

# Trajectory Imputation in Multi-Agent Sports with Derivative Accumulating Self-Ensemble

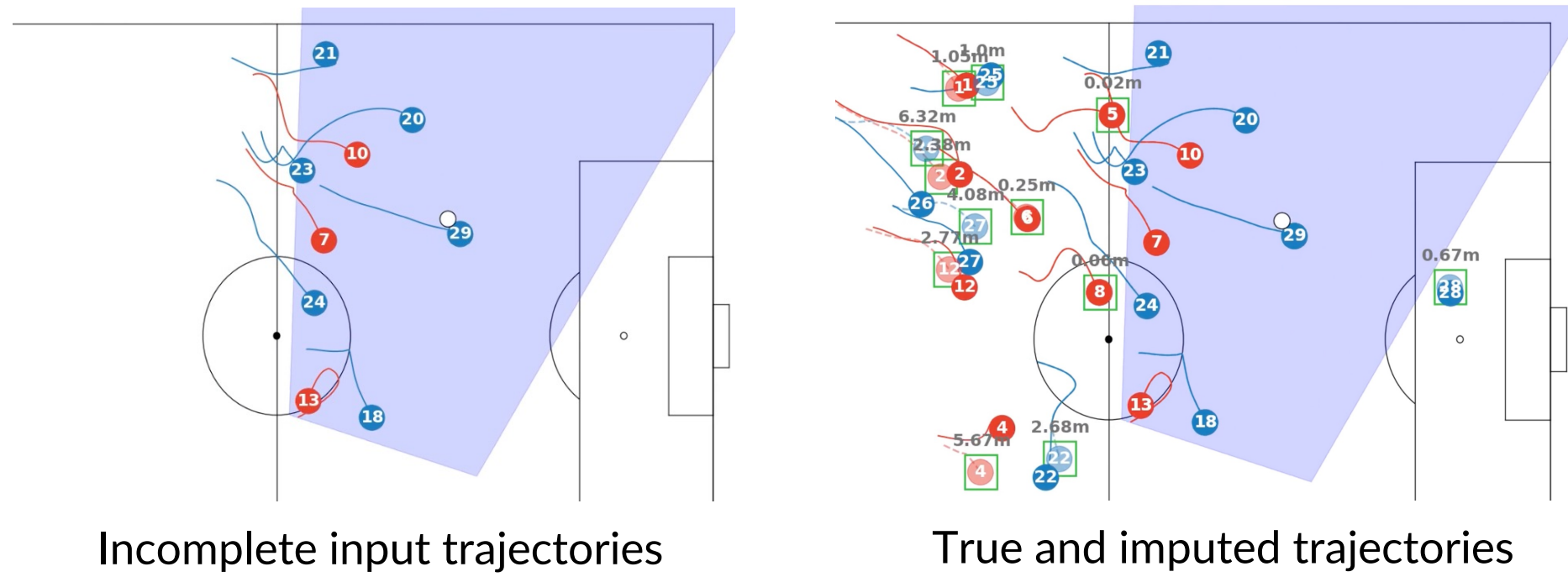
Han-Jun Choi<sup>1\*</sup>, Hyunsung Kim<sup>2,3\*</sup>, Minho Lee<sup>4</sup>, Minchul Jeong<sup>5</sup>, Changjo Kim<sup>3</sup>, Jinsung Yoon<sup>3</sup>, and Sang-Ki Ko<sup>6</sup>



## Background

- Sports tracking data often suffer from missing values due to various factors, such as player occlusion, frame drops, or low-quality sensor signals.
- Accurate and realistic imputation of missing trajectories is crucial for many downstream tasks in sports analytics.
- Existing time-series imputation methods performs poorly on sports data, as they do not account for the dynamic movements and interactions between players.

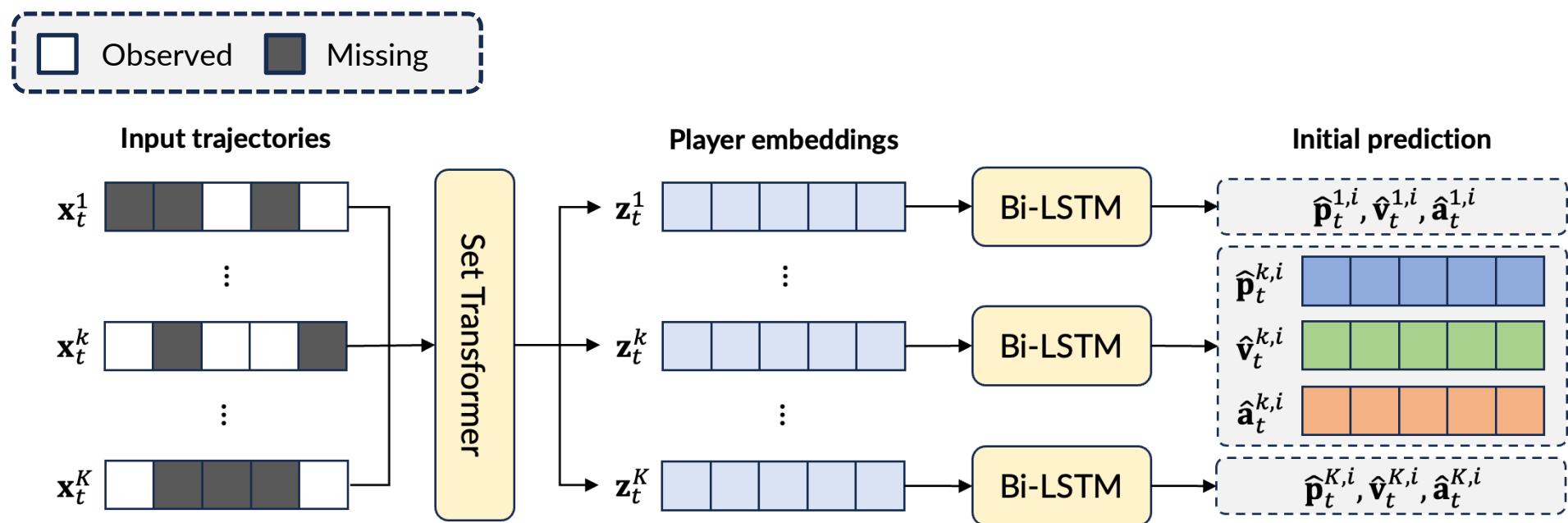
## Proposed Framework: MIDAS



- MIDAS: Multi-agent Imputer with Derivative-Accumulating Self-ensemble**
- Combines multiple predictions using a hybrid architecture

### Neural Network-Based Initial Prediction (IP)

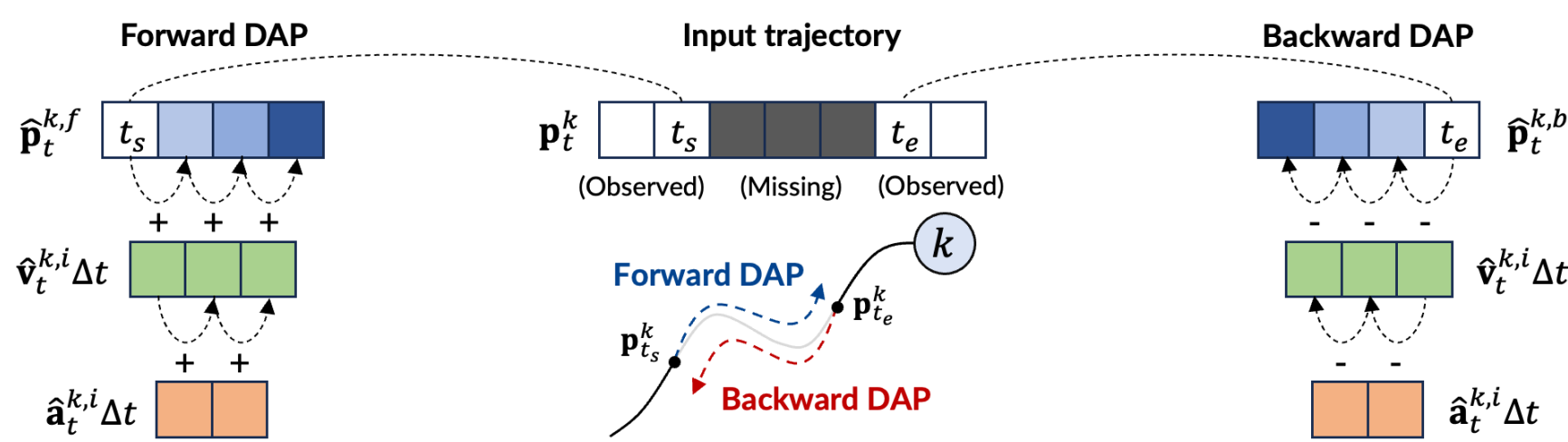
- Set Transformer encoder** for permutation-equivariant player embeddings
- Bi-LSTM** for player-wise temporal modeling
- Jointly predicts position, velocity, and acceleration for each time step



### Derivative-Accumulating Predictions (DAP)

- Produces alternative predictions  $\hat{\mathbf{p}}_t^{k,f}$  and  $\hat{\mathbf{p}}_t^{k,b}$  by **accumulating velocity and acceleration from the observed endpoints**  $t_s$  and  $t_e$ :

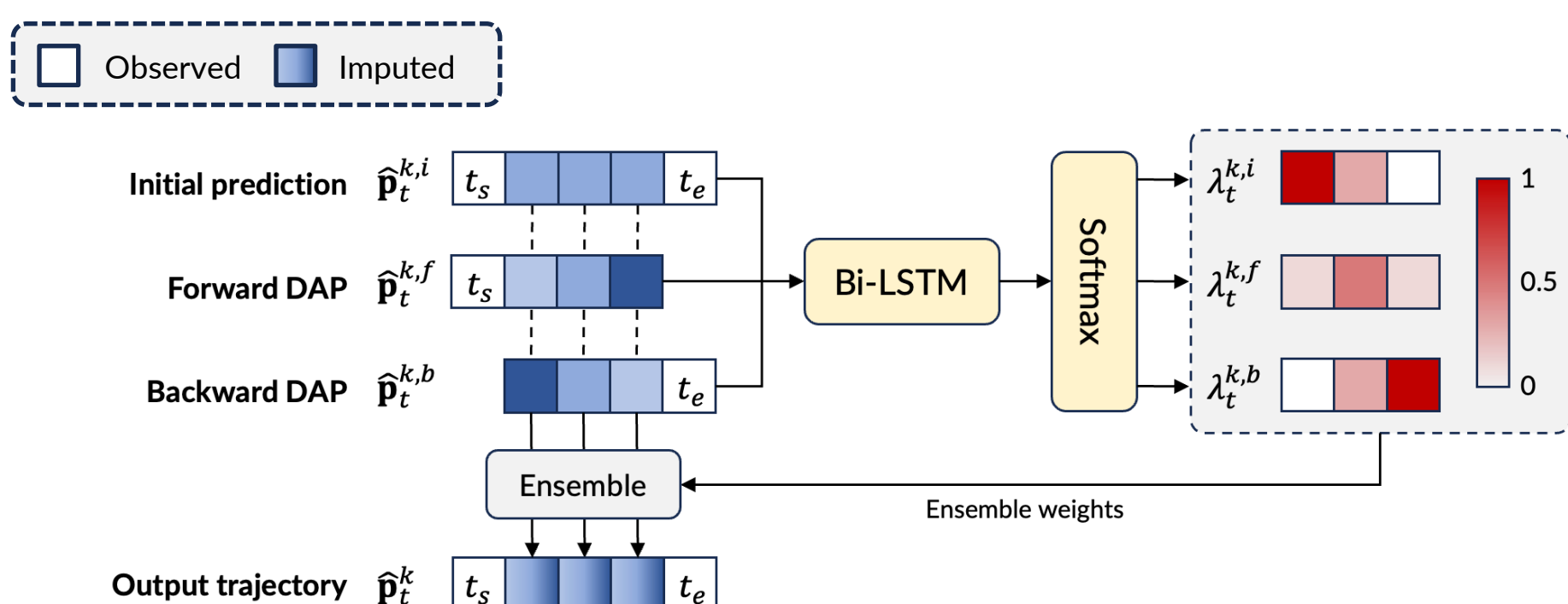
$$\begin{aligned} \hat{\mathbf{p}}_{t_s}^{k,f} &= \mathbf{p}_{t_s}^k, & \hat{\mathbf{p}}_{t+1}^{k,f} &\approx \hat{\mathbf{p}}_t^{k,f} + (\hat{\mathbf{v}}_t^{k,i} + \hat{\mathbf{a}}_t^{k,i} \Delta t) \Delta t \quad (t_s < t < t_e) \\ \hat{\mathbf{p}}_{t_e}^{k,b} &= \mathbf{p}_{t_e}^k, & \hat{\mathbf{p}}_{t-1}^{k,b} &\approx \hat{\mathbf{p}}_t^{k,b} - (\hat{\mathbf{v}}_t^{k,i} - \hat{\mathbf{a}}_t^{k,i} \Delta t) \Delta t \quad (t_s < t < t_e) \end{aligned}$$



### Dynamic Ensemble of Multiple Predictions

- Learns to dynamically weight and combine IP and forward/backward DAPs:

$$\hat{\mathbf{p}}_t^k = \lambda_t^{k,i} \hat{\mathbf{p}}_t^{k,i} + \lambda_t^{k,f} \hat{\mathbf{p}}_t^{k,f} + \lambda_t^{k,b} \hat{\mathbf{p}}_t^{k,b}$$

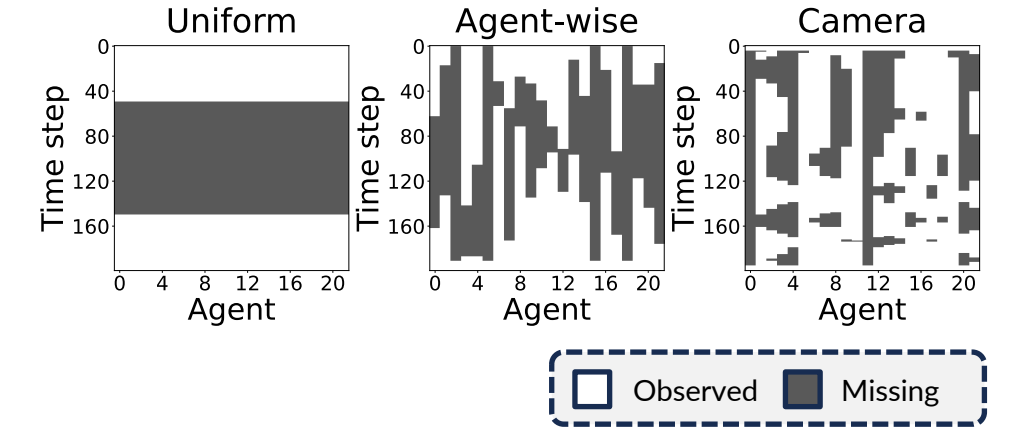


## Experiments

### Three sports datasets

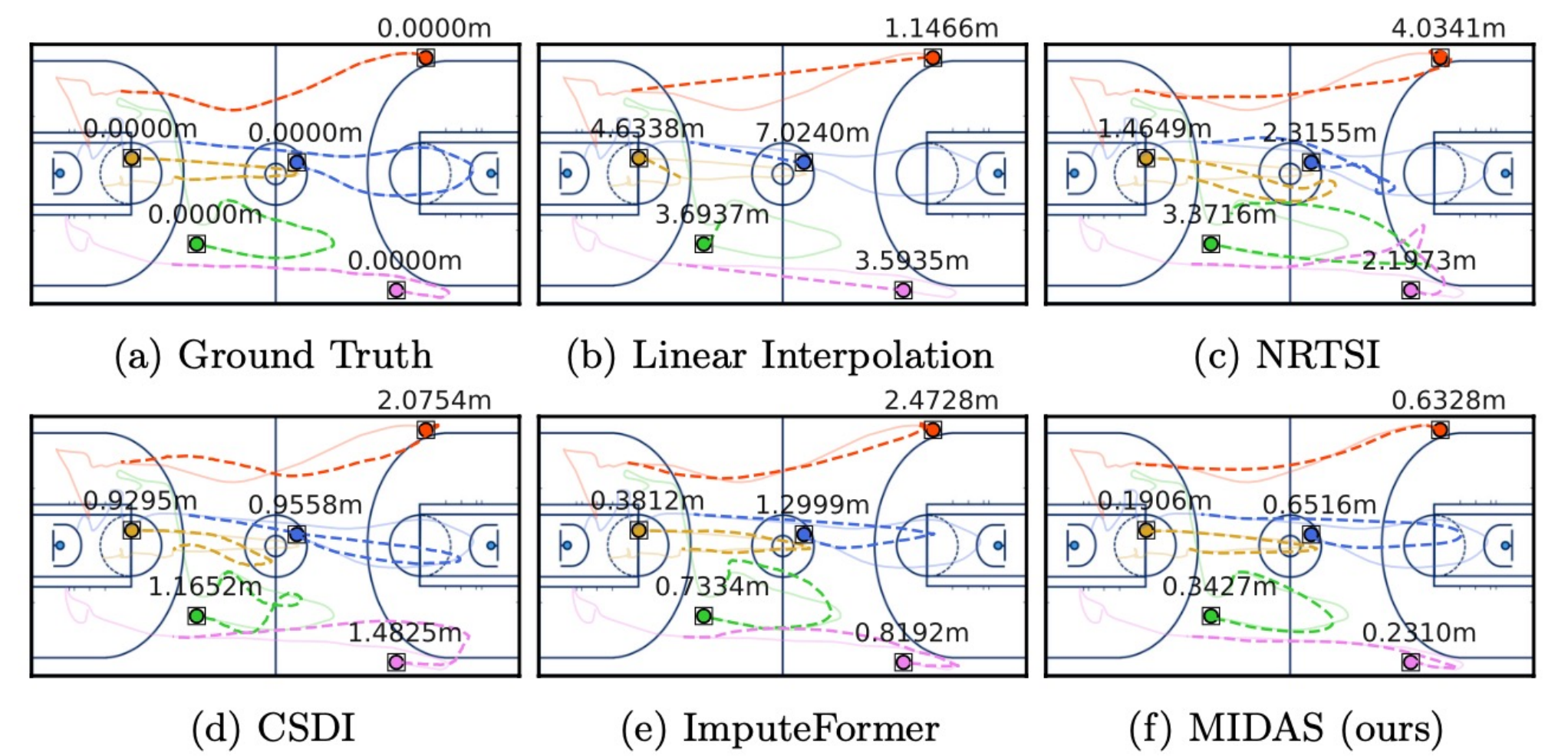
- Soccer: 2/0.5/0.5 matches
- Basketball: 70/10/20 matches
- American football: Preprocessed 5-second sequences

### Three missing scenarios



### Experimental results

Scenario	Metric	Method									
		LI	CS	BRITS	NAOMI	NRTSI	CSDI	GI	IF	IP	MIDAS
Soccer											
Uniform	PE	3.8406	2.2085	7.4859	4.5343	3.1791	3.4295	4.6511	2.0898	1.4563	<b>1.3205</b>
	SCE	0.1299	0.0867	3.9089	3.9793	0.0854	0.1586	0.1191	0.0815	0.1488	<b>0.0516</b>
Agent-wise	PE	5.0752	11.4647	5.7266	—	—	4.0279	5.6011	2.5798	2.0755	<b>1.9832</b>
	SCE	0.1631	0.2939	2.9627	—	—	0.1305	0.1508	0.0976	0.1057	<b>0.0535</b>
Camera	PE	3.1083	1.9209	7.4208	—	—	3.5181	3.6512	2.2151	1.4879	<b>1.2296</b>
	SCE	0.0993	0.0547	4.1967	—	—	0.2132	0.0934	0.3149	0.1554	<b>0.0374</b>
Basketball											
Uniform	PE	3.3481	2.3114	2.9085	1.5254	2.5291	2.2558	2.8305	1.3622	0.9801	<b>0.9727</b>
	SCE	0.1483	0.1025	1.0521	0.3230	0.0734	0.0631	0.1066	0.0531	<b>0.0432</b>	0.0438
Agent-wise	PE	4.4992	10.3857	2.4238	—	—	2.3471	2.5859	<b>1.3345</b>	1.3832	1.3862
	SCE	0.1787	0.2715	0.5397	—	—	0.0563	0.0700	0.0485	<b>0.0373</b>	0.0381
American Football											
Uniform	PE	0.8897	0.7448	1.7990	0.9692	0.5158	0.5558	0.8899	0.3673	0.2073	<b>0.1542</b>
	SCE	1.1063	0.9463	10.9459	2.3112	0.2989	0.4905	1.1023	0.2858	0.1990	<b>0.1126</b>
Agent-wise	PE	1.5128	1.2041	1.7527	—	—	0.6182	1.5128	0.3944	0.2383	<b>0.2104</b>
	SCE	1.0641	0.5306	10.6807	—	—	0.4288	1.0631	0.1869	0.1180	<b>0.0967</b>



### Analysis on missing length and ensemble behavior

Sports	Category	Missing Frames	$\hat{\mathbf{p}}_t^{k,i} (\hat{\lambda}_t^{k,i})$	$\hat{\mathbf{p}}_t^{k,f} (\hat{\lambda}_t^{k,f})$	$\hat{\mathbf{p}}_t^{k,b} (\hat{\lambda}_t^{k,b})$	$\hat{\mathbf{p}}_t^k$
Soccer	Short	33.30 ± 15.72	0.1379 (0.0001)	0.0742 (0.6501)	0.0783 (0.3498)	<b>0.0504</b>
	Medium	90.42 ± 19.17	0.7939 (0.0003)	0.7492 (0.6110)	0.7624 (0.3887)	<b>0.7004</b>
	Long	173.41 ± 21.70	2.7359 (0.0002)	2.7017 (0.5034)	2.7186 (0.4964)	<b>2.6082</b>
Basketball	Short	33.86 ± 16.11	0.0444 (0.1596)	0.0424 (0.5216)	0.0426 (0.3187)	<b>0.0376</b>
	Medium	90.65 ± 19.02	0.5395 (0.1783)	0.5483 (0.4809)	0.5482 (0.3406)	<b>0.5312</b>
	Long	172.60 ± 21.20	1.8104 (0.1801)	1.8288 (0.4313)	1.8258 (0.3985)	<b>1.8005</b>

## Downstream Applications

### Estimating physical metrics

Method	Distance (m)		Sprints	
	Mean	MAPE	Mean	MAPE
Ground Truth	11,093.5	—	41.49	—
Linear Interp.	10,167.8	8.46%	38.89	6.32%
Cubic Spline	10,686.3	3.73%	38.85	6.73%
BRITS	10,979.2	2.76%	59.89	53.62%
CSDI	11,343.0	2.77%	44.20	14.71%
Graph Imputer	8,972.1	19.15%	37.85	9.80%
ImputeFormer	11,441.7	3.22%	50.25	26.29%
MIDAS (ours)	<b>10,922.4</b>	<b>1.58%</b>	<b>40.71</b>	<b>4.95%</b>

### Estimating contextual metrics

