

This file has details about all of the scientific computation formula set used in this project. It should be obvious that I didn't derive these formula and relations. I have taken these books and online resources, and the selection is suitable for my particular system, setup and experiments. You will have to modify this to suit your needs.

Resources used :

- 1) National instruments on Strain gauge configuration types: <http://www.ni.com/white-paper/4172/en/>
- 2) Vishay Micromerements databooks on Stress analysis with Strain gauges : <http://www.vishaypg.com/micro-measurements/databooks/>
- 3) HBM strain gauge catalog - <http://www.hbm.com/en/menu/products/strain-gages-accessories/strain-gauge-catalog/>
- 4) "Experimental Stress analysis" textbook by Dr.Ramesh K, Professor, Indian Institute of Technology, Chennai, Tamilnadu, India.

To convert differential bridge output voltage to Strain :

- 1) Voltage ratio $V_r = (V_{\text{strained}} - V_{\text{unstrained}}) / (\text{Excitation voltage to bridge})$
- 2) Voltage to Strain conversion :

$$\text{strain } (\epsilon) = \frac{-4V_r}{GF(1 + 2V_r)} \times \left(1 + \frac{R_L}{R_g}\right)$$

GF = Gauge factor of the strain gauge (usually 2, refer manufacturer's datasheet)

R_g = Strain gauge resistance.

R_L = Lead resistance on the Strain gauge side of the bridge.

- 3) Repeat the formula above to calculate all 3 three strains in 3 corresponding directions.
- 4) To calculate strains on two Principal axes for 0,60,120 rosettes.

$$\epsilon_{P,Q} = \frac{\epsilon_1 + \epsilon_2 + \epsilon_3}{3} \pm \frac{\sqrt{2}}{3} \sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2}$$

- 5) To calculate principal stresses

$$\sigma_P = \frac{E}{1 - \nu^2} (\epsilon_P + \nu \epsilon_Q)$$

$$\sigma_Q = \frac{E}{1 - \nu^2} (\epsilon_Q + \nu \epsilon_P)$$

E= Young's modulus of the material.

ν =Poisson ratio of the material.

The above formula can be used only if the material is homogenous, isotropic in mechanical properties and if it is an biaxial system that obeys Hooke's law.

- 6) Angle from grid 1 to one principal axis

$$\phi_{P,Q} = -\theta = \frac{1}{2} \tan^{-1} \left(\frac{\sqrt{3} (\epsilon_2 - \epsilon_3)}{2\epsilon_1 - \epsilon_2 - \epsilon_3} \right)$$

The above is either Angle from Grid1 to P or to Q principal axis.

To find out which angle is obtained from the formula above, this method is used:

(for 0,60,120 rosette)

Strain 1 means the strain gauge output (in strain) in grid1 (the gage at 0 degrees in 0,60,120 rosette) and so on.

a) If $\text{Strain1} < (\text{strain2} + \text{strain3}) * 0.5$, then the calculated value is Angle P

b) Else, it is Angle Q.

c) If $\text{Strain1} = (\text{strain2} + \text{Strain3}) * 0.5$

And if $\text{strain1} > \text{strain2}$, Angle P is -45 degrees.

Else if $\text{strain2} > \text{strain1}$, Angle P is +45 degrees.

d) If all three strains are same, angle P,Q is indeterminate and the system has equal biaxial strain.

Condition 3 must be checked first, by the program, to avoid divide by zero error in Tan inverse formula. Then, remaining conditions are checked.

7) Shear strain (max) = Strain in Principal Axis P – Strain in principal Axis Q.

8) Shear stress formula

$$\tau_{xy} = G\gamma_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}$$

9) Von Mises yield criterion for failure:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = 3k^2 = \sigma_y^2 \quad \text{y denotes yield strength of the material.}$$

Left hand side of the above equation must always be less than the right hand side.

Sometimes, square root of the left hand side is called “Von Mises stress” for convenience.

And this square root (“Von mises stress”) is compared to Yield strength in the program.