## 1. Introduction

The goal of this project is to analyze and compare the heat transfer performance of rods with different cross-sectional shapes under identical surface area and length constraints. The analysis is done to determine which shape results in the fastest cooling (highest heat transfer rate, q) when exposed to natural (free) convection as well as forced convection in air and water, both externally and internally. The three shapes analyzed are:

* Circular Cylinder
* Square Cross-Section Rod
* Cone (A maximized internal volume was chosen since a minimized volume is degenerate)

All rods are solid, of equal length (6 inches = 0.1524 m), and with a fixed lateral surface area equivalent to that of the circular cylinder (~0.005 m2).

## 2. Material and Environment Parameters

**Initial Rod Temperature:** 100°C (373.15 K)  
**Ambient Temperature:** 20°C (293.15 K)

**Fluid Velocity (V) for Forced Convection Cases:**

**Properties of Rod (Table A.1)**

**Rod Material:** Aluminum  
**Thermal Conductivity (k):** 237 W/m-K  
**Density ()**: 2702 kg/m3  
**Specific Heat ()**: 903 J/kg-K  
**Thermal Diffusivity ()**:

Table 1 Properties of convective fluid.

|  |  |  |
| --- | --- | --- |
| **Fluid** | **Air** (Table A.4) | **Water** (Table A.6) |
| **Density (** |  |  |
| **Dynamic Viscosity (** |  |  |
| **Thermal Conductivity (** |  |  |
| **Prandtl Number ()** |  |  |

The above values were calculated via linear interpolation from the tables using the formula:

## 3. Modes of Heat Transfer

The primary mode of heat transfer is **convection**:

* **Forced convection**: heat transfer from externally driven air or water flow.
* **Internal forced convection**: heat transfer from the internal wall of hollow rods to a flowing fluid.
* **Free (natural) convection**: buoyancy-driven flow in quiescent air/water.

## 4. Methodology Overview

A structured Python program was developed to automate the analysis. Each rod shape was processed under consistent assumptions:

1. Constant surface temperature (100°C)
2. Uniform surface area (0.005 m²)
3. Use of empirical Nusselt number correlations for each convection regime
4. Geometric transformations to maintain equal surface area between shapes

**Key Steps**

1. **Film Temperature Calculation:** used to interpolate thermal properties of air and water.
2. **Geometry Calculations:** Maintain equal surface area for all shapes.
3. **Cone Optimization:** Use scipy.optimize.minimize to maximize volume under fixed surface area. The maximized cone condition was used since the minimized cone volume is the degenerative case (.
4. **Forced Convection (External and Internal):** Use Churchill-Bernstein correlation for external flows or constant Nusselt number for internal laminar flow.
5. **Free Convection:** Use empirical correlations (see Appendix I) for vertical plates, horizontal surfaces, and inclined surfaces.
6. **Cone Approximation:** Given that there are no readily available correlations for a cone, the shape way approximated in 2 ways.
   1. For the internal and external forced convection cases, the cone was discretized into N cylinders of decreasing diameter. Each cylinder was analyzed individually, and the heat transfer rates were summed.
   2. In the free convection case, the cone was approximated by a circular base and N inclined plates at an angle because correlations for these geometries are available in Table 9.3. The discretized cylinder method as detailed above was also used in the free convection case and yielded similar results.

## 5. Forced External Convection (Ch. 7)

1. Compute Reynolds number
2. Apply Churchill-Bernstein correlation (Eq. A5) to compute
3. Calculate (Eq. A.8) and then (Eq. A9)
   * For the cylinder, For the square rod,
   * For the cone, a series of N cylinders of height were used, each with its own and corresponding and . The total heat transfer rate was the sum over all cylinders.

Table 2

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Shape** | **Re** | **Nu** | **h (W/m²K)** | **q (W)** |
| **Cylinder** | 1500 | 19.59 | 59.15 | 21.58 |
| **Square** | 1178 | 17.33 | 66.60 | 24.30 |
| **Cone** | Varies | Varies | 40.05 (Average) | 9.66 |

In the case of forced external convection, the square rod cools the fastest. In forced convection with air at 20°C and *V* = 3 m/s, the square rod has the highest heat transfer rate due to stronger localized convection effects, followed by the cylinder, while the cone exhibits the lowest heat transfer rate due to its tapering geometry affecting flow characteristics. Below are the results when using water instead of air:

Table 3

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Shape** | **Re** | **Nu** | **h (W/m²K)** | **q (W)** |
| **Cylinder** | 60240 | 267.8 | 18380 | 6706 |
| **Square** | 47310 | 230.0 | 20099 | 7333 |
| **Cone** | Varies | Varies | 14035 (Average) | 3387 |

When water is used instead of air, all shapes experience significantly higher Reynolds numbers, Nusselt numbers, and heat transfer coefficients. Naturally, leading to increased convective heat transfer rates while preserving the rank order. The cone is the most challenging to analyze due to the lack of well-established forced convection correlations for varying cross-sections. Moreover, the cone’s heat transfer performance can be estimated by blending cylindrical and flat plate models, resulting in lower *h* and *q*, compared to the square and cylinder.

## 6. Forced Internal Convection (Ch. 8)

1. Use characteristic dimensions (diameter for cylinder and cone, width for square)
2. Compute and determine flow regime
3. Use appropriate Nusselt number correlation:
   * Laminar: Eq. A6
   * Transitional: Eq. A13 and Eq. A14
   * Turbulent: Eq. A7
4. Calculate and using Eq. A8 and Eq. A9, respectively.
   * Cone treated as a series of segments, each evaluated separately with the above method, and summed.

Table 4

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Shape** | **Re** | **Nu** | **h (W/m²K)** | **q (W)** |
| **Cylinder** | 1500 | 3.66 | 11.05 | 4.03 |
| **Square** | 1178 | 3.66 | 14.07 | 4.18 |
| **Cone** | Varies | Varies | 16.64 | 2.27 |

For forced internal convection, we see a similar trend as in the external convection case. The square cross section performs the best and transfers the most heat while the calculated heat transfer rate of the cone is significantly lower. Compared to the cylinder, the square rod benefits from enhanced flow interactions at its corners, improving convective exchange. The cone presents the greatest challenge in analysis due to its varying diameter, causing non-uniform velocity and temperature distributions, which contribute to its lower heat transfer rate.

Below are the results when using water instead of air:

Table 5

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Shape** | **Re** | **Nu** | **h (W/m²K)** | **q (W)** |
| **Cylinder** | 60238 | 237.3 | 16288 | 5942 |
| **Square** | 47311 | 195.6 | 17094 | 6237 |
| **Cone** | Varies | Varies | 7239 | 1747 |

Then water is used instead of air at the same velocity, all shapes experience higher heat transfer rates due to water’s superior thermal conductivity and lower viscosity, increasing the Reynolds and Nusselt numbers. The cone’s behavior remains difficult to model precisely, but estimations can be made using correlations for inclined plates combined with cylindrical assumptions. Based on these assumptions, the correlation values for the cone are likely to fall between those of a cylinder and a converging duct.

## 7. Free Convection (Ch. 9)

### 7.1. Circular Rod

1. Compute *Ra* with Eq. A1, using the diameter of the cylinder as the characteristic length.
2. Find empirical constants *c* and *n*, referring to Table 9.2.
3. Compute the Nusselt number, with the derived empirical constants.
4. Calculate the convective heat transfer coefficient, *h* (Eq. A8) with the diameter of the cylinder as the characteristic length.
5. Compute the total heat transfer rate, *q*, using Eq. A9.

### 7.2. Square Rod

The square rod is treated as four plates: two vertical, one top, and one bottom, each with different correlations.

1. Compute the Rayleigh number based on the characteristic length, *w,* (Eq. A11).
2. Calculate the Nusselt numbers:
   * Vertical plates: Eq. A2
   * Top plate: Eq. A3
   * Bottom plate: Eq. A4
3. Compute *h*, for each plate with Eq. A8 using their respective Nusselt number.
4. Calculate the total heat transfer rate: .

### 7.3. Cone

The cone is analyzed under a maximum volume condition. This approach involves optimizing geometric parameters while ensuring the same total surface area as the other shapes.

For the cone, the analysis is done two ways:

1. The geometry is viewed as a circular base with N inclined plates surrounding it.
2. Establish surface constraints:
   * Base: where
   * Inclined plates: where s is the slant height of the cone and thus,
3. Compute the Rayleigh number:
   * Base: Eq. A1
   * Inclined plates: Eq. A1 with replacing where .
4. Compute *Nu,* for the circular base and the inclined plates:
   * Circular base: Eq. A4
   * Lateral plates: Eq. A2
5. Calculate *h,* for the base and the plates using Eq. A8 with their corresponding .
6. Calculate *q*, for the base and plates using Eq A.9 with their respective areas.

Compute the total heat transfer rate:

1. The Geometry is viewed as N cylinders of decreasing diameter and height .

Table 6

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Shape** | **Ra** | **Nu** | **h (W/m²K)** | **q (W)** |
| **Cylinder** | 3864 | 4.02 | 12.12 | 4.42 |
| **Square** | 1872 |  |  | 4.77 |
| **Cone** | Varies | Varies | 12.44 | 2.66 |

Based on the free convection analysis, the square rod exhibits the highest total heat transfer rate, slightly outperforming the cylinder, while the cone has the lowest heat transfer rate. The square rod benefits from multiple surface orientations, allowing for greater convective exchange, while the cylinder maintains a more uniform flow around its curved surface. The cone, with its varying geometry, results in inconsistent Rayleigh and Nusselt numbers, leading to lower overall convective efficiency. Overall, given these results, the square rod has the largest heat transfer rate.

## 8. Results

The table below summarizes the heat transfer rates for a cylinder rod, square rod, and cone calculated for external flow, internal flow, and free convection cases.

Table 7

|  |  |  |  |
| --- | --- | --- | --- |
| **Shape** | **Ch7 -- Forced External (q)** | **Ch8 -- Forced Internal (q)** | **Ch9 -- Free Convection (q)** |
| **Cylinder** | 21.58 W | 4.03 W | 4.42 W |
| **Square** | 24.30 W | 4.18 W | 4.77 W |
| **Cone** | 9.66 W | 2.27 W | 2.66 W |

For all three convection conditions (External, Internal, and Free), the square rod has the highest heat transfer rate out of the three shapes analyzed. Therefore, it is safe to say that the square rod is the best shape to use if the goal is to transfer heat as quickly and efficiently as possible.

## 9. Application: Heat Exchanger Enhancement

There are multiple benefits for using a square profile in a heat exchanger. It is common to see circular tubing in heat exchangers because circular tubes are more cost-effective and lighter for the same structural strength. However, as shown above the heat transfer coefficient for forced external convection is 66.60 W/m²K for the square cross section and 59.15 W/m²K for the cylinder so square tubes are a viable option to increase the heat transfer rate. However, at the cost and strength.

## 10. Proposed Experiment

For analyzing the cone and finding c and m values for the Hilpert correlation computational fluid dynamics (CFD) is recommended for its ease of access and ability to perform quick iterations.

1. Model the shape, such as the cone, in a cad software package such as Design Modeler. Define the fluid domain surrounding the object (i.e., subtract the solid geometry from a larger fluid block) to simulate external flow. Create a “wind tunnel” for the shape making sure to increase the inlet length to achieve fully developed laminar flow and to increase the length of the outlet so that no backflow occurs.
2. Discretize (mesh) the shape in ANSYS Mesher, Refine the mesh near the object-fluid interface.

A blue rectangular object with a white triangle

AI-generated content may be incorrect.

Figure 1: Diagram of computational fluid dynamic analysis for square shape (2D model).

1. Use CFD software such as ANSYS Fluent. Input boundary conditions such as input fluid velocity and input fluid temperature defined by laminar flow.
2. Run the simulation.
3. Compute the bulk average fluid temperature, area average surface temperature of the cone-fluid boundary, and area average heat flux.
4. Compute the heat transfer coefficient. More info at [ANSYS Heat Transfer Coefficient Tutorial](https://innovationspace.ansys.com/courses/courses/topics-in-convective-heat-transfer-simulations/lessons/defining-heat-transfer-coefficient/).
5. Perform this for a variety of Reynolds numbers ranging from Re = 0 to Re = 400,000.
6. The coefficients (c and m) are found based on a least squares curve fit of measured data.

## 11. References

[1] Bergman, T., Lavine, A., Incropera, F. and Dewitt, D. (2011). *Fundamentals of Heat and Mass Transfer*. 8th ed. Hoboken: J. Wiley & Sons, Cop.

[2] Janjua, M. M., Khan, N. U., Khan, W. A., Khan, W. S., and Ali, H. M., 2020, “Numerical Study of forced convection heat transfer across a cylinder with various cross sections,” Journal of Thermal Analysis and Calorimetry, **143**(3), pp. 2039–2052.

[3] Sparrow, E. M., Abraham, J. P., and Tong, J. C. K., 2004, “Archival correlations for average heat transfer coefficients for non-circular and circular cylinders and for spheres in cross-flow,” International Journal of Heat and Mass Transfer, **47**(24), pp. 5285–5296.

[4] Wang, X., Bibeau, E., and Naterer, G. F., 2007, “Experimental correlation of forced convection heat transfer from a NACA airfoil,” Experimental Thermal and Fluid Science, **31**(8), pp. 1073–1082.

## Appendix I: Equations and Correlations Used

**Eq. A1:**

**Eq. A2:** for vertical plates

**Eq. A3:** for top plates

**Eq. A4:** for bottom plates

**Eq. A5:** Churchill-Bernstein equation for cross-flow cylinders

**Eq. A6:** for laminar internal flow

**Eq. A7:** for turbulent internal flow

**Eq. A8:** where is characteristic length

**Eq. A9:**

**Eq. A10:**

**Eq. A11:**  gives the side length of a square rod

**Eq. A12:**

**Eq. A13:**  flow friction factor for transitional flow

**Eq. A14:** for internal convection in transitional flow

## Appendix II: Python Script

<https://cooperunion-my.sharepoint.com/personal/gabriel_kret_cooper_edu/Documents/2024-25%20(Junior)/Spring%202025/ME-342%20Heat%20Transfer/Kret_Proj2_HeatTransfer_WorkingScript.py>