The transfer function of the motor-driver system is given by equation (1).

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{C_1}{C_2 s + 1} \tag{1}$$

From equation (1), one can solve for $V_{out}(t)$ in terms of $V_{in}(t)$:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{C_1}{C_2 s + 1} \rightarrow V_{out}(s) = \frac{C_1 V_{in}(s)}{C_2 s + 1} = \frac{C_1}{C_2}\frac{V_{in}(s)}{s + \frac{1}{C_2}} = \frac{C_1}{C_2}[\frac{V_{in}(s - \frac{1}{C_2})}{s}]|_{s \to s + \frac{1}{C_2}} \tag{2}$$

Apply the inverse Laplace transform to acquire $V_{out}(t)$ from $V_{out}(s)$.

$$V_{out}(t) = \frac{C_1}{C_2} e^{-\frac{t}{C_2}} \mathcal{L}^{-1}\{\frac{V_{in}(s - \frac{1}{C_2})}{s}\} = \frac{C_1}{C_2} e^{-\frac{t}{C_2}} \int_0^t \mathcal{L}^{-1}\{V_{in}(s - \frac{1}{C_2})\} dt' = \frac{C_1}{C_2} e^{-\frac{t}{C_2}} \int_0^t e^{\frac{t'}{C_2}} V_{in}(t') dt' \tag{3}$$

Substitute $C_1 = \frac{K}{B}$ and $C_2 = \frac{J}{B}$ to acquire the final expression:

$$V_{out}(t) = \frac{K}{J} e^{-\frac{Bt}{J}} \int_0^t e^{\frac{Bt'}{J}} V_{in}(t') dt' \tag{4}$$

The steady-state value is the value of $V_{out}(t)$ as t becomes large.

$$V_{out,ss} = \lim_{t \to \infty} \frac{K}{J} e^{-\frac{Bt}{J}} \int_0^t e^{\frac{Bt'}{J}} V_{in}(t') dt' \tag{5}$$

In this particular experiment, $V_{in}(t) = V_{in} 1(t)$. The step response can then be determined for this family of reference voltages.

$$V_{out}(t) = \frac{K}{J} e^{-\frac{Bt}{J}} \int_0^t e^{\frac{Bt'}{J}} V_{in} 1(t') dt' = \frac{K}{J} e^{-\frac{Bt}{J}} \int_0^t e^{\frac{Bt'}{J}} V_{in} dt' = \frac{K}{J} e^{-\frac{Bt}{J}} V_{in} \int_0^t e^{\frac{Bt'}{J}} dt' = \frac{K}{J} e^{-\frac{Bt}{J}} V_{in} \frac{J}{B}(e^{\frac{Bt}{J}} - 1) \tag{6}$$

So, the step response is given by:

$$V_{in}(t) = \frac{K}{B} V_{in}(1 - e^{-\frac{t}{(\frac{J}{B})}}) = C_1 V_{in}(1 - e^{-\frac{t}{C_2}}) \tag{7}$$

The steady state response can be determined by taking the limit in equation (5).

$$V_{in}(t) = \frac{K}{B} V_{in} = C_1 V_{in} \tag{8}$$

The time constant is determined by observing when the output attains $(1 - e^{-1})$ times its steady state value. The time at which this occurs is known as the time constant. By inspection, equation (7) implies that this occurs at $t = C_2 = \frac{J}{B}$.

$$\tau = C_2 = \frac{J}{B} \tag{9}$$