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# Trajectory Imputation in Multi-Agent Sports with Derivative Accumulating Self-Ensemble

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# Background: Sports Player Tracking

- Collected from wearable sensors or computer vision systems
- **Contain missing values for various reasons** (e.g., out-of-camera view, occlusion, etc.)
- Many downstream tasks require complete tracking data



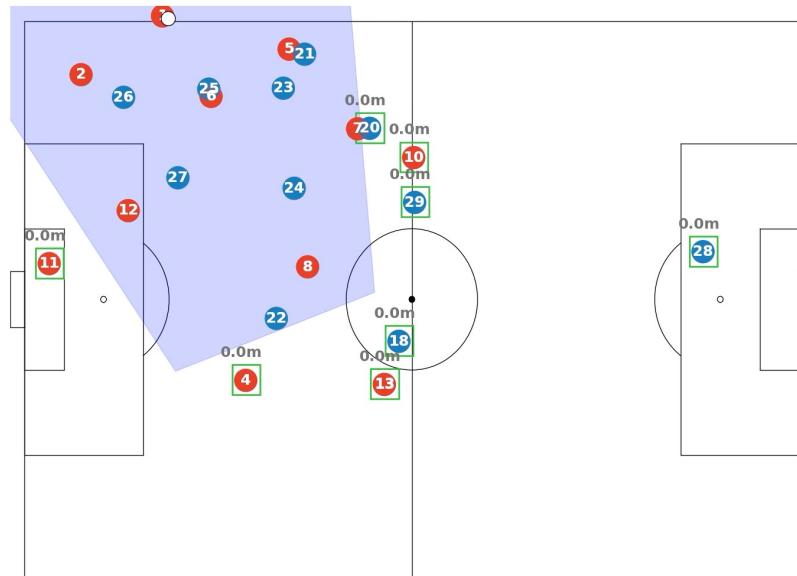
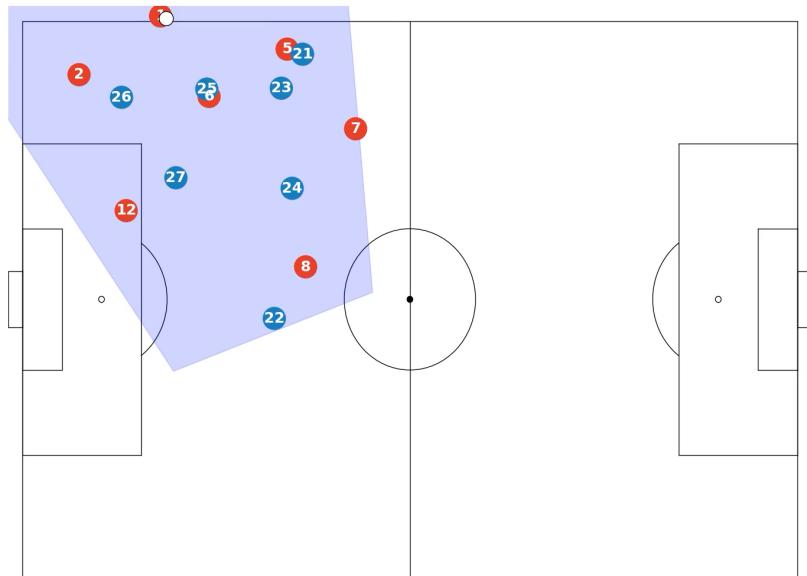
**Wearable tracking**  
(usually via GPS/IMU sensors)



**Optical tracking**  
(usually via cameras and CV techniques)

# Background: Sports Trajectory Imputation

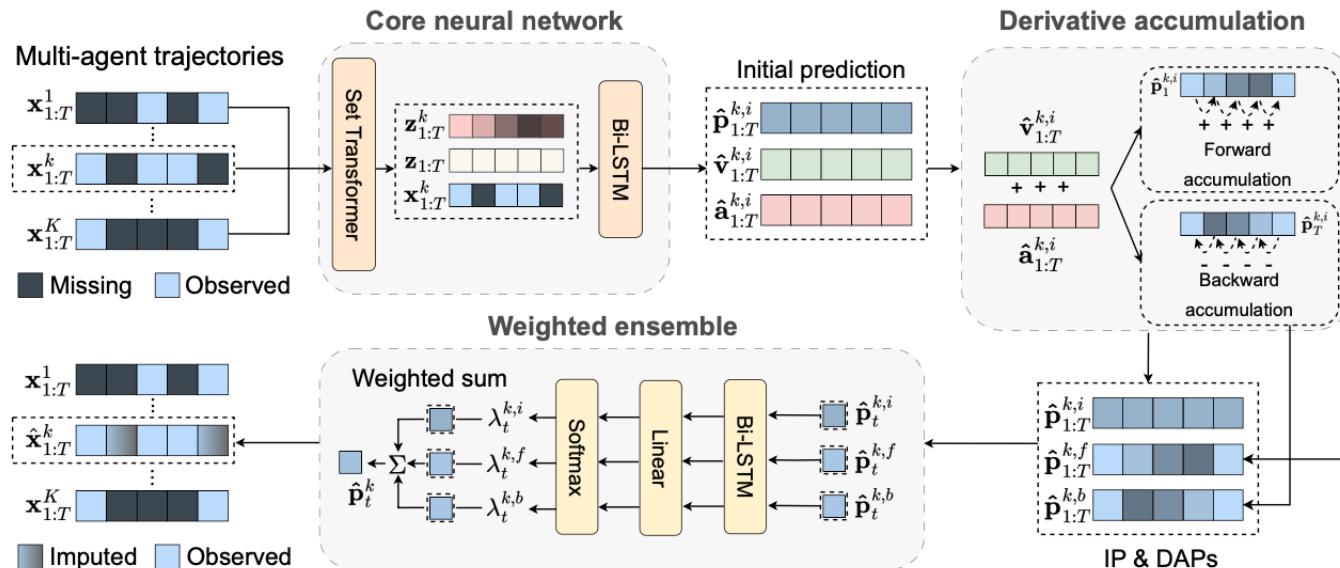
- Goal: Impute missing player trajectories in multi-agent sports
- Challenges compared to general time-series imputation
  - Dynamic inter-player relationships
  - Biomechanical constraints (e.g., limited speed and acceleration)
  - Data scarcity in real-world sports due to confidentiality and cost



# Proposed Framework: MIDAS

MIDAS: Multi-agent Imputer with Derivative-Accumulating Self-ensemble

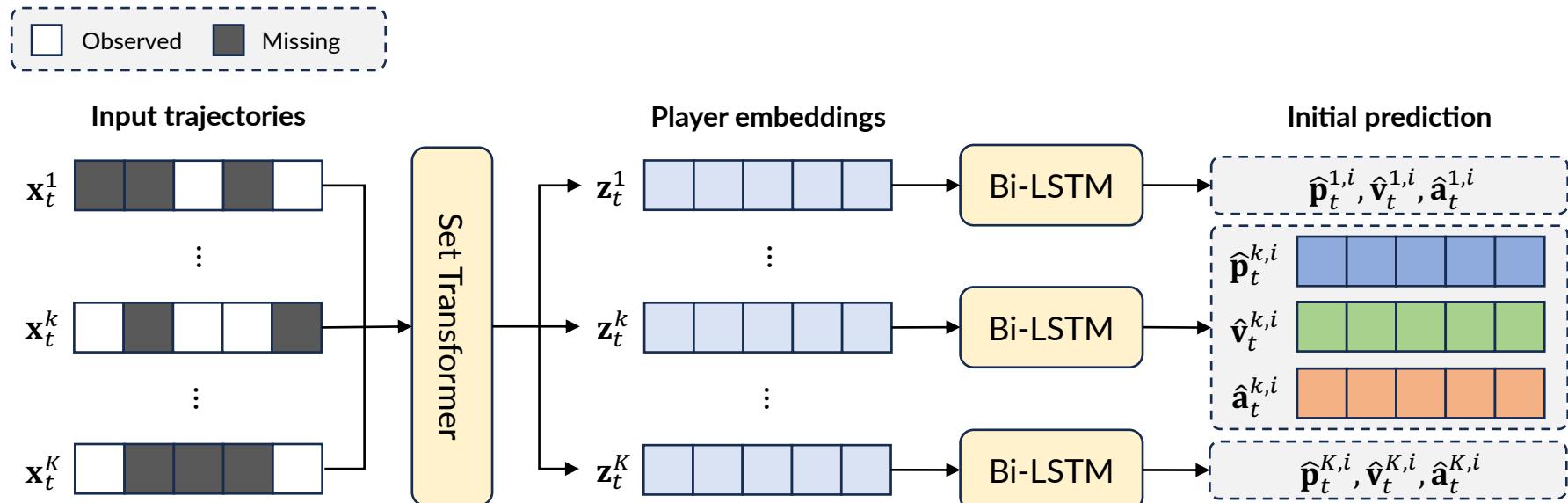
- **Initial prediction** of position, velocity, and acceleration using Set Transformer and Bi-LSTM
- **Alternative predictions** by forward/backward accumulation of velocity and acceleration values
- **Weighted ensemble** of the initial prediction and alternative predictions



# Neural Network-Based Initial Prediction (IP)

- Set Transformer encoder for permutation-equivariant player embeddings
- Bi-LSTM for player-wise temporal modeling
- Joint prediction of position, velocity, and acceleration for each time step:

$$\hat{\mathbf{x}}_t^{k,i} = (\hat{\mathbf{p}}_t^{k,i}, \hat{\mathbf{v}}_t^{k,i}, \hat{\mathbf{a}}_t^{k,i}) = (\hat{p}_{t,x}^{k,i}, \hat{p}_{t,y}^{k,i}, \hat{v}_{t,x}^{k,i}, \hat{v}_{t,y}^{k,i}, \hat{a}_{t,x}^{k,i}, \hat{a}_{t,y}^{k,i})$$



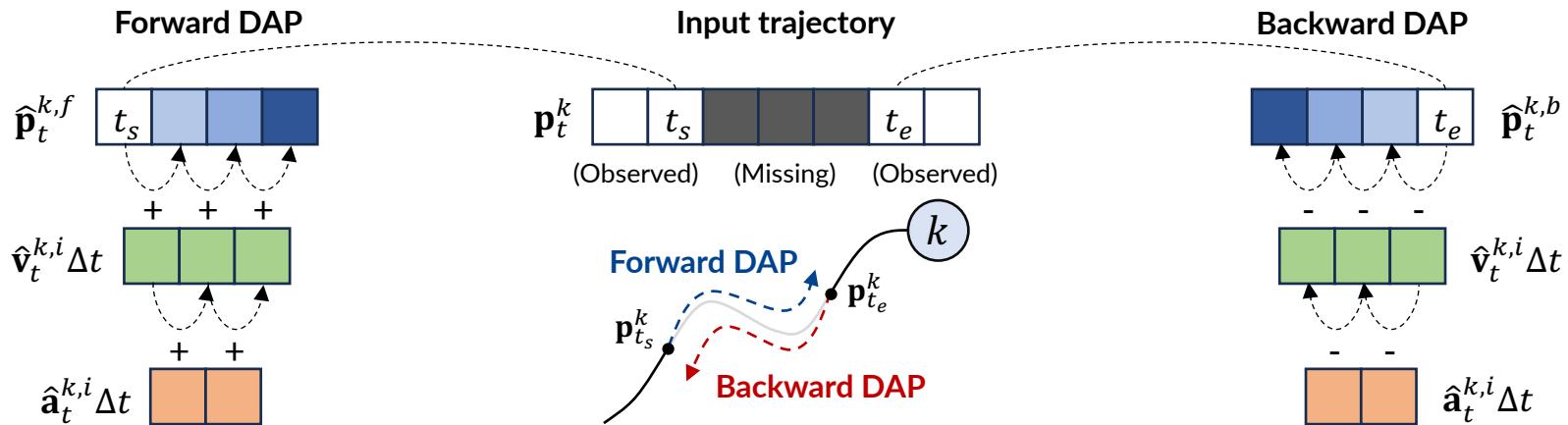
# Derivative-Accumulating Predictions (DAP)

- Physical relationships between position, velocity, and acceleration:

$$\mathbf{p}_{t+1}^k \approx \mathbf{p}_t^k + \mathbf{v}_{t+1}^k \Delta t \approx \mathbf{p}_t^k + (\mathbf{v}_t^k + \mathbf{a}_t^k \Delta t) \Delta t$$

- Initial prediction does not enforce this relationships.
- For each missing interval  $(t_s, t_e)$ , the model produces alternative predictions  $\hat{\mathbf{p}}_t^{k,f}$  and  $\hat{\mathbf{p}}_t^{k,b}$  by **accumulating velocity and acceleration values on the position** at either observed endpoint.

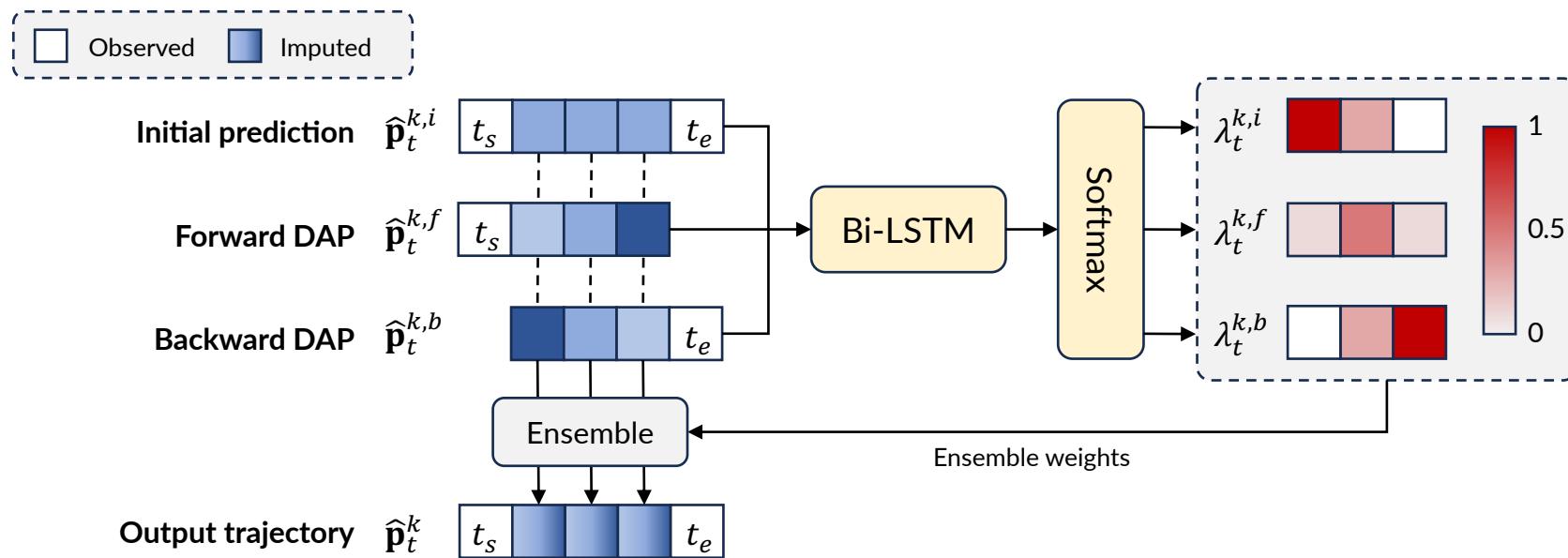
$$\begin{aligned}\hat{\mathbf{p}}_{t_s}^{k,f} &= \mathbf{p}_{t_s}^k, & \hat{\mathbf{p}}_{t+1}^{k,f} &\approx \hat{\mathbf{p}}_t^{k,f} + (\hat{\mathbf{v}}_t^{k,i} + \hat{\mathbf{a}}_t^{k,i} \Delta t) \Delta t \quad (t_s < t < t_e) \\ \hat{\mathbf{p}}_{t_e}^{k,b} &= \mathbf{p}_{t_e}^k, & \hat{\mathbf{p}}_{t-1}^{k,b} &\approx \hat{\mathbf{p}}_t^{k,b} - (\hat{\mathbf{v}}_t^{k,i} - \hat{\mathbf{a}}_t^{k,i} \Delta t) \Delta t \quad (t_s < t < t_e)\end{aligned}$$



# Dynamic Ensemble of Multiple Predictions

- Trade-off between IP and DAP
  - IP does not enforce relationships between physical quantities and thus lacks stability.
  - DAP suffers from error compounding problem, especially for long missing intervals.
- **Dynamic ensemble by learnable weights** to mix IP and forward/backward DAPs:

$$\hat{\mathbf{p}}_t^k = \lambda_t^{k,i} \hat{\mathbf{p}}_t^{k,i} + \lambda_t^{k,f} \hat{\mathbf{p}}_t^{k,f} + \lambda_t^{k,b} \hat{\mathbf{p}}_t^{k,b}$$



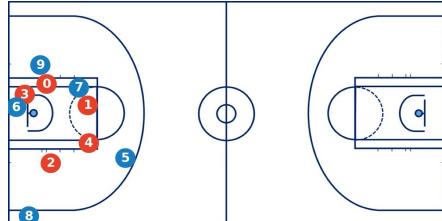
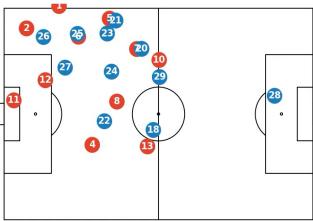
# Experiments

## Datasets

- Soccer: 3 matches provided by Metrica Sports
- Basketball: 100 NBA matches provided by SportsVU
- American football: Preprocessed NFL sequences released with a Kaggle competition

Table 1: Details on the three sports datasets.

Split	Soccer		Basketball		A. Football	
	Matches	Frames	Matches	Frames	Matches	Frames
Training	2	65,014	70	1,621,835	—	425,000
Validation	0.5	20,104	10	216,118	—	52,150
Test	0.5	21,242	20	468,885	—	—

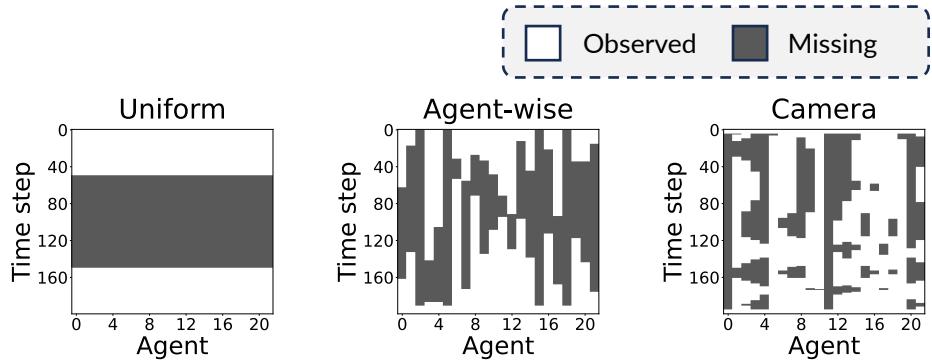


Soccer

Basketball

## Missing Scenarios

- Uniform: Same missing interval for all players
- Agent-wise: Different missing intervals per player
- Camera: Players outside camera view are missing



# Experiments

- Results

Scenario	Metric	Method									
		LI	CS	BRITS	NAOMI	NRTSI	CSDI	GI	IF	IP	MIDAS
<b>Soccer</b>											
Uniform	PE	3.8406	2.2085	7.4859	4.5343	3.1791	3.4295	4.6511	2.0898	1.4563	<b>1.3205</b>
	SCE	0.1299	0.0867	3.9089	3.9793	0.0854	0.1586	0.1191	0.0815	0.1488	<b>0.0516</b>
Agent-wise	PE	5.0752	11.4647	5.7266	—	—	4.0279	5.6011	2.5798	2.0755	<b>1.9832</b>
	SCE	0.1631	0.2939	2.9627	—	—	0.1305	0.1508	0.0976	0.1057	<b>0.0535</b>
Camera	PE	3.1083	1.9209	7.4208	—	—	3.5181	3.6512	2.2151	1.4879	<b>1.2296</b>
	SCE	0.0993	0.0547	4.1967	—	—	0.2132	0.0934	0.3149	0.1554	<b>0.0374</b>
<b>Basketball</b>											
Uniform	PE	3.3481	2.3114	2.9085	1.5254	2.5291	2.2558	2.8305	1.3622	0.9801	<b>0.9727</b>
	SCE	0.1483	0.1025	1.0521	0.3230	0.0734	0.0631	0.1066	0.0531	<b>0.0432</b>	0.0438
Agent-wise	PE	4.4992	10.3857	2.4238	—	—	2.3471	2.5859	<b>1.3345</b>	1.3832	1.3862
	SCE	0.1787	0.2715	0.5397	—	—	0.0563	0.0700	0.0485	<b>0.0373</b>	0.0381
<b>American Football</b>											
Uniform	PE	0.8897	0.7448	1.7990	0.9692	0.5158	0.5558	0.8899	0.3673	0.2073	<b>0.1542</b>
	SCE	1.1063	0.9463	10.9459	2.3112	0.2989	0.4905	1.1023	0.2858	0.1990	<b>0.1126</b>
Agent-wise	PE	1.5128	1.2041	1.7527	—	—	0.6182	1.5128	0.3944	0.2383	<b>0.2104</b>
	SCE	1.0641	0.5306	10.6807	—	—	0.4288	1.0631	0.1869	0.1180	<b>0.0967</b>
<b>Limited Data</b>											
	IF	IP	MIDAS								
	1.6741	1.1868	<b>1.1438</b>								
	0.0628	0.1483	<b>0.0493</b>								
	1.8876	1.6414	<b>1.5994</b>								
	0.0645	0.0452	<b>0.0439</b>								

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Agent-wise	PE	1.5128	1.2041	1.7527	—	—	0.6182	1.5128	0.3944	0.2383	<b>0.2104</b>
	SCE	1.0641	0.5306	10.6807	—	—	0.4288	1.0631	0.1869	0.1180	<b>0.0967</b>

Limited Data		
IF	IP	MIDAS
1.6741	1.1868	<b>1.1438</b>
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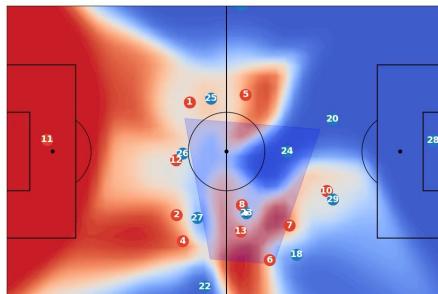
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# Applications

- Estimating physical metrics

Method	Distance (m)		Sprints	
	Mean	MAPE	Mean	MAPE
Ground Truth	11,093.5	—	41.49	—
Linear Interp.	10,167.8	8.46%	38.89	6.32%
Cubic Spline	10,686.3	3.73%	38.85	6.73%
BRITS	10,979.2	2.76%	59.89	53.62%
CSDI	11,343.0	2.77%	44.20	14.71%
Graph Imputer	8,972.1	19.15%	37.85	9.80%
ImputeFormer	11,441.7	3.22%	50.25	26.29%
MIDAS (ours)	<b>10,922.4</b>	<b>1.58%</b>	<b>40.71</b>	<b>4.95%</b>

- Estimating pass success probability maps (Pitch Control)



Ground truth



CSDI



ImputeFormer



MIDAS (ours)

# Conclusions

- **Main contributions**

- Imputation framework that exploits physical relationships
- Experiments on various missing scenarios and datasets
- Demonstrating applicability to real-world applications

- **Future work**

- Adapting MIDAS to other domains
- Developing metrics for quantitative evaluation in real-world applications
- Applying MIDAS to downstream tasks in an end-to-end manner

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## Thank you for listening!

Contact information

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