ratio, j*ratio );
    end;
end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
% get response of exponential forces and moments of end point
of      % pipe element

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% get the force of up and down direction of end point
for j = 1:n;
    for i = 1:account(2);
        expForcer2(j,i) = int( expInputZ1(i)*N3*L, theta, (j-
1)*ratio, j*ratio );
    end;
end;

% get the moment of up and down rotation of end point
for j = 1:n;

```

```

        for i = 1:account(2);
            expMomentumr2(j,i) = int( expInputZ1(i)*N4*L, theta, (j-1)*ratio, j*ratio );
        end;
    end;

% get the force of lateral direction of end point
for j = 1:n;
    for i = 1:account(2);
        expForcelat2(j,i) = int( expInputY1(i)*N3*L, theta, (j-1)*ratio, j*ratio );
    end;
end;

% get the moment of lateral rotation of end point
for j = 1:n;
    for i = 1:account(2);
        expMomentlat2(j,i) = int( expInputY1(i)*N4*L, theta, (j-1)*ratio, j*ratio );
    end;
end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%get forces and moments of starting point of drogue
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% get the force of up and down direction of starting point of drogue
expForceDr1= int( expInputZ1 * N1*L, theta, Lpipe/L, 1 );

% get the moment of up and down rotation of starting point of drogue
expMomentDr1 = int( expInputZ1 * N2*L, theta, Lpipe/L, 1);

% get the force of lateral direction of starting point of drogue
expForceDrLat1 = int( expInputY1 * N1*L, theta, Lpipe/L, 1 );

% get the moment of lateral direction of starting point of drogue
expMomentDrLat1 = int( expInputY1 * N2*L, theta, Lpipe/L, 1);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
% get forces and moments of end point of drogue

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% get the force of up and down direction of end point of
drogue
    expForceDr2 = int( expInputZ1 * N3*L, theta, Lpipe/L, 1);

% get the moment of up and down rotation of end point of
drogue
    expMomentDr2 = int( expInputZ1* N4*L, theta, Lpipe/L, 1 );

% get the force of lateral direction of end point of drogue
    expForceDrLat2 = int( expInputY1 * N3*L, theta, Lpipe/L, 1
);

% get the moment of lateral direction of end point of drogue
    expMomentDrLat2 = int( expInputY1 * N4*L, theta, Lpipe/L,
1);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% get the whole force and moment response of system under the
influence of exponential force
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% extend force and moment vector
expForcer1 = [ expForcer1; zeros( 2, account(2) ) ];
expMomenr1 = [ expMomenr1; zeros( 2, account(2) ) ];
expForcelat1 = [ expForcelat1; zeros( 2, account(2) ) ];
expMomenlat1 = [ expMomenlat1; zeros( 2, account(2) ) ];

expForcer2 = [ zeros(1,account(2) ); expForcer2; zeros( 1,
account(2) ) ];
expMomenr2 = [ zeros(1,account(2) ); expMomenr2; zeros( 1,
account(2) ) ];
expForcelat2 = [ zeros(1,account(2) ); expForcelat2; zeros( 1,
account(2) ) ];
expMomenlat2 = [ zeros(1,account(2) ); expMomenlat2; zeros(
1, account(2) ) ];

expForceDr = [zeros( n, account(2) ); expForceDr1;
expForceDr2];
expMomentDr = [zeros( n, account(2) ); expMomentDr1;
expMomentDr2];
expForceDrLat = [zeros( n, account(2) ); expForceDrLat1;
expForceDrLat2];
expMomentDrLat = [zeros( n, account(2) ); expMomentDrLat1;
expMomentDrLat2];

```

```

% get the whole response
expForce = expForcer1 + expForcer2 + expForceDr;
expMoment = expMomentr1 + expMomentr2 + expMomentDr;
expForceLat = expForcelat1 + expForcelat2 + expForceDrLat;
expMomentLat = expMomentlat1 + expMomentlat2 +
expMomentDrLat;

% get the whole Force and Moment vector
for j = 1 : account(2);

    for i = 0:n+1;
        expFM1(4*i+1,j) = expForce(i+1,j);
        expFM1(4*i+2,j) = expMoment(i+1,j);
        expFM1(4*i+3,j) = expForceLat(i+1,j);
        expFM1(4*(i+1),j) = expMomentLat(i+1,j);
    end;

end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% apply another exponenetial force only on the drogue

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
% define the exponential force on the drogue only
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

expInput2 = -1./exp(t);

expInput2 = 1 + expInput2;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%get forces and moments of starting point of drogue
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% as there is no force distribution in the lateral direction,
the force % response is 0

```



```

% get the force of up and down direction of starting point of
drogue
exp2ForceDr1 = int( expInput2 * N1*L, theta, Lpipe/L, 1 );

% get the moment of up and down rotation of starting point of
drogue
exp2MomentDr1 = int( expInput2 * N2*L, theta, Lpipe/L, 1
);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
% get forces and moments of end point of drogue

% get the force of up and down direction of end point of
drogue
exp2ForceDr2 = int( expInput2 * N3*L, theta, Lpipe/L, 1
);

% get the moment of up and down rotation of end point of
drogue
exp2MomentDr2 = int( expInput2 * N4*L, theta, Lpipe/L, 1
);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%

% get the whole force and moment response of system under the
influence % of exponential force

exp2ForceP = zeros( n+2, account(2) );
exp2MomentP = zeros( n+2, account(2) );

exp2ForceDr = [zeros( n, account(2) ); exp2ForceDr1;
exp2ForceDr2];
exp2MomentDr = [zeros( n, account(2) ); exp2MomentDr1;
exp2MomentDr2];

% get the whole response
exp2Force = exp2ForceP + exp2ForceDr;
exp2Moment = exp2MomentP + exp2MomentDr;

exp2Forcelat = zeros( n+2, account(2) );
exp2Momentlat = zeros( n+2, account(2) );

% get the whole Force and Moment vector
for j = 1 : account(2);

    for i = 0:n+1;

```

```

        expFM2(4*i+1,j) = exp2Force(i+1,j);
        expFM2(4*i+2,j) = exp2Moment(i+1,j);
        expFM2(4*i+3,j) = exp2Forcelat(i+1,j);
        expFM2(4*(i+1),j) = exp2Momentlat(i+1,j);
    end;

end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% get the state space function of the system

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% as the starting end of the pipe is fixed, so the displace
equals
% zero

Mr = M(1:4, 1:4);
Ms = M(5:4*(n+2), 5:4*(n+2));
Msinv = inv(Ms);

Kr = K(1:4, 1:4);
Ks = K(5:4*(n+2), 5:4*(n+2));

Cr = C(1:4, 1:4);
Cs = C(5:4*(n+2), 5:4*(n+2));

FMr = FM(1:4,1);
FMs = FM(5:4*(n+2),1);

expFM1r = expFM1(1:4,:);
expFM1s = expFM1(5:4*(n+2),:);

expFM2r = expFM2(1:4,:);
expFM2s = expFM2(5:4*(n+2),:);

sz = size(Ms);

A = [ zeros(sz) eye(sz); -Msinv*Ks -Msinv*Cs ];

for i = 1: account(2);
    B2(:,i) = Msinv*expFM1s(:,i);
end;

```

```

for i = 1: account(2);
    B3(:,i) = Msinv*expFM2s(:,i);
end;

% calculate the response at t = 1
B = [ zeros(sz(1), 3); Msinv*FMs B2(:,2) B3(:,2) ];

D = [zeros(1, 2*sz(2))];

E = zeros( 1, 3 );

```

APPENDIX IV

Vertical displacements for each node. Set $\alpha = 1$, and $\beta = 0.001$, including the effect of the exponential force.

The displacement at the 9th node with the previously defined parameters is shown by Figure 1

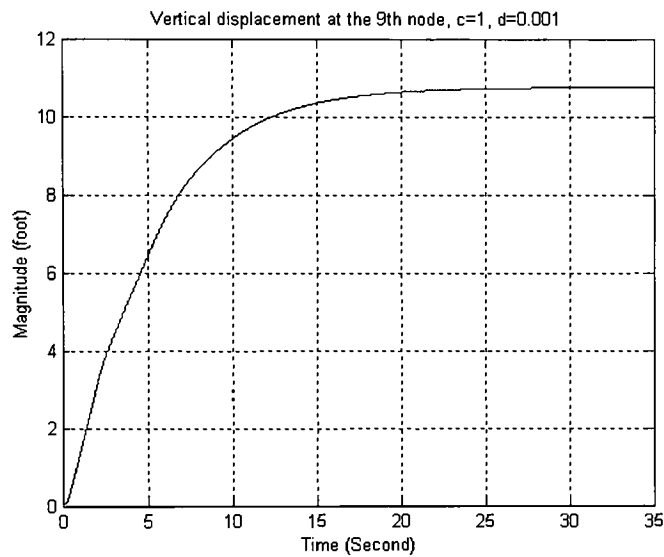


Figure 1 Vertical displacement at the 9th node, c= 1 d=0.001

The displacement at the 8th node with the previously defined parameters is shown by Figure 2

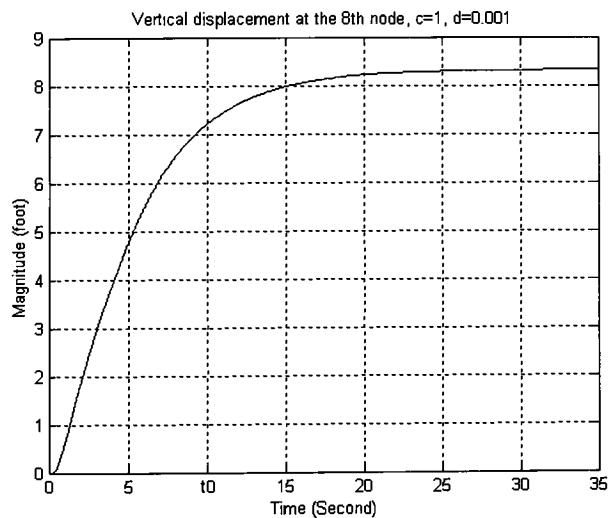


Figure 2 Vertical displacement at the 8th node, c= 1 d=0.001

The displacement at the 7th node with the previously defined parameters is shown by Figure 3

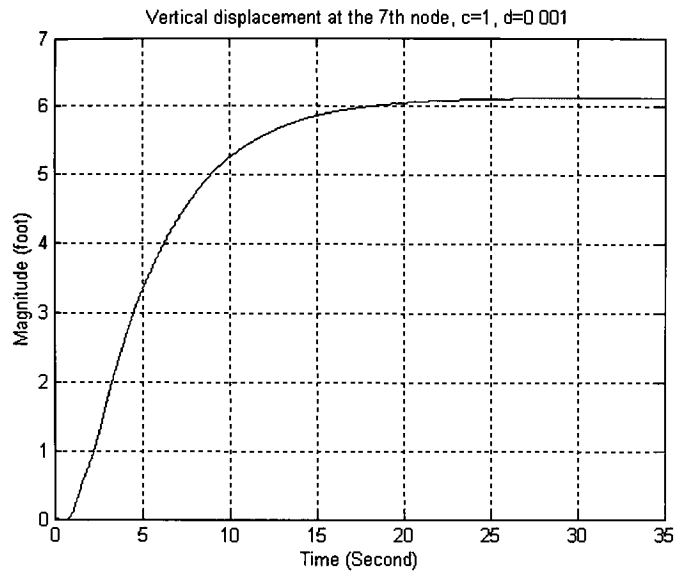


Figure 3 Vertical displacement at the 7th node, c= 1 d=0.001

The displacement at the 6th node with the previously defined parameters is shown by Figure 4

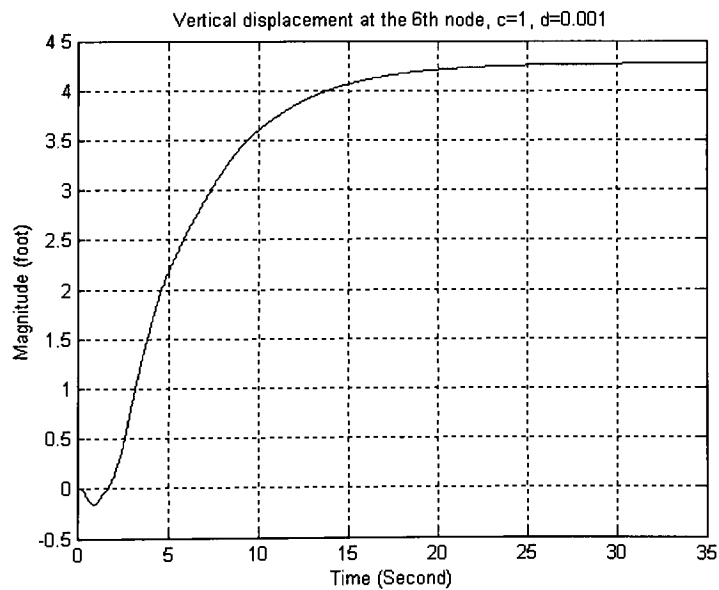


Figure 4 Vertical displacement at the 6th node, c= 1 d=0.001

The displacement at the 5th node with the previously defined parameters is shown by Figure 5

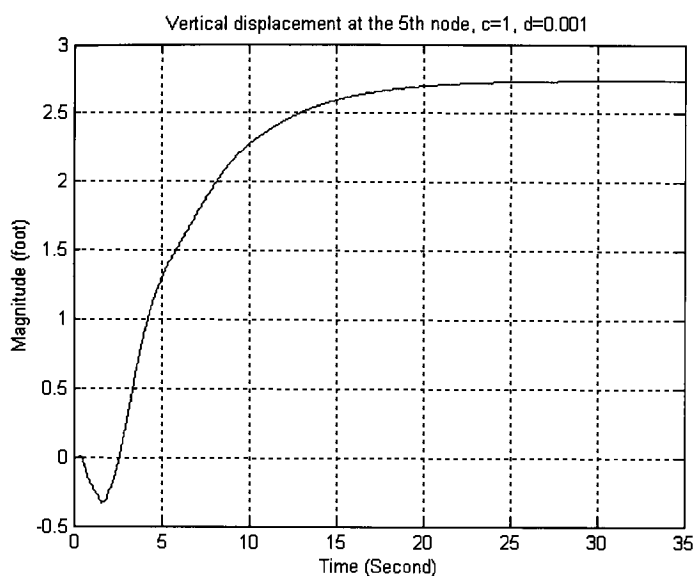


Figure 5 Vertical displacement at the 5th node, c= 1 d=0.001

The displacement at the 4th with the previously defined parameters is shown by Figure 6

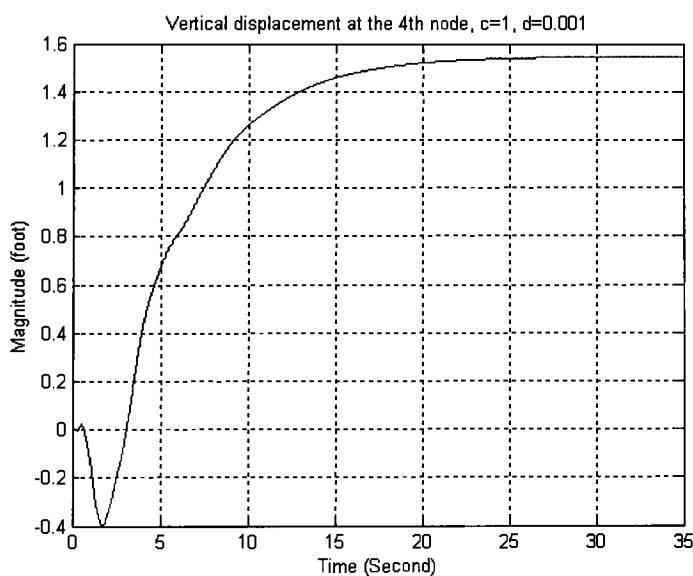


Figure 6 Vertical displacement at the 4th node, c= 1 d=0.001

The displacement at the 3rd node with the previously defined parameters is shown by Figure 7

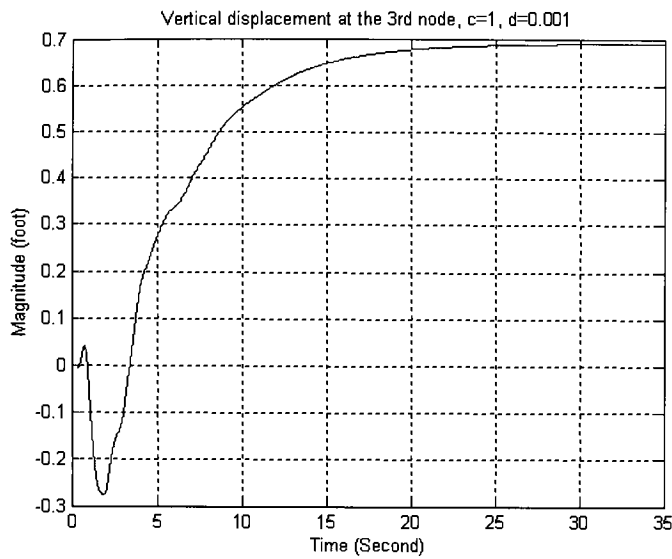


Figure 7 Vertical displacement at the 3rd node, c= 1 d=0.001

The displacement at the 2nd with the previously defined parameters is shown by Figure 8

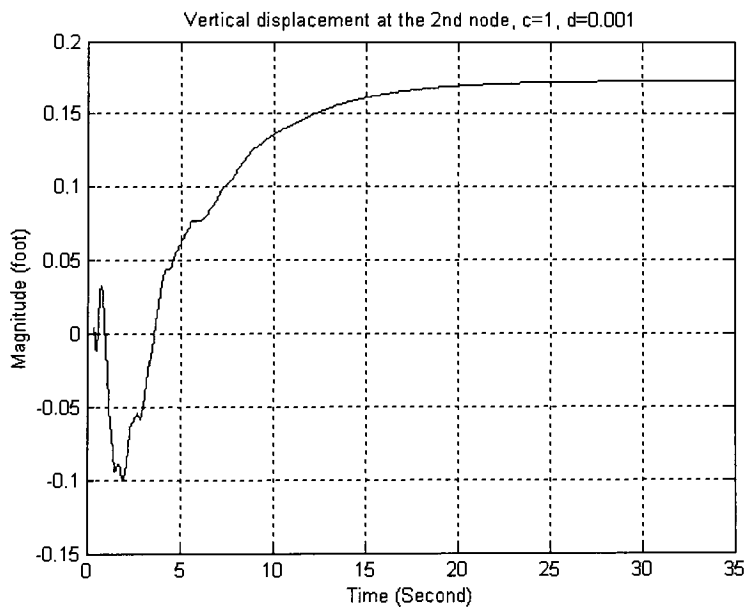


Figure 8 Vertical displacement at the 2nd node, c= 1 d=0.001