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# Trajectory Imputation in Multi-Agent Sports with Derivative Accumulating Self-Ensemble

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Han-Jun Choi<sup>1\*</sup>, Hyunsung Kim<sup>2,3\*</sup>, Minho Lee<sup>4</sup>, Minchul Jeong<sup>5</sup>,  
Changjo Kim<sup>3</sup>, Jinsung Yoon<sup>3</sup>, and Sang-Ki Ko<sup>6</sup>



1. KETI  
Seongnam, South Korea



2. KAIST  
Daejeon, South Korea



3. Fitotogether Inc.  
Seoul, South Korea



4. Saarland University  
Saarbrücken, Germany



5. Weflo Inc.  
Seoul, South Korea



6. University of Seoul  
Seoul, South Korea

\*Equal contribution

# Background: Sports Player Tracking

- Collected from wearable sensors or computer vision systems
- **Contain missing values for various reasons** (e.g., out-of-camera view, occlusion, etc.)
- Many downstream tasks require complete tracking data



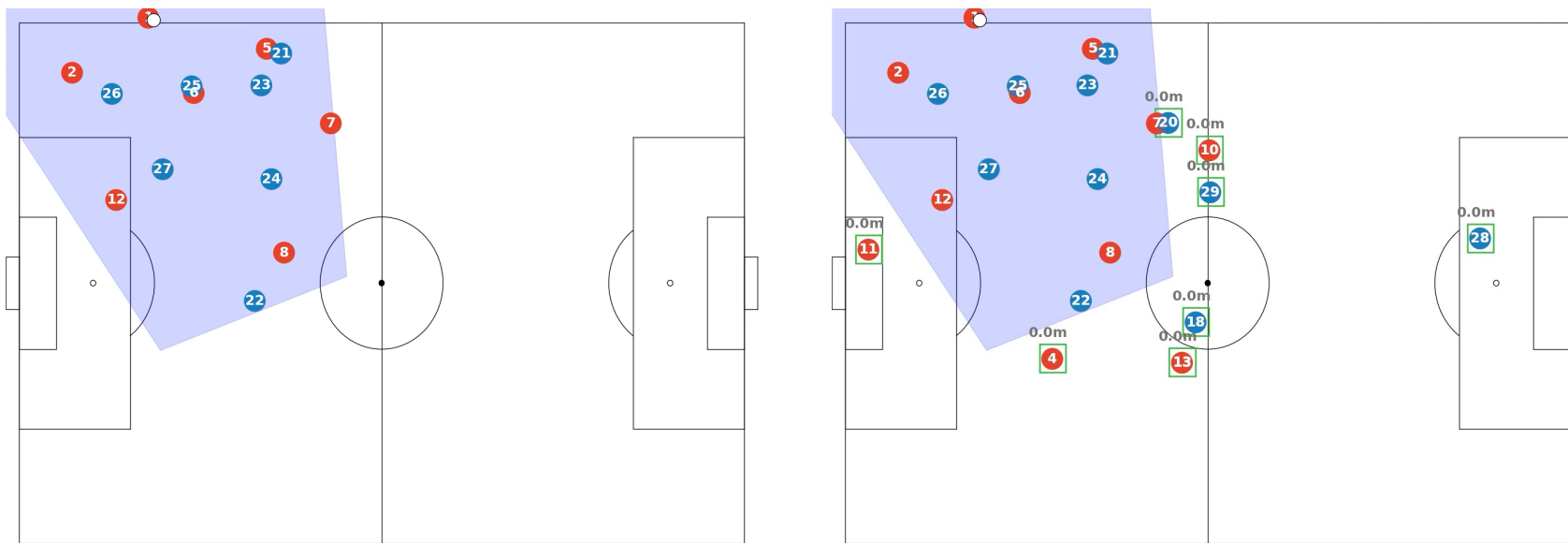
**Wearable tracking**  
(usually via GPS/IMU sensors)



**Optical tracking**  
(usually via cameras and CV techniques)

# Background: Sports Trajectory Imputation

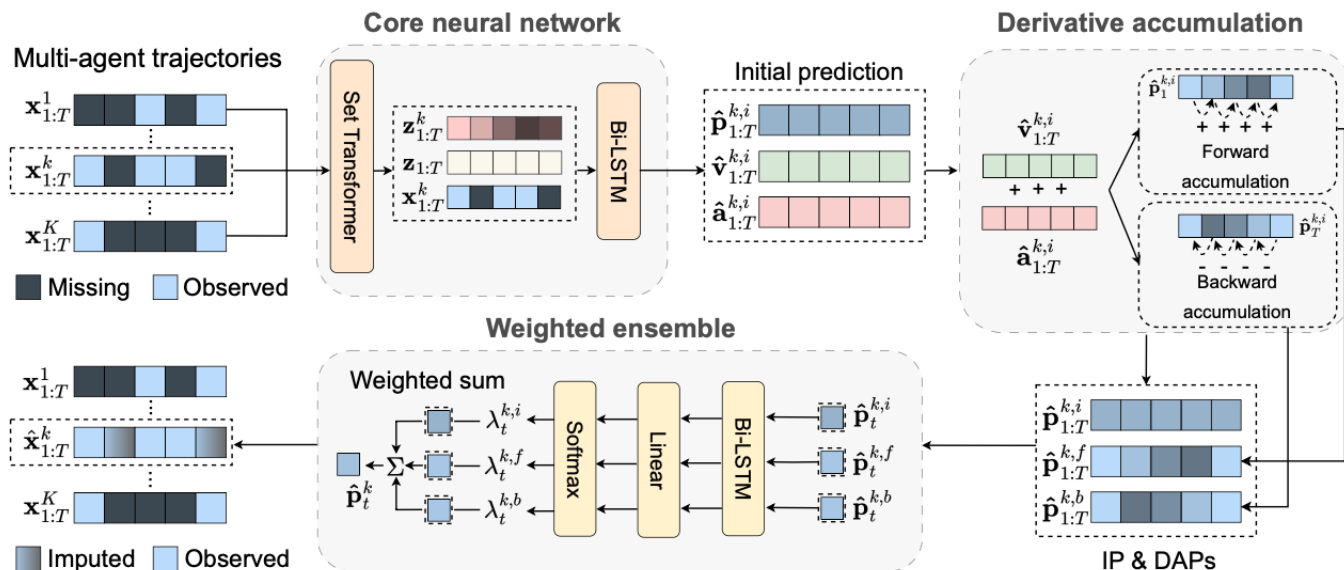
- **Goal: Impute missing player trajectories in multi-agent sports**
- Challenges compared to general time-series imputation
  - Dynamic inter-player relationships
  - Biomechanical constraints (e.g., limited speed and acceleration)
  - Data scarcity in real-world sports due to confidentiality and cost



# Proposed Framework: MIDAS

MIDAS: Multi-agent Imputer with Derivative-Accumulating Self-ensemble

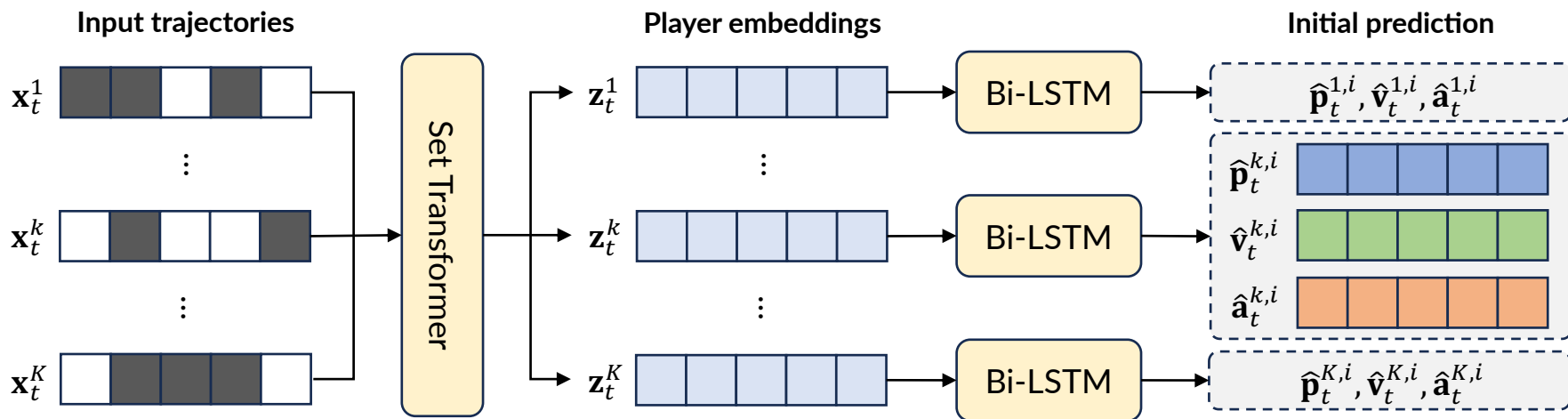
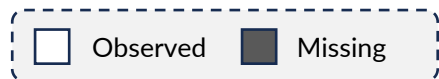
- **Initial prediction** of position, velocity, and acceleration using Set Transformer and Bi-LSTM
- **Alternative predictions** by forward/backward accumulation of velocity and acceleration values
- **Weighted ensemble** of the initial prediction and alternative predictions



# Neural Network-Based Initial Prediction (IP)

- **Set Transformer encoder** for permutation-equivariant player embeddings
- **Bi-LSTM** for player-wise temporal modeling
- Joint prediction of position, velocity, and acceleration for each time step:

$$\hat{\mathbf{x}}_t^{k,i} = (\hat{\mathbf{p}}_t^{k,i}, \hat{\mathbf{v}}_t^{k,i}, \hat{\mathbf{a}}_t^{k,i}) = (\hat{p}_{t,x}^{k,i}, \hat{p}_{t,y}^{k,i}, \hat{v}_{t,x}^{k,i}, \hat{v}_{t,y}^{k,i}, \hat{a}_{t,x}^{k,i}, \hat{a}_{t,y}^{k,i})$$



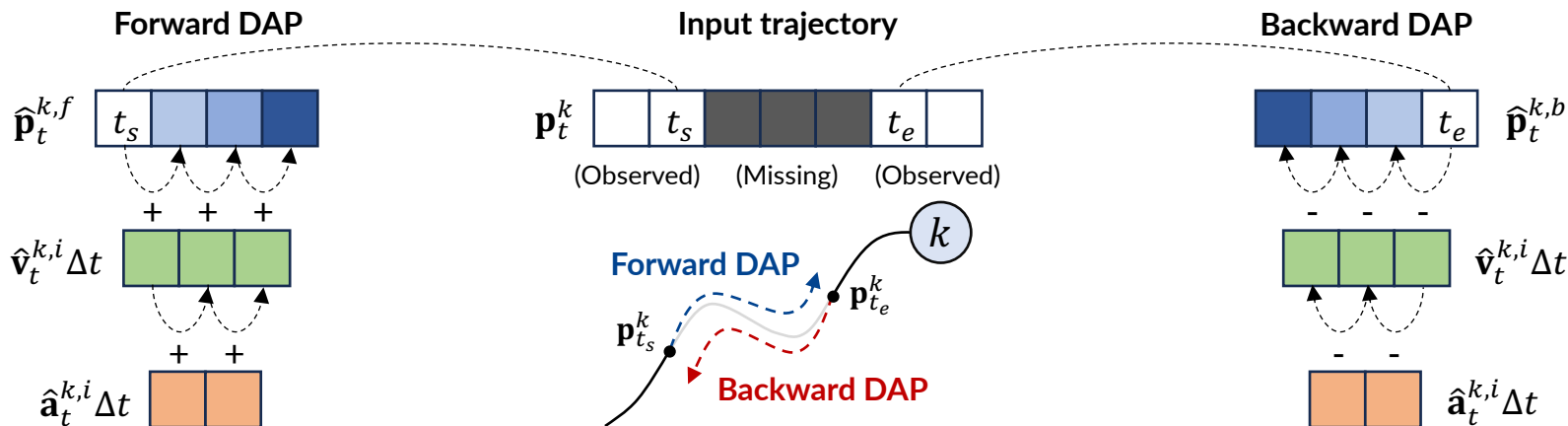
# Derivative-Accumulating Predictions (DAP)

- Physical relationships between position, velocity, and acceleration:

$$\mathbf{p}_{t+1}^k \approx \mathbf{p}_t^k + \mathbf{v}_{t+1}^k \Delta t \approx \mathbf{p}_t^k + (\mathbf{v}_t^k + \mathbf{a}_t^k \Delta t) \Delta t$$

- Initial prediction does not enforce this relationships.
- For each missing interval  $(t_s, t_e)$ , the model produces alternative predictions  $\hat{\mathbf{p}}_t^{k,f}$  and  $\hat{\mathbf{p}}_t^{k,b}$  by **accumulating velocity and acceleration values on the position** at either observed endpoint.

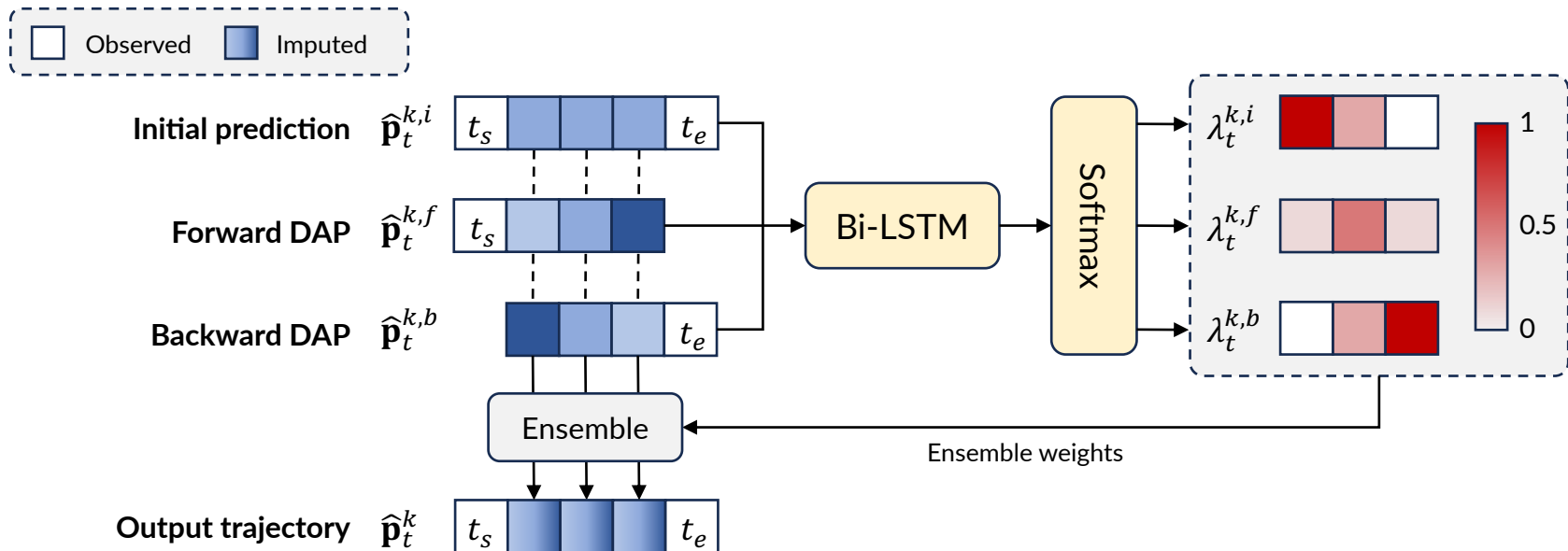
$$\begin{aligned} \hat{\mathbf{p}}_{t_s}^{k,f} &= \mathbf{p}_{t_s}^k, & \hat{\mathbf{p}}_{t+1}^{k,f} &\approx \hat{\mathbf{p}}_t^{k,f} + (\hat{\mathbf{v}}_t^{k,i} + \hat{\mathbf{a}}_t^{k,i} \Delta t) \Delta t \quad (t_s < t < t_e) \\ \hat{\mathbf{p}}_{t_e}^{k,b} &= \mathbf{p}_{t_e}^k, & \hat{\mathbf{p}}_{t-1}^{k,b} &\approx \hat{\mathbf{p}}_t^{k,b} - (\hat{\mathbf{v}}_t^{k,i} - \hat{\mathbf{a}}_t^{k,i} \Delta t) \Delta t \quad (t_s < t < t_e) \end{aligned}$$



# Dynamic Ensemble of Multiple Predictions

- Trade-off between IP and DAP
  - IP does not enforce relationships between physical quantities and thus lacks stability.
  - DAP suffers from error compounding problem, especially for long missing intervals.
- Dynamic ensemble by learnable weights** to mix IP and forward/backward DAPs:

$$\hat{\mathbf{p}}_t^k = \lambda_t^{k,i} \hat{\mathbf{p}}_t^{k,i} + \lambda_t^{k,f} \hat{\mathbf{p}}_t^{k,f} + \lambda_t^{k,b} \hat{\mathbf{p}}_t^{k,b}$$



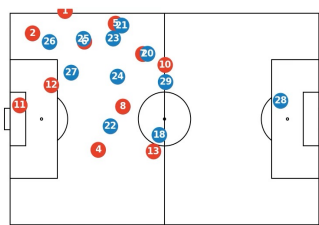
# Experiments

## ■ Datasets

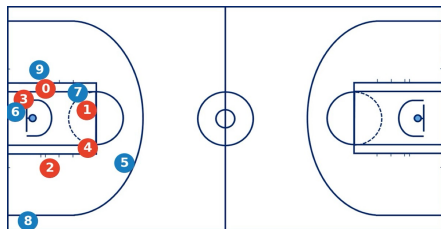
- Soccer: 3 matches provided by Metrica Sports
- Basketball: 100 NBA matches provided by SportsVU
- American football: Preprocessed NFL sequences released with a Kaggle competition

Table 1: Details on the three sports datasets.

Split	Soccer		Basketball		A. Football	
	Matches	Frames	Matches	Frames	Matches	Frames
<b>Training</b>	2	65,014	70	1,621,835	—	425,000
<b>Validation</b>	0.5	20,104	10	216,118	—	52,150
<b>Test</b>	0.5	21,242	20	468,885	—	—



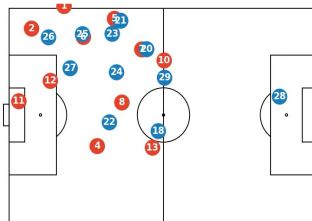
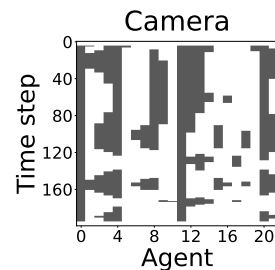
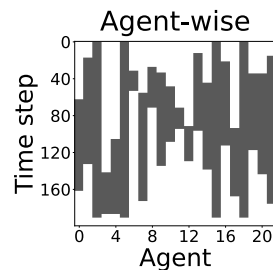
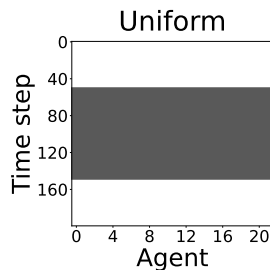
Soccer



Basketball

## ■ Missing Scenarios

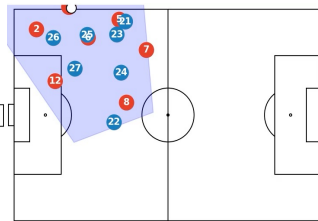
- Uniform: Same missing interval for all players
- Agent-wise: Different missing intervals per player
- Camera: Players outside camera view are missing



Uniform missing



Agent-wise missing



Broadcasting camera



# Experiments

## Results

Scenario	Metric	Method									
		LI	CS	BRITS	NAOMI	NRTSI	CSDI	GI	IF	IP	MIDAS
Soccer											
Uniform	PE	3.8406	2.2085	7.4859	4.5343	3.1791	3.4295	4.6511	2.0898	1.4563	<b>1.3205</b>
	SCE	0.1299	0.0867	3.9089	3.9793	0.0854	0.1586	0.1191	0.0815	0.1488	<b>0.0516</b>
Agent-wise	PE	5.0752	11.4647	5.7266	—	—	4.0279	5.6011	2.5798	2.0755	<b>1.9832</b>
	SCE	0.1631	0.2939	2.9627	—	—	0.1305	0.1508	0.0976	0.1057	<b>0.0535</b>
Camera	PE	3.1083	1.9209	7.4208	—	—	3.5181	3.6512	2.2151	1.4879	<b>1.2296</b>
	SCE	0.0993	0.0547	4.1967	—	—	0.2132	0.0934	0.3149	0.1554	<b>0.0374</b>
Basketball											
Uniform	PE	3.3481	2.3114	2.9085	1.5254	2.5291	2.2558	2.8305	1.3622	0.9801	<b>0.9727</b>
	SCE	0.1483	0.1025	1.0521	0.3230	0.0734	0.0631	0.1066	0.0531	<b>0.0432</b>	0.0438
Agent-wise	PE	4.4992	10.3857	2.4238	—	—	2.3471	2.5859	<b>1.3345</b>	1.3832	1.3862
	SCE	0.1787	0.2715	0.5397	—	—	0.0563	0.0700	0.0485	<b>0.0373</b>	0.0381
American Football											
Uniform	PE	0.8897	0.7448	1.7990	0.9692	0.5158	0.5558	0.8899	0.3673	0.2073	<b>0.1542</b>
	SCE	1.1063	0.9463	10.9459	2.3112	0.2989	0.4905	1.1023	0.2858	0.1990	<b>0.1126</b>
Agent-wise	PE	1.5128	1.2041	1.7527	—	—	0.6182	1.5128	0.3944	0.2383	<b>0.2104</b>
	SCE	1.0641	0.5306	10.6807	—	—	0.4288	1.0631	0.1869	0.1180	<b>0.0967</b>

Limited Data		
IF	IP	MIDAS
1.6741	1.1868	<b>1.1438</b>
0.0628	0.1483	<b>0.0493</b>
1.8876	1.6414	<b>1.5994</b>
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# Experiments

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	SCE	1.0641	0.5306	10.6807	—	—	0.4288	1.0631	0.1869	0.1180	0.0967	

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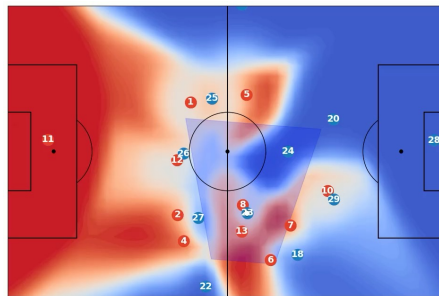
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# Applications

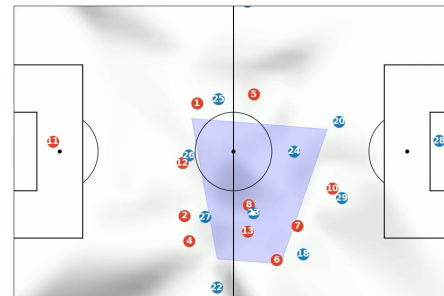
- Estimating physical metrics

Method	Distance (m)		Sprints	
	Mean	MAPE	Mean	MAPE
Ground Truth	11,093.5	—	41.49	—
Linear Interp.	10,167.8	8.46%	38.89	6.32%
Cubic Spline	10,686.3	3.73%	38.85	6.73%
BRITS	10,979.2	2.76%	59.89	53.62%
CSDI	11,343.0	2.77%	44.20	14.71%
Graph Imputer	8,972.1	19.15%	37.85	9.80%
ImputeFormer	11,441.7	3.22%	50.25	26.29%
MIDAS (ours)	<b>10,922.4</b>	<b>1.58%</b>	<b>40.71</b>	<b>4.95%</b>

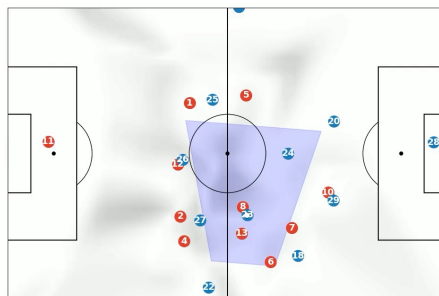
- Estimating pass success probability maps (Pitch Control)



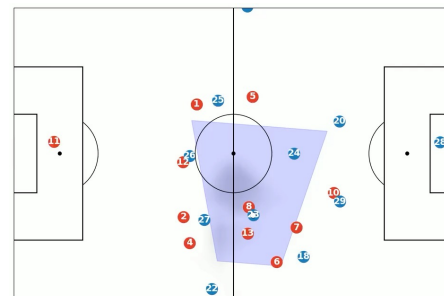
Ground truth



CSDI



ImputeFormer



MIDAS (ours)

# Conclusions

## ▪ Main contributions

- Imputation framework that exploits physical relationships
- Experiments on various missing scenarios and datasets
- Demonstrating applicability to real-world applications

## ▪ Future work

- Adapting MIDAS to other domains
- Developing metrics for quantitative evaluation in real-world applications
- Applying MIDAS to downstream tasks in an end-to-end manner

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**Thank you for listening!**

Contact information

Han-Jun Choi (co-first author): [hanjun\\_c@keti.re.ke](mailto:hanjun_c@keti.re.ke)

Hyunsung Kim (co-first author): [hyunsung.kim@kaist.ac.kr](mailto:hyunsung.kim@kaist.ac.kr)

Sang-Ki Ko (corresponding): [sangkiko@uos.ac.kr](mailto:sangkiko@uos.ac.kr)