CS771A: Assignment 1

ML Gods
Anshu Kumar, 200150
Dhruv Khandelwal, 200331
Rashmi G R, 200772
Sudhanshu Kumar, 201010
Sweta Kumari, 201036

Abstract

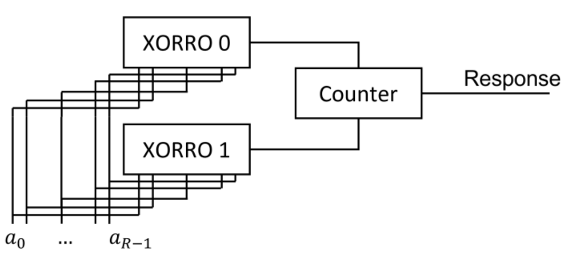
This pdf file contains submissions for the Question 1, Question 2 and Question 4 parts of the assignment.

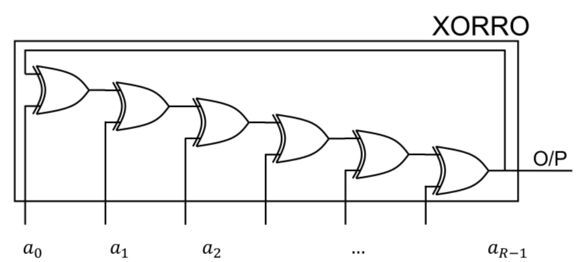
1 Question 1

Given: Simple XORRO PUF has two XORROs and has no select bits and no multiplexers. A single XORRO PUF has R XORs. The challenge is an R-bit string that is fed into both XORROs as their configuration bits:

$$a \overset{\text{def}}{=} [a_0, a_1, ..a_{R-1}]$$

Now, we will find the time delay for i^{th} XOR.





Here a_i denotes the config bit for the i^{th} XOR. Let $\delta^i_{00}, \delta^i_{01}, \delta^i_{10}, \delta^i_{11}$ be the time that i^{th} XOR gate takes before giving its output. When the input to that gate is $00, 01, 10$ and 11, respectively. Let D_i be the time delay for the i^{th} XOR.

When $\mu_i = 0$,

$\Delta_i^{\mu_i=0}$ = $a_i\delta^i_{01}$ + $(1 - a_i)\delta^i_{00}$ ——(1)

When $\mu_i = 1$,

$\Delta_i^{\mu_i=1}$ = $a_i\delta^i_{11}$ + $(1 - a_i)\delta^i_{10}$ ——(2)

From (1) and (2), we will get the time delay of i^{th} XOR.

Δ_i = $\mu_i\Delta_i^{\mu_i=1}$ + $(1 - \mu_i)\Delta_i^{\mu_i=0}$

Preprint. Under review.

$\Delta_i = \mu_i(a_i\delta_{11}^i + (1-a_i)\delta_{10}^i) + (1-\mu_i)(a_i\delta_{01}^i + (1-a_i)\delta_{00}^i)$

Time Delays:

For 0^{th} XOR,

When $\mu_i = 0$,

$\Delta_0^{\mu_0=0} = a_0\delta_{01}^0 + (1-a_0)\delta_{00}^0$ ——(3)

When $\mu_0 = 1$,

$\Delta_0^{\mu_0=1} = a_0\delta_{11}^0 + (1-a_0)\delta_{10}^0$ ——(4)

Adding (3) and (4), we get,

$\Delta_0^{\mu_0=0} + \Delta_0^{\mu_0=1} = a_0(\delta_{01}^0 + \delta_{11}^0 - \delta_{00}^0 - \delta_{10}^0) + (\delta_{00}^0 + \delta_{10}^0)$

For 1^{st} XOR, When $\mu_0^r = 0$, We know that when one of the two inputs to XOR gate is 0, it acts as an identity i.e, it will give the 2^{nd} input as output.

$\therefore \mu_1 = a_0$

$\Delta_1^{\mu_0=0} = \mu_1(a_1\delta_{11}^1 + (1-a_1)\delta_{10}^1) + (1-\mu_1)(a_1\delta_{01}^1 + (1-a_1)\delta_{00}^1)$ ——(5)

When $\mu_0 = 1$, We know that when one of the inputs to XOR gate is 1, it acts as an inverter i.e, it will give the negation of the 2^{nd} input as output.
$\therefore \mu_1 = 1 - a_0$

$\Delta_1^{\mu_0=1} = (1-a_0)(a_1\delta_{11}^1 + (1-a_1)\delta_{10}^1) + a_0(a_1\delta_{01}^1 + (1-a_1)\delta_{00}^1)$ ——(6)

Adding (5) and (6), we get,

$\Delta_1^{\mu_0=0} + \Delta_1^{\mu_0=1} = a_1(\delta_{01}^1 + \delta_{11}^1 - \delta_{00}^1 - \delta_{10}^1) + \delta_{00}^1 + \delta_{10}^1$

Similarly, For the 3^{rd} XOR,

When $\mu_0 = 0$,

$\mu_2 = (1-a_0)a_1 + a_0(1-a_1)$

$\Delta_2^{\mu_0=0} = \mu_2(a_2\delta_{11}^2 + (1-a_2)\delta_{10}^2) + (1-\mu_2)(a_2\delta_{01}^2 + (1-a_2)\delta_{00}^2)$ ——(7)

When $\mu_0 = 1$,

$\mu_2 = [1 - [(1-a_0)a_1 + a_0(1-a_1)]]$

$\Delta_2^{\mu_0=1} = [1 - [(1-a_0)a_1 + a_0(1-a_1)]](a_2\delta_{11}^2 + (1-a_2)\delta_{10}^2) + [(1-a_0)a_1 + a_0(1-a_1)](a_2\delta_{01}^2 + (1-a_2)\delta_{00}^2)$ ——(8)

Adding (7) and (8), we get,

$\Delta_2^{\mu_0=0} + \Delta_2^{\mu_0=1} = a_2(\delta_{01}^2 + \delta_{11}^2 - \delta_{00}^2 - \delta_{10}^2) + \delta_{00}^2 + \delta_{10}^2$

Now, From the pattern, we can write,

$\Delta_i^{\mu_0=0} + \Delta_i^{\mu_0=1} = a_i(\delta_{01}^i + \delta_{11}^i - \delta_{00}^i - \delta_{10}^i) + \delta_{00}^i + \delta_{10}^i$

Now, $\Sigma_{i=0}^{1=R-1}(\Delta_2^{\mu_0=0} + \Delta_2^{\mu_0=1}) = \Sigma a_i(\delta_{01}^i + \delta_{11}^i - \delta_{00}^i - \delta_{10}^i) + \Sigma(\delta_{00}^i + \delta_{10}^i)$ We know that,

$\Sigma_{i=0}^{1=R-1}\Delta_i^{\mu_0=0} = t_0$ and $\Sigma_{i=0}^{1=R-1}\Delta_i^{\mu_0=1} = t_1$

$\therefore t_0 + t_1$ = $\Sigma a_i(\delta^i_{01} + \delta^i_{11} - \delta^i_{00} - \delta^i_{10})$ + $\Sigma(\delta^i_{00} + \delta^i_{10})$

For Upper XORRO, or XORRO2,

$(t_0 + t_1)^{upper}$ = $[\Sigma a_i(\delta^i_{01} + \delta^i_{11} - \delta^i_{00} - \delta^i_{10}) + \Sigma(\delta^i_{00} + \delta^i_{10})]^{upper}$ —(9)

For Lower XORRO, or XORRO1,

$(t_0 + t_1)^{lower}$ = $[\Sigma a_i(\delta^i_{01} + \delta^i_{11} - \delta^i_{00} - \delta^i_{10}) + \Sigma(\delta^i_{00} + \delta^i_{10})]^{lower}$ —(10)

Subtracting (10) from (9), we get,

$(t_0 + t_1)^{upper}$ - $(t_0 + t_1)^{lower}$ = $[\Sigma a_i(\delta^i_{01} + \delta^i_{11} - \delta^i_{00} - \delta^i_{10}) + \Sigma(\delta^i_{00} + \delta^i_{10})]^{upper}$ - $[\Sigma a_i(\delta^i_{01} + \delta^i_{11} - \delta^i_{00} - \delta^i_{10}) + \Sigma(\delta^i_{00} + \delta^i_{10})]^{lower}$

$\implies (t_0+t_1)^{upper}$ - $(t_0+t_1)^{lower}$ = $[\Sigma a_i[(\delta^i_{01}+\delta^i_{11}-\delta^i_{00}-\delta^i_{10})^{upper} - (\delta^i_{01}+\delta^i_{11}-\delta^i_{00}-\delta^i_{10})^{lower}]$
+ $\Sigma[(\delta^i_{00} + \delta^i_{10})^{upper} - (\delta^i_{00} + \delta^i_{10})^{lower}]$

Given: If upper XORRO has higher frequency, then the output in 1.

$\implies (t_0 + t_1)^{upper}$ - $(t_0 + t_1)^{lower} < 0$ Response = 1
and,
$\implies (t_0 + t_1)^{upper}$ - $(t_0 + t_1)^{lower} > 0$ Response = 0

$\implies$ output = $\frac{1+sign((t_0+t_1)^{lower}-(t_0+t_1)^{upper})}{2}$

$\implies$ output = $\frac{1+sign([\Sigma[(\delta^i_{01}+\delta^i_{11}-\delta^i_{00}-\delta^i_{10})^{lower}-(\delta^i_{01}+\delta^i_{11}-\delta^i_{00}-\delta^i_{10})^{upper}]a_i+\Sigma[(\delta^i_{00}+\delta^i_{10})^{lower}-(\delta^i_{00}+\delta^i_{10})^{upper}])}{2}$

Comparing the above equation with, $\implies$ output = $\frac{1+sign(w^T\phi(c)+b}{2}$

We see that,

$w_i = [(\delta^i_{01} + \delta^i_{11} - \delta^i_{00} - \delta^i_{10})^{lower} - (\delta^i_{01} + \delta^i_{11} - \delta^i_{00} - \delta^i_{10})^{upper}]$

$b = \Sigma[(\delta^i_{00} + \delta^i_{10})^{lower} - (\delta^i_{00} + \delta^i_{10})^{upper}]$

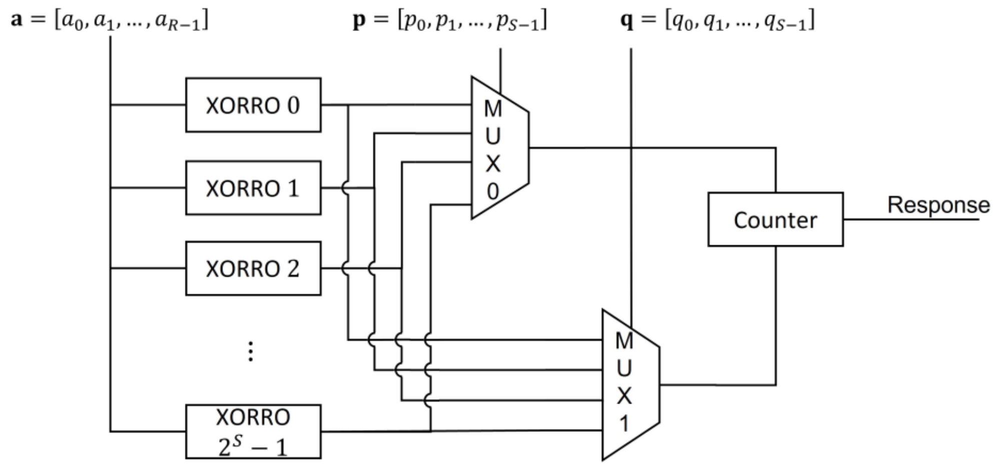
$X = a$

Given the map $\phi : [0, 1]^R \to R^D$

$\implies D = R$ and $\phi(c) = c$.

2 Question 2

Given: Advanced XORRO contains 2^s XORROs and 2 Multiplexers.
Input for MUX0: P = $[p_0, p_1, ..., p_{s-1}]$
Input for MUX1: Q = $[q_0, q_1, ..., q_{s-1}]$



To find the XORRO chosen by MUX0 and MUX1, we convert the given bits p and q into their corresponding decimal number, say P and Q.

The upper XORRO: $P = \Sigma_{i=0}^{s-1} 2^i . p_{s-(i+1)}$

The lower XORRO: $Q = \Sigma_{i=0}^{s-1} 2^i . q_{s-(i+1)}$

The total number of unique pairs (P, Q) possible is N = $2^s(2^s - 1)$

For a given (P_0, Q_0), the response for (Q_0, P_0) will be the negation of the response for (P_0, Q_0). Hence, we can reduce the number of linear models to M = $2^{s-1}(2^s - 1)$ by switching the response of (Q_0, P_0) while training the model of (P_0, Q_0).

To solve the problem of the advanced XORRO PUF, we will train M models now. Each model will be trained by dividing the data set into M parts in which all the CRPs having (P_0, Q_0) and (Q_0, P_0) as their Pand Q will be grouped together. We can reduce this problem to M simple XORRO PUF models as follows:

For a given P, Q:

$w_i = [(\delta^i_{01} + \delta^i_{11} - \delta^i_{00} - \delta^i_{10})^{Q^{th} XORRO} - (\delta^i_{01} + \delta^i_{11} - \delta^i_{00} - \delta^i_{10})^{P^{th} XORRO}]$

$b = \Sigma[(\delta^i_{00} + \delta^i_{10})^{Q^{th} XORRO} - (\delta^i_{00} + \delta^i_{10})^{P^{th} XORRO}]$

$X = a$

where $P, Q \in [0, 2^{s-1}]$ and $P \neq Q$

3 Question 3

The python file for question 3 is included in the folder as submit.py.

4 Question 4

4.1 Question 4a

LinearSVC:

loss	Accuracy (%)	Time (secs)
Square hinge	94.7375	4.735
Hinge	93.9725	4.222

4.2 Question 4b

LinearSVC:

C	Accuracy (%)	Time (secs)
0.01	90.3975	3.3247
1	94.74	4.7953
100	93.935	4.3577

Logistic regression:

C	Accuracy (%)	Time (secs)
0.01	82.5175	4.089
1	93.9175	4.7354
100	94.9425	6.85

4.3 Question 4c

LinearSVC:

tol	Accuracy (%)	Time (secs)
$1e^{-8}$	93.9175	4.5361
$1e^{-4}$	93.9175	4.728
$1e^{0}$	93.78	3.9921

Logistic regression:

tol	Accuracy (%)	Time (secs)
$1e^{-8}$	94.725	4.855
$1e^{-4}$	94.74	4.7947
$1e^{0}$	94.5425	3.391

4.4 Question 4d

LinearSVC:

penalty	Accuracy (%)	Time (secs)
l1	94.7325	4.7435
l2	94.7425	4.788

Logistic regression:

penalty	Accuracy (%)	Time (secs)
l1	93.9175	4.565
l2	93.9175	4.719