

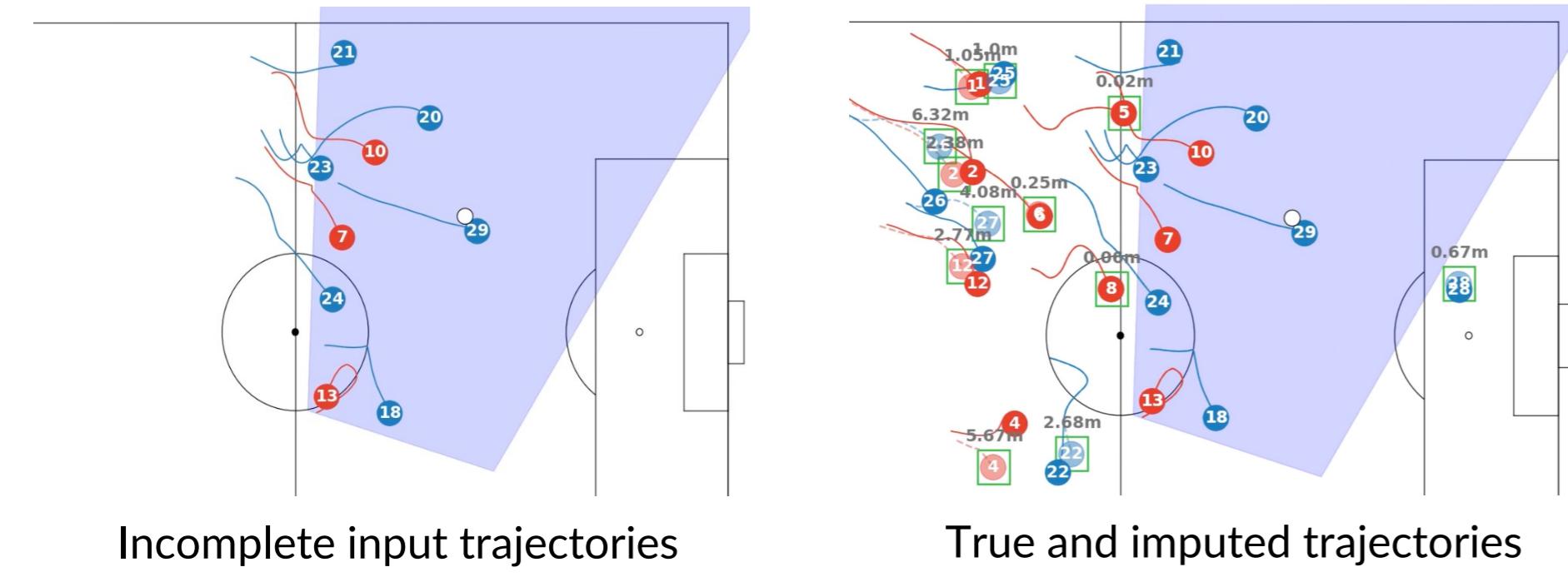
Trajectory Imputation in Multi-Agent Sports with Derivative Accumulating Self-Ensemble

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Background

- Sports tracking data often suffer from missing values due to various factors, such as player occlusion, frame drops, or low-quality sensor signals.
- Accurate and realistic imputation of missing trajectories is crucial for many downstream tasks in sports analytics.
- Existing time-series imputation methods performs poorly on sports data, as they do not account for the dynamic movements and interactions between players.

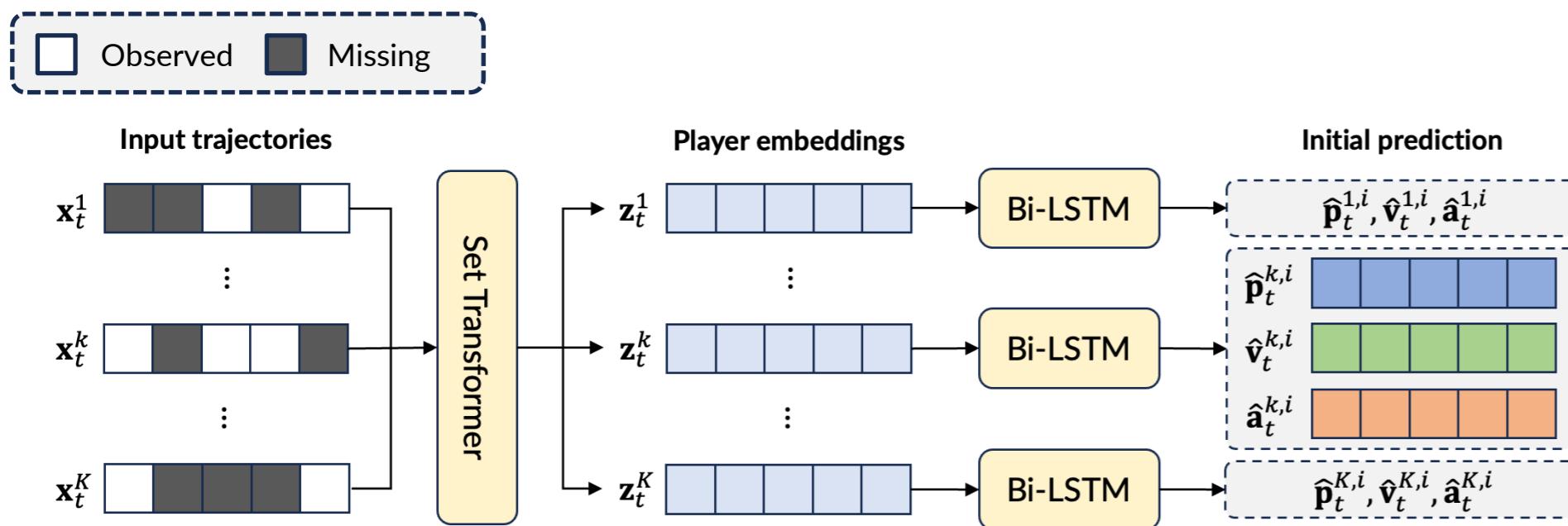
Proposed Framework: MIDAS



- MIDAS: Multi-agent Imputer with Derivative-Accumulating Self-ensemble**
- Combines multiple predictions using a hybrid architecture

Neural Network-Based Initial Prediction (IP)

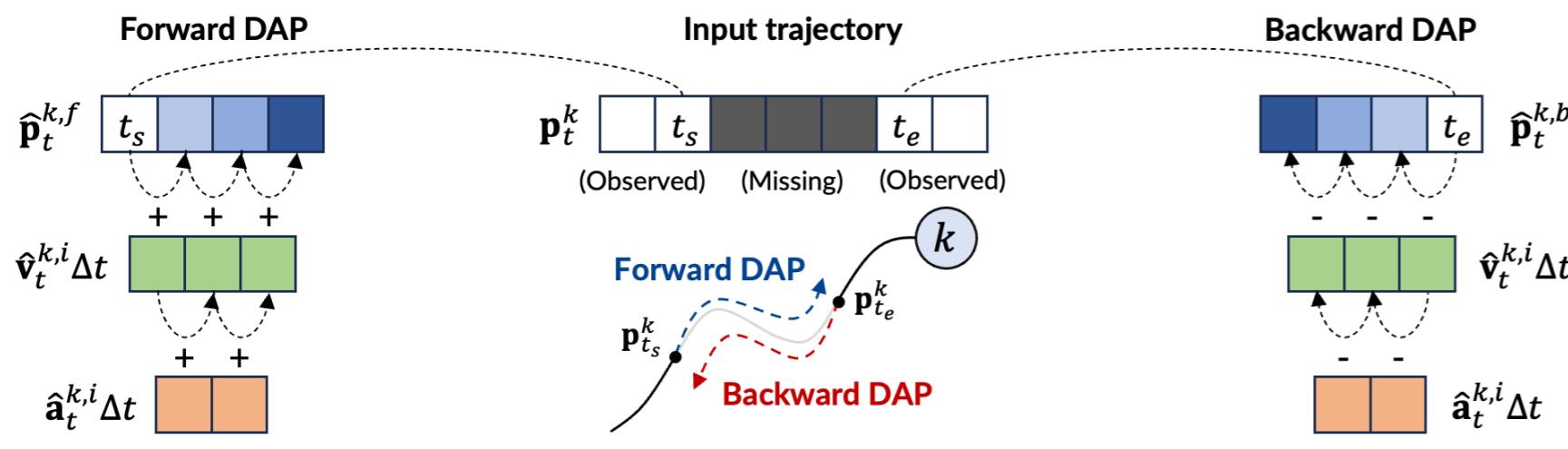
- Set Transformer encoder for permutation-equivariant player embeddings
- Bi-LSTM for player-wise temporal modeling
- Jointly predicts position, velocity, and acceleration for each time step



Derivative-Accumulating Predictions (DAP)

- Produces alternative predictions $\hat{p}_t^{k,f}$ and $\hat{p}_t^{k,b}$ by **accumulating velocity and acceleration from the observed endpoints** t_s and t_e :

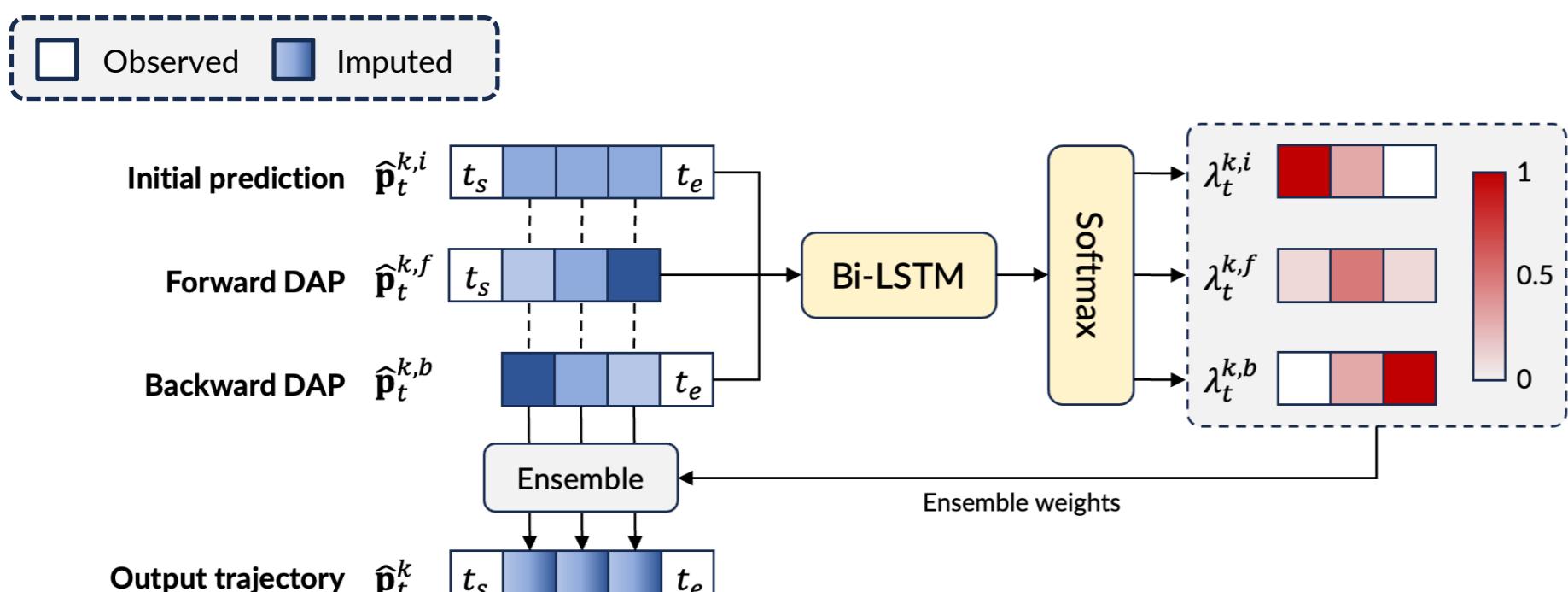
$$\begin{aligned} \hat{p}_t^{k,f} &= \mathbf{p}_{t_s}^k, & \hat{p}_t^{k,f} &\approx \hat{p}_t^{k,f} + (\hat{v}_t^{k,i} + \hat{a}_t^{k,i} \Delta t) \Delta t \quad (t_s < t < t_e) \\ \hat{p}_t^{k,b} &= \mathbf{p}_{t_e}^k, & \hat{p}_t^{k,b} &\approx \hat{p}_t^{k,b} - (\hat{v}_t^{k,i} - \hat{a}_t^{k,i} \Delta t) \Delta t \quad (t_s < t < t_e) \end{aligned}$$



Dynamic Ensemble of Multiple Predictions

- Learns to dynamically weight and combine IP and forward/backward DAPs:

$$\hat{\mathbf{p}}_t^k = \lambda_t^{k,i} \hat{\mathbf{p}}_t^{k,i} + \lambda_t^{k,f} \hat{\mathbf{p}}_t^{k,f} + \lambda_t^{k,b} \hat{\mathbf{p}}_t^{k,b}$$

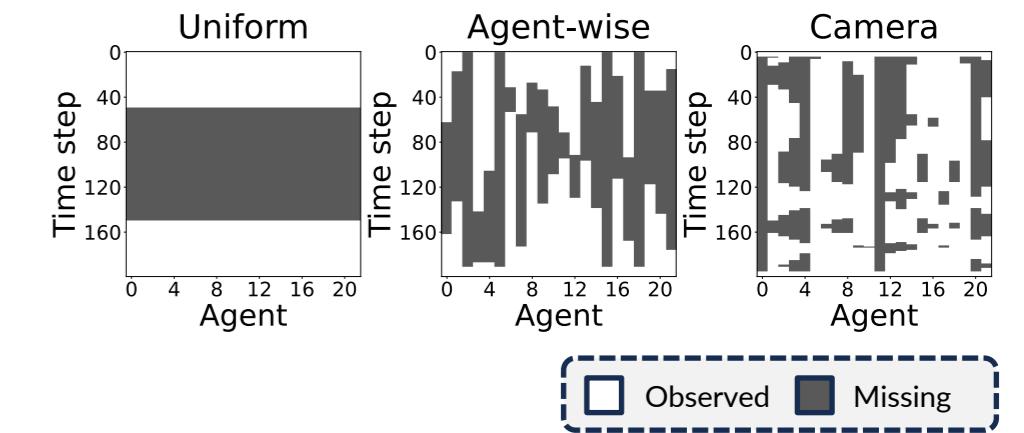


Experiments

Three sports datasets

- Soccer: 2/0.5/0.5 matches
- Basketball: 70/10/20 matches
- American football: Preprocessed 5-second sequences

Three missing scenarios



Experimental results

| Scenario | Metric | Method | | | | | | | | | |
|-------------------|------------|--------|--------|---------|---------|--------|--------|--------|--------|----------------------|----------------------|
| | | LI | CS | BRITS | NAOMI | NRTSI | CSDI | GI | IF | IP | MIDAS |
| Soccer | Uniform | PE | 3.8406 | 2.2085 | 7.4859 | 4.5343 | 3.1791 | 3.4295 | 4.6511 | 2.0898 | 1.4563 1.3205 |
| | | SCE | 0.1299 | 0.0867 | 3.9089 | 3.9793 | 0.0854 | 0.1586 | 0.1191 | 0.0815 | 0.1488 0.0516 |
| | Agent-wise | PE | 5.0752 | 11.4647 | 5.7266 | — | — | 4.0279 | 5.6011 | 2.5798 | 2.0755 1.9832 |
| | | SCE | 0.1631 | 0.2939 | 2.9627 | — | — | 0.1305 | 0.1508 | 0.0976 | 0.1057 0.0535 |
| Camera | PE | 3.1083 | 1.9209 | 7.4208 | — | — | 3.5181 | 3.6512 | 2.2151 | 1.4879 | 1.2296 |
| | | SCE | 0.0993 | 0.0547 | 4.1967 | — | — | 0.2132 | 0.0934 | 0.3149 | 0.1554 0.0374 |
| Basketball | Uniform | PE | 3.3481 | 2.3114 | 2.9085 | 1.5254 | 2.5291 | 2.2558 | 2.8305 | 1.3622 | 0.9801 0.9727 |
| | | SCE | 0.1483 | 0.1025 | 1.0521 | 0.3230 | 0.0734 | 0.0631 | 0.1066 | 0.0531 | 0.0432 0.0438 |
| | Agent-wise | PE | 4.4992 | 10.3857 | 2.4238 | — | — | 2.3471 | 2.5859 | 1.3345 1.3832 | 1.3862 |
| | | SCE | 0.1787 | 0.2715 | 0.5397 | — | — | 0.0563 | 0.0700 | 0.0485 | 0.0373 0.0381 |
| American Football | Uniform | PE | 0.8897 | 0.7448 | 1.7990 | 0.9692 | 0.5158 | 0.5558 | 0.8899 | 0.3673 | 0.2073 0.1542 |
| | | SCE | 1.1063 | 0.9463 | 10.9459 | 2.3112 | 0.2989 | 0.4905 | 1.1023 | 0.2858 | 0.1990 0.1126 |
| | Agent-wise | PE | 1.5128 | 1.2041 | 1.7527 | — | — | 0.6182 | 1.5128 | 0.3944 | 0.2383 0.2104 |
| | | SCE | 1.0641 | 0.5306 | 10.6807 | — | — | 0.4288 | 1.0631 | 0.1869 | 0.1180 0.0967 |

Analysis on missing length and ensemble behavior

| Sports | Category | Missing Frames | $\hat{p}_t^{k,f}(\hat{\lambda}_t^{k,i})$ | $\hat{p}_t^{k,f}(\hat{\lambda}_t^{k,f})$ | $\hat{p}_t^{k,b}(\hat{\lambda}_t^{k,b})$ | \hat{p}_t^k |
|------------|----------|--------------------|------------------------------------------|------------------------------------------|------------------------------------------|---------------|
| Soccer | Short | 33.30 ± 15.72 | 0.1379 (0.0001) | 0.0742 (0.6501) | 0.0783 (0.3498) | 0.0504 |
| | Medium | 90.42 ± 19.17 | 0.7939 (0.0003) | 0.7492 (0.6110) | 0.7624 (0.3887) | 0.7004 |
| | Long | 173.41 ± 21.70 | 2.7359 (0.0002) | 2.7017 (0.5034) | 2.7186 (0.4964) | 2.6082 |
| Basketball | Short | 33.86 ± 16.11 | 0.0444 (0.1596) | 0.0424 (0.5216) | 0.0426 (0.3187) | 0.0376 |
| | Medium | 90.65 ± 19.02 | 0.5395 (0.1783) | 0.5483 (0.4809) | 0.5482 (0.3406) | 0.5312 |
| | Long | 172.60 ± 21.20 | 1.8104 (0.1801) | 1.8288 (0.4313) | 1.8258 (0.3985) | 1.8005 |

Downstream Applications

Estimating physical metrics

| Method | Distance (m) | | Sprints | |
|----------------------|-----------------|--------------|--------------|--------------|
| | Mean | MAPE | Mean | MAPE |
| Ground Truth | 11,093.5 | — | 41.49 | — |
| Linear Interpolation | 10,167.8 | 8.46% | 38.89 | 6.32% |
| Cubic Spline | 10,686.3 | 3.73% | 38.85 | 6.73% |
| BRITS | 10,979.2 | 2.76% | 59.89 | 53.62% |
| CSDI | 11,343.0 | 2.77% | 44.20 | 14.71% |
| Graph Imputer | 8,972.1 | 19.15% | 37.85 | 9.80% |
| ImputeFormer | 11,441.7 | 3.22% | 50.25 | 26.29% |
| MIDAS (ours) | 10,922.4 | 1.58% | 40.71 | 4.95% |

Estimating contextual metrics

