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DS6014

Homework 1:
Probability Review and Priors

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1 (15)

You are a data Scientist and are choosing between three approaches, A, B, and C to a problem. With approach A you will spend a total of four days coding and running an algorithm and it will not produce useful results. With approach B you will spend a total of three days coding and running an algorithm and it will not produce useful results. With approach C you will spend over day coding and running an algorithm and it will give the results you are looking for. You are equality likely to choose among unselected options. What is the expected time in days for you to obtain the results you are looking for? What is the variance on this time?

<u>Response:</u>

Considering all the possible approach chains

If starting with approach A:

- A-B-C: 4+3+1 = 8 days
- A-C: 4+1 = 5 days

If starting with approach B:

- B-A-C: 3+4+1= 8 days
- B-C : 3+1 = 4 days

If staring with approach C

- C: need one day = 1 days

Let E to represent the days to solve the question = {8, 5, 8, 4, 1} and the mean value is 5.2

The variance of time based on the variance formula:

$S^2 = \frac{\sum(x_i - \bar{x})^2}{n-1} = \frac{34.8}{4} =$ 8.7, the sigma = 2.95

2 (15)

Suppose if it is sunny or not in Charlottesville depends on the weather of the last three days. Show how this can be modeled as Markov chain.

Response:

We can think about "Markov chain" in this way

- The next term in a sequence could depend on all the previous terms
    - If it only depends on the previous term it is called "first-order" Markov
    - If it depends on the two previous terms it is "second-order" Markov

We can form a Markov chain as follows, take the weather states R(rain), (nice), S(Sunny) to form transition probabilities.

$$P = \begin{bmatrix} P11 & P12 & P13 \\ P21 & P22 & P23 \\ P31 & P32 & P33 \end{bmatrix}$$

$$P_{ij} \geq 0, i = 1, \ldots \ldots.. n, j = 1, \ldots..n \; and \sum_{j=1}^{n} P_{ij} = 1$$

For example:

$$\begin{array}{cc} & \begin{array}{ccc} R & N & S \end{array} \\ P = \begin{array}{c} R \\ N \\ S \end{array} & \begin{pmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{pmatrix} \end{array}$$

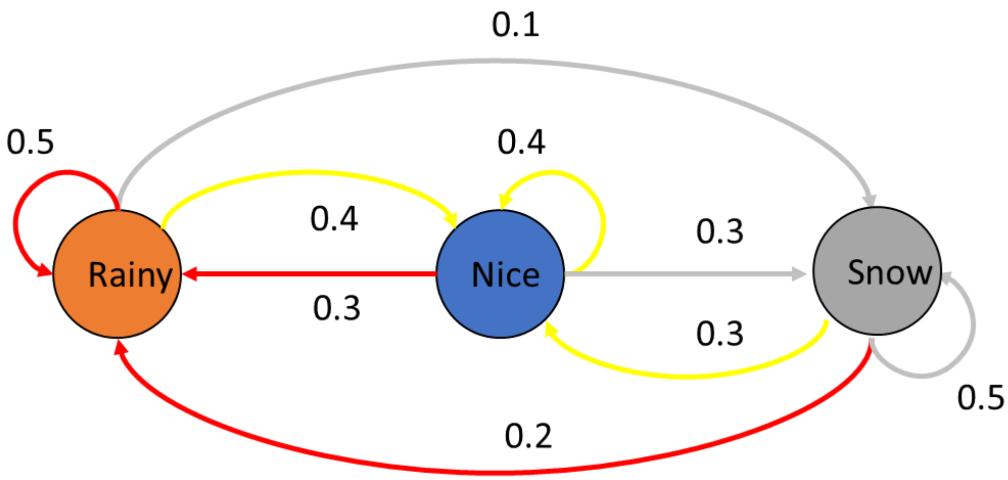
The entries in the first row of the matrix P represent the probabilities for the various kinds of weather following a rainy day. Similarly, the entries in the second and third rows represent the probabilities for the various kinds of weather following nice and snowy day, respectively.

Considering given the chain is in state i today, it will be in state j two days from now given this probability by p_{ij}

Markov property: The state of the system at time t+1 only depends on thee state of the system at time t

$P[X_{t+1} = x_{t+1} | X_t = x_t, X_{t-1} = x_{t-1,\ldots\ldots\ldots\ldots\ldots}, X_1 = x_1, X_0 = x_0] =$

$P[X_{t+1} = x_{t+1} | X_t = x_t]$



Weather forecasting

- Two days: $\begin{pmatrix}0.5 & 0.4 & 0.1\\ 0.3 & 0.4 & 0.3\\ 0.2 & 0.3 & 0.5\end{pmatrix}\begin{pmatrix}0.5 & 0.4 & 0.1\\ 0.3 & 0.4 & 0.3\\ 0.2 & 0.3 & 0.5\end{pmatrix} = \begin{pmatrix}0.39 & 0.39 & 0.22\\ 0.33 & 0.37 & 0.30\\ 0.29 & 0.35 & 0.36\end{pmatrix}$
- Four days:

$$\begin{pmatrix}0.5 & 0.4 & 0.1\\ 0.3 & 0.4 & 0.3\\ 0.2 & 0.3 & 0.5\end{pmatrix}^2\begin{pmatrix}0.5 & 0.4 & 0.1\\ 0.3 & 0.4 & 0.3\\ 0.2 & 0.3 & 0.5\end{pmatrix}^2 = \begin{pmatrix}0.3446 & 0.3734 & 0.2820\\ 0.3378 & 0.3706 & 0.2916\\ 0.3330 & 0.3686 & 0.2984\end{pmatrix}$$

The graph 3 (15)

Assume a Gaussian distribution for observations, $X_{i,\ i=1,\ldots\ldots\ldots N}$ with unknown mean, M and known variance 5. Suppose the prior for M is Gaussian with variance 10. How large a random sample must be taken (i.e., what is the minimum value for N) to specify an interval having unit length of 1 such that the probability that M lies in this interval is 0.95?

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<u>Response:</u>

Gaussian with Unknown mean and known variance

From lecture

- Likelihood with N trials, $x = (x_1, \dots \dots \dots, x_N)$ with unknown mean M and know variance σ^2
  - $f(x|m, \sigma^2) \propto \frac{1}{\sigma} \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{N}(x_i - m)^2\right)$
- Prior for M is $N(\mu_0, \sigma_0^2)$
- Posterior is $N(\mu_N \ and\ \sigma_{post}^2) where$
  - $\mu_N = \frac{\mu_0\, \sigma^2 + N\bar{x}\ \sigma_0^2}{\sigma^2 + N\sigma_0^2}$
  - $\sigma_{post}^2 = \frac{\sigma_0^2\sigma^2}{\sigma^2 + N\sigma_0^2}$

The variance of the Gaussian distribution is 5 (σ) and the variance for the prior is 10 (σ_0^2)

Since the posterior is a function of N, we need to find an N that makes this variance (here is 5) that the probability M is centered at the posterior mean $\mu_N = 0.95$

$$\sigma_{post}^2 = \frac{\sigma_0^2\sigma^2}{\sigma^2 + N\sigma_0^2} \rightarrow \sigma_{post}^2 = \frac{10*5}{5+N10} = \frac{50}{5+10N}$$

$$CDF = \frac{1}{2}\left[1 + erf\left(\frac{x - \mu}{\sigma\sqrt{2}}\right)\right]$$

$$0.975 = \frac{1}{2}\left[1 + erf\left(\frac{0.5}{\sqrt{\frac{50}{5+10N}}\sqrt{2}}\right)\right]$$

$$1.3859 * \left(\sqrt{\frac{50}{5+10N}}\sqrt{2}\right) = 0.5$$

$$1.9207 * \left(\frac{100}{5+10N}\right) = 0.25\ ,\ \mathrm{N} = 77$$

4 (15)

You have started an online business selling books that are of interest to your customers. A publisher has just given you a large book with photos from famous 20th century photographers. You think this book will appeal to people who have bought art books, history books and coffee table books. In an initial offering of the new book you collect data on purchases of the new book and combine these data with data from the past purchases (see ArtHistBooks.csv).

Use Baysesian analysis to give the posterior probabilities for purchases of art books, history books and coffee table books, as well as, the separate probabilities for purchases of new books given each possible combination of prior purchases of art books, history books and coffee table books. Do this by first using beta priors with values of the hyperparameters that represent lack of prior information. Then compute these probabilities again with beta priors that show strong weighting for low likelihood of a book purchase. Compare your results.

Response: see the notebook in Appendix

5 (15)

The data set CHDdata.csv contains cases of coronary heart disease(CHD) and variables associated with the patient's condition: systolic blood pressure, yearly tobacco use (in kg), how density lipoprotein (ldl), adiposity, family history (0 or 1), type A personality score (typea), obesity (body mass index), alcohol use, and the diagnosis of CHD (0 or 1). Perform a Bayesian analysis of these data that finds the posterior marginal probability distributions for the means for the data of patients with and without CHD. You should first standard scale (substract the man and divide by the standard deviation) all the numeric variables (remove family history and do not scale CHD). Then separate the data into two sets, one for patients with CHD and one for patients without CHD.

Your priors for both groups should assume means of 0 for all variables and a correlation of 0 between all pairs of variables. You should assume all variances for the variables are 1. Use a prior alpha equal to one plus the number of predictor variables. Compute and compare the Bayesian estimates for the posterior means for each group.

For 5 extra credit points, compute the probability of observing a point at least as extreme as the posterior mean of patients without coronary heart disease under the posterior distribution for the patients with coronary heart disease. Then compute the probability of observing a point at least as extreme as the posterior mean of patients with coronary heart disease under the posterior distribution for the patients without coronary heart disease

Response: see the notebook in Appendix

6 (10)

For each of the following types of distributions, state the support type (single or multivariable and discrete or continuous), the formula for the PMP or PDF, the parameters, the support, the mean, and some typical uses of the distribution. You may use whatever source(s) you want, including for example Wikipedia.

(a) Bernoulli Distribution
(b) Binomial Distribution
(c) Poisson Distribution
(d) Uniform Distribution
(e) Beta Distribution
(f) Gamma Distribution
(g) Gaussian Distribution
(h) t Distribution
(i) Cauchy Distribution
(j) Multinomial Distribution
(k) Dirichlet Distribution
(l) Multivariate Gaussian Distribution
(m) Multivariate t Distribution
(n) Wishart Distribution

Distribution, PMP, PDF, support type

a	Bernoulli Distribution	Discreate probability distribution Support k $\in \{0, 1\}$ PMF $\begin{cases} q = 1 - p \; if \; k = 0 \\ p \; if \; k = 1 \end{cases}$ Mean = p Bernoulli is the discrete probability distribution of a random variable which takes the value 1 with probability p and the value 0 with probability q=1-p. A Bernoulli distribution can be thought of as a model for the set of possible outcomes of any single experiment that asks a yes/true/one with probability p, and failure/no/false/zero with probability q such as coin toss problem.
b	Binomial Distribution	Binomial distribution is the discrete probability distribution. Support k $\in \{0, 1, \ldots \ldots \ldots . n\}$ – number of successes PMF $\binom{n}{k} p^k q^{n-k}$ Mean np Thee binomial distribution is frequently used to model the number of successes in a sample of size n drawn with replacement from a population of size N.
c	Poisson Distribution	The Poisson distribution is a discrete probability distribution Support k $\in N_0$, Natural numbers starting from 0 PMF = $\frac{\lambda^k e^{-\lambda}}{k!}$ Mean = λ For instance: expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate and independently of the time since the last event.
d	Uniform Distribution	Uniform distribution is the continuous uniform distribution or rectangular distribution Support $x \in [a, b]$

		PDF $\begin{cases} \frac{1}{b-a} \; for\ x \in [a,b] \\ 0\ otherwise \end{cases}$ Mean ½(a+b) For instance, density function, uniform probability density function
e	Beta Distribution	The beta distribution is a family of continuous probability distributions defined on the interval[0, 1] parameterized by two positive shape parameters, denoted by α and β, that appear as exponents of the random variable and control t he shape of the distribution. Support x ∈ $[0,1]\ or\ x \in (0,1)$ PDF$\frac{X^{\alpha-1}(1-X)^{\beta-1}}{B(\alpha,\beta)}$ Where B(α, β) =$\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ and Γ is the Gamma function Mean: $E[X] = \frac{\alpha}{\alpha+\beta}$ $E[\ln X] = \varphi(\alpha) - \varphi(\alpha+\beta)$ $E[XlnX] = \frac{\alpha}{\alpha+\beta}[\varphi(\alpha+1) - \varphi(\alpha+\beta+1)]$
f	Gamma Distribution	Gamma distribution is a two-parameter family of continuous probability distributions. Support $x \in (0,\infty)$ PDF: $f(x) = \frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-\frac{x}{\theta}}$ (k>0, $\theta > 0$) $f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$ (α>0, $\theta > 0$) Mean :kθ (k>0, $\theta > 0$) $\frac{\alpha}{\beta}$ (α>0, $\theta > 0$) For instance: in life testing, the waiting time until death is a random variable that is frequently modeled with a gamma distribution.
g	Gaussian Distribution	Gaussian distribution is a type of continuous probability distribution Support $x \in R$ PDF = $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ Mean σ
h	t Distribution	t-distribution is an member of family of continuous probability distribution support $x \in (-\infty,\ \infty)$

		PDF $\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)}(1+\frac{x^2}{\nu})^{-\frac{\nu+1}{2}}$ Mean 0 for $\nu > 1$
i	Cauchy Distribution	The Cauchy distribution is a continuous probability distribution. Support $x \in (-\infty, \infty)$ PDF = $\frac{1}{\pi\gamma\left[1+\left(\frac{x-x_0}{\gamma}\right)^2\right]}$ Mean undefined
j	Multinomial Distribution	In a probability theory, the multinominal distribution is a generalization of the binomial distribution. For example, it models the probability of counts for each side of a k-sided die rolled n times. For n independent trials each of which leads to a success for exactly one of k categories, with each category having a given fixed success probability, the multinomial distribution gives the probability of any particular combination of numbers of successes for the various categories. Support $x_i \in \{0, \ldots.. n\}, i \in \{1, \ldots \ldots k\}$ $$\sum x_i = n$$ PMF $\frac{n!}{x_1!\ldots\ldots x_k!} p_1^{x_1} \ldots \ldots \ldots .. p_k^{x_k}$ Mean E(X_i) = np_i
k	Dirichlet Distribution	The Dirichlet distribution is a family of continuous multivariance probability distributions parametrized by a vector α of positive reals. Support $x_1, \ldots \ldots \ldots x_K$ where $x_i \in (0,1) and \ \sum_{i=1}^{K} x_i = 1$ PDF: $\frac{1}{B(\alpha)} \prod_{i=1}^{K} x_i^{\alpha_i - 1}$ Where B(α) = $\frac{\prod_{i=1}^{K}\Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^{K}\alpha_i)}$ Where α = $(\alpha_1, \ldots \ldots \ldots \alpha_k)$ Mean $$E[X_i] = \frac{\alpha_i}{\Gamma(\sum_{k=1}^{K}\alpha_k)}$$ $$E[lnX_i] = \varphi(\alpha_i) - \varphi(\sum_k \alpha_k)$$

l	Multivariate Gaussian Distribution	In probability theory, the multivariate normal distribution is a generalization of the one-dimensional normal distribution to higher dimensions. Support $x \in \mu + span(\Sigma) \subseteq R^k$ PDF $(2\pi)^{-\frac{k}{2}} \det(\Sigma)^{-\frac{1}{2}} e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)}$ exists only when Σ is positive-define Mean μ
m	Multivariate t Distribution	In statistics, the multivariate t-distribution is a multivariate probability distribution. Support $x \in R^p$ PDF $\frac{\Gamma[(\nu+p)/2]}{\Gamma(\nu/2)\nu^{p/2}\pi^{p/2}\lvert\Sigma\rvert^{1/2}}\left[1+\frac{1}{\nu}(x-\mu)^T\Sigma^{-1}(x-\mu)\right]^{-(\nu+p)/2}$ Mean μ if $\nu > 2$, else undefined.
n	Wishart Distribution	In statistics, the Wishart distribution is a generalization to multiple dimensions of the gamma distribution. Support X($p \times p$) PDF $f_x(x) = \frac{\lvert x\rvert^{(n-p-1)/2} e^{-tr(V^{-1}x)/2}}{2^{\frac{np}{2}} \lvert V\rvert^{\frac{n}{2}} \Gamma_p(\frac{n}{2})}$ Mean $E[X]$ = nV

7 (10)

Using the Python Notebook http://www.kaggle.com/billbasener/pt2-probabilities-likelihoods-and-bayes-theorem, complete the challenge questions from Seciton6: Modify the code from Section 5 to add the ability to use the posterior_from_conjugate_prior function to output the posterior probability parameters given parameters and for a Gaussian Likelihood with known variance σ^2, and use your modified function to create the Prior, Likelihood, Posterior plots as in Section 5 of the notebook.

Response: see the notebook in Appendix

Appendix

Problem 4

In [93]:
```
### Summary table (a, b) = (1, 1)
```

In [94]:
```
## adopting a=1, b=1
```

In [95]:
```
import pandas as pd
import numpy as np
from scipy.stats import binom
from scipy.stats import beta
import seaborn as sns
import matplotlib.pyplot as plt
fname = 'ArtHistBooks.csv'
```

In [96]:
```
df = pd.read_csv(fname)
```

In [97]:
```
df
```

Out[97]:

	ArtBooks	HistoryBooks	TableBooks	Purchase
0	0	0	1	0
1	0	1	0	0
2	0	0	0	0
3	1	0	1	0
4	1	1	1	0
...	...	...	...	...
995	1	1	0	1
996	0	1	0	0
997	1	0	1	0
998	1	1	0	0
999	0	1	0	0

1000 rows × 4 columns

In [98]:
```
df.ArtBooks
```

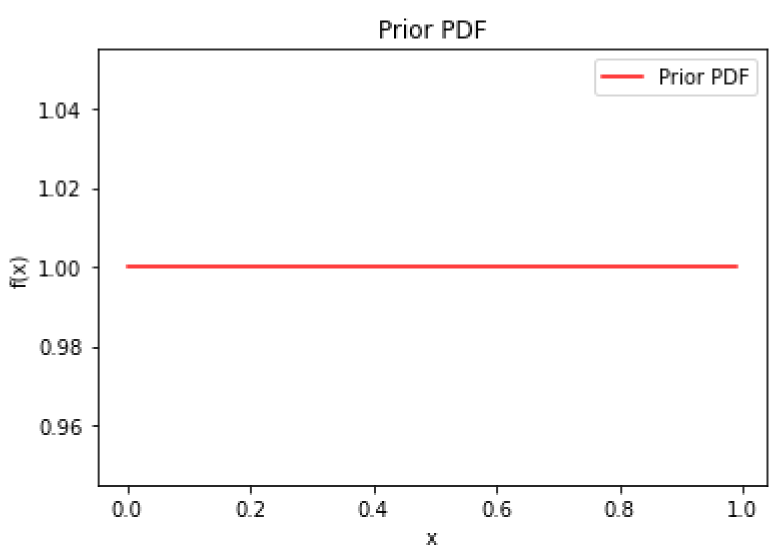
Out[98]:
```
0      0
1      0
2      0
3      1
4      1
      ..
995    1
996    0
997    1
998    1
999    0
Name: ArtBooks, Length: 1000, dtype: int64
```

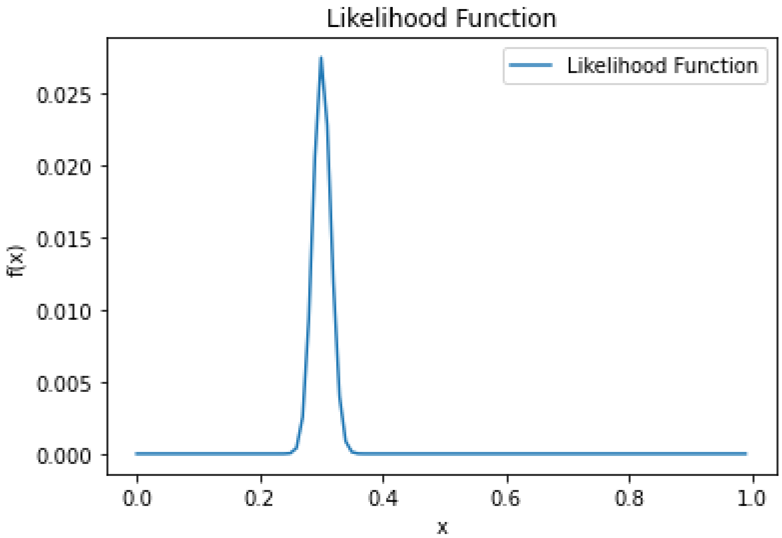
In [99]:
```
def posterior_from_conjugate_prior(**kwargs):
    if kwargs['Likelihood_Dist_Type'] == 'Binomial':
        # Get the parameters for the likelihood and prior distribution from the k
        x = kwargs['x'] # possible values for p, range across [0, 1]
        n = kwargs['n'] # number of trials (number of customers)
        k = kwargs['k'] # number of successes (purchases)
        a = kwargs['a'] # alpha parameters on the prior
        b = kwargs['b'] # beta parameter on the prior

        print(f'a_prime = {k + a}.')
        print(f'b_prime = {n - k + b}.')
        Likelihood = binom.pmf(p=x, n=n, k=k)
        Prior = beta.pdf(x=x, a=a, b=b)
        Posterior = beta.pdf(x=x, a=k+a, b=n-k+b)

        return [Prior, Likelihood, Posterior]

    else:
        print('Distribution type not supported.')
```

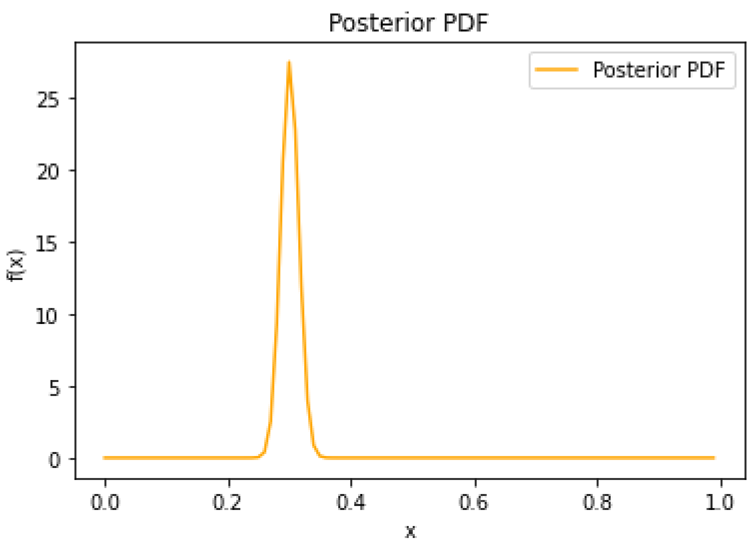
Prior: Artbooks

In [100]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df.ArtBooks) #num_trials = 1000
num_successes = np.sum(df.ArtBooks > 0)

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=1)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 302.
b_prime = 700.
```







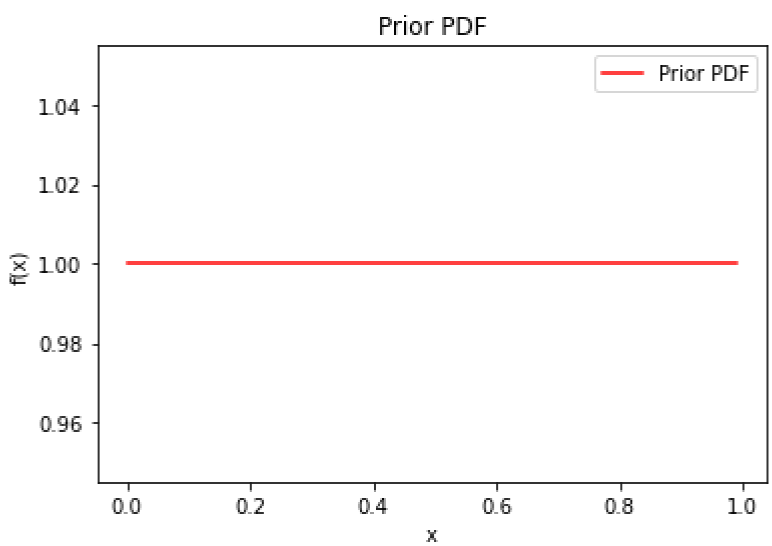
In [101]:
```
art_weight = np.argmax(Posterior)
art_weight
```

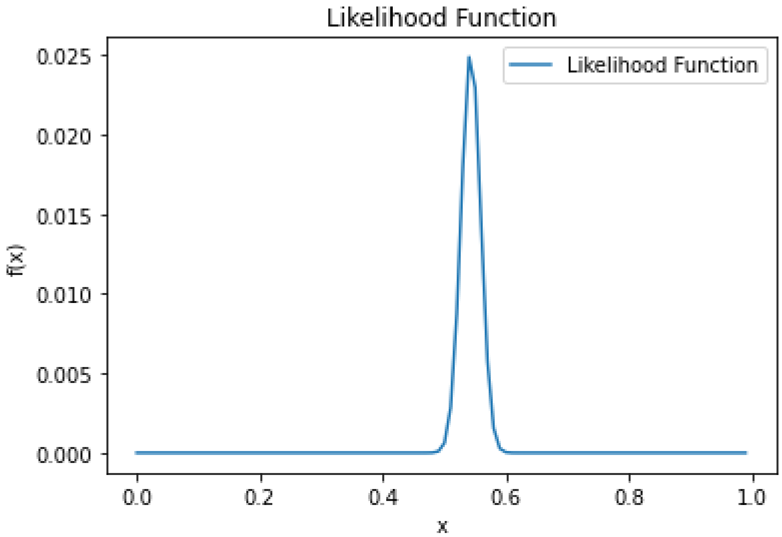
Out[101]: 30

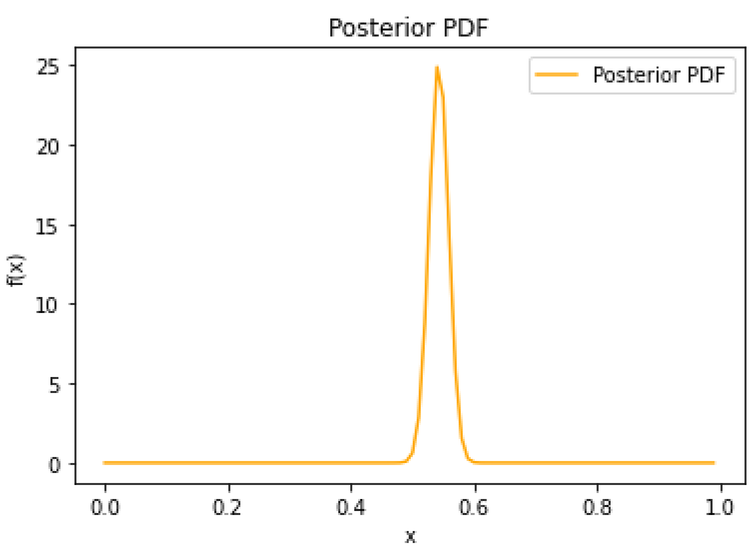
Prior: HistoryBooks

In [102]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df.HistoryBooks) #num_trials = 1000
num_successes = np.sum(df.HistoryBooks > 0)

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=1)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 544.
b_prime = 458.
```




Likelihood Function
Likelihood Function
f(x)
x
0.025
0.020
0.015
0.010
0.005
0.000
0.0
0.2
0.4
0.6
0.8
1.0


Posterior PDF
Posterior PDF
f(x)
x
25
20
15
10
5
0
0.0
0.2
0.4
0.6
0.8
1.0

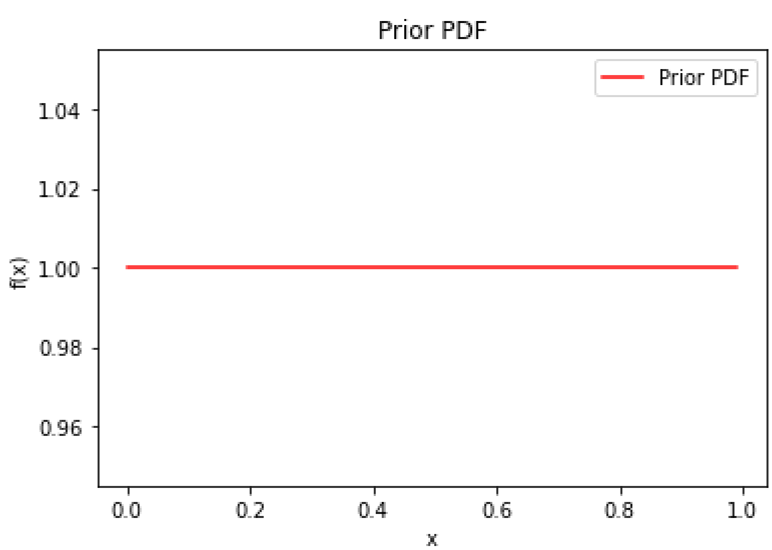
In [103]:
```
h_weight = np.argmax(Posterior)
h_weight
```

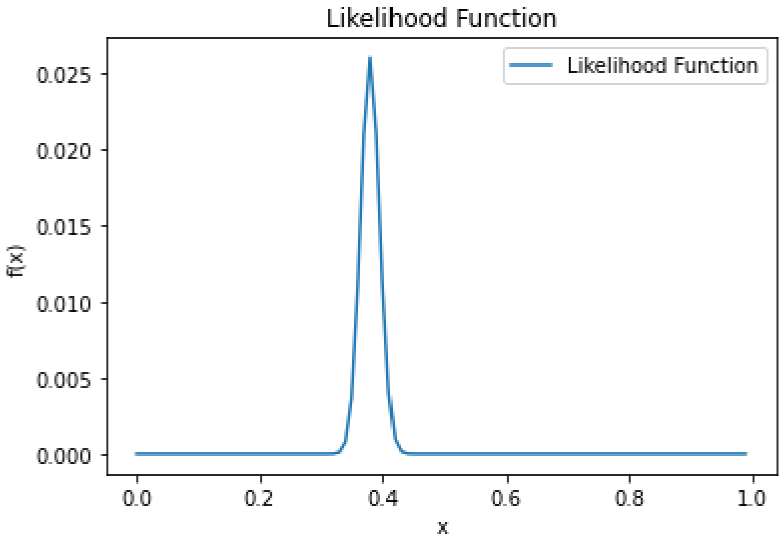
Out[103]: 54

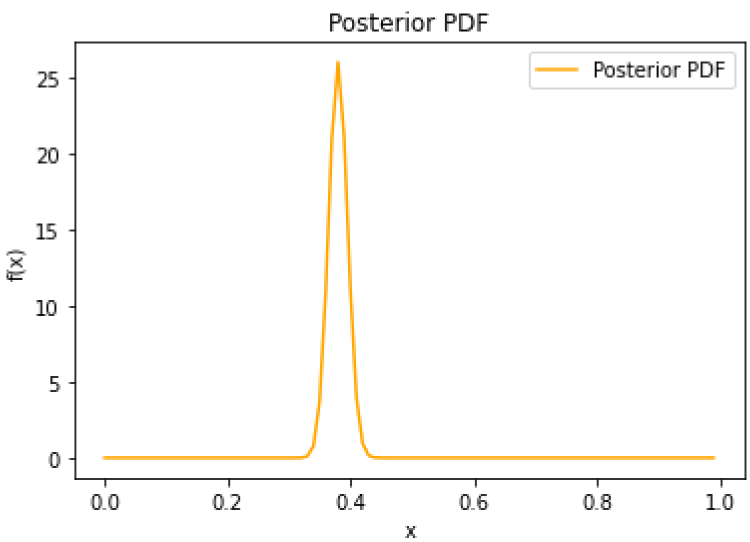
Prior: TableBooks

In [104]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df.TableBooks) #num_trials = 1000
num_successes = np.sum(df.TableBooks > 0)

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=1)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 381.
b_prime = 621.
```







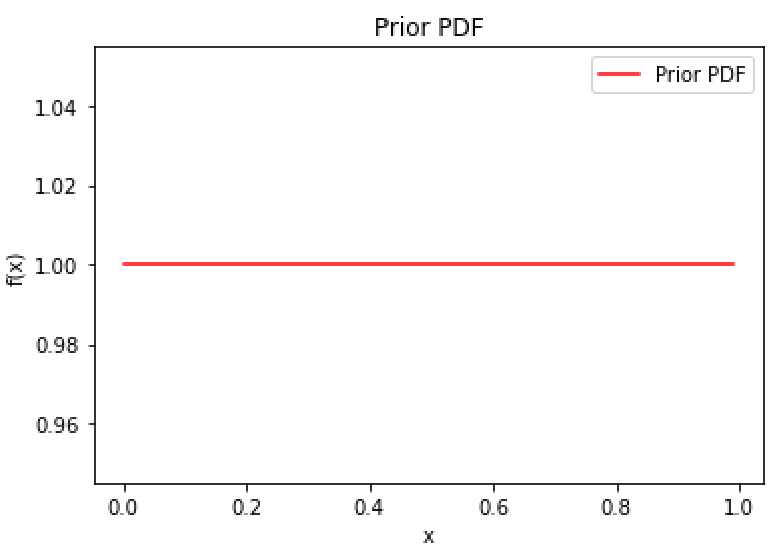
In [105]:
```
t_weight = np.argmax(Posterior)
t_weight
```
Out[105]: 38

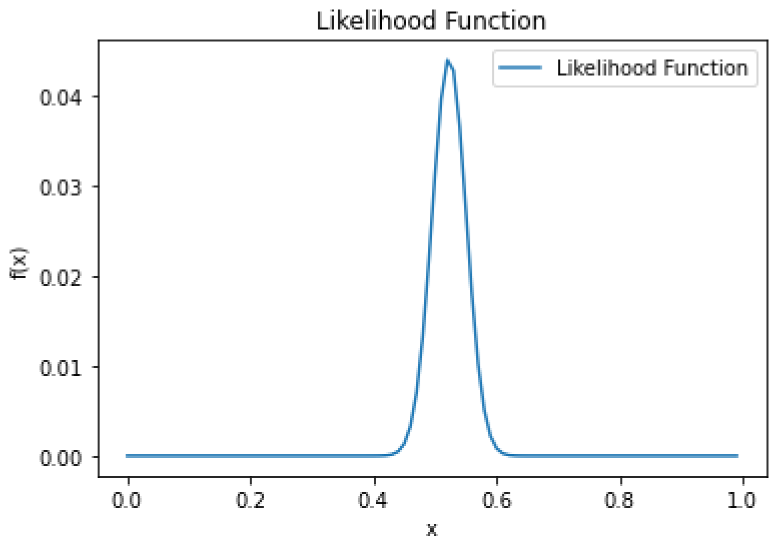
In [106]:
```
num_trials = len(df[(df["ArtBooks"] == 0) & (df["HistoryBooks"] == 0) & (df["Tabl
```

Prior: Artboks and HistoryBooks

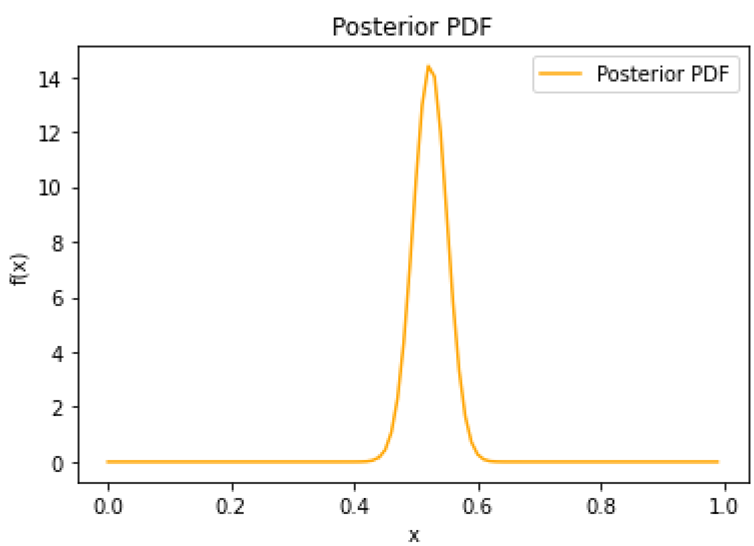
In [107]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials = len(df[(df["ArtBooks"] == 0) & (df["HistoryBooks"] == 0)]) #num_tri
num_successes = np.sum((df.ArtBooks > 0) & (df.HistoryBooks))

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=1)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 172.
b_prime = 157.
```





Likelihood Function
Likelihood Function
0.04
0.03
0.02
0.01
0.00
f(x)
0.0
0.2
0.4
0.6
0.8
1.0
x



Posterior PDF
Posterior PDF
14
12
10
8
6
4
2
0
f(x)
0.0
0.2
0.4
0.6
0.8
1.0
x

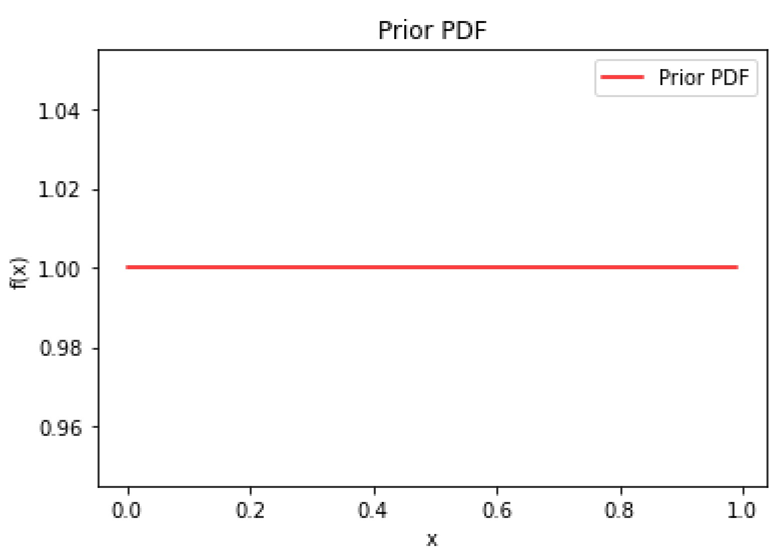
In [108]:
```
ah_weight = np.argmax(Posterior)
ah_weight
```

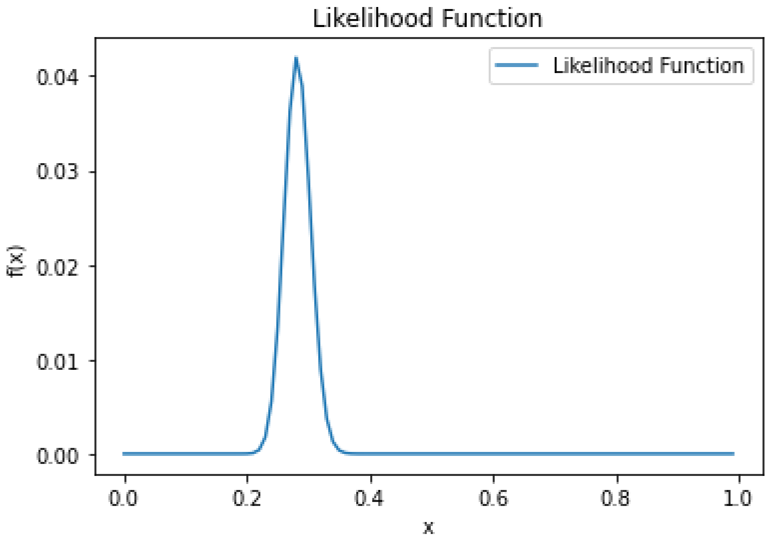
Out[108]:
```
52
```

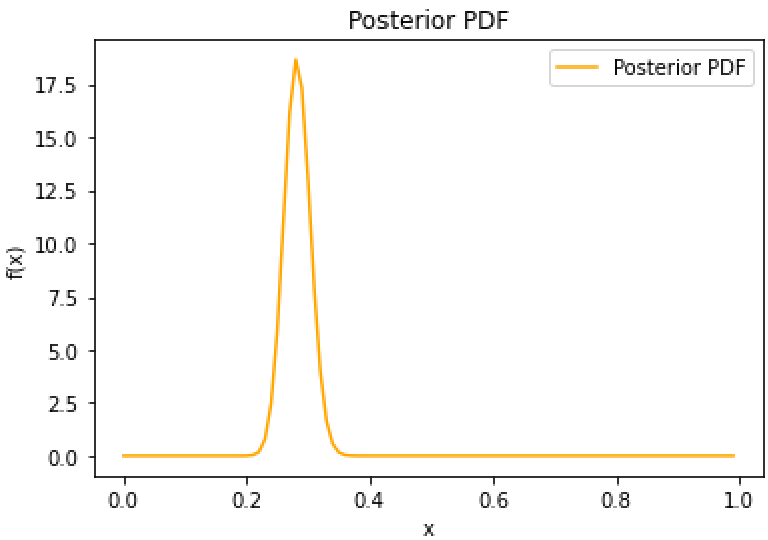
Prior: Artboks and TableBooks

In [109]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df[(df["ArtBooks"] == 0)  & (df["TableBooks"] == 0)])#num_trials
num_successes = np.sum((df.ArtBooks > 0) & (df.TableBooks))

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=1)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 126.
b_prime = 320.
```







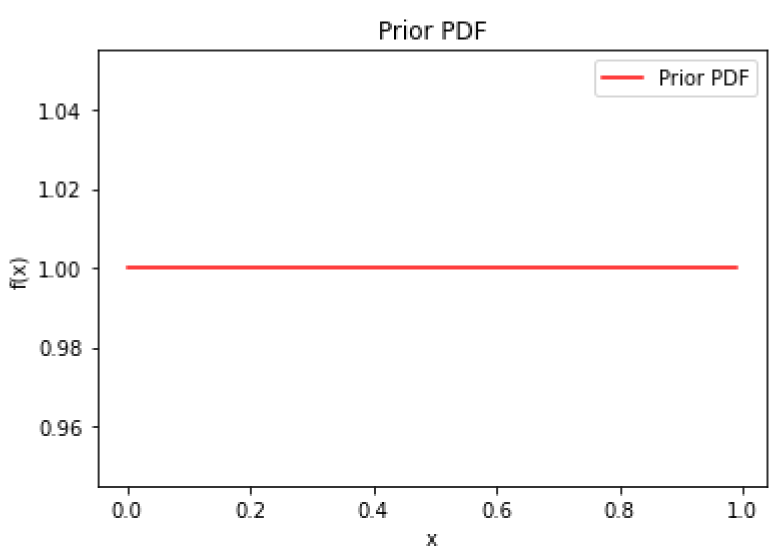
In [110]:
```
at_weight = np.argmax(Posterior)
at_weight
```

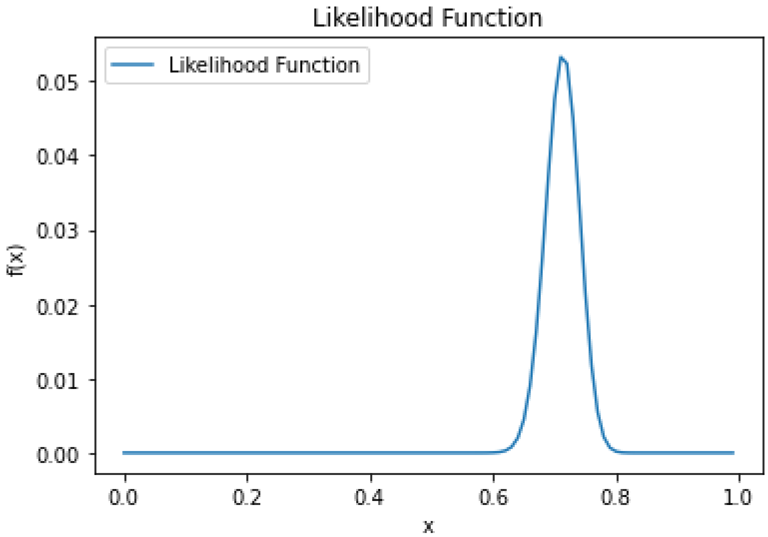
Out[110]: 28

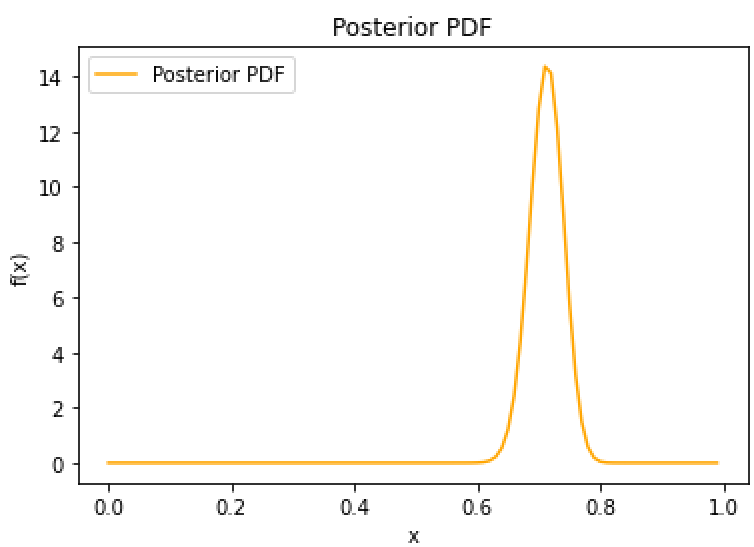
Prior: HistoryBooks and TableBooks

In [111]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df[(df["HistoryBooks"] == 0) & (df["TableBooks"] == 0)]) #num_tri
num_successes = np.sum((df.HistoryBooks > 0) & (df.TableBooks > 0 ))

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=1)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 193.
b_prime = 78.
```




Likelihood Function
Likelihood Function
0.05
0.04
0.03
0.02
0.01
0.00
f(x)
0.0
0.2
0.4
0.6
0.8
1.0
x


Posterior PDF
Posterior PDF
14
12
10
8
6
4
2
0
f(x)
0.0
0.2
0.4
0.6
0.8
1.0
x

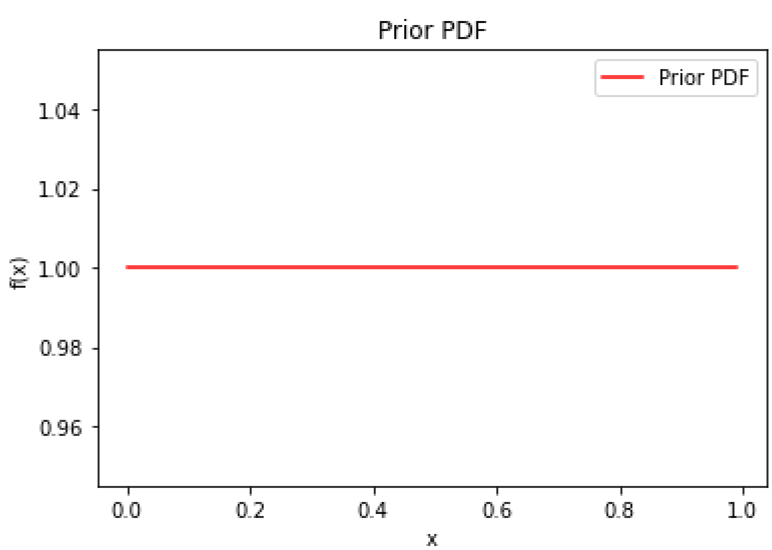
In [112]:
```
ht_weight = np.argmax(Posterior)
ht_weight
```

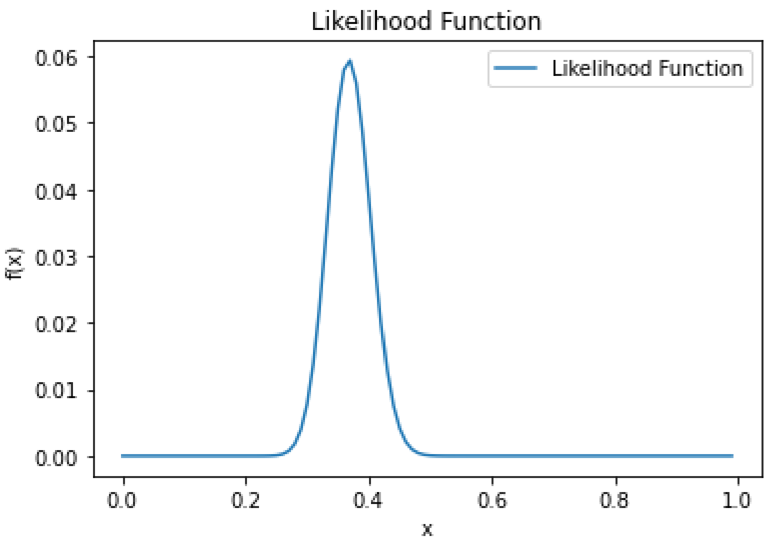
Out[112]: 71

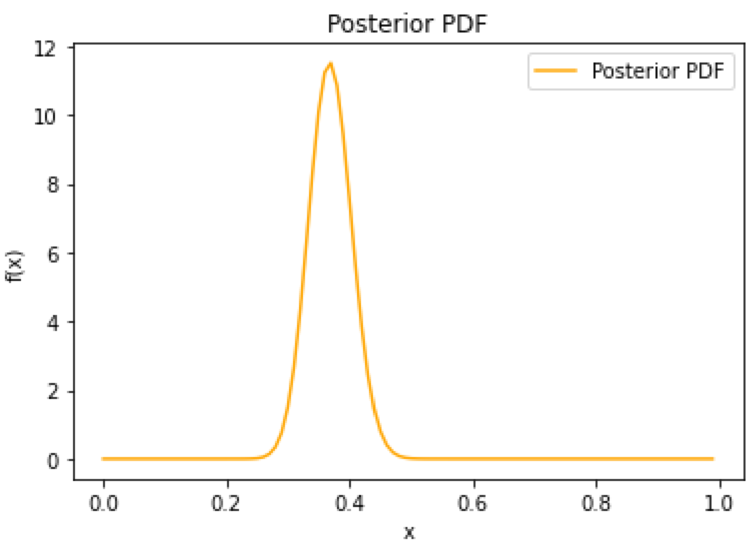
Prior: ArtBooks and HistoryBooks and TableBooks

In [113]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df[(df["ArtBooks"] == 0) & (df["HistoryBooks"] == 0) & (df["Table
num_successes = np.sum((df.HistoryBooks > 0) & (df.TableBooks > 0) & (df.ArtBooks

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=1)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 72.
b_prime = 123.
```







In [114]:
```
aht_weight = np.argmax(Posterior)
aht_weight
```

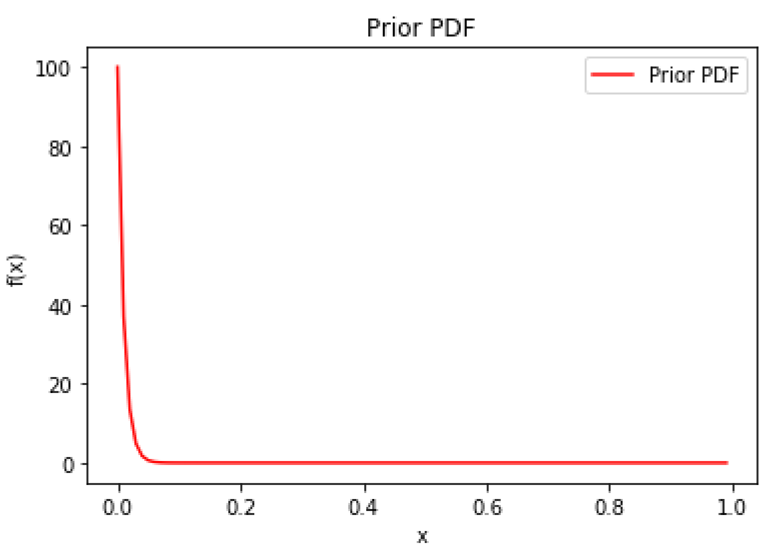
Out[114]: 37

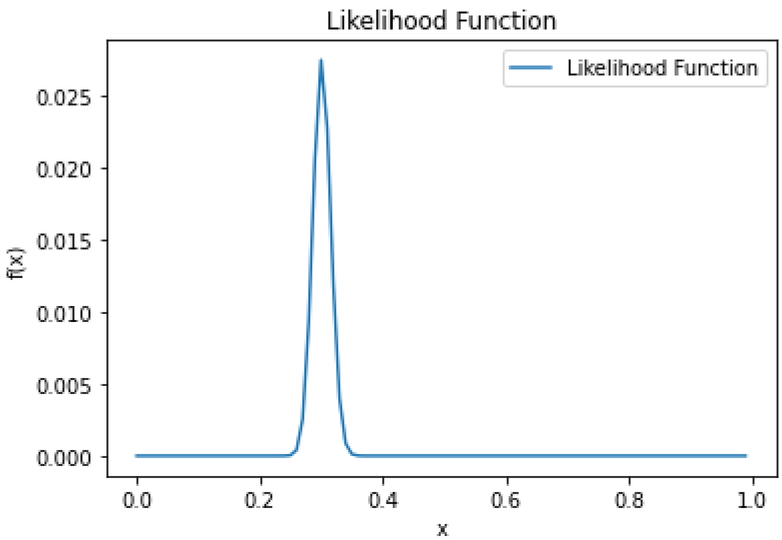
In [115]:
```
## a = 1, b= 100
```

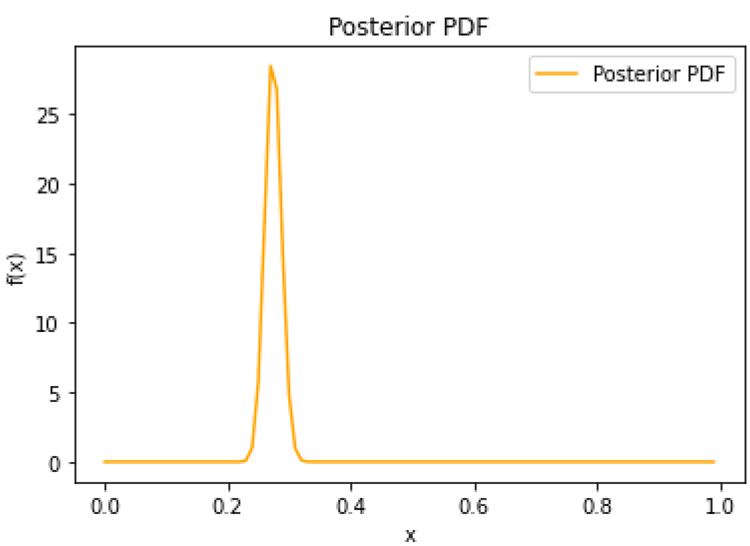
Prior: Artbooks

In [116]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df.ArtBooks) #num_trials = 1000
num_successes = np.sum(df.ArtBooks > 0)

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=100)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 302.
b_prime = 799.
```




Likelihood Function
Likelihood Function
f(x)
x
0.000
0.005
0.010
0.015
0.020
0.025
0.0
0.2
0.4
0.6
0.8
1.0


Posterior PDF
Posterior PDF
f(x)
x
0
5
10
15
20
25
0.0
0.2
0.4
0.6
0.8
1.0

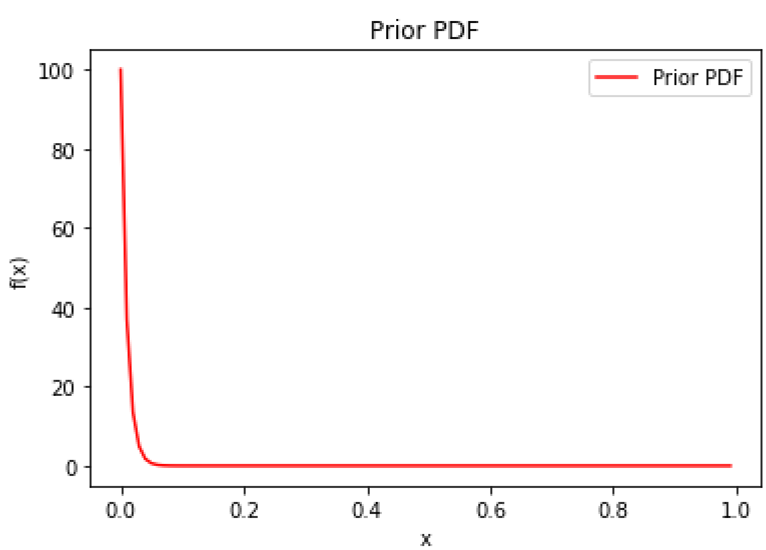
In [117]:
```
art_weight_s = np.argmax(Posterior)
art_weight_s
```

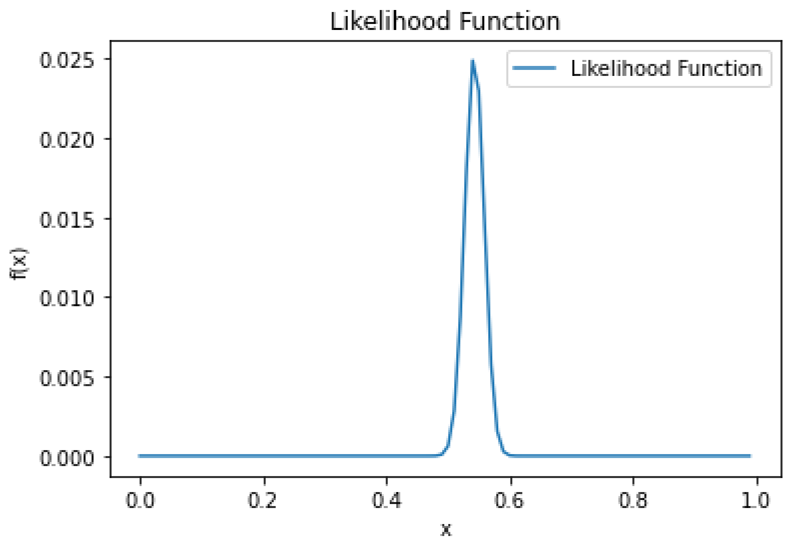
Out[117]: 27

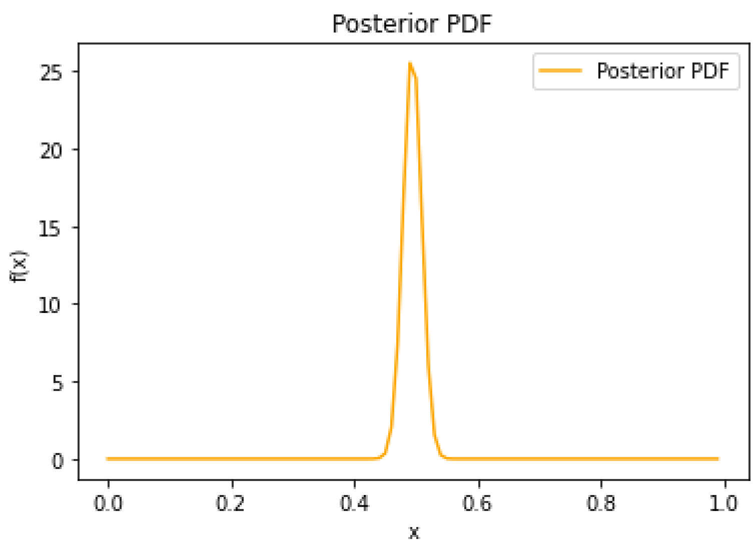
Prior: HistoryBooks

In [118]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df.HistoryBooks) #num_trials = 1000
num_successes = np.sum(df.HistoryBooks > 0)

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=100)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 544.
b_prime = 557.
```







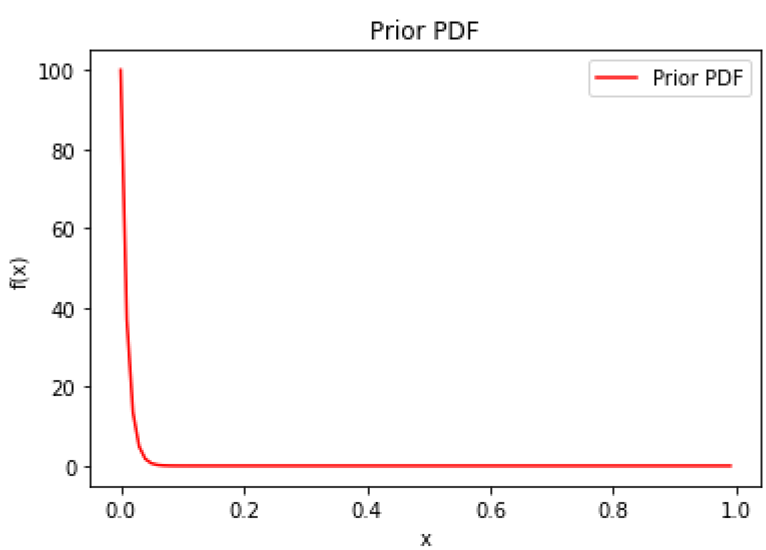
In [119]:
```
h_weight_s = np.argmax(Posterior)
h_weight_s
```

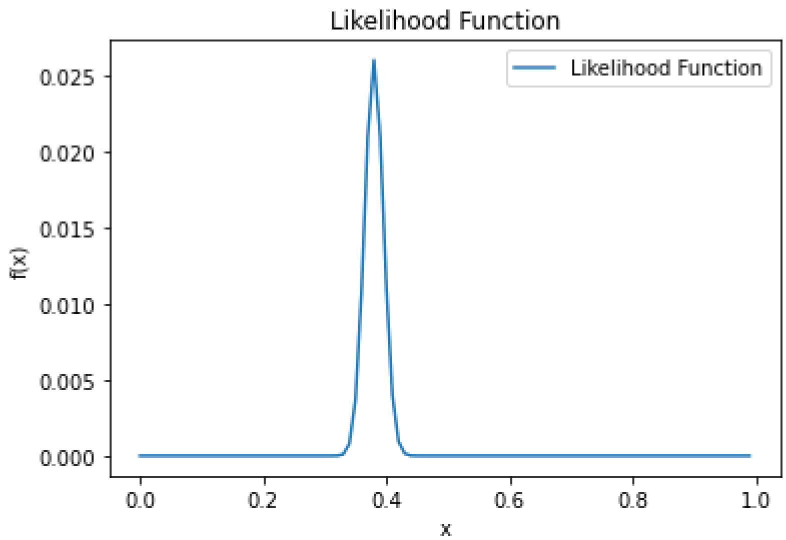
Out[119]: 49

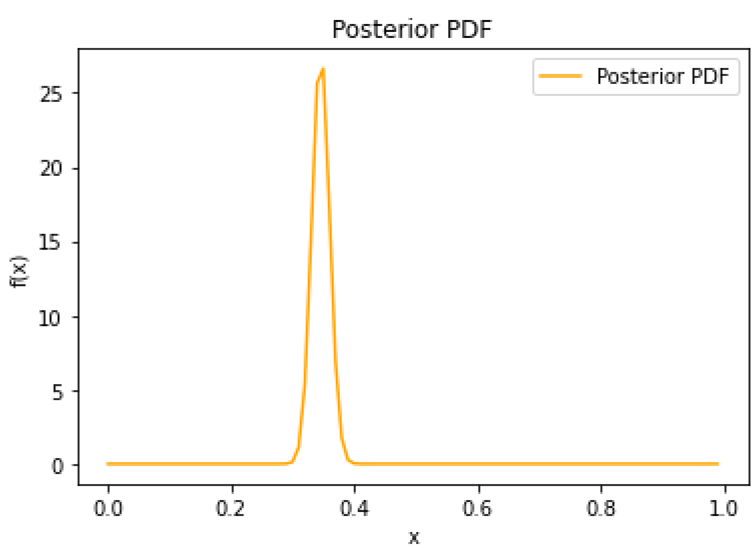
Prior: TableBooks

In [120]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df.TableBooks) #num_trials = 1000
num_successes = np.sum(df.TableBooks > 0)

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=100)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 381.
b_prime = 720.
```




Likelihood Function
Likelihood Function
0.025
0.020
0.015
0.010
0.005
0.000
f(x)
0.0
0.2
0.4
0.6
0.8
1.0
x


Posterior PDF
Posterior PDF
25
20
15
10
5
0
f(x)
0.0
0.2
0.4
0.6
0.8
1.0
x

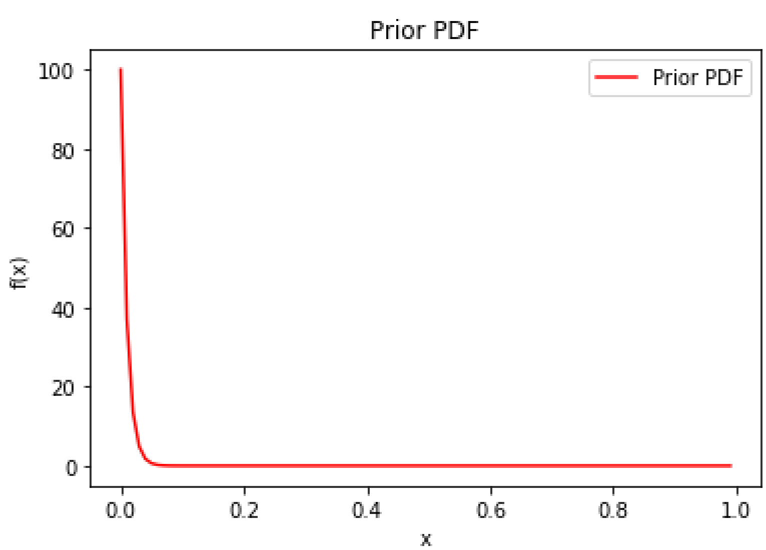
In [121]:
```
t_weight_s = np.argmax(Posterior)
t_weight_s
```

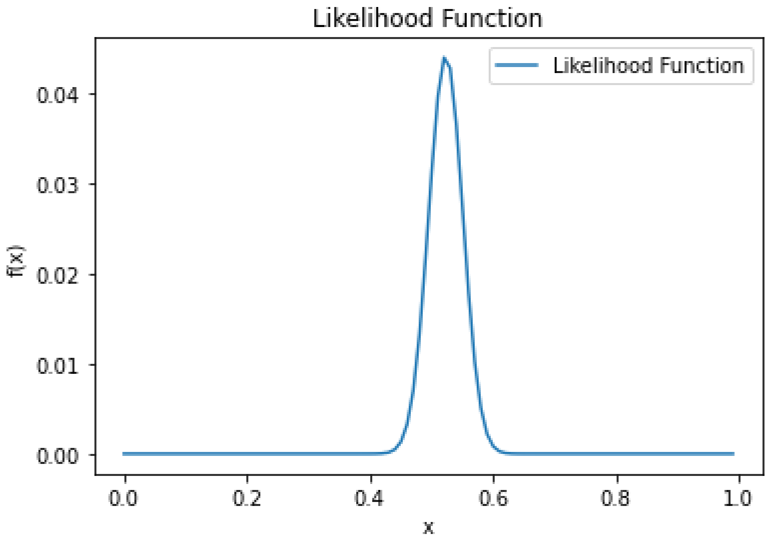
Out[121]:
```
35
```

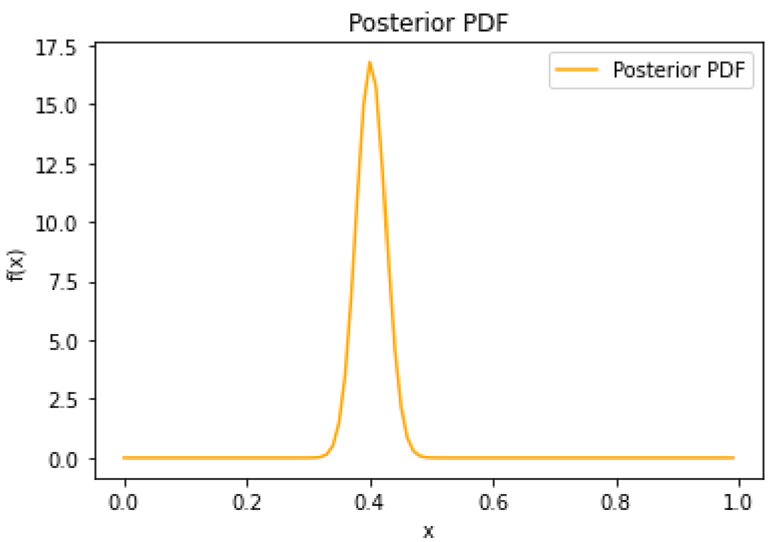
Prior: Artboks and HistoryBooks

In [122]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df[(df["ArtBooks"] == 0) & (df["HistoryBooks"] == 0)]) #num_trial
num_successes = np.sum((df.ArtBooks > 0) & (df.HistoryBooks))

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=100)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 172.
b_prime = 256.
```







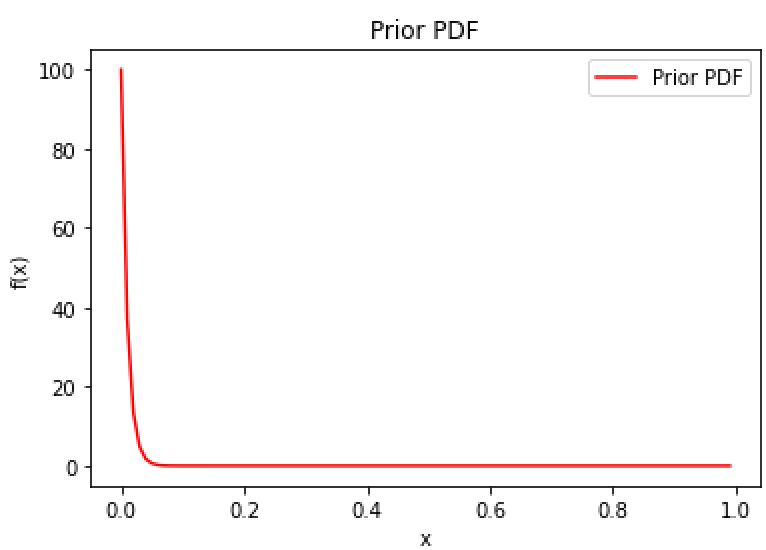
In [123]:
```
ah_weight_s = np.argmax(Posterior)
ah_weight_s
```

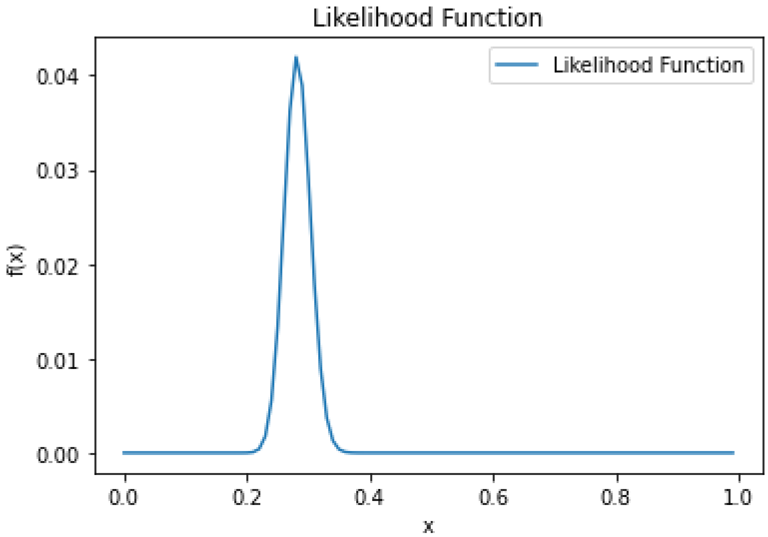
Out[123]: 40

Prior: Artboks and TableBooks

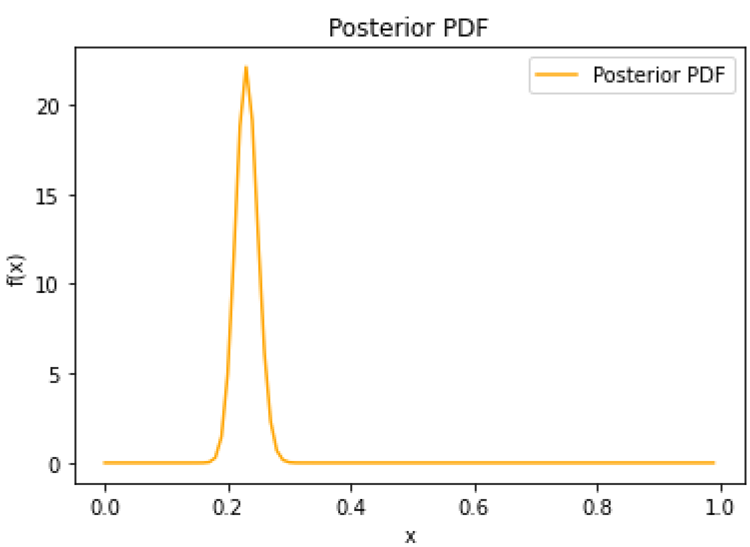
In [124]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df[(df["ArtBooks"] == 0) & (df["TableBooks"] == 0)]) #num_trials
num_successes = np.sum((df.ArtBooks > 0) & (df.TableBooks))

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=100)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 126.
b_prime = 419.
```





Likelihood Function
Likelihood Function
f(x)
0.04
0.03
0.02
0.01
0.00
0.0
0.2
0.4
0.6
0.8
1.0
x



Posterior PDF
Posterior PDF
f(x)
20
15
10
5
0
0.0
0.2
0.4
0.6
0.8
1.0
x

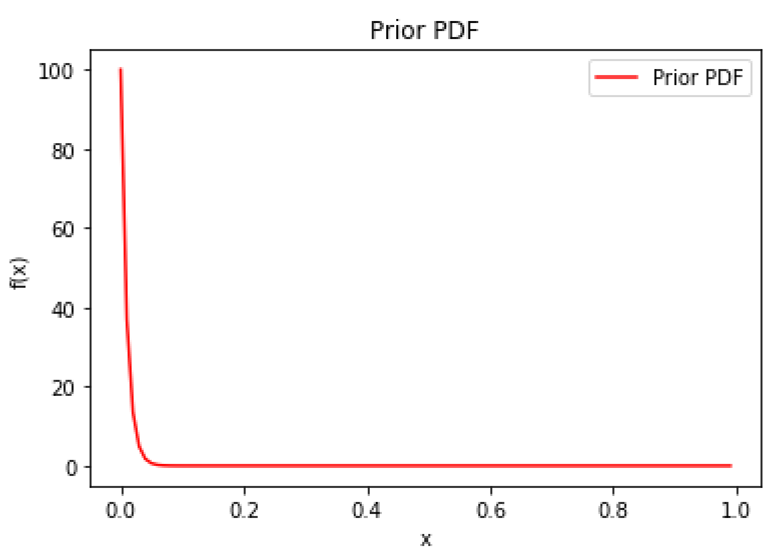
In [125]:
```
at_weight_s = np.argmax(Posterior)
at_weight_s
```

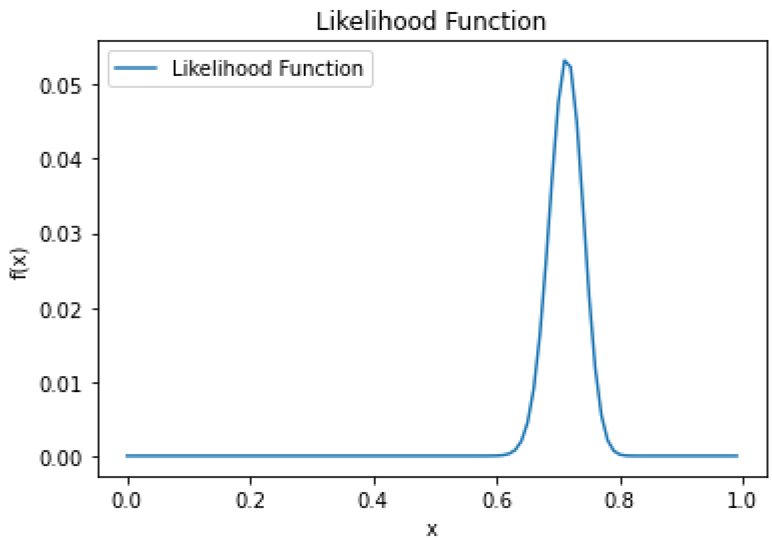
Out[125]:
```
23
```

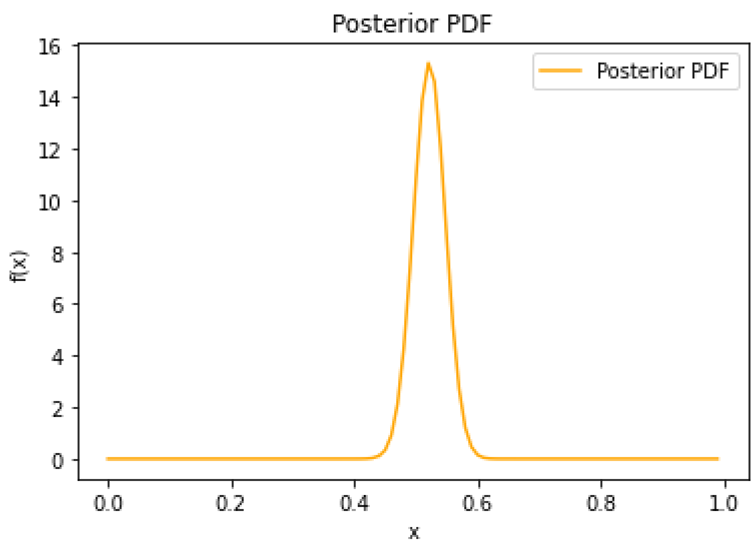
Prior: HistoryBooks and TableBooks

In [126]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df[ (df["HistoryBooks"] == 0) & (df["TableBooks"] == 0)]) #num_tr
num_successes = np.sum((df.HistoryBooks > 0) & (df.TableBooks > 0 ))

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=100)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 193.
b_prime = 177.
```







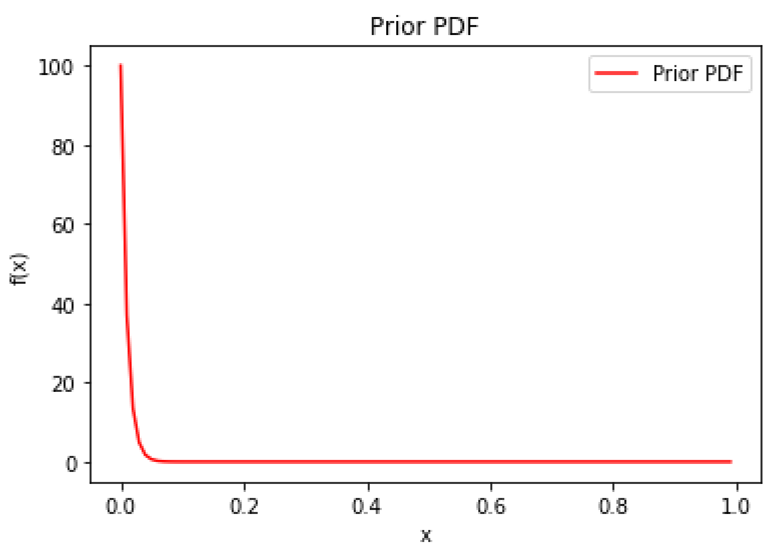
In [127]:
```
ht_weight_s = np.argmax(Posterior)
ht_weight_s
```

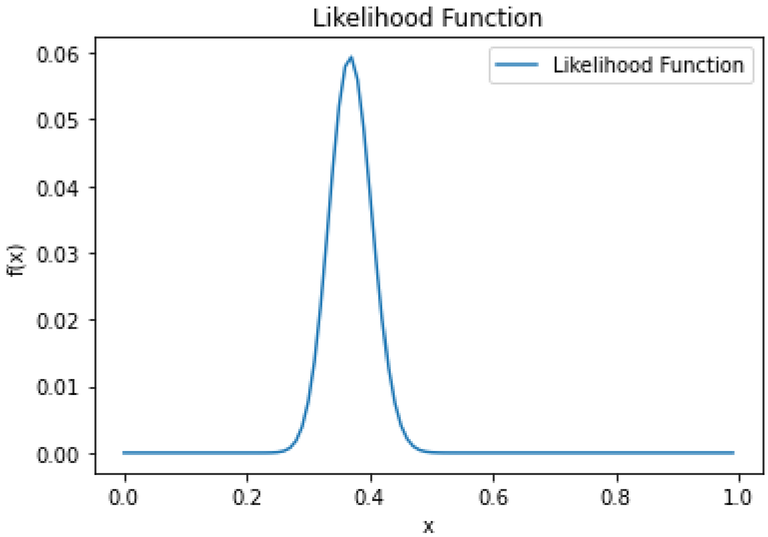
Out[127]: 52

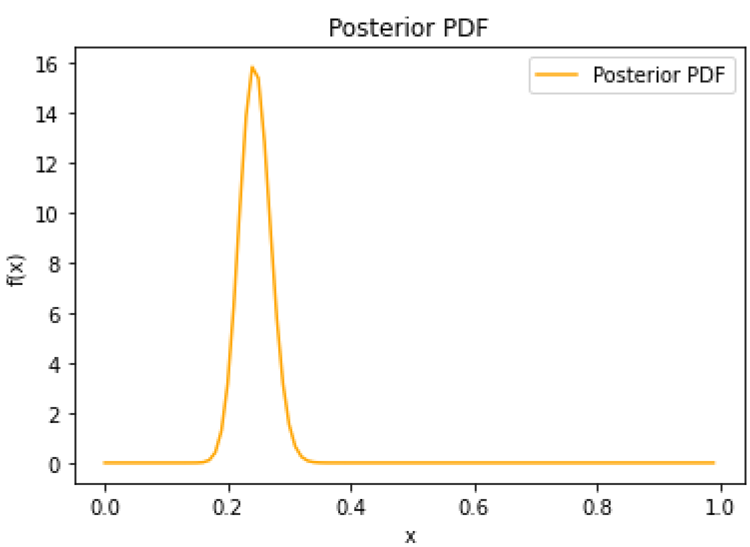
Prior: ArtBooks and HistoryBooks and TableBooks

In [128]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df[(df["ArtBooks"] == 0) & (df["HistoryBooks"] == 0) & (df["Table
num_successes = np.sum((df.HistoryBooks > 0) & (df.TableBooks > 0) & (df.ArtBooks

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=100)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 72.
b_prime = 222.
```




Likelihood Function
Likelihood Function
f(x)
x
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0.0
0.2
0.4
0.6
0.8
1.0


Posterior PDF
Posterior PDF
f(x)
x
16
14
12
10
8
6
4
2
0
0.0
0.2
0.4
0.6
0.8
1.0

In [129]:
```
aht_weight_s = np.argmax(Posterior)
aht_weight_s
```

Out[129]: 24

In [130]:
```
data = {'MaxPosterior':['Art', 'His', 'Tab', 'ArtHis', 'HisTab', 'ArtTab', 'ArtHi
        'a:b =1:1':[30, 54, 38, 52, 28, 71, 37] , 'a:b =1:100':[27, 49, 35, 40, 2
```

In [131]:
```
df = pd.DataFrame(data)
df
```

Out[131]:

	MaxPosterior	a:b =1:1	a:b =1:100
0	Art	30	27
1	His	54	49
2	Tab	38	35
3	ArtHis	52	40
4	HisTab	28	23
5	ArtTab	71	52
6	ArtHisTab	37	24

Response:

When we use strong weighting for low likelihood, i.e. a: b = 1:100,
We can see the maximum posterior shift to the lower values for all case.
This is consistent the undestanding of posteior equation is the propotional factor of likelihood.
I use a:b = 1:1 and a:b = 1:100 for this exercise, the posterior of beta function a:b = 1:100 is smaller that of a:b =1:1.
This is consitent to our understanding of likelihood.

Problem 5

From the lecture slide f(m|w) = N(μ0, vw), V>0
f(w) is Wishart with α degrees of freedom and
precision matrix r, with α > k − 1

Likelihood: N(M, W)
Posterior: f(m|x, w) ~ N (μ ,(v + N)w)
Posterior: f(w|x) is Wishart with α + N degrees of
freedom & precision matrix r
Posterior f(m}x) is a multivariate t distribution with α + N − k + 1
location parameter, μ∗ and precision (v + N)(α + N − k + 1)(r∗)−1
In our case, we have 8 variables, meaning k = 8, Prior α = 8+1 = 9. v is 462 for this case. This is multivariate
Gaussian with unknown mean & variance-covariance matrix. Meahwhile, f(w) is Wishart with multiple dimensions of the
gamma distribution.

In [132]:
```
import numpy as np
import pandas as pd
chd_data = pd.read_csv("CHDdata.csv")
chd_data.describe()
```

Out[132]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	
count	462.000000	462.000000	462.000000	462.000000	462.000000	462.000000	462.000000	462.00
mean	138.326840	3.635649	4.740325	25.406732	53.103896	26.044113	17.044394	42.81
std	20.496317	4.593024	2.070909	7.780699	9.817534	4.213680	24.481059	14.60
min	101.000000	0.000000	0.980000	6.740000	13.000000	14.700000	0.000000	15.00
25%	124.000000	0.052500	3.282500	19.775000	47.000000	22.985000	0.510000	31.00
50%	134.000000	2.000000	4.340000	26.115000	53.000000	25.805000	7.510000	45.00
75%	148.000000	5.500000	5.790000	31.227500	60.000000	28.497500	23.892500	55.00
max	218.000000	31.200000	15.330000	42.490000	78.000000	46.580000	147.190000	64.00

In [133]:
```
#drop "famhist" column
chd_data = chd_data.drop(columns= "famhist")
#check that it worked
chd_data.head()
```

Out[133]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	age	chd
0	160	12.00	5.73	23.11	49	25.30	97.20	52	1
1	144	0.01	4.41	28.61	55	28.87	2.06	63	1
2	118	0.08	3.48	32.28	52	29.14	3.81	46	0
3	170	7.50	6.41	38.03	51	31.99	24.26	58	1
4	134	13.60	3.50	27.78	60	25.99	57.34	49	1

In [134]:
```
for key in chd_data.keys()[0:8]:
    print("Standardizing "+key+".")
    chd_data[key] = chd_data[key] - np.mean(chd_data[key])
    chd_data[key] = chd_data[key] / np.std(chd_data[key])
```

```
Standardizing sbp.
Standardizing tobacco.
Standardizing ldl.
Standardizing adiposity.
Standardizing typea.
Standardizing obesity.
Standardizing alcohol.
Standardizing age.
```

In [135]:
```
# Check that it worked
chd_data.describe()
```

Out[135]:

	sbp	tobacco	ldl	adiposity	typea	obesity	
count	4.620000e+02	4.620000e+02	4.620000e+02	4.620000e+02	4.620000e+02	4.620000e+02	4
mean	-2.571296e-16	5.022437e-16	-3.963040e-15	1.559599e-15	1.153478e-17	-5.286776e-15	
std	1.001084e+00	1.001084e+00	1.001084e+00	1.001084e+00	1.001084e+00	1.001084e+00	1
min	-1.823123e+00	-7.924170e-01	-1.817753e+00	-2.401708e+00	-4.089354e+00	-2.695129e+00	
25%	-6.997535e-01	-7.809742e-01	-7.047170e-01	-7.245926e-01	-6.224081e-01	-7.267824e-01	
50%	-2.113321e-01	-3.565020e-01	-1.935182e-01	9.112757e-02	-1.059418e-02	-5.680824e-02	
75%	4.724579e-01	4.063492e-01	5.074164e-01	7.489145e-01	7.031887e-01	5.828745e-01	2
max	3.891408e+00	6.007857e+00	5.119082e+00	2.197976e+00	2.538631e+00	4.878906e+00	5

In [136]:
```
chd_positive = chd_data[chd_data.chd == 1]
chd_negative = chd_data[chd_data.chd == 0]
```

In [137]:
```
# Check that it worked
chd_positive.describe()
```

Out[137]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	
count	160.000000	160.000000	160.000000	160.000000	160.000000	160.000000	160.000000	160.00
mean	0.264268	0.411771	0.361398	0.349128	0.141722	0.137517	0.085909	0.51
std	1.156458	1.212965	1.075607	0.908099	1.044840	1.043288	1.070602	0.72
min	-1.774281	-0.792417	-1.542213	-2.060752	-3.375571	-2.695129	-0.696983	-1.76
25%	-0.528806	-0.465481	-0.386879	-0.250150	-0.545931	-0.572356	-0.677559	-0.00
50%	-0.015964	0.107747	0.156949	0.385765	0.193344	0.102370	-0.356351	0.69
75%	0.985300	0.994834	0.890513	1.052558	0.805158	0.649991	0.308250	1.10
max	3.891408	6.007857	4.553501	2.197976	2.538631	4.674587	5.321938	1.45

In [138]:
```
# Lets check the mean of each class to get a first look at the seperation
print("Mean for CHD Positive:")
print(np.array([chd_positive.mean()[0:8]]))
print("Mean for CHD Negative:")
print(np.array([chd_negative.mean()[0:8]]))
```

```
Mean for CHD Positive:
[[0.26426823 0.41177089 0.36139839 0.34912802 0.14172199 0.13751694
  0.0859086  0.51241433]]
Mean for CHD Negative:
[[-0.14000966 -0.21815676 -0.19146935 -0.18496849 -0.0750845  -0.07285666
  -0.04551449 -0.27147779]]
```

In [139]:
```
chd_positive
```

Out[139]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	age	chd
0	1.058564	1.823073	0.478412	-0.295503	-0.418470	-0.176786	3.277738	0.629336	1
1	0.277089	-0.790237	-0.159680	0.412140	0.193344	0.671373	-0.612745	1.383115	1
3	1.546985	0.842264	0.807126	1.624141	-0.214532	1.412621	0.295062	1.040488	1
4	-0.211332	2.171805	-0.599577	0.305351	0.703189	-0.012856	1.647775	0.423760	1
7	-1.188175	0.096850	-0.072667	-1.390421	0.907127	-0.697085	-0.422187	1.040488	1
...	...	...	...	...	...	...	...	...	...
453	-0.699754	-0.443685	1.198683	1.836434	-1.744067	1.296207	-0.696983	0.560810	1
454	0.374774	-0.652924	0.038515	0.336230	0.703189	0.490812	-0.360440	-0.261494	1
455	-0.504385	-0.304192	-0.923457	0.138089	-0.520439	-0.495142	1.242125	-1.083798	1
458	2.133091	0.123004	-0.159680	0.861173	-0.112563	0.609602	0.068519	0.629336	1
461	-0.309016	-0.792417	0.038515	1.029720	0.907127	-2.695129	-0.696983	0.218184	1

160 rows × 9 columns

In [140]:
```
chd_negative_new = chd_negative.iloc[ : ,[0,1, 2, 3, 4, 5, 6, 7]]
```

In [141]:
```
chd_positive_new = chd_positive.iloc[ : ,[0,1, 2, 3, 4, 5, 6, 7]]
```

In [142]:
```
## For patients withouot chd
v = 462
alpha = 9
k = 8
xmean = np.mean(chd_negative_new)
N = len(chd_negative_new)
mu0 = np.zeros(8)
mu_star = (v*mu0+ N *xmean)/(v+N)
print("the posterior mean for patients without CHD is:\n", mu_star)
```

```
the posterior mean for patients without CHD is:
 sbp         -0.055344
tobacco     -0.086235
ldl         -0.075686
adiposity   -0.073116
typea       -0.029680
obesity     -0.028799
alcohol     -0.017991
age         -0.107312
dtype: float64
```

In [143]:
```
## S-matrix:
S_matrix = pd.DataFrame(0, index=chd_negative_new.columns, columns = chd_negative
for index, row in chd_negative_new.iterrows():
    tmp = (row-xmean).to_frame()
    tmpT=tmp.T
    each_i = tmp.dot(tmpT)
    S_matrix+=each_i
```

In [144]:
```
S_matrix
```

Out[144]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	
sbp	232.260049	38.244868	36.897715	108.277837	-19.633118	79.008160	42.595168	107
tobacco	38.244868	186.564280	49.194077	77.992896	4.809790	39.289941	44.962429	11
ldl	36.897715	49.194077	246.079082	110.856114	2.591264	85.059945	10.736726	85
adiposity	108.277837	77.992896	110.856114	301.046673	-26.657035	207.981738	61.931813	202
typea	-19.633118	4.809790	2.591264	-26.657035	283.505089	9.629805	25.318168	-36
obesity	79.008160	39.289941	85.059945	207.981738	9.629805	284.307565	42.598878	100
alcohol	42.595168	44.962429	10.736726	61.931813	25.318168	42.598878	277.949377	45
age	107.072330	111.987969	85.897294	202.736423	-36.878619	100.765087	45.104219	313

In [145]:
```
mu0_xmean = (-xmean).to_frame().dot((-xmean).to_frame().T)
```

In [146]:
```
r_star = np.eye(8)+Smatrix+mu0_xmean*v*N/(v+N)
print("Posterior precision matrix is:\n")
r_star
```

```
Posterior precision matrix is:
```

Out[146]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	
sbp	217.225808	38.755896	17.754005	38.571500	-14.158151	23.946275	17.735802	46
tobacco	38.755896	243.625445	-4.576305	26.616119	-22.851166	7.285175	40.968083	55
ldl	17.754005	-4.576305	191.647090	68.207638	7.847611	58.016730	-32.176979	22
adiposity	38.571500	26.616119	68.207638	138.366582	-2.849924	113.776741	-21.383196	51
typea	-14.158151	-22.851166	7.847611	-2.849924	175.608281	20.789706	-9.426105	-24
obesity	23.946275	7.285175	58.016730	113.776741	20.789706	175.033026	-21.036718	20
alcohol	17.735802	40.968083	-32.176979	-21.383196	-9.426105	-21.036718	183.622480	-6
age	46.335853	55.234929	22.319061	51.836311	-24.578114	20.400289	-6.903003	99

In [147]:
```
## Degree of freedom
alpha+len(chd_negative_new)
## Posterior marginal alpha+N-k+1
alpha+N-k+1
## Parameter
param = (v+N)*(alpha+N-k+1)
## r_star inversion
r_star_inv.dot(r_star)
```

Out[147]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	age
sbp	1.069377	0.108618	0.020314	-0.000588	-0.019139	-0.052472	0.050636	0.085350
tobacco	0.151827	1.543612	-0.184024	0.047481	-0.164963	0.000047	0.192738	0.149879
ldl	0.015897	-0.216758	0.816598	0.071226	0.012547	0.046530	-0.085224	-0.025689
adiposity	-0.130916	-0.088725	0.097584	0.569991	0.091600	0.094388	-0.213406	-0.071258
typea	-0.011122	-0.158342	0.022480	0.022370	0.633017	0.049156	-0.120558	-0.050050
obesity	-0.076090	0.037253	-0.028544	0.064939	0.020501	0.625602	-0.012682	-0.001009
alcohol	-0.057739	-0.030678	-0.119325	-0.188493	-0.086557	-0.163993	0.692124	-0.088249
age	-0.176169	-0.350871	-0.142269	-0.240033	0.000619	-0.164768	-0.056321	0.278358

In [148]:
```
## Given by the lecture, Posterior f(m|x) is multivariaate t distribution with al
## and precision (v+N)(allpha+N-k+1)(r_star)_inv
## r_star_posterior
r_star_posterior = param*r_star_inv
r_star_posterior
```

Out[148]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcc
sbp	1253.888388	10.451143	21.292419	-175.786542	43.504717	-119.126753	-97.938
tobacco	10.451143	1611.998553	-153.425600	-20.656145	-93.254071	51.280758	-165.824
ldl	21.292419	-153.425600	1144.366556	-313.173423	-53.211475	-87.417415	73.113
adiposity	-175.786542	-20.656145	-313.173423	2737.489658	167.903060	-1438.041061	-182.268
typea	43.504717	-93.254071	-53.211475	167.903060	856.879146	-150.444767	-96.075
obesity	-119.126753	51.280758	-87.417415	-1438.041061	-150.444767	1781.554879	12.410
alcohol	-97.938426	-165.824903	73.113516	-182.268326	-96.075750	12.410417	911.752
age	-276.347333	-551.809764	-68.656640	-1109.369643	75.267817	393.331393	47.379

In [149]:
```
## ## For patients with chd
v = 462
alpha = 9
k = 8
xmean = np.mean(chd_positive_new)
N = len(chd_positive_new)
mu0 = np.zeros(8)
mu_star = (v*mu0+ N *xmean)/(v+N)
print("the posterior mean for patients with CHD is:\n", mu_star)
```

```
the posterior mean for patients with CHD is:
 sbp          0.067979
tobacco      0.105922
ldl          0.092964
adiposity    0.089808
typea        0.036456
obesity      0.035374
alcohol      0.022099
age          0.131811
dtype: float64
```

In [150]:
```
## S-matrix:
S_matrix = pd.DataFrame(0, index=chd_positive_new.columns, columns = chd_positive
for index, row in chd_positive_new.iterrows():
    tmp = (row-xmean).to_frame()
    tmpT=tmp.T
    each_i = tmp.dot(tmpT)
    S_matrix+=each_i
```

In [151]:
```
S_matrix
```

Out[151]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	
sbp	212.645903	33.177849	12.858328	33.842043	-16.077986	22.083404	16.572042	39
tobacco	33.177849	233.933981	-12.204530	19.246891	-25.842566	4.382533	39.154765	44
ldl	12.858328	-12.204530	183.952036	61.739898	5.222153	55.469172	-33.768471	12
adiposity	33.842043	19.246891	61.739898	131.118438	-5.386242	111.315680	-22.920653	42
typea	-16.077986	-25.842566	5.222153	-5.386242	173.578711	19.790684	-10.050207	-28
obesity	22.083404	4.382533	55.469172	111.315680	19.790684	173.063646	-21.642302	16
alcohol	16.572042	39.154765	-33.768471	-22.920653	-10.050207	-21.642302	182.244164	-9
age	39.394441	44.419132	12.826375	42.665925	-28.300661	16.788195	-9.159525	84

In [152]:
```
mu0_xmean = (-xmean).to_frame().dot((-xmean).to_frame().T)
```

In [153]:
```
r_star = np.eye(8)+Smatrix+mu0_xmean*v*N/(v+N)
print("Posterior precision matrix is:\n")
r_star
```

```
Posterior precision matrix is:
```

Out[153]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	
sbp	221.945586	46.110041	24.208508	44.806855	-11.627025	26.402300	19.270112	55
tobacco	46.110041	255.084343	5.480811	36.331770	-18.907279	11.112042	43.358776	69
ldl	24.208508	5.480811	200.473905	76.734760	11.309037	61.375451	-30.078742	34
adiposity	44.806855	36.331770	76.734760	146.604188	0.493978	117.021426	-19.356200	63
typea	-11.627025	-18.907279	11.309037	0.493978	176.965676	22.106825	-8.603283	-19
obesity	26.402300	11.112042	61.375451	117.021426	22.106825	176.311064	-20.238310	25
alcohol	19.270112	43.358776	-30.078742	-19.356200	-8.603283	-20.238310	184.121255	-3
age	55.487470	69.494567	34.834298	63.926624	-19.670277	25.162505	-3.927986	116

In [154]:
```
## Degree of freedom
alpha+len(chd_positive_new)
## Posterior marginal alpha+N-k+1
alpha+N-k+1
## Parameter
param = (v+N)*(alpha+N-k+1)
## r_star inversion
r_star_inv.dot(r_star)
```

Out[154]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	age
sbp	1.078740	0.123206	0.033117	0.011781	-0.014118	-0.047600	0.053680	0.103504
tobacco	0.174951	1.579642	-0.152401	0.078030	-0.152562	0.012080	0.200255	0.194716
ldl	0.031140	-0.193008	0.837443	0.091364	0.020721	0.054462	-0.080269	0.003866
adiposity	-0.128646	-0.085188	0.100688	0.572990	0.092817	0.095570	-0.212668	-0.066857
typea	-0.000083	-0.141142	0.037576	0.036953	0.638937	0.054900	-0.116970	-0.028646
obesity	-0.085143	0.023147	-0.040925	0.052978	0.015646	0.620891	-0.015625	-0.018563
alcohol	-0.060867	-0.035552	-0.123603	-0.192625	-0.088234	-0.165621	0.691108	-0.094314
age	-0.161610	-0.328185	-0.122358	-0.220798	0.008427	-0.157191	-0.051588	0.306588

In [155]:
```
## Given by the lecture, Posterior f(m|x) is multivariaate t distribution with al
## and precision (v+N)(allpha+N-k+1)(r_star)_inv
## r_star_posterior
r_star_posterior = param*r_star_inv
r_star_posterior
```

Out[155]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol
sbp	543.998043	4.534216	9.237692	-76.264790	18.874472	-51.683006	-42.490474
tobacco	4.534216	699.363729	-66.563521	-8.961645	-40.458172	22.248098	-71.942945
ldl	9.237692	-66.563521	496.482122	-135.869932	-23.085738	-37.925945	31.720215
adiposity	-76.264790	-8.961645	-135.869932	1187.656758	72.844551	-623.892470	-79.076905
typea	18.874472	-40.458172	-23.085738	72.844551	371.756038	-65.270290	-41.682354
obesity	-51.683006	22.248098	-37.925945	-623.892470	-65.270290	772.925547	5.384245
alcohol	-42.490474	-71.942945	31.720215	-79.076905	-41.682354	5.384245	395.562686
age	-119.892974	-239.402035	-29.786605	-481.298751	32.654856	170.646375	20.555446

Problem 7

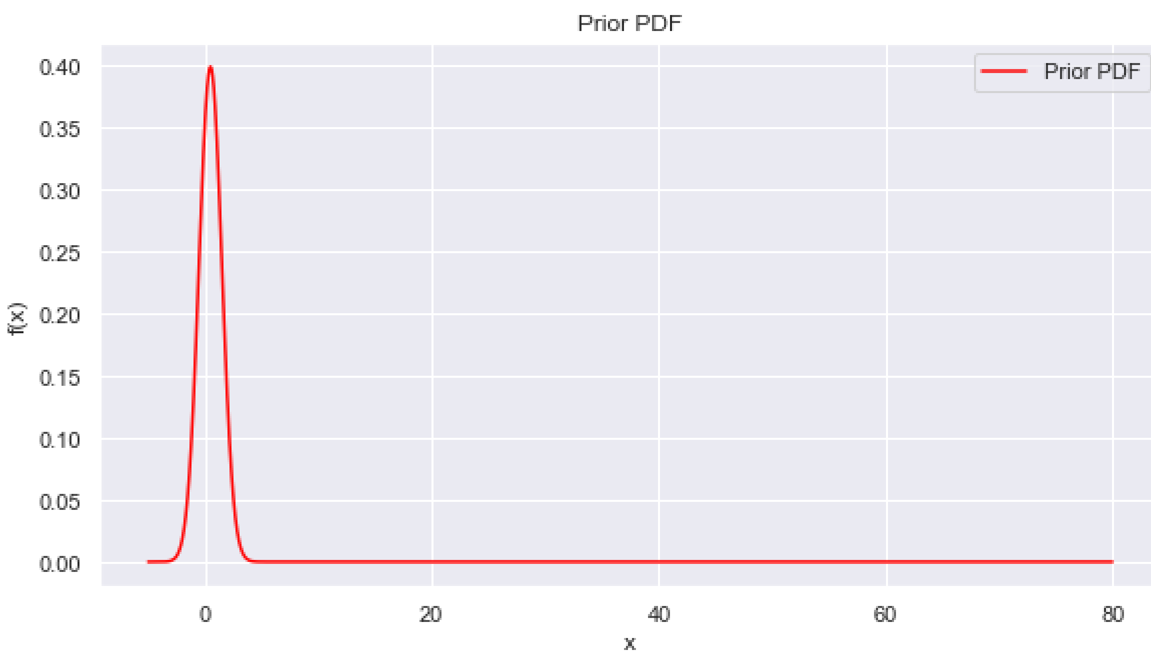
In [156]:
```
from scipy.stats import beta
from scipy.stats import norm

def posterior_from_conjugate_prior(**kwargs):
    if kwargs['Likelihood_Dist_Type'] == 'Binomial':
        # Get the parameters for the likelihood and prior distribution from the k
        x = kwargs['x']
        n = kwargs['n']
        k = kwargs['k']
        a = kwargs['a']
        b = kwargs['b']

        print(f'a_prime = {k + a}.')
        print(f'b_prime = {n - k + b}.')
        Likelihood = binom.pmf(p=x, n=n, k=k)
        Prior = beta.pdf(x=x, a=a, b=b)
        Posterior = beta.pdf(x=x, a=k+a, b=n-k+b)

        return [Prior, Likelihood, Posterior]

    elif kwargs['Likelihood_Dist_Type'] == 'Gaussian_Known_Variance':
        # Get the parameters for the likelihood and prior distribution from the k
        x = kwargs['x']
        n= len(x)
        mu = kwargs['mu']
        var = kwargs['var']
        prior_mu = kwargs['prior_mu']
        prior_var = kwargs['prior_var']
        print(kwargs)


        # To answer the challenge question, modify this section with the correct
        x_bar = np.mean(x)
        mu_prime = (prior_mu*var + n*x_bar*prior_var)/(var+n*prior_var)
        var_prime =(prior_var*var)/(var+n*prior_var)
        print(f'mu_prime = {mu_prime:.2f}.')
        print(f'var_prime = {var_prime: .2f}.')
        Likelihood = norm.pdf(x, loc = mu, scale=var**(.5))
        Prior = norm.pdf(x= x, loc = prior_mu, scale = prior_var**(.5))
        Posterior = norm.pdf(x= x, loc = mu_prime, scale = var_prime**(.50))

        return [Prior, Likelihood, Posterior]

    else:
        print('Distribution type not supported.')
        return -1, -1, -1
```

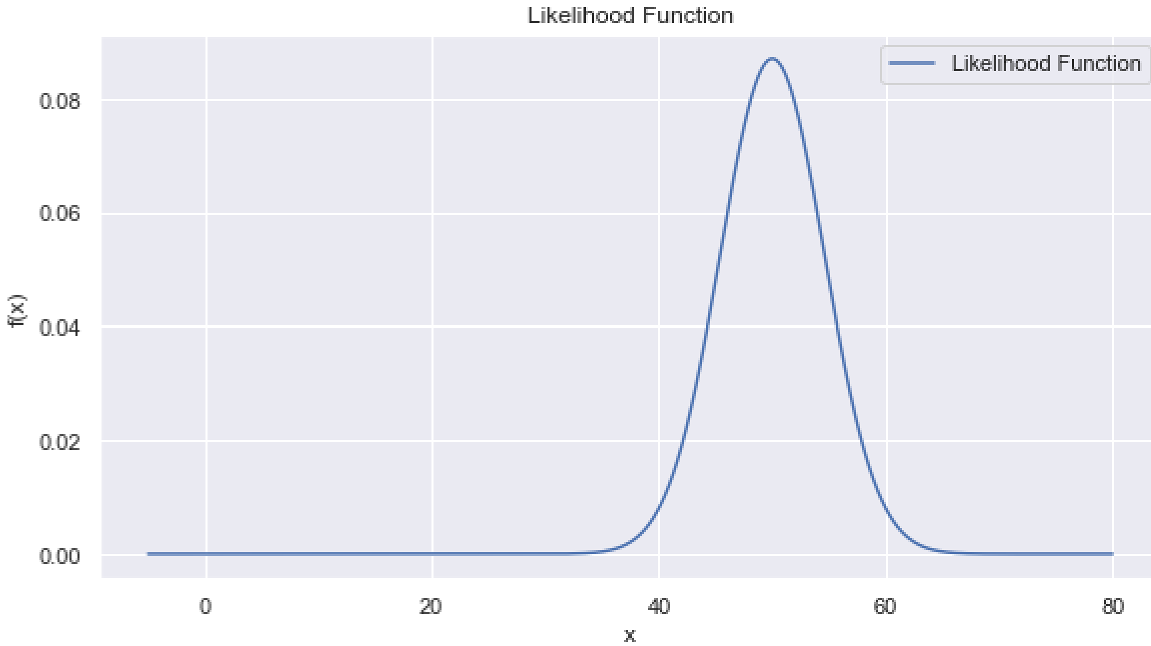
In [157]:
```
import numpy as np
x = np.arange(-5, 80, 0.01)
Prior, Likelihood, Posterior = posterior_from_conjugate_prior(Likelihood_Dist_Typ
```

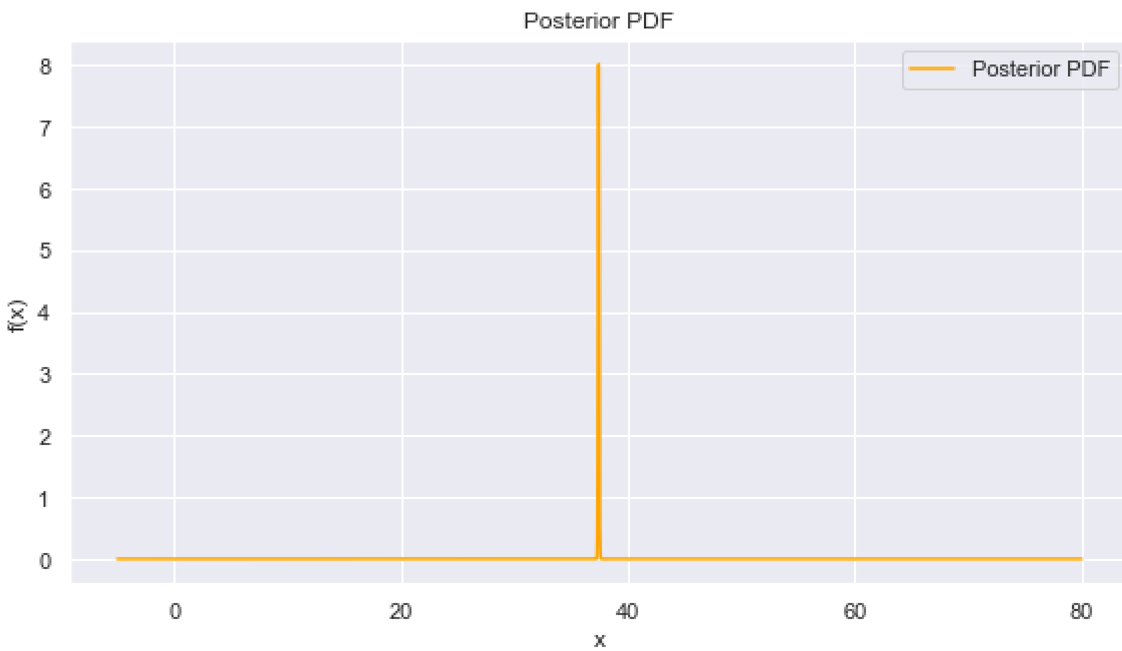
```
{'Likelihood_Dist_Type': 'Gaussian_Known_Variance', 'x': array([-5.  , -4.99, -
4.98, ..., 79.97, 79.98, 79.99]), 'mu': 50, 'var': 21, 'prior_mu': 0.5, 'prior_
var': 1}
mu_prime = 37.40.
var_prime =  0.00.
```

In [158]:
```
import numpy as np
import pandas as pd

# import matplotlib
import matplotlib.pyplot as plt
# import seaborn
import seaborn as sns
# settings for seaborn plotting style
sns.set(color_codes=True)
# settings for seaborn plot sizes
sns.set(rc={'figure.figsize':(9.5,5)})

x = np.arange(-5, 80, 0.01)
Prior, Likelihood, Posterior = posterior_from_conjugate_prior(Likelihood_Dist_Typ

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
{'Likelihood_Dist_Type': 'Gaussian_Known_Variance', 'x': array([-5.  , -4.99, -
4.98, ..., 79.97, 79.98, 79.99]), 'theta': 5, 'n': 100, 'k': 21, 'mu': 50, 'va
r': 21, 'prior_mu': 0.5, 'prior_var': 1}
mu_prime = 37.40.
var_prime =  0.00.
```




Likelihood Function
Likelihood Function
f(x)
0.08
0.06
0.04
0.02
0.00
0
20
40
60
80
x


Posterior PDF
Posterior PDF
f(x)
8
7
6
5
4
3
2
1
0
0
20
40
60
80
x