Problem 4

In [93]:
```
### Summary table (a, b) = (1, 1)
```

In [94]:
```
## adopting a=1, b=1
```

In [95]:
```
import pandas as pd
import numpy as np
from scipy.stats import binom
from scipy.stats import beta
import seaborn as sns
import matplotlib.pyplot as plt
fname = 'ArtHistBooks.csv'
```

In [96]:
```
df = pd.read_csv(fname)
```

In [97]:
```
df
```

Out[97]:

	ArtBooks	HistoryBooks	TableBooks	Purchase
0	0	0	1	0
1	0	1	0	0
2	0	0	0	0
3	1	0	1	0
4	1	1	1	0
...	...	...	...	...
995	1	1	0	1
996	0	1	0	0
997	1	0	1	0
998	1	1	0	0
999	0	1	0	0

1000 rows × 4 columns

In [98]:
```
df.ArtBooks
```

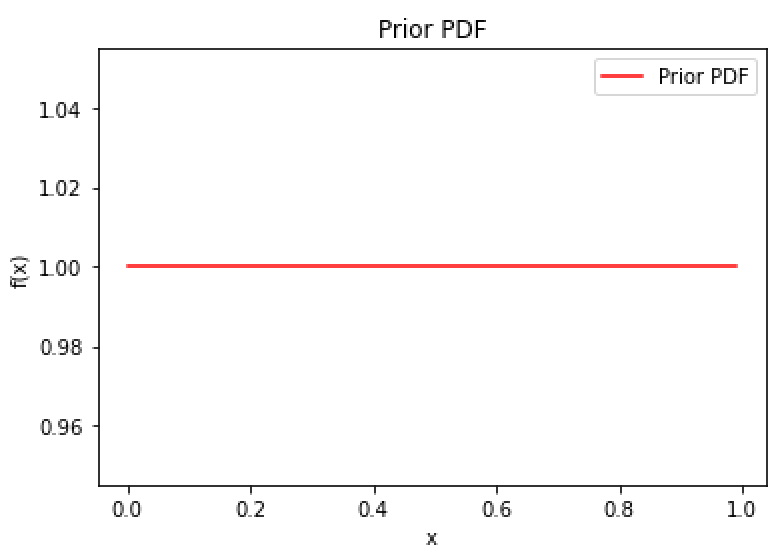
Out[98]:
```
0      0
1      0
2      0
3      1
4      1
      ..
995    1
996    0
997    1
998    1
999    0
Name: ArtBooks, Length: 1000, dtype: int64
```

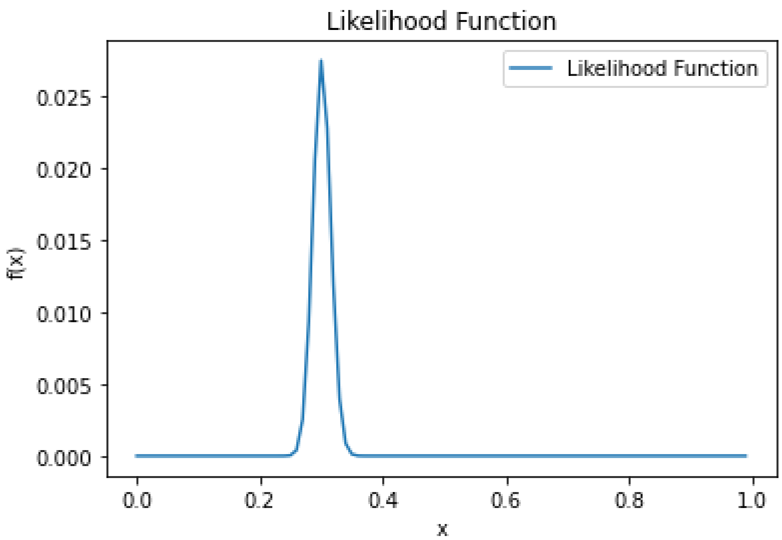
In [99]:
```
def posterior_from_conjugate_prior(**kwargs):
    if kwargs['Likelihood_Dist_Type'] == 'Binomial':
        # Get the parameters for the likelihood and prior distribution from the k
        x = kwargs['x'] # possible values for p, range across [0, 1]
        n = kwargs['n'] # number of trials (number of customers)
        k = kwargs['k'] # number of successes (purchases)
        a = kwargs['a'] # alpha parameters on the prior
        b = kwargs['b'] # beta parameter on the prior

        print(f'a_prime = {k + a}.')
        print(f'b_prime = {n - k + b}.')
        Likelihood = binom.pmf(p=x, n=n, k=k)
        Prior = beta.pdf(x=x, a=a, b=b)
        Posterior = beta.pdf(x=x, a=k+a, b=n-k+b)

        return [Prior, Likelihood, Posterior]

    else:
        print('Distribution type not supported.')
```

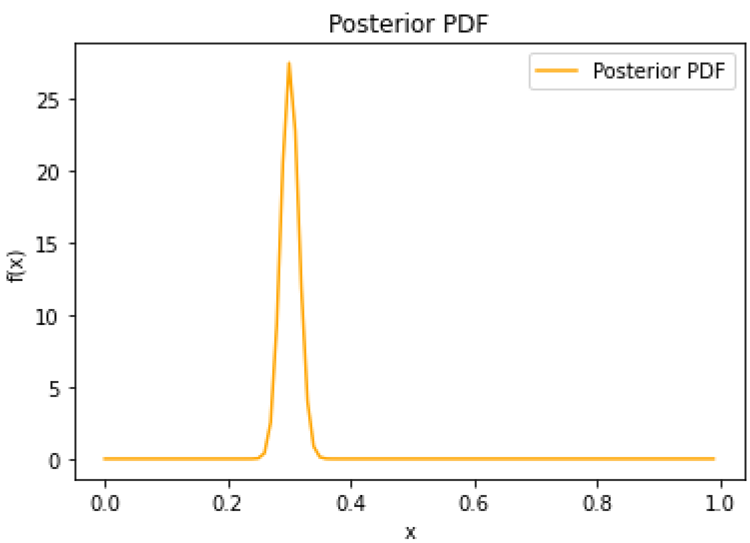
Prior: Artbooks

In [100]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df.ArtBooks) #num_trials = 1000
num_successes = np.sum(df.ArtBooks > 0)

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=1)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 302.
b_prime = 700.
```







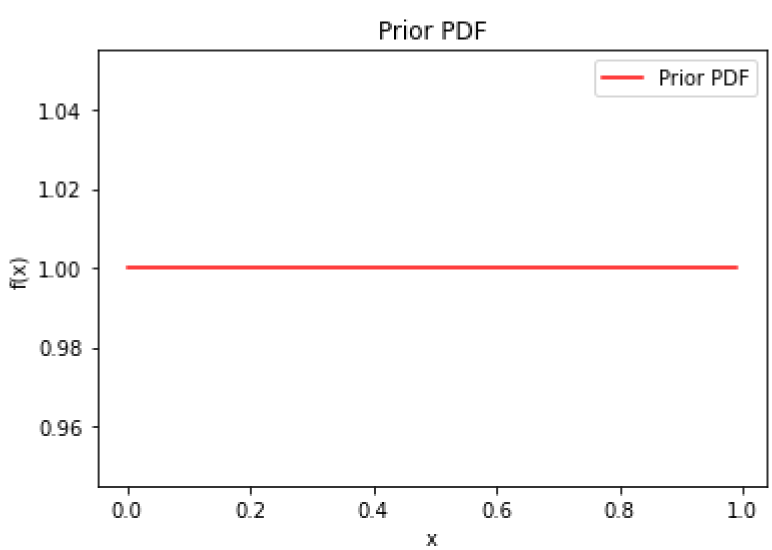
In [101]:
```
art_weight = np.argmax(Posterior)
art_weight
```

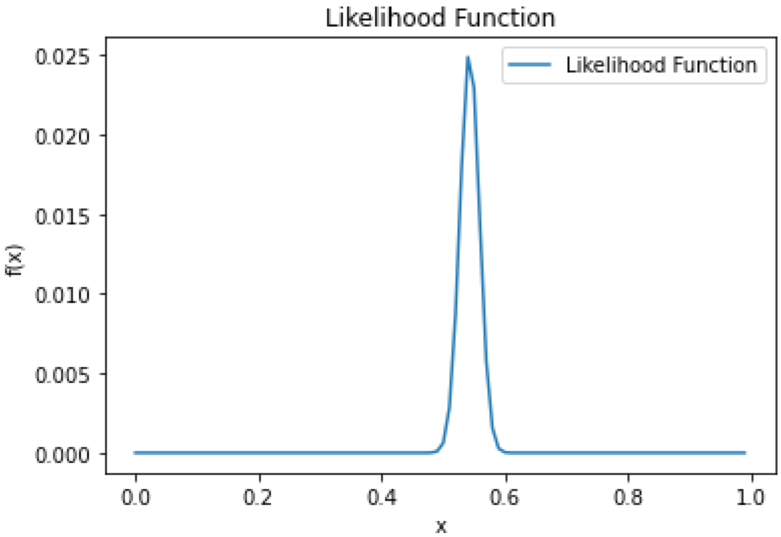
Out[101]: 30

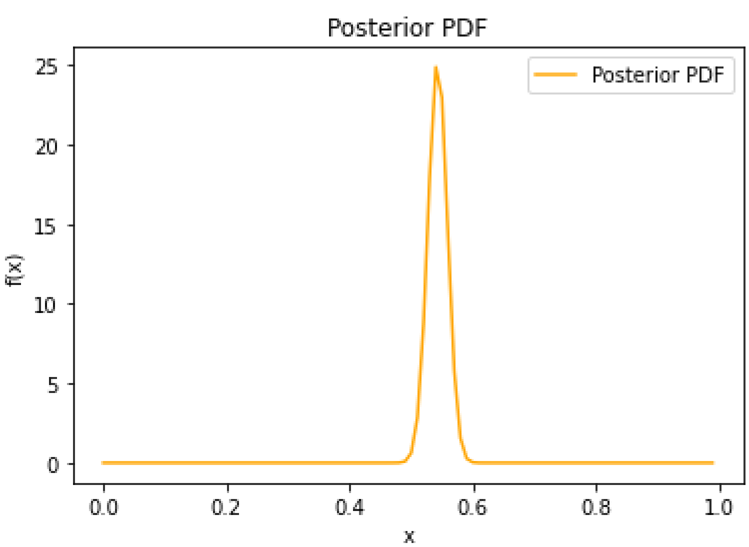
Prior: HistoryBooks

In [102]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df.HistoryBooks) #num_trials = 1000
num_successes = np.sum(df.HistoryBooks > 0)

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=1)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 544.
b_prime = 458.
```




Likelihood Function
Likelihood Function
f(x)
x
0.025
0.020
0.015
0.010
0.005
0.000
0.0
0.2
0.4
0.6
0.8
1.0


Posterior PDF
Posterior PDF
f(x)
x
25
20
15
10
5
0
0.0
0.2
0.4
0.6
0.8
1.0

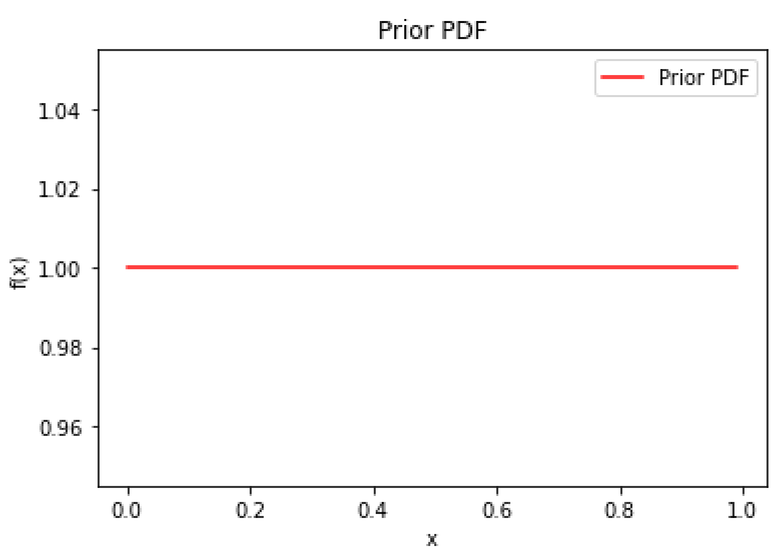
In [103]:
```
h_weight = np.argmax(Posterior)
h_weight
```

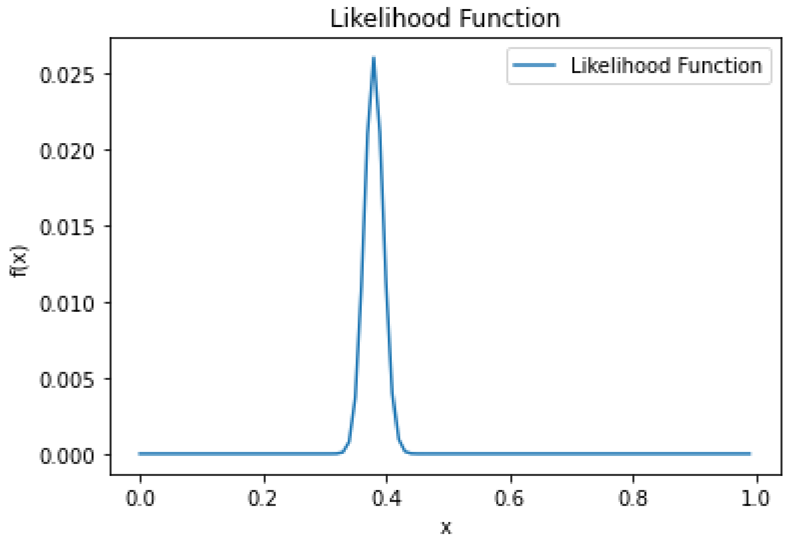
Out[103]: 54

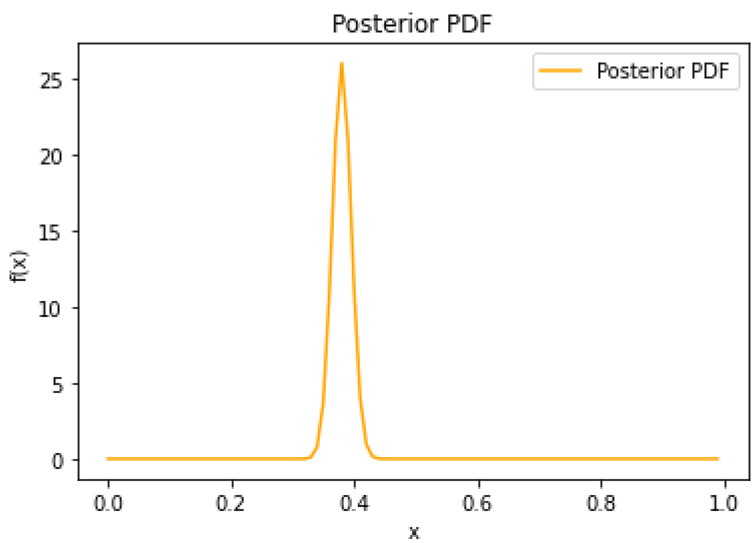
Prior: TableBooks

In [104]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df.TableBooks) #num_trials = 1000
num_successes = np.sum(df.TableBooks > 0)

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=1)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 381.
b_prime = 621.
```







In [105]:
```
t_weight = np.argmax(Posterior)
t_weight
```

Out[105]: 38

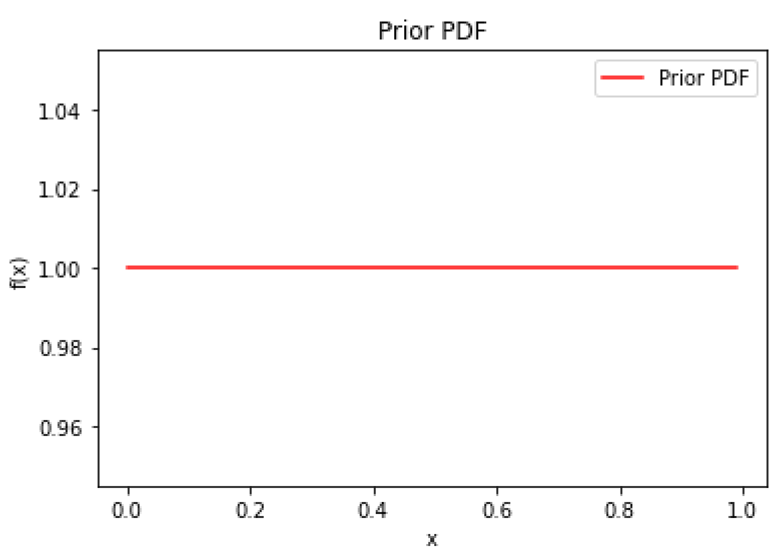
In [106]:
```
num_trials = len(df[(df["ArtBooks"] == 0) & (df["HistoryBooks"] == 0) & (df["Tabl
```

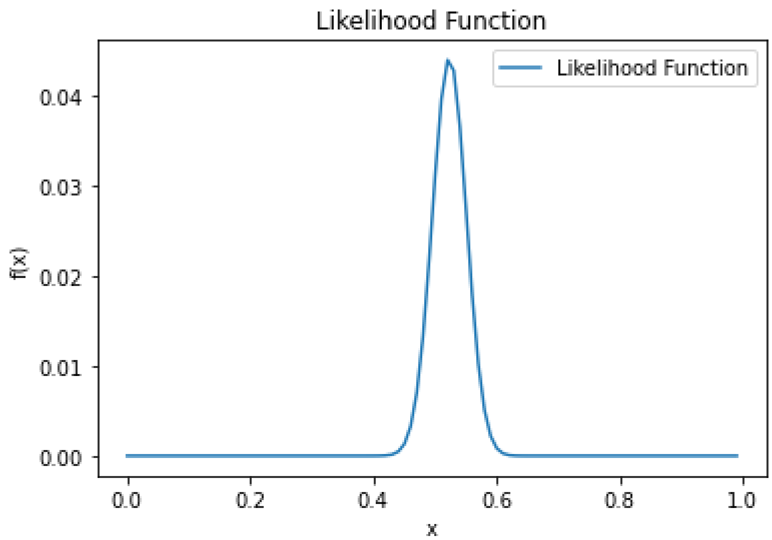
Prior: Artboks and HistoryBooks

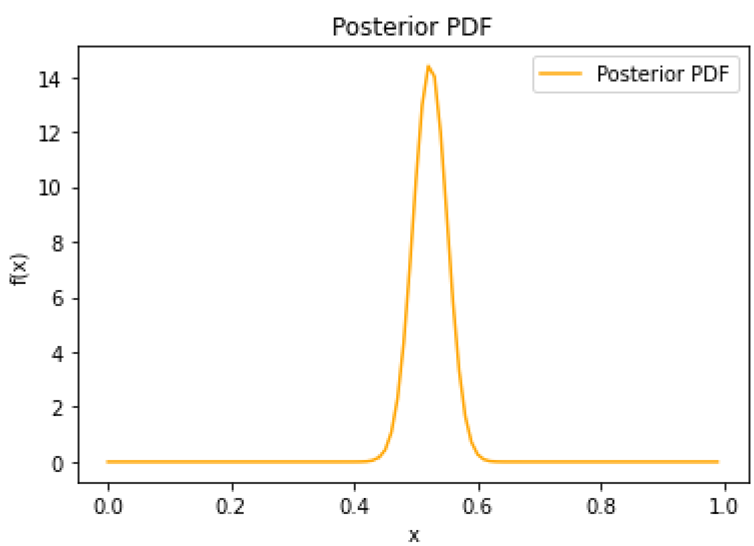
In [107]:

```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials = len(df[(df["ArtBooks"] == 0) & (df["HistoryBooks"] == 0)]) #num_tric
num_successes = np.sum((df.ArtBooks > 0) & (df.HistoryBooks))

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=1)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 172.
b_prime = 157.
```




Likelihood Function
Likelihood Function
0.04
0.03
0.02
0.01
0.00
f(x)
0.0
0.2
0.4
0.6
0.8
1.0
x


Posterior PDF
Posterior PDF
14
12
10
8
6
4
2
0
f(x)
0.0
0.2
0.4
0.6
0.8
1.0
x

In [108]:
```
ah_weight = np.argmax(Posterior)
ah_weight
```

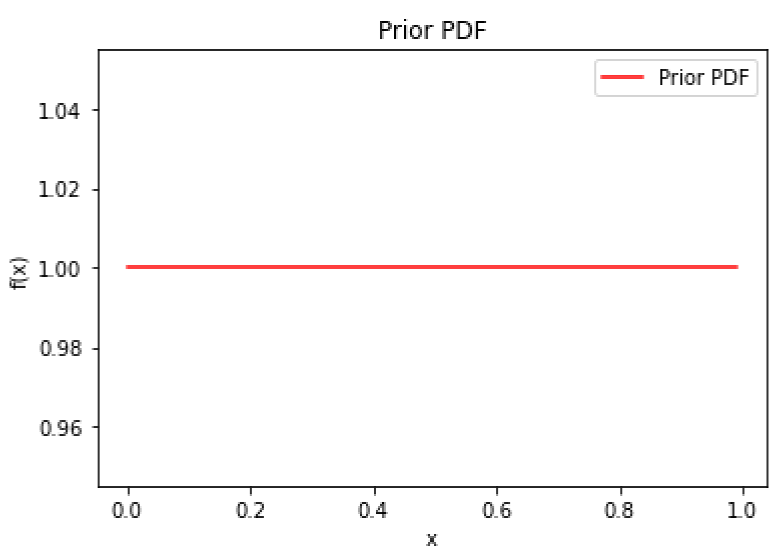
Out[108]:
```
52
```

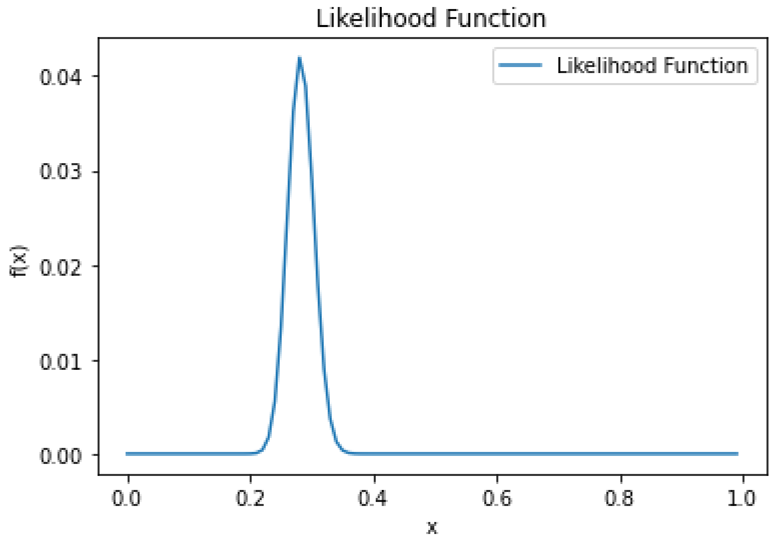
Prior: Artboks and TableBooks

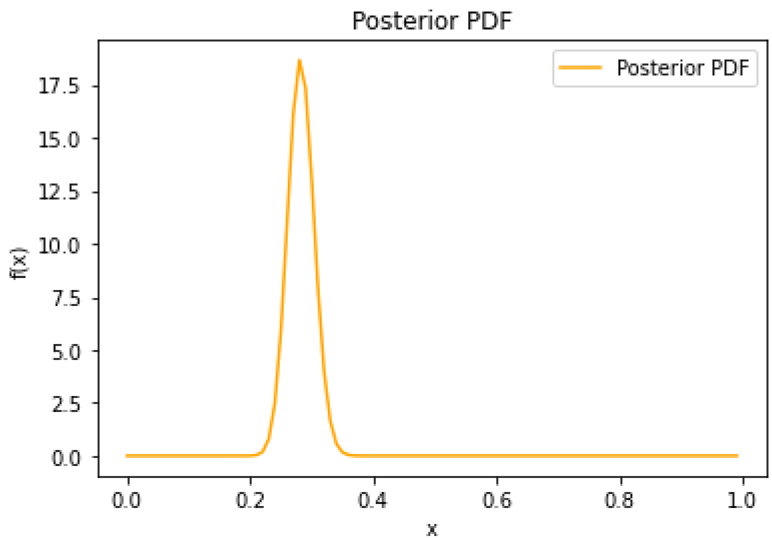
In [109]:

```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df[(df["ArtBooks"] == 0)  & (df["TableBooks"] == 0)])#num_trials
num_successes = np.sum((df.ArtBooks > 0) & (df.TableBooks))

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=1)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 126.
b_prime = 320.
```







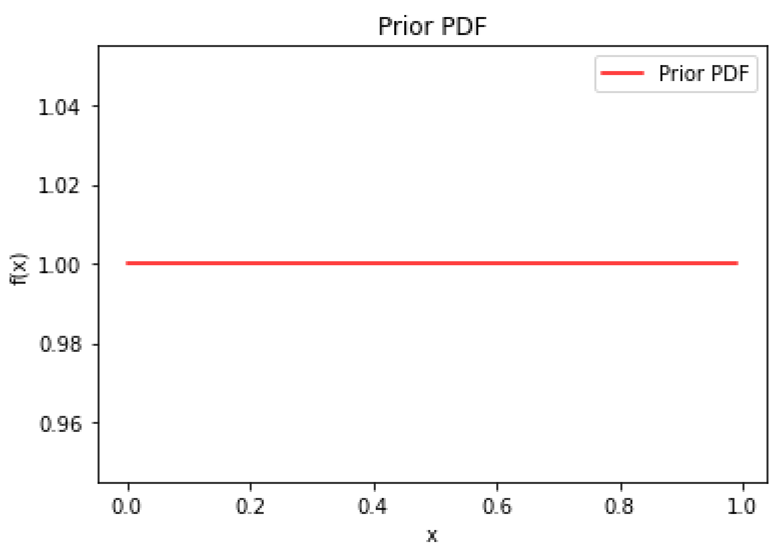
In [110]:
```
at_weight = np.argmax(Posterior)
at_weight
```

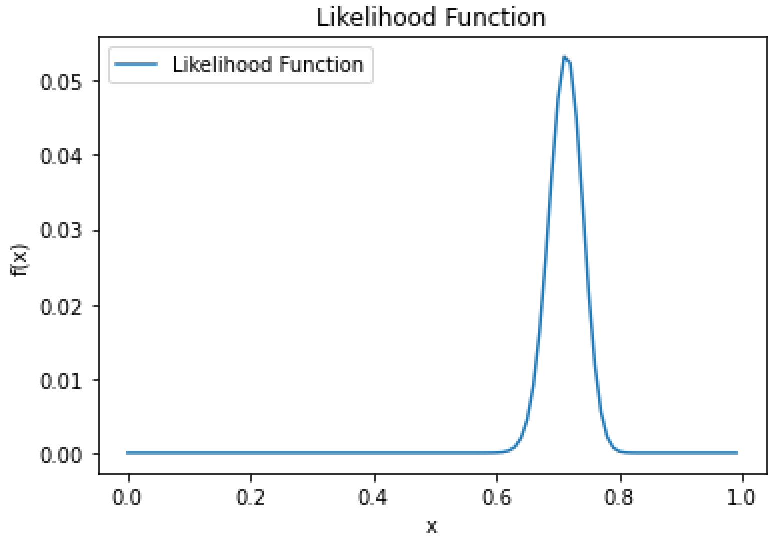
Out[110]: 28

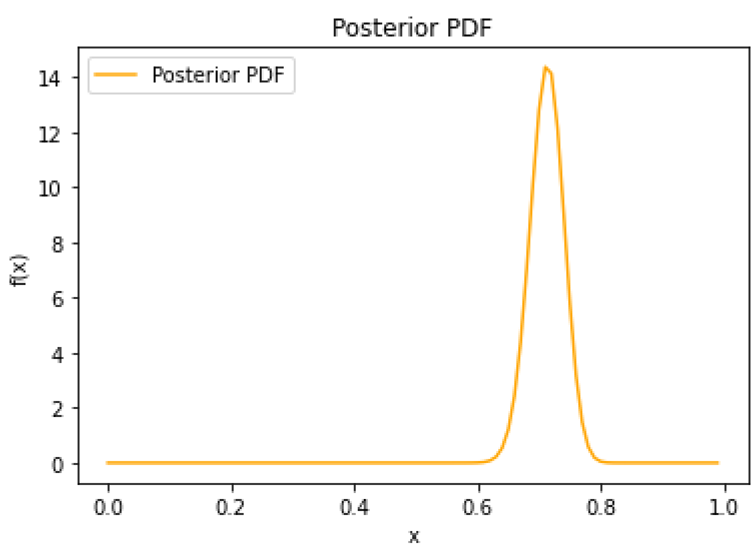
Prior: HistoryBooks and TableBooks

In [111]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df[(df["HistoryBooks"] == 0) & (df["TableBooks"] == 0)]) #num_tri
num_successes = np.sum((df.HistoryBooks > 0) & (df.TableBooks > 0 ))

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=1)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 193.
b_prime = 78.
```




Likelihood Function
Likelihood Function
0.05
0.04
0.03
0.02
0.01
0.00
f(x)
0.0
0.2
0.4
0.6
0.8
1.0
x


Posterior PDF
Posterior PDF
14
12
10
8
6
4
2
0
f(x)
0.0
0.2
0.4
0.6
0.8
1.0
x

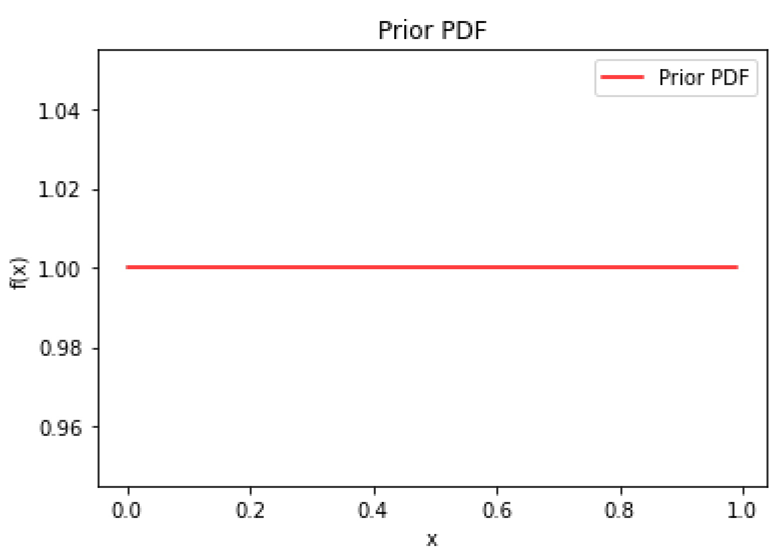
In [112]:
```
ht_weight = np.argmax(Posterior)
ht_weight
```

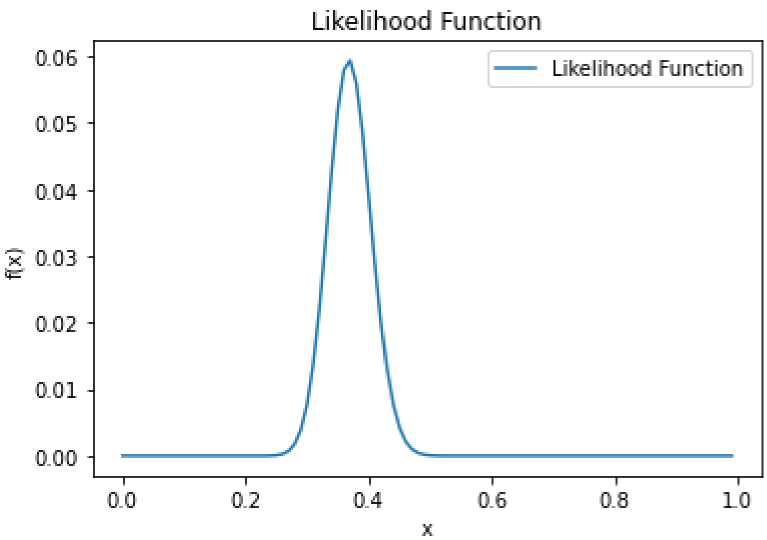
Out[112]: 71

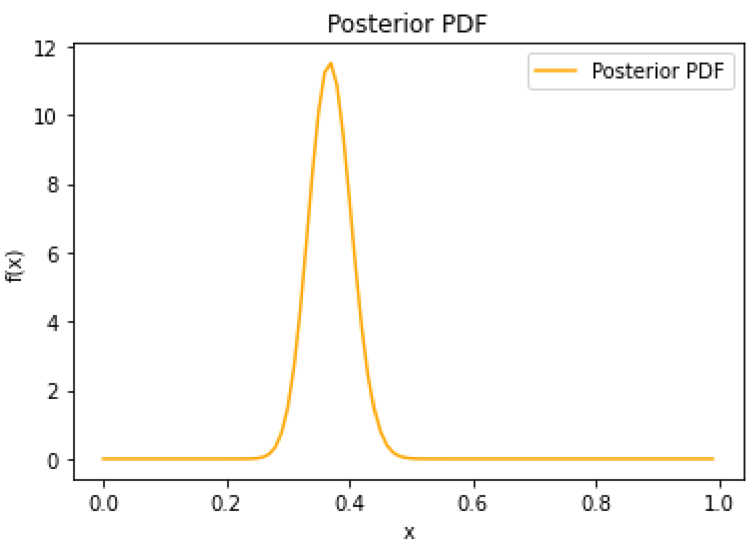
Prior: ArtBooks and HistoryBooks and TableBooks

In [113]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df[(df["ArtBooks"] == 0) & (df["HistoryBooks"] == 0) & (df["Table
num_successes = np.sum((df.HistoryBooks > 0) & (df.TableBooks > 0) & (df.ArtBooks

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=1)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 72.
b_prime = 123.
```







In [114]:
```
aht_weight = np.argmax(Posterior)
aht_weight
```

Out[114]: 37

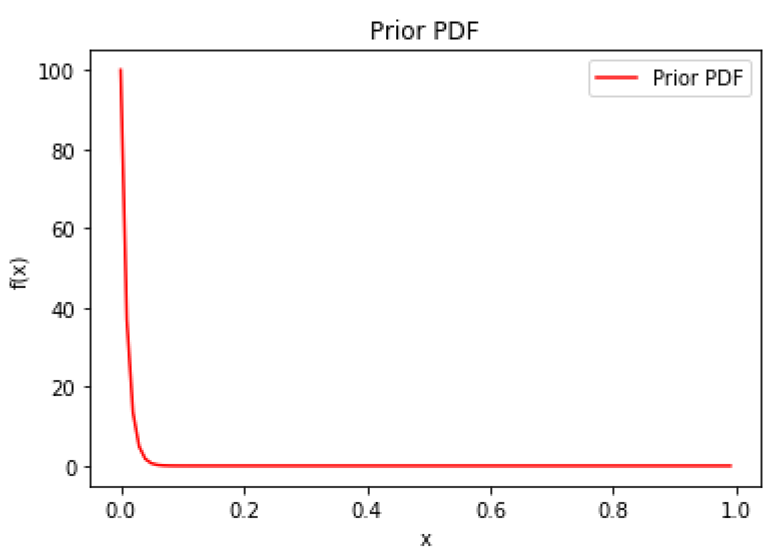
In [115]:
```
## a = 1, b= 100
```

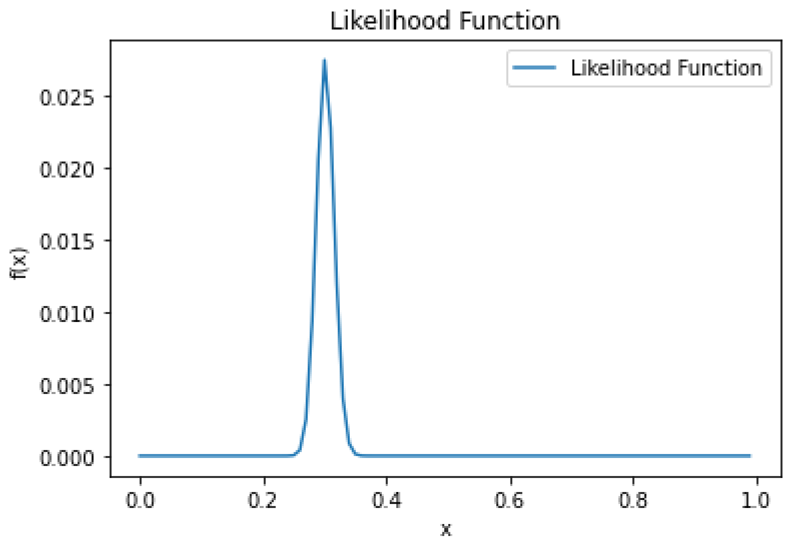
Prior: Artbooks

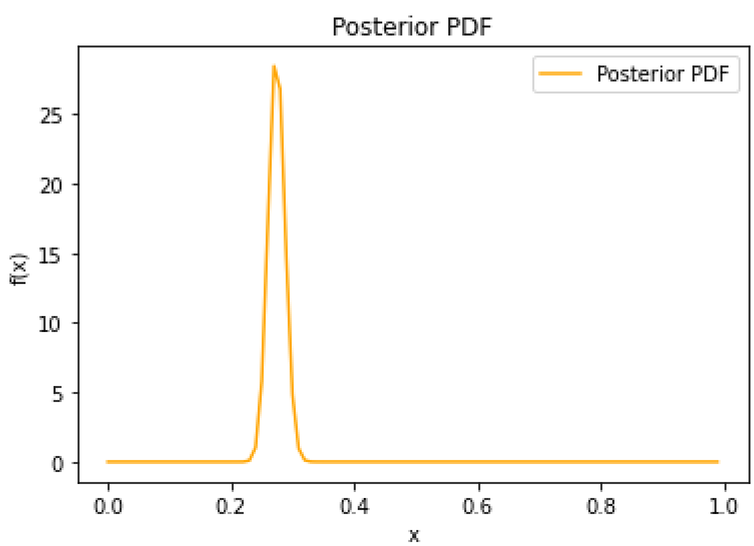
In [116]:

```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df.ArtBooks) #num_trials = 1000
num_successes = np.sum(df.ArtBooks > 0)

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=100)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 302.
b_prime = 799.
```




Likelihood Function
Likelihood Function
0.025
0.020
0.015
0.010
0.005
0.000
f(x)
0.0
0.2
0.4
0.6
0.8
1.0
x


Posterior PDF
Posterior PDF
25
20
15
10
5
0
f(x)
0.0
0.2
0.4
0.6
0.8
1.0
x

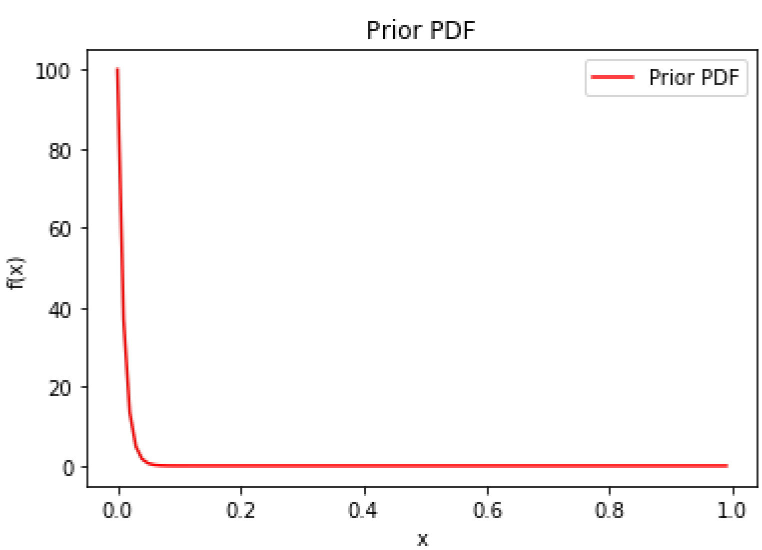
In [117]:
```
art_weight_s = np.argmax(Posterior)
art_weight_s
```

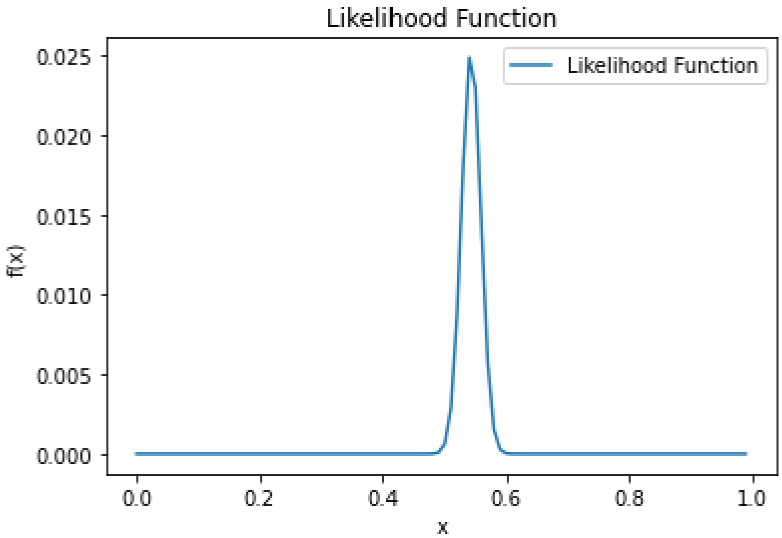
Out[117]: 27

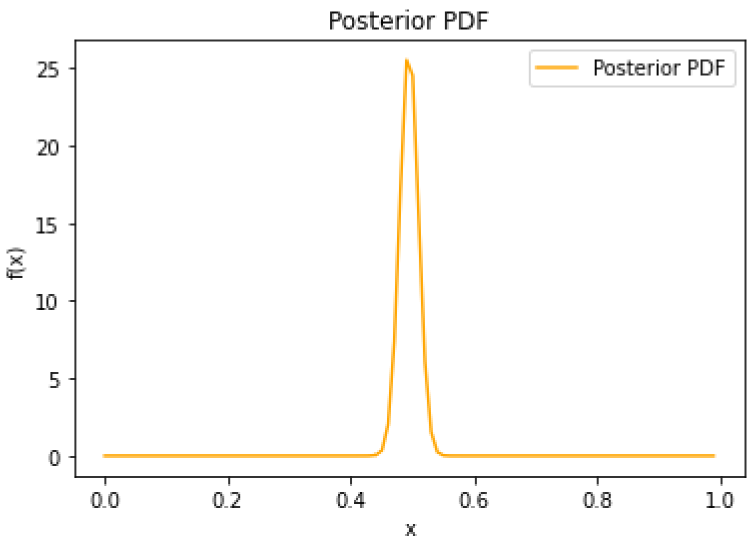
Prior: HistoryBooks

In [118]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df.HistoryBooks) #num_trials = 1000
num_successes = np.sum(df.HistoryBooks > 0)

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=100)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 544.
b_prime = 557.
```







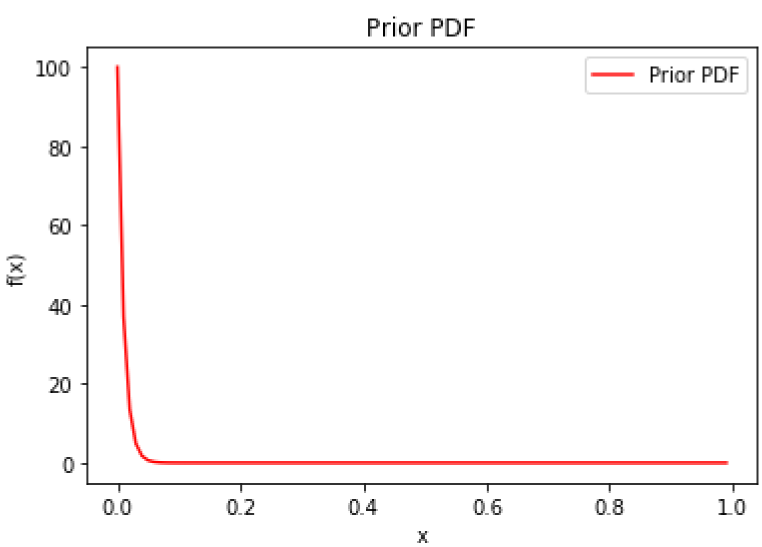
In [119]:
```
h_weight_s = np.argmax(Posterior)
h_weight_s
```

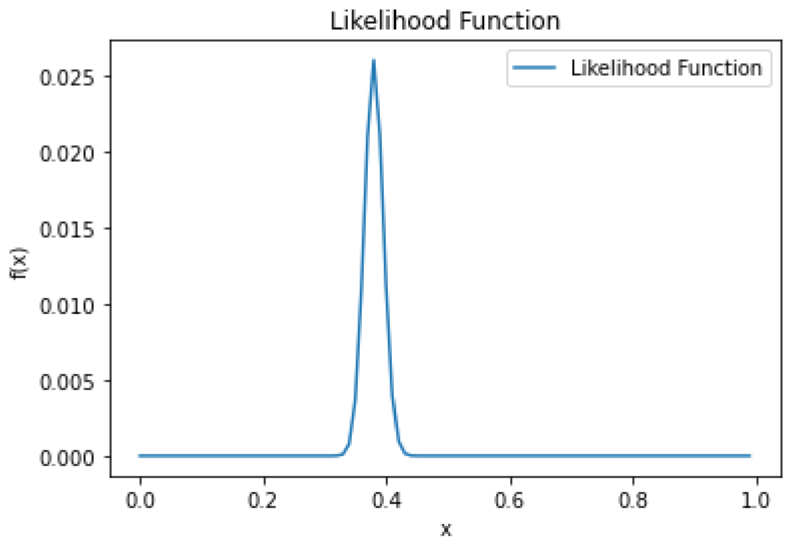
Out[119]: 49

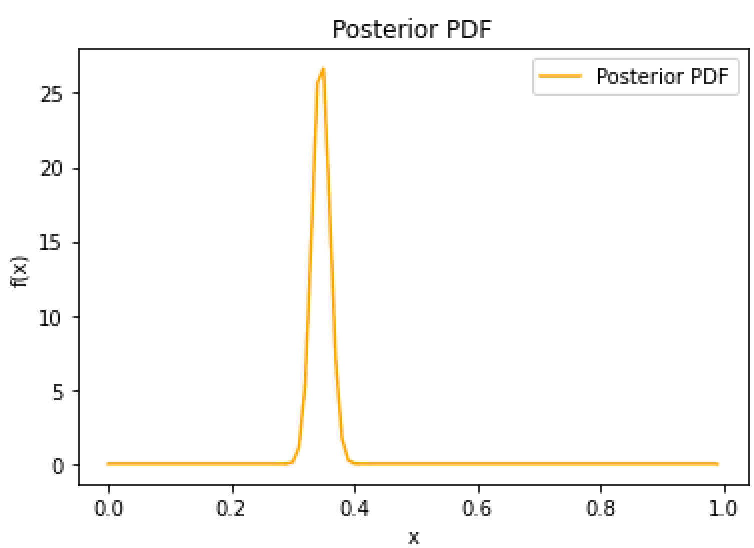
Prior: TableBooks

In [120]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df.TableBooks) #num_trials = 1000
num_successes = np.sum(df.TableBooks > 0)

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=100)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 381.
b_prime = 720.
```




Likelihood Function
Likelihood Function
0.025
0.020
0.015
0.010
0.005
0.000
f(x)
0.0
0.2
0.4
0.6
0.8
1.0
x


Posterior PDF
Posterior PDF
25
20
15
10
5
0
f(x)
0.0
0.2
0.4
0.6
0.8
1.0
x

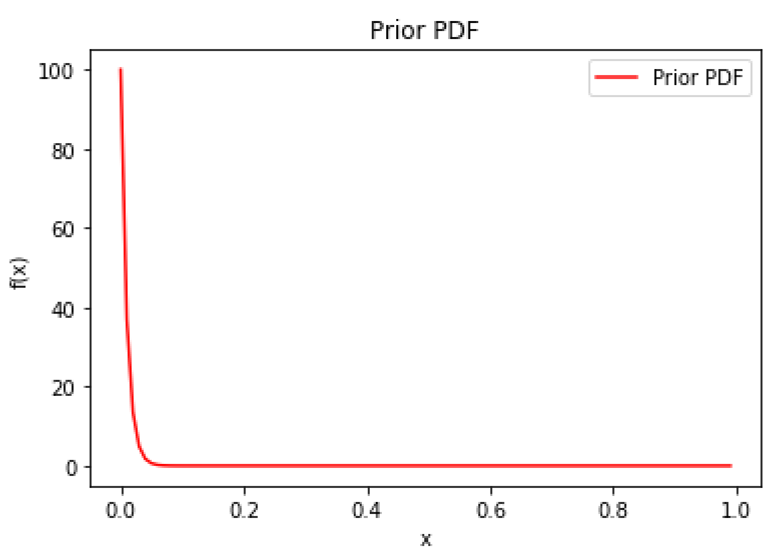
In [121]:
```
t_weight_s = np.argmax(Posterior)
t_weight_s
```

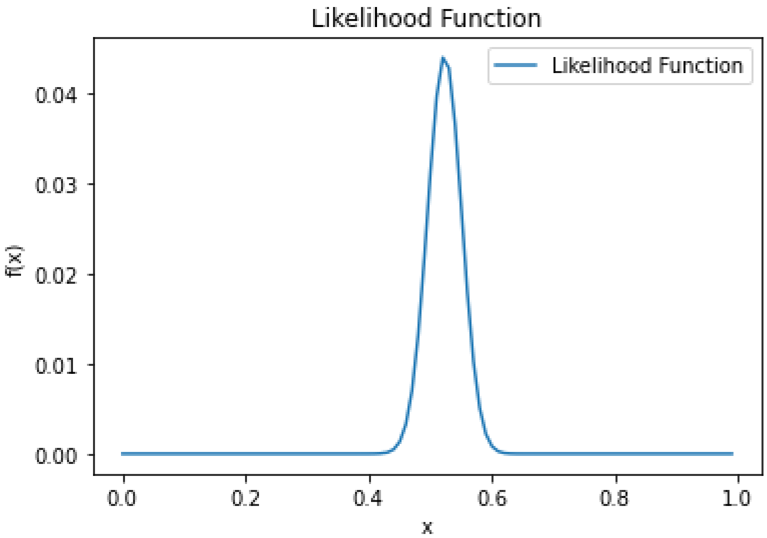
Out[121]:
```
35
```

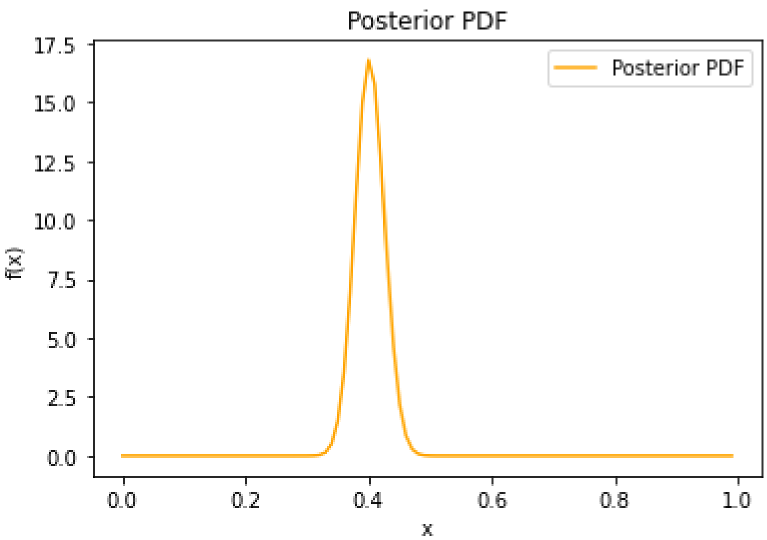
Prior: Artboks and HistoryBooks

In [122]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df[(df["ArtBooks"] == 0) & (df["HistoryBooks"] == 0)]) #num_trial
num_successes = np.sum((df.ArtBooks > 0) & (df.HistoryBooks))

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=100)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 172.
b_prime = 256.
```







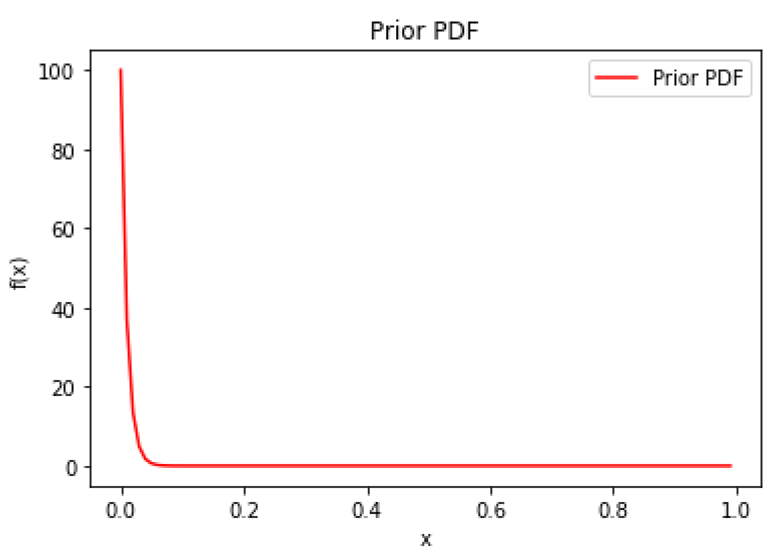
In [123]:
```
ah_weight_s = np.argmax(Posterior)
ah_weight_s
```

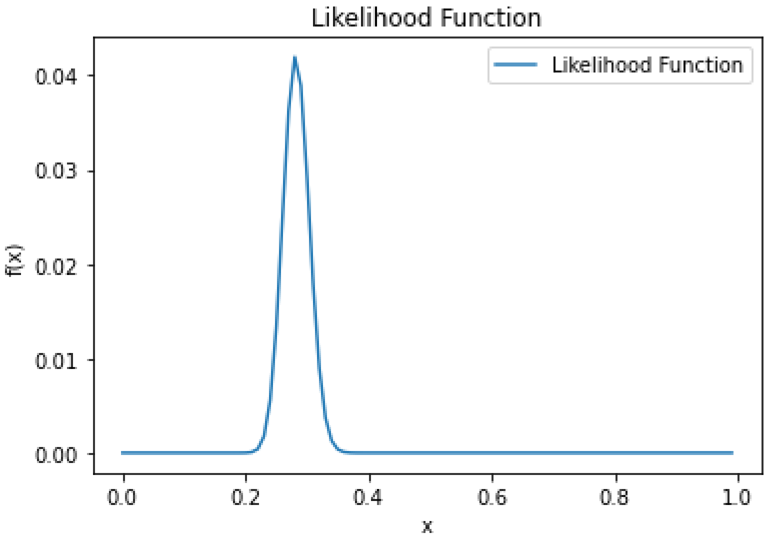
Out[123]: 40

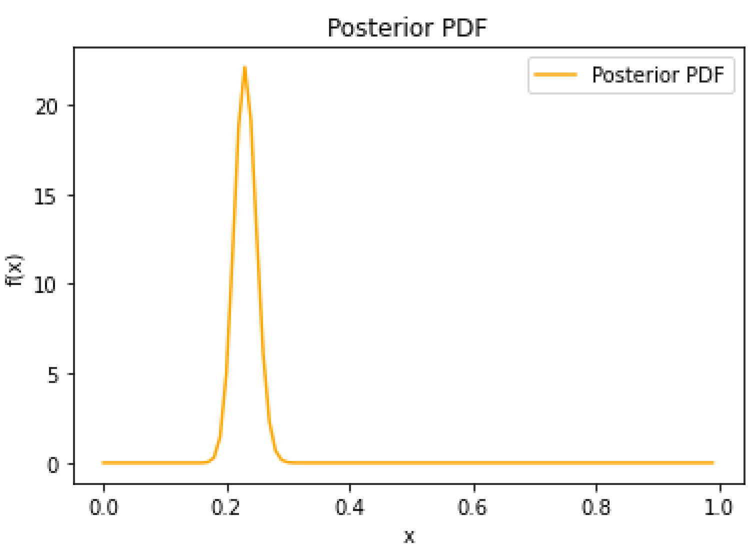
Prior: Artboks and TableBooks

In [124]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df[(df["ArtBooks"] == 0) & (df["TableBooks"] == 0)]) #num_trials
num_successes = np.sum((df.ArtBooks > 0) & (df.TableBooks))

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=100)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 126.
b_prime = 419.
```




Likelihood Function
Likelihood Function
f(x)
x
0.00
0.01
0.02
0.03
0.04
0.0
0.2
0.4
0.6
0.8
1.0


Posterior PDF
Posterior PDF
f(x)
x
0
5
10
15
20
0.0
0.2
0.4
0.6
0.8
1.0

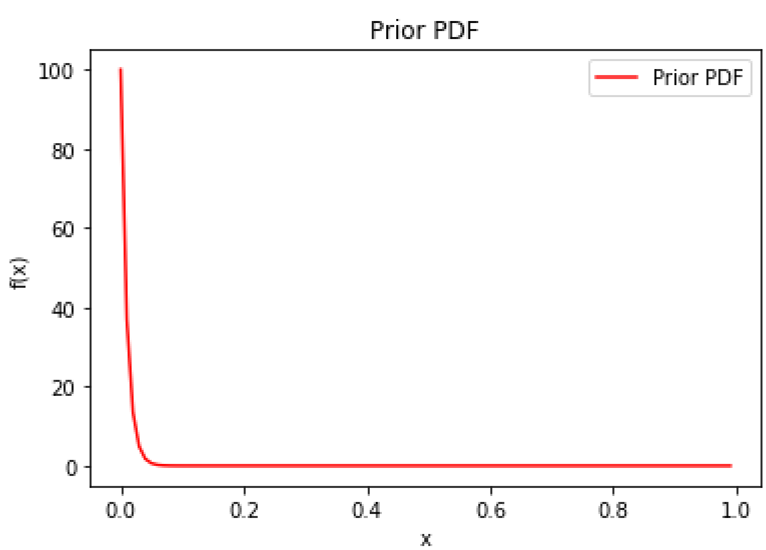
In [125]:
```
at_weight_s = np.argmax(Posterior)
at_weight_s
```

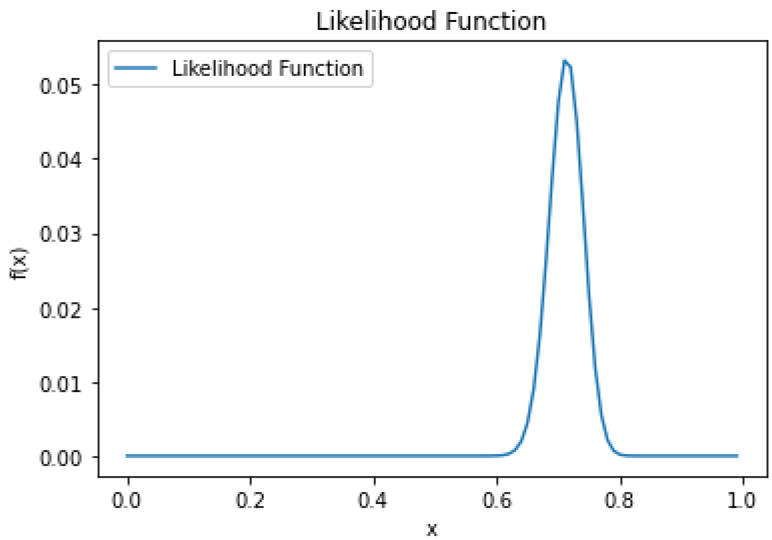
Out[125]: 23

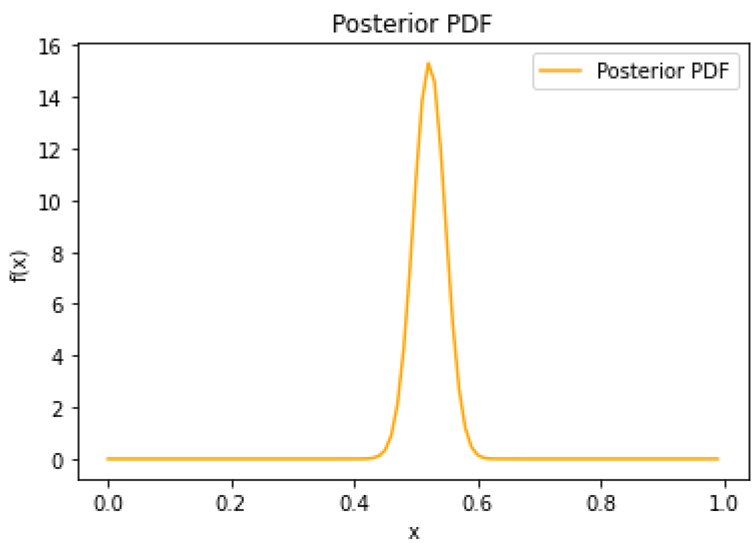
Prior: HistoryBooks and TableBooks

In [126]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df[ (df["HistoryBooks"] == 0) & (df["TableBooks"] == 0)]) #num_tr
num_successes = np.sum((df.HistoryBooks > 0) & (df.TableBooks > 0 ))

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=100)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 193.
b_prime = 177.
```







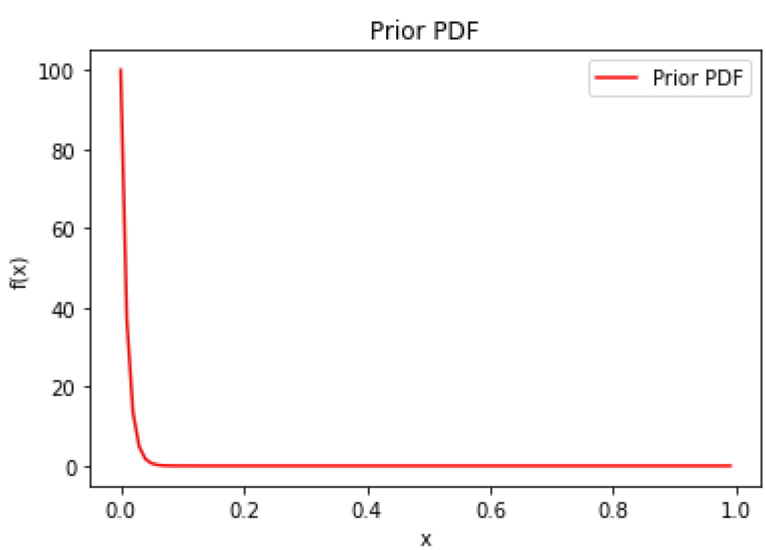
In [127]:
```
ht_weight_s = np.argmax(Posterior)
ht_weight_s
```

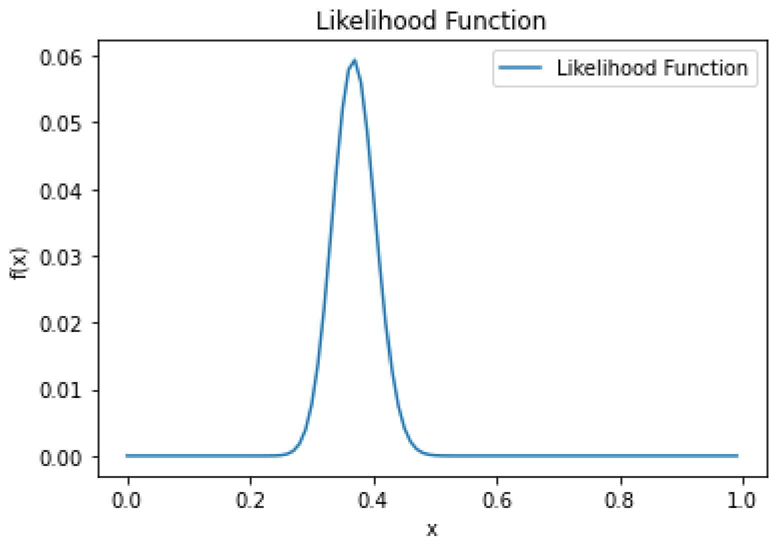
Out[127]: 52

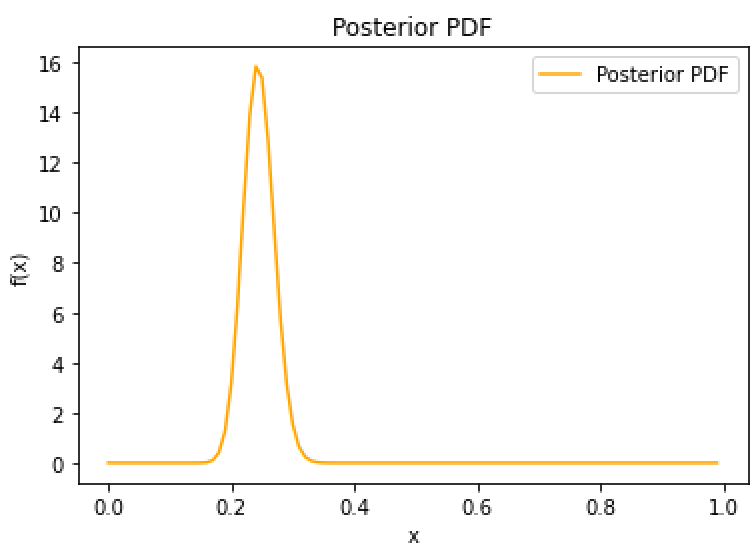
Prior: ArtBooks and HistoryBooks and TableBooks

In [128]:
```
import numpy as np
x = np.arange(0, 1, 0.01)
num_trials= len(df[(df["ArtBooks"] == 0) & (df["HistoryBooks"] == 0) & (df["Table
num_successes = np.sum((df.HistoryBooks > 0) & (df.TableBooks > 0) & (df.ArtBooks

Prior, Likelihood, Posterior = posterior_from_conjugate_prior(
    Likelihood_Dist_Type='Binomial',
    x=x,
    n=num_trials,
    k=num_successes,
    a=1,
    b=100)

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
a_prime = 72.
b_prime = 222.
```




Likelihood Function
Likelihood Function
0.06
0.05
0.04
0.03
0.02
0.01
0.00
f(x)
0.0
0.2
0.4
0.6
0.8
1.0
x


Posterior PDF
Posterior PDF
16
14
12
10
8
6
4
2
0
f(x)
0.0
0.2
0.4
0.6
0.8
1.0
x

In [129]:
```
aht_weight_s = np.argmax(Posterior)
aht_weight_s
```

Out[129]: 24

In [130]:
```
data = {'MaxPosterior':['Art', 'His', 'Tab', 'ArtHis', 'HisTab', 'ArtTab', 'ArtHi
        'a:b =1:1':[30, 54, 38, 52, 28, 71, 37] , 'a:b =1:100':[27, 49, 35, 40, 2
```

In [131]:
```
df = pd.DataFrame(data)
df
```

Out[131]:

	MaxPosterior	a:b =1:1	a:b =1:100
0	Art	30	27
1	His	54	49
2	Tab	38	35
3	ArtHis	52	40
4	HisTab	28	23
5	ArtTab	71	52
6	ArtHisTab	37	24

Response:

When we use strong weighting for low likelihood, i.e. a: b = 1:100,
We can see the maximum posterior shift to the lower values for all case.
This is consistent the undestanding of posteior equation is the propotional factor of likelihood.
I use a:b = 1:1 and a:b = 1:100 for this exercise, the posterior of beta function a:b = 1:100 is smaller that of a:b =1:1.
This is consitent to our understanding of likelihood.

Problem 5

From the lecture slide f(m|w) = N(μ0, vw), V>0
f(w) is Wishart with α degrees of freedom and
precision matrix r, with α > k − 1

Likelihood: N(M, W)
Posterior: f(m|x, w) ~ N (μ ,(v + N)w)
Posterior: f(w|x) is Wishart with α + N degrees of
freedom & precision matrix r
Posterior f(m|x) is a multivariate t distribution with α + N − k + 1
location parameter, μ∗ and precision (v + N)(α + N − k + 1)(r∗)−1
In our case, we have 8 variables, meaning k = 8, Prior α = 8+1 = 9. v is 462 for this case. This is multivariate
Gaussian with unknown mean & variance-covariance matrix. Meahwhile, f(w) is Wishart with multiple dimensions of the
gamma distribution.

In [132]:
```
import numpy as np
import pandas as pd
chd_data = pd.read_csv("CHDdata.csv")
chd_data.describe()
```

Out[132]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	
count	462.000000	462.000000	462.000000	462.000000	462.000000	462.000000	462.000000	462.00
mean	138.326840	3.635649	4.740325	25.406732	53.103896	26.044113	17.044394	42.81
std	20.496317	4.593024	2.070909	7.780699	9.817534	4.213680	24.481059	14.60
min	101.000000	0.000000	0.980000	6.740000	13.000000	14.700000	0.000000	15.00
25%	124.000000	0.052500	3.282500	19.775000	47.000000	22.985000	0.510000	31.00
50%	134.000000	2.000000	4.340000	26.115000	53.000000	25.805000	7.510000	45.00
75%	148.000000	5.500000	5.790000	31.227500	60.000000	28.497500	23.892500	55.00
max	218.000000	31.200000	15.330000	42.490000	78.000000	46.580000	147.190000	64.00

In [133]:
```
#drop "famhist" column
chd_data = chd_data.drop(columns= "famhist")
#check that it worked
chd_data.head()
```

Out[133]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	age	chd
0	160	12.00	5.73	23.11	49	25.30	97.20	52	1
1	144	0.01	4.41	28.61	55	28.87	2.06	63	1
2	118	0.08	3.48	32.28	52	29.14	3.81	46	0
3	170	7.50	6.41	38.03	51	31.99	24.26	58	1
4	134	13.60	3.50	27.78	60	25.99	57.34	49	1

In [134]:
```
for key in chd_data.keys()[0:8]:
    print("Standardizing "+key+".")
    chd_data[key] = chd_data[key] - np.mean(chd_data[key])
    chd_data[key] = chd_data[key] / np.std(chd_data[key])
```

```
Standardizing sbp.
Standardizing tobacco.
Standardizing ldl.
Standardizing adiposity.
Standardizing typea.
Standardizing obesity.
Standardizing alcohol.
Standardizing age.
```

In [135]:
```
# Check that it worked
chd_data.describe()
```

Out[135]:

	sbp	tobacco	ldl	adiposity	typea	obesity	
count	4.620000e+02	4.620000e+02	4.620000e+02	4.620000e+02	4.620000e+02	4.620000e+02	4
mean	-2.571296e-16	5.022437e-16	-3.963040e-15	1.559599e-15	1.153478e-17	-5.286776e-15	
std	1.001084e+00	1.001084e+00	1.001084e+00	1.001084e+00	1.001084e+00	1.001084e+00	1
min	-1.823123e+00	-7.924170e-01	-1.817753e+00	-2.401708e+00	-4.089354e+00	-2.695129e+00	
25%	-6.997535e-01	-7.809742e-01	-7.047170e-01	-7.245926e-01	-6.224081e-01	-7.267824e-01	
50%	-2.113321e-01	-3.565020e-01	-1.935182e-01	9.112757e-02	-1.059418e-02	-5.680824e-02	
75%	4.724579e-01	4.063492e-01	5.074164e-01	7.489145e-01	7.031887e-01	5.828745e-01	2
max	3.891408e+00	6.007857e+00	5.119082e+00	2.197976e+00	2.538631e+00	4.878906e+00	5

In [136]:
```
chd_positive = chd_data[chd_data.chd == 1]
chd_negative = chd_data[chd_data.chd == 0]
```

In [137]:
```
# Check that it worked
chd_positive.describe()
```

Out[137]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	
count	160.000000	160.000000	160.000000	160.000000	160.000000	160.000000	160.000000	160.00
mean	0.264268	0.411771	0.361398	0.349128	0.141722	0.137517	0.085909	0.51
std	1.156458	1.212965	1.075607	0.908099	1.044840	1.043288	1.070602	0.72
min	-1.774281	-0.792417	-1.542213	-2.060752	-3.375571	-2.695129	-0.696983	-1.76
25%	-0.528806	-0.465481	-0.386879	-0.250150	-0.545931	-0.572356	-0.677559	-0.00
50%	-0.015964	0.107747	0.156949	0.385765	0.193344	0.102370	-0.356351	0.69
75%	0.985300	0.994834	0.890513	1.052558	0.805158	0.649991	0.308250	1.10
max	3.891408	6.007857	4.553501	2.197976	2.538631	4.674587	5.321938	1.45

In [138]:
```
# Lets check the mean of each class to get a first look at the seperation
print("Mean for CHD Positive:")
print(np.array([chd_positive.mean()[0:8]]))
print("Mean for CHD Negative:")
print(np.array([chd_negative.mean()[0:8]]))
```

```
Mean for CHD Positive:
[[0.26426823 0.41177089 0.36139839 0.34912802 0.14172199 0.13751694
  0.0859086  0.51241433]]
Mean for CHD Negative:
[[-0.14000966 -0.21815676 -0.19146935 -0.18496849 -0.0750845  -0.07285666
  -0.04551449 -0.27147779]]
```

In [139]:
```
chd_positive
```

Out[139]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	age	chd
0	1.058564	1.823073	0.478412	-0.295503	-0.418470	-0.176786	3.277738	0.629336	1
1	0.277089	-0.790237	-0.159680	0.412140	0.193344	0.671373	-0.612745	1.383115	1
3	1.546985	0.842264	0.807126	1.624141	-0.214532	1.412621	0.295062	1.040488	1
4	-0.211332	2.171805	-0.599577	0.305351	0.703189	-0.012856	1.647775	0.423760	1
7	-1.188175	0.096850	-0.072667	-1.390421	0.907127	-0.697085	-0.422187	1.040488	1
...	...	...	...	...	...	...	...	...	...
453	-0.699754	-0.443685	1.198683	1.836434	-1.744067	1.296207	-0.696983	0.560810	1
454	0.374774	-0.652924	0.038515	0.336230	0.703189	0.490812	-0.360440	-0.261494	1
455	-0.504385	-0.304192	-0.923457	0.138089	-0.520439	-0.495142	1.242125	-1.083798	1
458	2.133091	0.123004	-0.159680	0.861173	-0.112563	0.609602	0.068519	0.629336	1
461	-0.309016	-0.792417	0.038515	1.029720	0.907127	-2.695129	-0.696983	0.218184	1

160 rows × 9 columns

In [140]:
```
chd_negative_new = chd_negative.iloc[ : ,[0,1, 2, 3, 4, 5, 6, 7]]
```

In [141]:
```
chd_positive_new = chd_positive.iloc[ : ,[0,1, 2, 3, 4, 5, 6, 7]]
```

In [142]:
```
## For patients withouot chd
v = 462
alpha = 9
k = 8
xmean = np.mean(chd_negative_new)
N = len(chd_negative_new)
mu0 = np.zeros(8)
mu_star = (v*mu0+ N *xmean)/(v+N)
print("the posterior mean for patients without CHD is:\n", mu_star)
```

```
the posterior mean for patients without CHD is:
 sbp          -0.055344
tobacco      -0.086235
ldl          -0.075686
adiposity    -0.073116
typea        -0.029680
obesity      -0.028799
alcohol      -0.017991
age          -0.107312
dtype: float64
```

In [143]:
```
## S-matrix:
S_matrix = pd.DataFrame(0, index=chd_negative_new.columns, columns = chd_negative
for index, row in chd_negative_new.iterrows():
    tmp = (row-xmean).to_frame()
    tmpT=tmp.T
    each_i = tmp.dot(tmpT)
    S_matrix+=each_i
```

In [144]:
```
S_matrix
```

Out[144]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	
sbp	232.260049	38.244868	36.897715	108.277837	-19.633118	79.008160	42.595168	107
tobacco	38.244868	186.564280	49.194077	77.992896	4.809790	39.289941	44.962429	11
ldl	36.897715	49.194077	246.079082	110.856114	2.591264	85.059945	10.736726	8
adiposity	108.277837	77.992896	110.856114	301.046673	-26.657035	207.981738	61.931813	20
typea	-19.633118	4.809790	2.591264	-26.657035	283.505089	9.629805	25.318168	-3
obesity	79.008160	39.289941	85.059945	207.981738	9.629805	284.307565	42.598878	10
alcohol	42.595168	44.962429	10.736726	61.931813	25.318168	42.598878	277.949377	4
age	107.072330	111.987969	85.897294	202.736423	-36.878619	100.765087	45.104219	31

In [145]:
```
mu0_xmean = (-xmean).to_frame().dot((-xmean).to_frame().T)
```

In [146]:
```
r_star = np.eye(8)+Smatrix+mu0_xmean*v*N/(v+N)
print("Posterior precision matrix is:\n")
r_star
```

```
Posterior precision matrix is:
```

Out[146]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	
sbp	217.225808	38.755896	17.754005	38.571500	-14.158151	23.946275	17.735802	46
tobacco	38.755896	243.625445	-4.576305	26.616119	-22.851166	7.285175	40.968083	55
ldl	17.754005	-4.576305	191.647090	68.207638	7.847611	58.016730	-32.176979	22
adiposity	38.571500	26.616119	68.207638	138.366582	-2.849924	113.776741	-21.383196	51
typea	-14.158151	-22.851166	7.847611	-2.849924	175.608281	20.789706	-9.426105	-24
obesity	23.946275	7.285175	58.016730	113.776741	20.789706	175.033026	-21.036718	20
alcohol	17.735802	40.968083	-32.176979	-21.383196	-9.426105	-21.036718	183.622480	-6
age	46.335853	55.234929	22.319061	51.836311	-24.578114	20.400289	-6.903003	99

In [147]:
```
## Degree of freedom
alpha+len(chd_negative_new)
## Posterior marginal alpha+N-k+1
alpha+N-k+1
## Parameter
param = (v+N)*(alpha+N-k+1)
## r_star inversion
r_star_inv.dot(r_star)
```

Out[147]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	age
sbp	1.069377	0.108618	0.020314	-0.000588	-0.019139	-0.052472	0.050636	0.085350
tobacco	0.151827	1.543612	-0.184024	0.047481	-0.164963	0.000047	0.192738	0.149879
ldl	0.015897	-0.216758	0.816598	0.071226	0.012547	0.046530	-0.085224	-0.025689
adiposity	-0.130916	-0.088725	0.097584	0.569991	0.091600	0.094388	-0.213406	-0.071258
typea	-0.011122	-0.158342	0.022480	0.022370	0.633017	0.049156	-0.120558	-0.050050
obesity	-0.076090	0.037253	-0.028544	0.064939	0.020501	0.625602	-0.012682	-0.001009
alcohol	-0.057739	-0.030678	-0.119325	-0.188493	-0.086557	-0.163993	0.692124	-0.088249
age	-0.176169	-0.350871	-0.142269	-0.240033	0.000619	-0.164768	-0.056321	0.278358

In [148]:
```
## Given by the lecture, Posterior f(m|x) is multivariaate t distribution with al
## and precision (v+N)(allpha+N-k+1)(r_star)_inv
## r_star_posterior
r_star_posterior = param*r_star_inv
r_star_posterior
```

Out[148]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alco
sbp	1253.888388	10.451143	21.292419	-175.786542	43.504717	-119.126753	-97.938
tobacco	10.451143	1611.998553	-153.425600	-20.656145	-93.254071	51.280758	-165.824
ldl	21.292419	-153.425600	1144.366556	-313.173423	-53.211475	-87.417415	73.113
adiposity	-175.786542	-20.656145	-313.173423	2737.489658	167.903060	-1438.041061	-182.268
typea	43.504717	-93.254071	-53.211475	167.903060	856.879146	-150.444767	-96.075
obesity	-119.126753	51.280758	-87.417415	-1438.041061	-150.444767	1781.554879	12.410
alcohol	-97.938426	-165.824903	73.113516	-182.268326	-96.075750	12.410417	911.752
age	-276.347333	-551.809764	-68.656640	-1109.369643	75.267817	393.331393	47.379

In [149]:
```
## ## For patients with chd
v = 462
alpha = 9
k = 8
xmean = np.mean(chd_positive_new)
N = len(chd_positive_new)
mu0 = np.zeros(8)
mu_star = (v*mu0+ N *xmean)/(v+N)
print("the posterior mean for patients with CHD is:\n", mu_star)
```

```
the posterior mean for patients with CHD is:
 sbp          0.067979
tobacco      0.105922
ldl          0.092964
adiposity    0.089808
typea        0.036456
obesity      0.035374
alcohol      0.022099
age          0.131811
dtype: float64
```

In [150]:
```
## S-matrix:
S_matrix = pd.DataFrame(0, index=chd_positive_new.columns, columns = chd_positive
for index, row in chd_positive_new.iterrows():
    tmp = (row-xmean).to_frame()
    tmpT=tmp.T
    each_i = tmp.dot(tmpT)
    S_matrix+=each_i
```

In [151]:
```
S_matrix
```

Out[151]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	
sbp	212.645903	33.177849	12.858328	33.842043	-16.077986	22.083404	16.572042	39
tobacco	33.177849	233.933981	-12.204530	19.246891	-25.842566	4.382533	39.154765	44
ldl	12.858328	-12.204530	183.952036	61.739898	5.222153	55.469172	-33.768471	12
adiposity	33.842043	19.246891	61.739898	131.118438	-5.386242	111.315680	-22.920653	42
typea	-16.077986	-25.842566	5.222153	-5.386242	173.578711	19.790684	-10.050207	-28
obesity	22.083404	4.382533	55.469172	111.315680	19.790684	173.063646	-21.642302	16
alcohol	16.572042	39.154765	-33.768471	-22.920653	-10.050207	-21.642302	182.244164	-9
age	39.394441	44.419132	12.826375	42.665925	-28.300661	16.788195	-9.159525	84

In [152]:
```
mu0_xmean = (-xmean).to_frame().dot((-xmean).to_frame().T)
```

In [153]:
```
r_star = np.eye(8)+Smatrix+mu0_xmean*v*N/(v+N)
print("Posterior precision matrix is:\n")
r_star
```

```
Posterior precision matrix is:
```

Out[153]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	
sbp	221.945586	46.110041	24.208508	44.806855	-11.627025	26.402300	19.270112	55
tobacco	46.110041	255.084343	5.480811	36.331770	-18.907279	11.112042	43.358776	69
ldl	24.208508	5.480811	200.473905	76.734760	11.309037	61.375451	-30.078742	34
adiposity	44.806855	36.331770	76.734760	146.604188	0.493978	117.021426	-19.356200	63
typea	-11.627025	-18.907279	11.309037	0.493978	176.965676	22.106825	-8.603283	-19
obesity	26.402300	11.112042	61.375451	117.021426	22.106825	176.311064	-20.238310	25
alcohol	19.270112	43.358776	-30.078742	-19.356200	-8.603283	-20.238310	184.121255	-3
age	55.487470	69.494567	34.834298	63.926624	-19.670277	25.162505	-3.927986	116

In [154]:
```
## Degree of freedom
alpha+len(chd_positive_new)
## Posterior marginal alpha+N-k+1
alpha+N-k+1
## Parameter
param = (v+N)*(alpha+N-k+1)
## r_star inversion
r_star_inv.dot(r_star)
```

Out[154]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol	age
sbp	1.078740	0.123206	0.033117	0.011781	-0.014118	-0.047600	0.053680	0.103504
tobacco	0.174951	1.579642	-0.152401	0.078030	-0.152562	0.012080	0.200255	0.194716
ldl	0.031140	-0.193008	0.837443	0.091364	0.020721	0.054462	-0.080269	0.003866
adiposity	-0.128646	-0.085188	0.100688	0.572990	0.092817	0.095570	-0.212668	-0.066857
typea	-0.000083	-0.141142	0.037576	0.036953	0.638937	0.054900	-0.116970	-0.028646
obesity	-0.085143	0.023147	-0.040925	0.052978	0.015646	0.620891	-0.015625	-0.018563
alcohol	-0.060867	-0.035552	-0.123603	-0.192625	-0.088234	-0.165621	0.691108	-0.094314
age	-0.161610	-0.328185	-0.122358	-0.220798	0.008427	-0.157191	-0.051588	0.306588

In [155]:
```
## Given by the lecture, Posterior f(m|x) is multivariaate t distribution with al
## and precision (v+N)(allpha+N-k+1)(r_star)_inv
## r_star_posterior
r_star_posterior = param*r_star_inv
r_star_posterior
```

Out[155]:

	sbp	tobacco	ldl	adiposity	typea	obesity	alcohol
sbp	543.998043	4.534216	9.237692	-76.264790	18.874472	-51.683006	-42.490474
tobacco	4.534216	699.363729	-66.563521	-8.961645	-40.458172	22.248098	-71.942945
ldl	9.237692	-66.563521	496.482122	-135.869932	-23.085738	-37.925945	31.720215
adiposity	-76.264790	-8.961645	-135.869932	1187.656758	72.844551	-623.892470	-79.076905
typea	18.874472	-40.458172	-23.085738	72.844551	371.756038	-65.270290	-41.682354
obesity	-51.683006	22.248098	-37.925945	-623.892470	-65.270290	772.925547	5.384245
alcohol	-42.490474	-71.942945	31.720215	-79.076905	-41.682354	5.384245	395.562686
age	-119.892974	-239.402035	-29.786605	-481.298751	32.654856	170.646375	20.555446

Problem 7

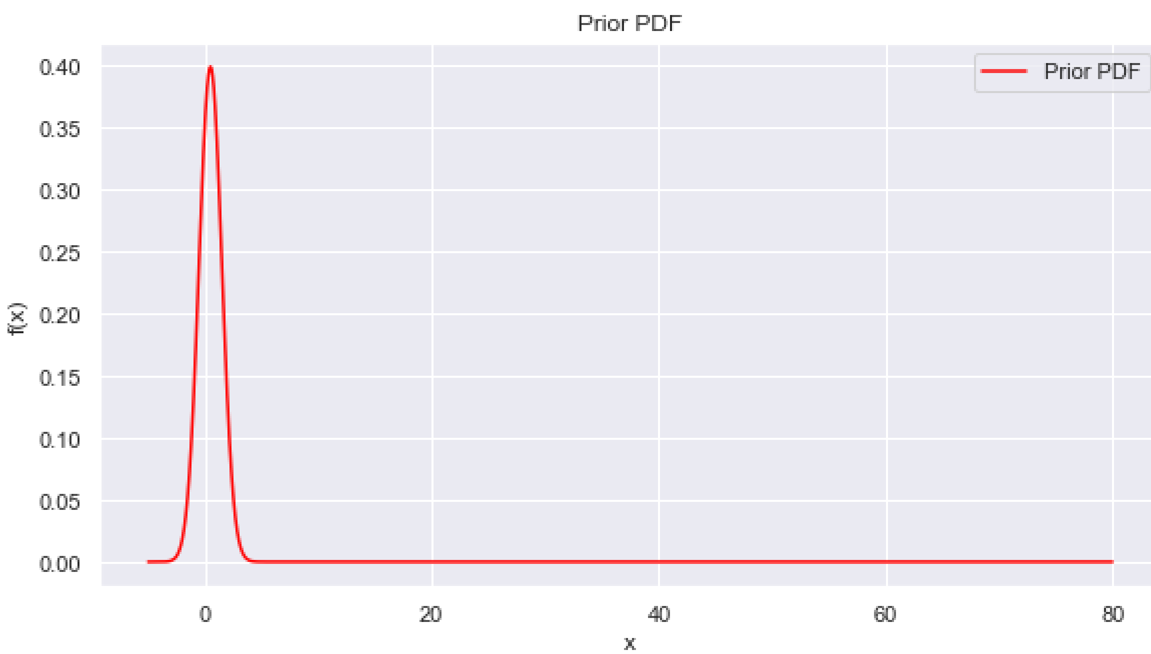
In [156]:
```
from scipy.stats import beta
from scipy.stats import norm

def posterior_from_conjugate_prior(**kwargs):
    if kwargs['Likelihood_Dist_Type'] == 'Binomial':
        # Get the parameters for the likelihood and prior distribution from the k
        x = kwargs['x']
        n = kwargs['n']
        k = kwargs['k']
        a = kwargs['a']
        b = kwargs['b']

        print(f'a_prime = {k + a}.')
        print(f'b_prime = {n - k + b}.')
        Likelihood = binom.pmf(p=x, n=n, k=k)
        Prior = beta.pdf(x=x, a=a, b=b)
        Posterior = beta.pdf(x=x, a=k+a, b=n-k+b)

        return [Prior, Likelihood, Posterior]

    elif kwargs['Likelihood_Dist_Type'] == 'Gaussian_Known_Variance':
        # Get the parameters for the likelihood and prior distribution from the k
        x = kwargs['x']
        n= len(x)
        mu = kwargs['mu']
        var = kwargs['var']
        prior_mu = kwargs['prior_mu']
        prior_var = kwargs['prior_var']
        print(kwargs)


        # To answer the challenge question, modify this section with the correct
        x_bar = np.mean(x)
        mu_prime = (prior_mu*var + n*x_bar*prior_var)/(var+n*prior_var)
        var_prime =(prior_var*var)/(var+n*prior_var)
        print(f'mu_prime = {mu_prime:.2f}.')
        print(f'var_prime = {var_prime: .2f}.')
        Likelihood = norm.pdf(x, loc = mu, scale=var**(.5))
        Prior = norm.pdf(x= x, loc = prior_mu, scale = prior_var**(.5))
        Posterior = norm.pdf(x= x, loc = mu_prime, scale = var_prime**(.50))

        return [Prior, Likelihood, Posterior]

    else:
        print('Distribution type not supported.')
        return -1, -1, -1
```

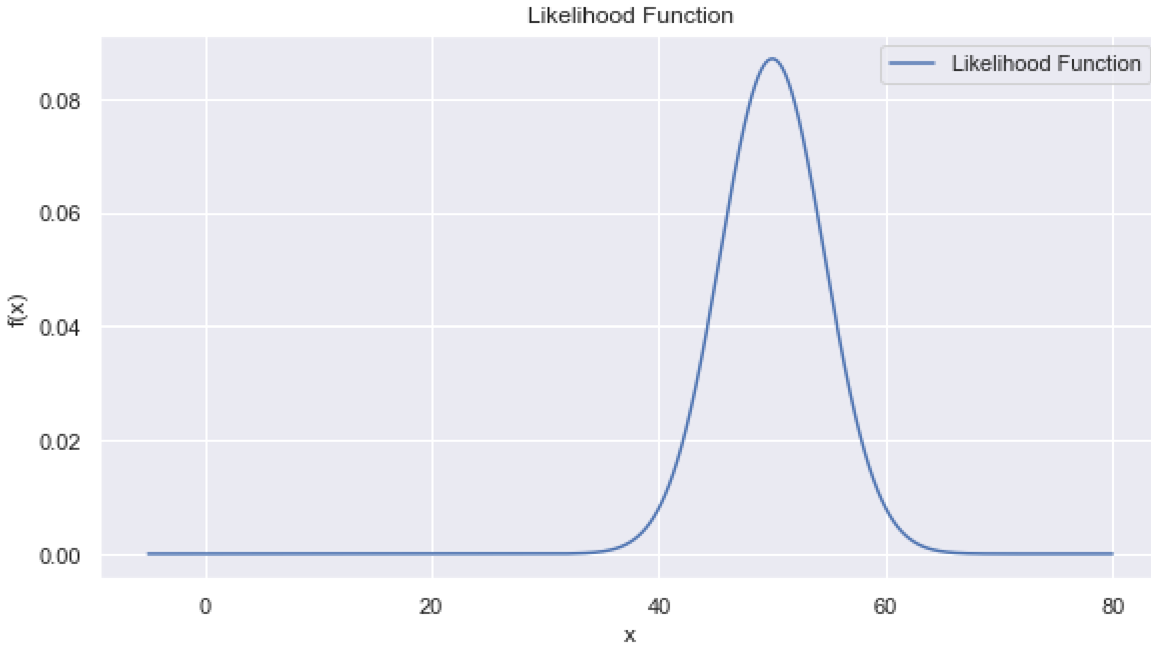
In [157]:
```
import numpy as np
x = np.arange(-5, 80, 0.01)
Prior, Likelihood, Posterior = posterior_from_conjugate_prior(Likelihood_Dist_Typ
```

```
{'Likelihood_Dist_Type': 'Gaussian_Known_Variance', 'x': array([-5.  , -4.99, -
4.98, ..., 79.97, 79.98, 79.99]), 'mu': 50, 'var': 21, 'prior_mu': 0.5, 'prior_
var': 1}
mu_prime = 37.40.
var_prime =  0.00.
```

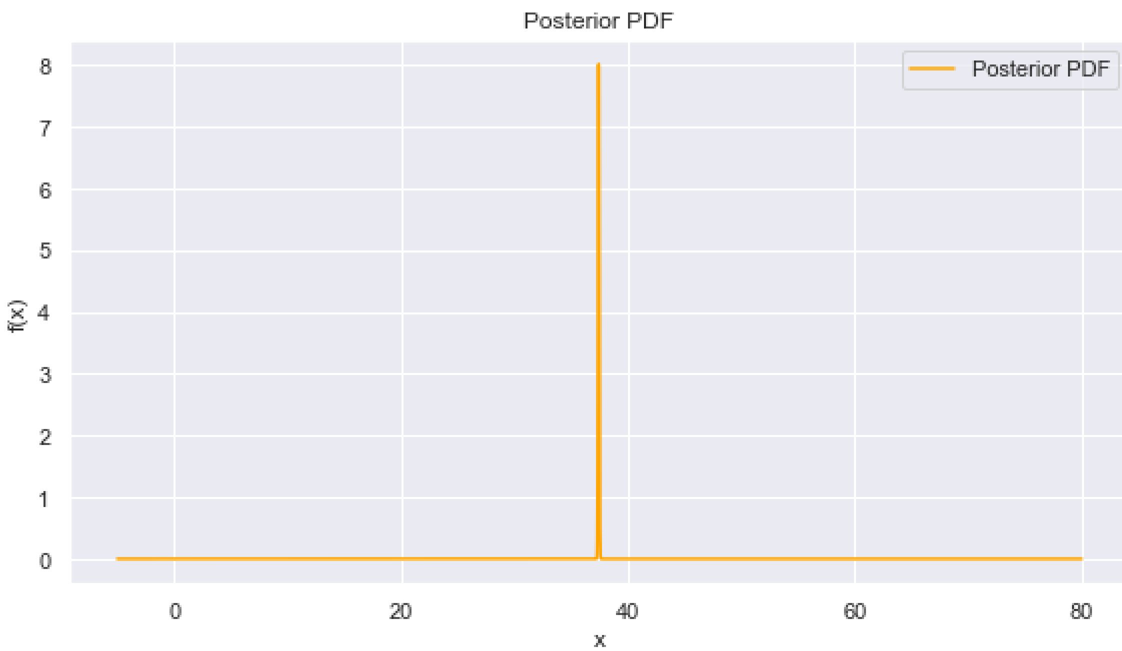
In [158]:
```
import numpy as np
import pandas as pd

# import matplotlib
import matplotlib.pyplot as plt
# import seaborn
import seaborn as sns
# settings for seaborn plotting style
sns.set(color_codes=True)
# settings for seaborn plot sizes
sns.set(rc={'figure.figsize':(9.5,5)})

x = np.arange(-5, 80, 0.01)
Prior, Likelihood, Posterior = posterior_from_conjugate_prior(Likelihood_Dist_Typ

ax1 = sns.lineplot(x, Prior, color='red')
ax1.set(xlabel='x', ylabel='f(x)', title=f'Prior PDF');
plt.legend(labels=['Prior PDF']);
plt.show()

ax2 = sns.lineplot(x, Likelihood)
ax2.set(xlabel='x', ylabel='f(x)', title=f'Likelihood Function');
plt.legend(labels=['Likelihood Function']);
plt.show()

ax3 = sns.lineplot(x, Posterior, color='orange')
ax3.set(xlabel='x', ylabel='f(x)', title=f'Posterior PDF');
plt.legend(labels=['Posterior PDF']);
plt.show()
```

```
{'Likelihood_Dist_Type': 'Gaussian_Known_Variance', 'x': array([-5.  , -4.99, -
4.98, ..., 79.97, 79.98, 79.99]), 'theta': 5, 'n': 100, 'k': 21, 'mu': 50, 'va
r': 21, 'prior_mu': 0.5, 'prior_var': 1}
mu_prime = 37.40.
var_prime =  0.00.
```





Likelihood Function
Likelihood Function
0.08
0.06
0.04
0.02
0.00
f(x)
0
20
40
60
80
x



Posterior PDF
Posterior PDF
8
7
6
5
4
3
2
1
0
f(x)
0
20
40
60
80
x