Muhammed Göktepe
2017400162

Question 1

a)

Muhammed Göktepe

1a) $a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j(2\pi k/T_0)t}\, dt$

In this question $T_0 = 10$ seconds $\quad x(t) = 0.2t$ for $-5 < t < 5$

$a_k = \frac{1}{10}\int_{-5}^{5} 0.2t\, e^{-j(2\pi k/10)t}\, dt$ $\quad$ Lets focus on integral part.

$\int 0.2t\, e^{\frac{-j(\pi k)t}{5}}\, dt = \frac{1}{5}\int t\, e^{\frac{-j\pi k t}{5}}\, dt$

Lets substitu $u = \frac{-t}{5}$ $\quad dt = -5du$

$\int t e^{\frac{-j\pi k t}{5}}\, dt = 25\int u e^{\pi j k u}\, du$

Lets substitude $f = u \quad dv = e^{\pi j k u} du$

$1 = du \quad v = \frac{e^{\pi j k u}}{\pi j k}$

$\int u e^{\pi j k u} du = \frac{u e^{\pi j k u}}{\pi j k} - \int \frac{e^{\pi j k u}}{\pi j k} du$

$= \frac{u e^{\pi j k u}}{\pi j k} - \frac{e^{\pi j k u}}{(\pi j k)^2} \Rightarrow 25\int u^{\pi j k u} du = \frac{25 u e^{\pi j k u}}{\pi j k} - \frac{25 e^{\pi j k u}}{(\pi j k)^2}$

Lets undo substitution $u = \frac{-t}{5}$

$= \frac{-5t e^{\frac{-j\pi k t}{5}}}{j\pi k} - \frac{25 e^{\frac{-j\pi k t}{5}}}{(j\pi k)^2}$

$= \frac{-t e^{-j\pi k t/5}}{j\pi k} - \frac{5 e^{-j\pi k t/5}}{(j\pi k)^2} = -\frac{(j\pi k t + 5)\, e^{-j\pi k t/5}}{j^2\pi^2 k^2}$

$a_k = \frac{1}{10}\int_{-5}^{5} 0.2t\, e^{-j(2\pi k/10)t}\, dt = -\frac{1}{10}\left(\frac{(j\pi k t + 5)\, e^{-j\pi k t/5}}{j^2\pi^2 k^2}\Bigg|_{-5}^{5}\right)$

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$$-\frac{1}{10}\left(\frac{(5\pi jk+5)e^{-\pi jk}}{\pi^2 j^2 k^2}-\frac{(-5\pi jk+5)e^{j\pi k}}{\pi^2 j^2 k^2}\right)=\frac{-1}{10}\left(\frac{(5\pi jk+5)e^{-\pi jk}+(5\pi jk-5)e^{j\pi k}}{(\pi j k)^2}\right)$$

$$\boxed{=-\frac{e^{-j\pi k}\left((\pi jk-1)e^{2\pi jk}+\pi jk+1\right)}{2\pi^2 j^2 k^2}=a_k}=-\frac{(-1)^k(2\pi jk)}{2\pi^2 j^2 k^2}=-\frac{(-1)^k}{\pi jk}$$

Substitude with these values

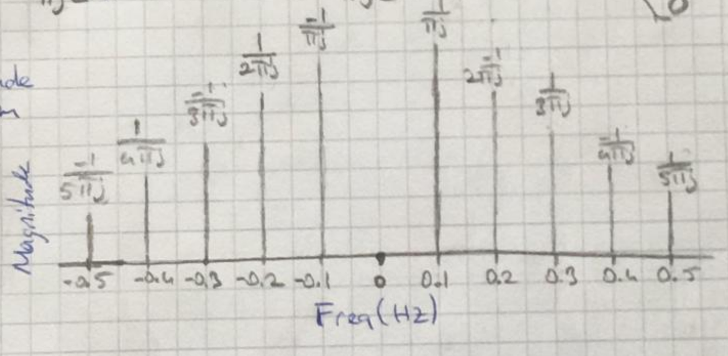
$$a_0=\frac{1}{10}\int_{-5}^{5}0.2t\,dt=\frac{1}{10}\left(\frac{0.2t^2}{2}\Big|_{-5}^{5}\right)=\frac{1}{10}\cdot\frac{1}{10}\left(t^2\Big|_{-5}^{5}\right)=\frac{1}{100}\cdot(25-25)=\underline{\underline{0}}$$

$$e^{-j\pi}=e^{j\pi}=-1 \qquad e^{-j2\pi}=e^{j2\pi}=1$$

$$a_k=\begin{cases}\frac{1}{\pi jk} & k=\pm1,\pm3,\pm5\ldots\\ \frac{-1}{\pi jk} & k=\pm2,\pm4,\pm6\ldots\\ 0 & k=0\end{cases}$$

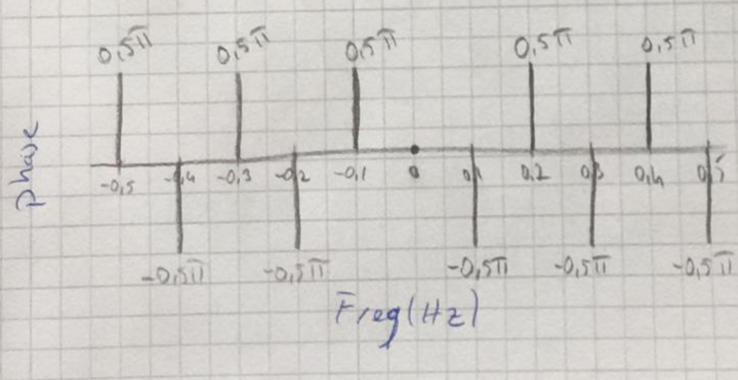
$$a_{odd}=\frac{1}{\pi jk}\ // \qquad a_{even}=\frac{-1}{\pi jk}\ //$$

Magnitude Diagram



$T_0=10\,sec$

$f=\frac{1}{10}=0.1$ Hertz



Phasor Diagram

Muhammed Göktepe
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b)

```
%%%%%%%%%%%%%%%%%% This part creates original signal %%%%%%%%%%%%%%%%%%%%%%
fs = 100;
t = -10:1/fs:10-1/fs;
x = sawtooth(2*pi*0.1*t+pi);
%%%%%%%%%%%%%%%%%% This part creates original signal %%%%%%%%%%%%%%%%%%%%%%

h3 = sin(0*t);  % 0 signal for sum of first 3 harmonics
h5 = sin(0*t);  % 0 signal for sum of first 5 harmonics
h15 = sin(0*t); % 0 signal for sum of first 15 harmonics

% This loop computes the first 15 harmonics
for c = 1:15
    % Since the fourier coefficents are different for even and odd terms
    % If it is a odd term harmonic then it's coefficient ak = (1/(pi*j*k)
    if mod(c,2) == 1
        h = (1/(pi*j*c)*exp(j*2*pi*c*0.1*t)) + (-1/(pi*j*c)*exp(j*2*pi*c*(-
0.1)*t));
    % If it is a even term harmonic then it's coefficient ak = (-1/(pi*j*k)
    else
        h = (-1/(pi*j*c)*exp(j*2*pi*c*0.1*t)) + (1/(pi*j*c)*exp(j*2*pi*c*(-
0.1)*t));
    end

    % This if statement sums the first 3 harmonics
    if c < 4
        h3 = h3 + h;
    % These two if statements add 4th and 5th harmonics to h3.
    % h5 is the sum of first 5 harmonics
    elseif c == 4
        h5 = h3 + h;
    elseif c == 5
        h5 = h5 + h;
    % These last two if statements sums the remaning harmonics until 15th
    % h15 is the sum of first 15 harmonics
    elseif c == 6
        h15 = h5 + h;
    else
        h15 = h15 + h;
    end
end

subplot(1,1,1);
plot(t,x,t,h3,t,h5,t,h15); % Plots the waves to the figure
legend("original","sum of first 3 harmonics","sum of first 5 harmonics","sum
of first 15 harmonics"); % shows which color represents which wave
xlabel('time');
ylabel('amplitude');
title('Harmonic Summation');
```

First, I found original signal as given in the pdf file. Then I created three zero amplitude sin wave for hormonic sums. Then with a for loop I computed first fifteen harmonics. While calculating these harmonics I used the Fourier coefficients that I found in part a. I multiplied the coefficients with exponential term for each harmonic. Finally, I plot the waves to the figure and added a legend to show which color represents which wave.

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Question 2

```
[y,Fs] = audioread('faultyphone.wav');   % Creating audio signal from audio
file
info = audioinfo('faultyphone.wav');      % Storing the audio info of audio
file
% sound(y,Fs);  % playing the sound

ytran = fft(y); %discrete fourier transform
n = length(y);  % number of samples
fshift = (-n/2:n/2-1)*(Fs/n);    % 0-centered frequency range
yshift = fftshift(ytran);        % shift y values

%%%%%%%%%%%% Plotting the signal w.r.t zero centered frequency %%%%%%%%%%%%
subplot(3,1,1)
plot(fshift,abs(yshift)/n)
xlabel('Frequency (Hz)')
ylabel('Amplitude')
title('faultyphone.wav');
%%%%%%%%%%%% Plotting the signal w.r.t zero centered frequency %%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%% Plotting the signal w.r.t frequency %%%%%%%%%%%%%%%%%%
% The tall lines at the beginning and at the end are the places that we
% need to filter. And these values are 1736 and 187535. So in ytran wave we
% need to make zero the ytran(1736) and ytran(1875535).
subplot(3,1,2)
plot(abs(ytran))
xlabel('Frequency (Hz)')
ylabel('Amplitude')
%%%%%%%%%%%%%%%%%%%% Plotting the signal w.r.t frequency %%%%%%%%%%%%%%%%%%


ytran(1736)=0;  % We are making the power of this frequency zero to eliminate
noise.
ytran(187535)=0;% We are making the power of this frequency zero to eliminate
noise.
yfinal=ifft(ytran); % Then we are moving from frequency domain to time domain.
% sound(yfinal,Fs);
audiowrite('filteredphone.wav',yfinal,Fs); % Writing filtered sound to a file

filtered_y = fft(yfinal);    % discrete fourier transform for filtered sound
new_N = length(yfinal);      % number of samples
filtered_shift = (-new_N/2:new_N/2-1)*(Fs/new_N);    % 0-centered frequency
range
filtered_yshift = fftshift(filtered_y);             % shift y values

%%%%%%%% Plotting the filtered signal w.r.t zero centered frequency %%%%%%%
subplot(3,1,3)
plot(fshift,abs(filtered_yshift)/new_N)
xlabel('Frequency (Hz)')
ylabel('Amplitude')
title('After Filtering');
%%%%%%%% Plotting the filtered signal w.r.t zero centered frequency %%%%%%%
```

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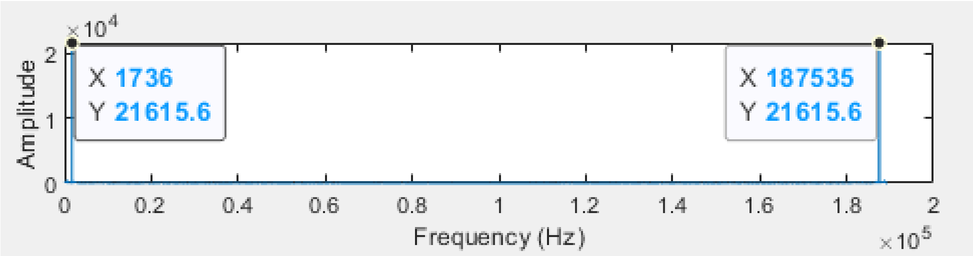
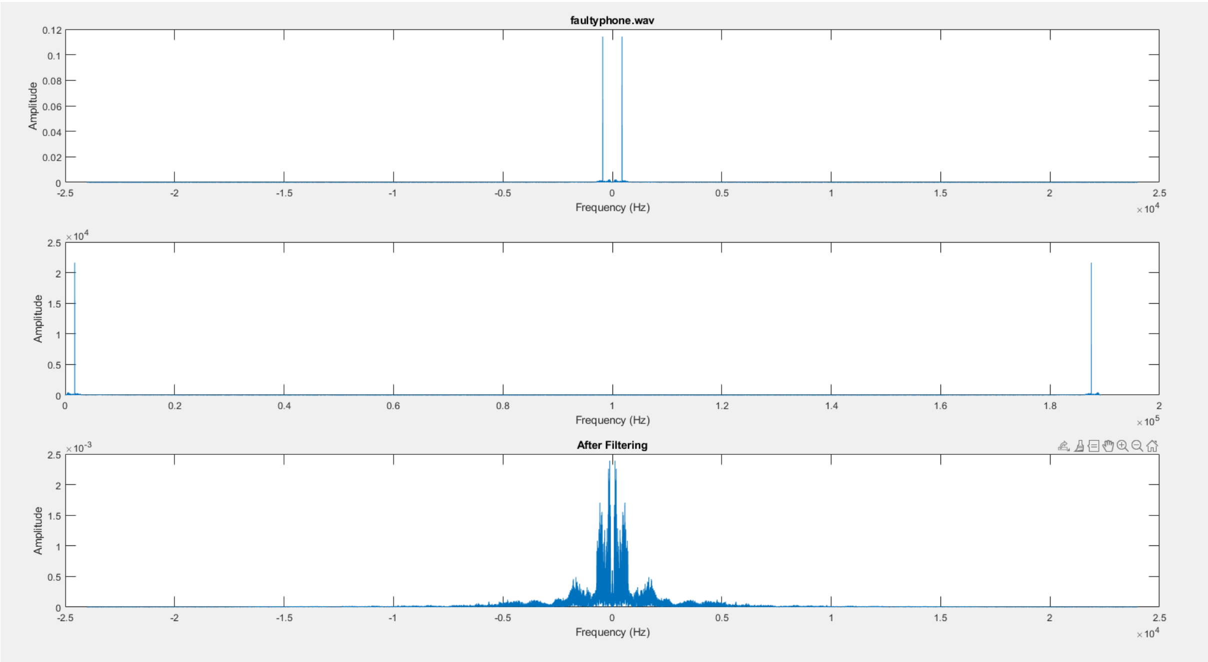
In this question firstly, we need to read the sound wave from the file. Then I made a Fourier transform and shift them to get zero centered frequency plot. I also plotted the not shifted version so that we can see which frequencies make the noises. As we can see in the



picture 1736 and 187535 are the certain frequencies that we need to make their power zero. After making these frequencies power zero, I moved from frequency domain to time domain. Then I write the audio to a file to listen. Lastly, I made another Fourier transform and shift them to get zero centered frequency plot for filtered signal.

I listened the filtered sound and realized that the la note was gone and remaining was voice record. Also I realized that the magnitude of the recording was smaller than original noise.



Question 3

In this question firstly I read the sound from the file. Then I designed a 50-point average time filter. To obtain this filter I created a coefficient vector "b" which has 50 term equals to 1/50. Then I filtered the sound and obtained filtered sound that I will be use. From this filtered sound I plotted the spectrogram and try to find the fundamental frequency. It placed in the left side of the figure. It has revealed itself with a big amplitude at a low frequency. So, it is a yellow line (which is the highest amplitude's color) in the left. Then I zoomed in the yellow line read the value of 0.0091. this is normalized frequency. To find the fundamental frequency we

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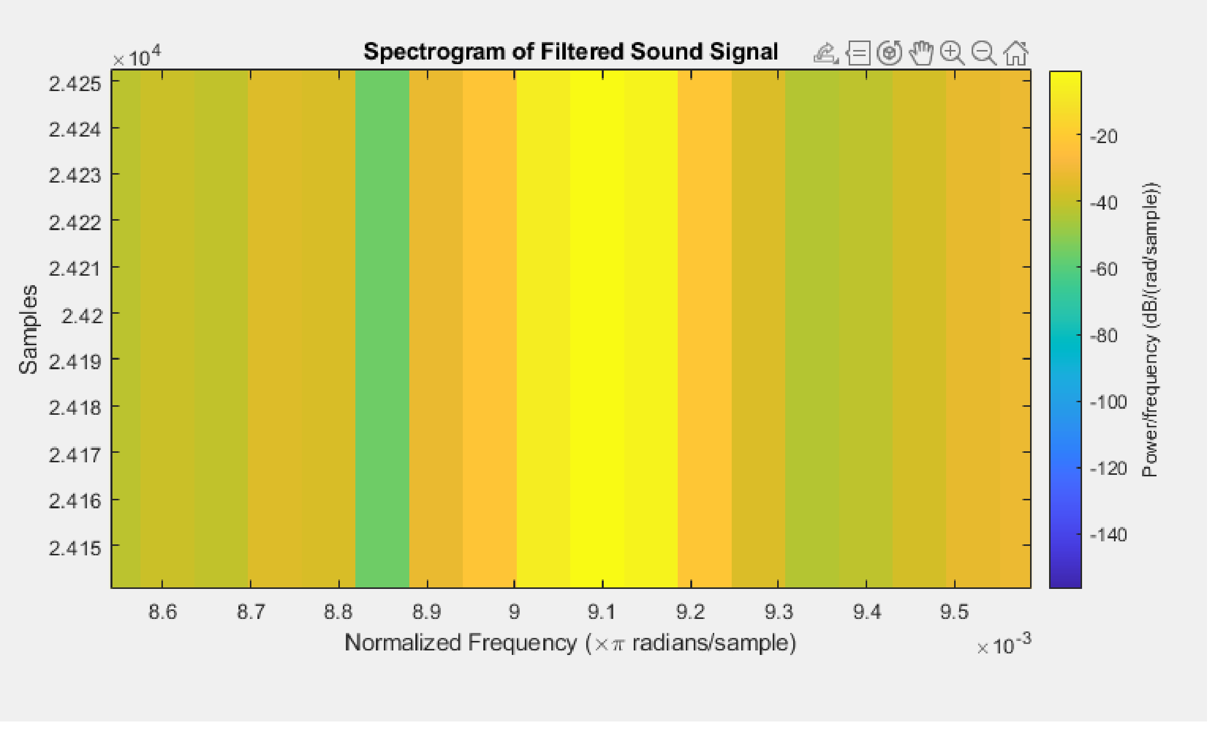
need to multiply this value with sampling frequency and divide it to 2. Thus, it will become $f_0 = 0.0091 * \frac{48000}{2} = 218.4\ Hz$

Then we implied the given formula which is $N_{RPM} = \frac{1}{2}\frac{N_{stroke}}{N_{cylinder}}\frac{60[RPM]}{1[Hz]}f_0[Hz]$

$N_{cylinder} = N_{stroke} = 4\ given\ in\ the\ question$

Thus $N_{RPM} = \frac{1}{2}\frac{4}{4}\frac{60[RPM]}{1[Hz]}218.4[Hz] = 6552[RPM]$



Here is the spectrogram of the filtered sound and in the middle, you can see the normalized frequency that we will use.

```
[x,Fs] = audioread('enginesound.m4a');   % Creating audio signal from audio
file
info = audioinfo('enginesound.m4a');      % Storing the audio info of audio
file
% sound(x,Fs);   % playing the sound
t = 0:seconds(1/Fs):seconds(info.Duration); % finding the duration of the
sound
t = t(1:end-1);

%%%%%%%%%%%%%%%%%%%%%%% Plots original sound signal%%%%%%%%%%%%%%%%%%%%%%
subplot(2,2,1);
plot(t,x);
xlabel('Time')
ylabel('Amplitude')
title('Original Sound Signal');
%%%%%%%%%%%%%%%%%%%%%%% Plots original sound signal%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%% Filtering the original sound %%%%%%%%%%%%%%%%%%%%%%
```

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```
% Creating the filter coefficient vectors. Since it is a 50 point avarager
% we need [1/50 1/50 1/50 1/50 .....   1/50] vector with size 50
a = 1;
c = 1/50;
b = c(ones(1, 50));
avg_x = filter(b,a,x);  % This function will filter the sound.
% sound(avg_x,Fs);
%%%%%%%%%%%%%%%%%%%%%%% Filtering the original sound %%%%%%%%%%%%%%%%%%%%%%


%%%%%%%%%%%%%%%%%%%%%%% Plots filtered sound and spectrogram%%%%%%%%%%%%%%
subplot(2,2,2);
plot(t,avg_x);
xlabel('Time')
ylabel('Amplitude')
title('Filtered Sound Signal');

subplot(2,2,3);
spectrogram(avg_x);
title('Spectrogram of Filtered Sound Signal');
%%%%%%%%%%%%%%%%%%%%%%% Plots filtered sound and spectrogram%%%%%%%%%%%%%%

y = fft(avg_x);
n = length(avg_x);
fshift = (-n/2:n/2-1)*(Fs/n);
yshift = fftshift(y);

subplot(2,2,4)
plot(fshift,abs(yshift)/n)
xlabel('Frequency (Hz)')
ylabel('Amplitude')

f0 = 0.0091 * Fs /2;     % Fundamental frequency obtained from spectrogram
N_stroke = 4;            % Given in question
N_cylinder = 4;          % given in question

N_rpm = (1/2) * (N_stroke/N_cylinder) * 60 * f0;     % formula to calculate
RPM
```