## 1. Introduction

The goal of this project is to analyze and compare the heat transfer performance of rods with different cross-sectional shapes under identical surface area and length constraints. The analysis is done to determine which shape results in the fastest cooling (highest heat transfer rate, q) when exposed to natural (free) convection as well as forced convection in air and water, both externally and internally. The three shapes analyzed are:

* Circular Cylinder
* Square Cross-Section Rod
* Cone (A maximized internal volume was chosen since a minimized volume is degenerate)

All rods are solid, of equal length (6 inches = 0.1524 m), and with a fixed lateral surface area equivalent to that of the circular cylinder (~0.005 m2).

## 2. Material and Environment Parameters

All material properties, fluid properties and equations are attributed to (Bergman et. al).

**Initial Rod Temperature:** 100°C (373.15 K)  
**Ambient Temperature:** 20°C (293.15 K)

**Fluid Velocity (V) for Forced Convection Cases:**

**Properties of Rod (Table A.1)**

**Rod Material:** Aluminum  
**Thermal Conductivity (k):** 237 W/m-K  
**Density ()**: 2702 kg/m3  
**Specific Heat ()**: 903 J/kg-K  
**Thermal Diffusivity ()**:

Table 1: Properties of convective fluid.

|  |  |  |
| --- | --- | --- |
| **Fluid** | **Air** (Table A.4) | **Water** (Table A.6) |
| **Density (** |  |  |
| **Dynamic Viscosity (** |  |  |
| **Thermal Conductivity (** |  |  |
| **Prandtl Number ()** |  |  |

The above values were calculated via linear interpolation from the tables using the formula:

## 3. Modes of Heat Transfer

The primary mode of heat transfer is **convection**:

* **Forced convection**: heat transfer from externally driven air or water flow.
* **Internal forced convection**: heat transfer from the internal wall of hollow rods to a flowing fluid.
* **Free (natural) convection**: buoyancy-driven flow in quiescent air/water.

## 4. Methodology Overview

A structured Python program was developed to automate the analysis. Each rod shape was processed under consistent assumptions:

1. Constant surface temperature (100°C)
2. Uniform surface area (~0.005 m²)
3. Use of empirical Nusselt number correlations for each convection regime
4. Geometric transformations to maintain equal surface area between shapes

**Key Steps**

1. **Film Temperature Calculation:** used to interpolate thermal properties of air and water.
2. **Geometry Calculations:** Maintain equal surface area for all shapes.
3. **Cone Optimization:** Use *scipy.optimize.minimize* to maximize volume under fixed surface area. The maximized cone condition was used since the minimized cone volume is the degenerative case (.
4. **Forced Convection (External and Internal):** Use Churchill-Bernstein correlation for external flows or constant Nusselt number for internal laminar flow.
5. **Free Convection:** Use empirical correlations (see Appendix I) for vertical plates, horizontal surfaces, and inclined surfaces.
6. **Cone Approximation:** Given that there are no readily available correlations for a cone, the shape was approximated via varying methods.

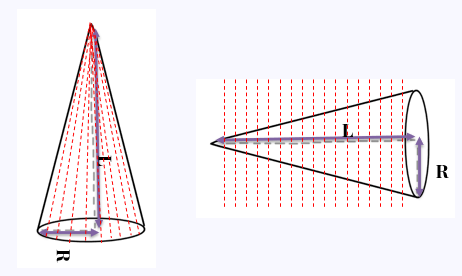


Figure 1: Two Discretization Methods of the Cylinder

* 1. For the internal and external forced convection cases, the cone was discretized into N cylinders of decreasing diameter. Each cylinder was analyzed individually, and the heat transfer rates were summed. This is represented in the right image of Figure 1.
  2. In the free convection case, the cone was approximated by 2 methods:
     1. A circular base and N inclined plates at an angle because correlations for these geometries are available in Table 9.3. This is represented in the left image of Figure 1.
     2. The discretized cylinder method, as detailed in part a, was also used in the free convection case and yielded similar results as the vertical plate method.

## 5. Forced External Convection (Ch. 7)

Direction of fluid flow

A drawing of a cone

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Figure 2: Direction of fluid flow across all three shapes, flow is perpendicular to the length of each shape, L.

1. Compute Reynolds number
2. Apply Churchill-Bernstein correlation (Eq. A5) to compute
3. Calculate (Eq. A.8) and then (Eq. A9)
   * For the cylinder, For the square rod, , where *w* can be found via Eq. A11.
   * For the cone, a series of N cylinders of height were used, each with its own and corresponding and . The total heat transfer rate was the sum over all cylinders.

Table 2: Calculated Reynolds and Nusselt numbers, heat transfer coefficients and heat transfer rates in forced external convection for air.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Shape** | **Re** | **Nu** | **h (W/m²K)** | **q (W)** |
| **Cylinder** | 1500 | 19.59 | 59.15 | 21.58 |
| **Square** | 1178 | 17.33 | 66.60 | 24.30 |
| **Cone** | Varies | Varies | 40.05 (Average) | 9.66 |

In the case of forced external convection, the square rod cools the fastest. At 20°C and *v* = 3 m/s, the square rod has the highest heat transfer rate due to stronger localized convection effects, followed by the cylinder. The cone exhibits the lowest heat transfer rate due to its tapering geometry affecting flow characteristics. Below are the results when using water instead of air:

Table 3: Calculated Reynolds and Nusselt numbers, heat transfer coefficients and heat transfer rates in forced external convection for water.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Shape** | **Re** | **Nu** | **h (W/m²K)** | **q (W)** |
| **Cylinder** | 60240 | 267.8 | 18380 | 6706 |
| **Square** | 47310 | 230.0 | 20099 | 7333 |
| **Cone** | Varies | Varies | 14035 (Average) | 3387 |

When water is used instead of air, all shapes experience significantly higher Reynolds numbers, Nusselt numbers, and heat transfer coefficients. This led to increased convective heat transfer rates. Moreover, the rank order where the cone, cylinder, and cone have the lowest to highest heat transfer rates respectively, remains consistent.

The cone is the most challenging to analyze due to the lack of well-established forced convection correlations for varying cross-sections. To estimate the heat transfer rate for the cone, its varying diameter was accounted for along the length by treating it as a series of N cylindrical segments, each with a small height. This approach allowed us to apply standard convection correlations to each segment separately to find the heat transfer coefficient. Summing the individual heat transfer contributions across all segments provided the total heat transfer rate, ensuring an accurate estimation despite the geometric complexity.

## 6. Forced Internal Convection (Ch. 8)

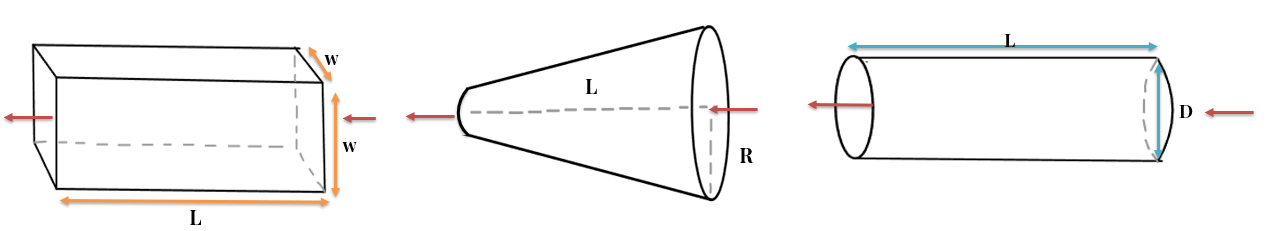


Figure 3: Direction of fluid flow indicated by red arrows for forced internal convection

1. Use characteristic dimensions (diameter for cylinder and cone, width for square)
   * Note: Each of the shapes *were assumed to be thin walled* such that the internal dimensions can be considered equal to the outside dimensions in Section 5 analysis. Thus, taking the internal surface area to be equal to the external surface area (~0.005 m²).
2. Compute and determine flow regime.
3. Use appropriate Nusselt number correlation:
   * Laminar: Eq. A6
   * Transitional: Eq. A13 and Eq. A14
   * Turbulent: Eq. A7
4. Calculate and using Eq. A8 and Eq. A9, respectively.
   * Cone treated as a series of segments, each evaluated separately with the above method, and summed.

Table 4: Calculated Reynolds and Nusselt numbers, heat transfer coefficients and heat transfer rates in forced internal convection for air.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Shape** | **Re** | **Nu** | **h (W/m²K)** | **q (W)** |
| **Cylinder** | 1500 | 3.66 | 11.05 | 4.03 |
| **Square** | 1178 | 3.66 | 14.07 | 4.18 |
| **Cone** | Varies | Varies | 16.64 | 2.27 |

For forced internal convection, we see a similar trend as in the external convection case. The square cross section performs the best and transfers the most heat while the calculated heat transfer rate of the cone is significantly lower. Compared to the cylinder, the square rod benefits from enhanced flow interactions at its corners, improving convective exchange. The cone presents the greatest challenge in analysis due to its varying diameter, causing non-uniform velocity and temperature distributions, which contribute to its lower heat transfer rate.

Below are the results when using water instead of air:

Table 5: Calculated Reynolds and Nusselt numbers, heat transfer coefficients and rates in forced external convection for water.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Shape** | **Re** | **Nu** | **h (W/m²K)** | **q (W)** |
| **Cylinder** | 60238 | 237.3 | 16288 | 5942 |
| **Square** | 47311 | 195.6 | 17094 | 6237 |
| **Cone** | Varies | Varies | 7239 | 1747 |

When water replaces air as the working fluid, heat transfer rates increase across all shapes due to water’s higher thermal conductivity and lower kinematic viscosity, leading to significantly higher Reynolds and Nusselt numbers. This results in enhanced convective cooling compared to air, particularly for the cylinder and square rod, which maintain stable flow characteristics. The cone, however, remains difficult to model due to its varying cross-section, which introduces non-uniform velocity distributions. To estimate its performance, it is treated as a series of N cylinders. Based on these assumptions, the correlation values for the cone are likely to fall between those of a cylinder and a converging duct (Wang et. al.).

## Free Convection (Ch. 9)

A drawing of a cone

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Figure 4: Orientation of shapes in free convection.

### 7.1. Circular Rod

1. Compute *Ra* with Eq. A1, using the diameter of the cylinder as the characteristic length.
2. Find empirical constants *c* and *n*, referring to Table 9.2.
3. Compute the Nusselt number, with the derived empirical constants.
4. Calculate the convective heat transfer coefficient, *h* (Eq. A8) with the diameter of the cylinder as the characteristic length.
5. Compute the total heat transfer rate, *q*, using Eq. A9.

### 7.2. Square Rod

The square rod is treated as four plates: two vertical, one top, and one bottom, each with different correlations.

1. Compute the Rayleigh number based on the characteristic length, *w,* (Eq. A11).
2. Calculate the Nusselt numbers:
   * Vertical plates: Eq. A2
   * Top plate: Eq. A3
   * Bottom plate: Eq. A4
3. Compute *h*, for each plate with Eq. A8 using their respective Nusselt number.
4. Calculate the total heat transfer rate: .

### 7.3. Cone

The cone is analyzed under a maximum volume condition. This approach involves optimizing geometric parameters while ensuring the same total surface area as the other shapes.

For the cone, the analysis is done two ways:

1. The geometry is viewed as a circular base with N inclined plates surrounding it.
2. Establish surface constraints:
   * Base: where
   * Inclined plates: where s is the slant height of the cone and thus,
3. Compute the Rayleigh number:
   * Base: Eq. A1
   * Inclined plates: Eq. A1 with replacing where .
4. Compute *Nu,* for the circular base and the inclined plates:
   * Circular base: Eq. A4
   * Lateral plates: Eq. A2
5. Calculate *h,* for the base and the plates using Eq. A8 with their corresponding .
6. Calculate *q*, for the base and plates using Eq A.9 with their respective areas.

Compute the total heat transfer rate:

1. The geometry is viewed as N cylinders of decreasing diameter and height .
   1. For this method, we individually analyze each discrete section of the cone, which is an approximate cylinder for small using the same analysis as in section 7.1.
   2. Compute with Eq. A1, using the diameter of the cylinder as the characteristic length for the cylinder.
   3. Compute the Nusselt number for the cylinder, with the empirical constants *c* and *n* from Table 9.2.
   4. Calculate the convective heat transfer coefficient for the cylinder, (Eq. A8) with the diameter of the cylinder as the characteristic length.
   5. Compute the heat transfer rate for each discrete cylinder and the total heat transfer rate by summing the discrete values such that .

Table 6: Calculated Reynolds and Nusselt numbers, heat transfer coefficients and heat transfer rates in free convection for air.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Shape** | **Ra** | **Nu** | **h (W/m²K)** | **q (W)** |
| **Cylinder** | 3864 | 4.02 | 12.12 | 4.42 |
| **Square** | 1872 |  |  | 4.77 |
| **Cone** | Varies | Varies | 12.44 | 2.66 for the angled plate analysis (or 2.58 for the stacked cylinders method) |

Based on the free convection analysis, the square rod exhibits the highest total heat transfer rate, slightly outperforming the cylinder, while the cone has the lowest heat transfer rate. The square rod benefits from multiple surface orientations, allowing for greater convective exchange, while the cylinder maintains a more uniform flow around its curved surface. The cone, with its varying geometry, results in inconsistent Rayleigh and Nusselt numbers, leading to lower overall convective efficiency. Overall, given these results, the square rod has the largest heat transfer rate. The orientation and discretization of the cone also affected the heat transfer rate as we see the method using the inclined plates (See Figure 1 and Table 6) resulted in a slightly higher heat transfer rate.

When oriented vertically, the cone is analyzed as N inclined plates plus one circular base. Unlike a square or cylindrical rod, which maintains a consistent cross-sectional area, the cone’s sloped surfaces cause variations in flow interaction, leading to non-uniform heat transfer distribution. The inclined plates, angled relative to the flow direction, experience less convection. Additionally, since each inclined plate has a different effective Rayleigh number, convective forces do not distribute evenly, resulting in a lower Nusselt number across the cone’s surface. The circular base, though treated as a flat plate, does not contribute significantly enough to compensate for the heat transfer limitations of the inclined sections, leading to an overall lower heat transfer rate.

When oriented horizontally, the cone is treated as N stacked cylindrical segments of increasing diameter. This segmentation introduces disruptions in flow uniformity as the air or fluid moves through sections of varying Reynolds numbers. This leads to lower convection coefficients and lower heat transfer rates.

## 8. Results

The table below summarizes the heat transfer rates for a cylinder rod, square rod, and cone calculated for external flow, internal flow, and free convection cases.

Table 7: Summary of heat transfer rates for a cylinder, square, and cone in forced external, forced internal, and free convection.

|  |  |  |  |
| --- | --- | --- | --- |
| **Shape** | **Ch7 -- Forced External (q)** | **Ch8 -- Forced Internal (q)** | **Ch9 -- Free Convection (q)** |
| **Cylinder** | 21.58 W | 4.03 W | 4.42 W |
| **Square** | 24.30 W | 4.18 W | 4.77 W |
| **Cone** | 9.66 W | 2.27 W | 2.66 W |

Across all convection regimes, the square rod consistently outperforms both the cylindrical and conical shapes in terms of total heat transfer rate, despite having the same surface area and length constraints. This performance can be attributed to its higher ratio of surface area to internal volume, particularly in the corners and flat faces that are further from the internal volume and promote stronger localized convection. The cylindrical rod comes in second in all cases. Due to its smooth, uniform surface, it experiences more streamlined flow behavior but lacks the turbulent enhancements seen in the square profile.

The cone, despite having the same surface area, shows the lowest performance in all three modes. This is largely due to its non-uniform geometry, which introduces complex flow patterns, inconsistent local Reynolds and Rayleigh numbers, and reduced average convective coefficients. The cone's design, partially due to the maximized internal volume constraint, is less efficient at transferring heat due to a combination of smaller effective cross-sectional areas along its length with larger internal volumes.

To better understand the performance difference, the following ratios (normalized to the square rod performance) can be analyzed:

|  |  |  |  |
| --- | --- | --- | --- |
| **Shape** | **Forced External** | **Forced Internal** | **Free Convection** |
| Cylinder | 0.89 | 0.96 | 0.93 |
| Square | **1.00** | **1.00** | **1.00** |
| Cone | 0.40 | 0.54 | 0.56 |

These normalized values clearly demonstrate the superiority of the square rod across all thermal performance categories. The cone’s shape, while perhaps optimal in some volumetric or structural contexts, proves thermally inefficient under convection-dominated conditions.

## 9. Application: Heat Exchanger Enhancement

Given the above findings, square rods represent a strong candidate for applications where heat removal is critical, such as compact heat exchangers. There are multiple benefits for using a square profile in a heat exchanger. It is common to see circular tubing in heat exchangers because circular tubes are more cost-effective and lighter for the same structural strength. However, as shown above in Section 5, the heat transfer coefficient for forced external convection is 66.60 W/m²K for the square cross section and 59.15 W/m²K for the cylinder so square tubes are a viable option to increase the heat transfer rate. Despite traditional reliance on circular tubing for structural benefits vs. cost, the square geometry offers a notable performance boost in convective heat transfer that may justify the trade-off in certain designs.

Diagram of a rectangular object with text

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Figure 5: Heat Exchanger Sample with Square Tubes

## 10. Proposed Experiment

For analyzing the cone and finding c and m values for the Hilpert correlation computational fluid dynamics (CFD) is recommended for its ease of access and ability to perform quick iterations.

1. Model the shape, such as the cone, in a cad software package such as Design Modeler. Define the fluid domain surrounding the object (i.e., subtract the solid geometry from a larger fluid block) to simulate external flow. Create a “wind tunnel” for the shape making sure to increase the inlet length to achieve fully developed laminar flow and to increase the length of the outlet so that no backflow occurs.
2. Discretize (mesh) the shape in ANSYS Mesher, Refine the mesh near the object-fluid interface.

A blue rectangular object with a white triangle

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Figure 6: Diagram of computational fluid dynamic analysis for square shape (2D model).

1. Use CFD software such as ANSYS Fluent. Input boundary conditions such as input fluid velocity and input fluid temperature defined by laminar flow.
2. Run the simulation.
3. Compute the bulk average fluid temperature, area average surface temperature of the cone-fluid boundary, and area average heat flux.
4. Compute the heat transfer coefficient. More info at [ANSYS Heat Transfer Coefficient Tutorial](https://innovationspace.ansys.com/courses/courses/topics-in-convective-heat-transfer-simulations/lessons/defining-heat-transfer-coefficient/).
5. Perform this for a variety of Reynolds numbers ranging from Re = 0 to Re = 400,000.
6. The coefficients (c and m) are found based on a least squares curve fit of measured data.

## 11. References

Bergman, T., Lavine, A., Incropera, F. and Dewitt, D. (2011). *Fundamentals of Heat and Mass Transfer*. 8th ed. Hoboken: J. Wiley & Sons, Cop.

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Sparrow, E. M., Abraham, J. P., and Tong, J. C. K., 2004, “Archival correlations for average heat transfer coefficients for non-circular and circular cylinders and for spheres in cross-flow,” International Journal of Heat and Mass Transfer, **47**(24), pp. 5285–5296.

Wang, X., Bibeau, E., and Naterer, G. F., 2007, “Experimental correlation of forced convection heat transfer from a NACA airfoil,” Experimental Thermal and Fluid Science, **31**(8), pp. 1073–1082.

## Appendix I: Equations and Correlations Used

**Eq. A1:** use for angled plates

**Eq. A2:** for vertical plates

**Eq. A3:** for top plates

**Eq. A4:** for bottom plates

**Eq. A5:** Churchill-Bernstein equation for cross-flow cylinders

**Eq. A6:** for laminar internal flow

**Eq. A7:** for turbulent internal flow

**Eq. A8:** where is characteristic length

**Eq. A9:**

**Eq. A10:**

**Eq. A11:**  gives the side length of a square rod

**Eq. A12:**

**Eq. A13:**  flow friction factor for transitional flow

**Eq. A14:** for internal convection in transitional flow

## Appendix II: Python Script

The python script used for the analysis can be found in the following GitHub Repository:

<https://github.com/gkret123/ME-342_HeatTransfer_FinalProject>