1. Adjacency Matrix

Adjacency Matrix keeps a value(1/0) for every pair of nodes, whether the edge exists or not.
The reason why we say it adjacency matrix, becasue it stores data in the format of matrix.
Adjacency Matrix can be expressed in the format shown below:
Considering |V| = n, |E| = m
A[i, j]
1 if (i, j) ∈ E
0 if (i, j) ∉ E
1. Get(i, j) - $\theta(1)$, query whether there is an edge from edge from i to j, it takes $\theta(1)$
2. Add(i, j, w) - $\theta(1)$, add edge
3. AllFrom(i) - this takes $\theta(n^2)$
The reasons are as as following:
If we want to check there is an edge from i to j, simply check if A[i, j] is 1 or 0, it is $\theta(1)$
If we want to add edge from i to j, then add new index in this matrix, it is $\theta(1)$
If we want to store a list of the edge with source i, we need to traverse all matrix, this is $\theta(n^2)$

In [19]:

```python
# add a new vertex in the graph
def add_vertex(v):
  global graph
  global num_vertex
  global vertices
  if v in vertices:
    print("Found Vertex ", v)
  else:
    num_vertex = num_vertex + 1
    vertices.append(v)
    if num_vertex > 1:
        for vertex in graph:
            vertex.append(0)
    nv = []
    for i in range(num_vertex):
        nv.append(0)
    graph.append(nv)

# v1(from) v2(to) e(weight)
def add_edge(v1, v2, e):
    global graph
    global num_vertex
    global vertices
    # check v1 in the graph or not
    if v1 not in vertices:
        print("cannot find Vertex ", v1)
    # check v2 in the graph or not
    elif v2 not in vertices:
        print("cannot find vertex", v2)
    else:
        index1 = vertices.index(v1)
        index2 = vertices.index(v2)
        graph[index1][index2] = e

# Print the graph
def print_graph():
  global graph
  global vertices_no
  for i in range(vertices_no):
    for j in range(vertices_no):
      if graph[i][j] != 0:
        print(vertices[i], " -> ", vertices[j], \
        " edge weight: ", graph[i][j])


vertices = []

num_vertex= 0
graph = []
add_vertex("a")
add_vertex("b")
add_vertex("c")
add_vertex("d")
add_vertex("e")
## add vertex from, to, weight
add_edge("a", "b", 1)
add_edge("a", "c", 1)
add_edge("b", "c", 3)
add_edge("c", "d", 4)
```

```
add_edge("d", "a", 5)
print_graph()
print("This graph: ", graph)
```

```
a  ->  b  edge weight:  1
a  ->  c  edge weight:  1
b  ->  c  edge weight:  3
c  ->  d  edge weight:  4
d  ->  a  edge weight:  5
This graph:  [[0, 1, 1, 0, 0], [0, 0, 3, 0, 0], [0, 0, 0, 4, 0], [5, 0, 0, 0, 0], [0, 0,
0, 0, 0]]
```

2. Adjacency list

Adjacency list only contains existing edges, this means its length is at most the number of edges (considering the number
of nodes in case there are fewer edges then nodes)

We can consider adjacency list as array with V element
One entry for each vertex, each component in the array contains the list of neighibors for the index vertex
Considering |V| = n, |E| = m
1. Get(i, j) - $\theta(1)$
2. Add(i, j, w) - $\theta(1)$, append the list
3. AllFrom(i) - it needs $\theta(n + m)$, need to traverse linkedlist of the vertex
We can consider to check whether the edge between i and j exist or not, we need to go to vertex i
then check if j is linked to i or not, this will be some constant, saying $\theta(1)$
If we want to add an edge from i, j, we can simply append the list, this should be $\theta(1)$
If we want to know all j from source i. We can consider there are two steps.
First, we need to deal with element in single vertex. Second, we need to deal with all vertices
For single vertex, the run time is $\theta(1 + deg(v1))$, this is up to how many edges exist in this vertex
For all vertices, sum the run time up, this is $\theta(|V| + deg(v1) + deg(v2)....) = \theta(|V| + |E|) = \theta(n + m)$

In [62]:

```python
# add vertex and check if vertex in the list or not
def add_vertex(v):
  global graph
  global num_vertex
  if v in graph:
    print("Found Vertex ", v)
  else:
    num_vertex= num_vertex + 1
    graph[v] = []

# add vertex v1(from) v2(to) e(weight)
def add_edge(v1, v2, e):
  global graph
  if v1 not in graph:
    print("cannot find vertex ", v1)
  elif v2 not in graph:
    print("cannot find Vertex ", v2)
  else:
    ## append v2, e to v1
    nv = [v2, e]
    graph[v1].append(nv)

def print_graph():
  global graph
  for vertex in graph:
    for edges in graph[vertex]:
      print("vertex", vertex,":" , "(%s, %s)" %( edges[0], edges[1]))

graph = {}
# identify the vertex in the graph
num_vertex = 0
add_vertex("a")
add_vertex("b")
add_vertex("c")
add_vertex("d")
add_vertex("e")
# add aagae from to to with weights
add_edge("a", "b", 1)
add_edge("a", "c", 6)
add_edge("a", "d", 7)
add_edge("b", "c", 2)
add_edge("c", "d", 3)
add_edge("d", "e", 4)
add_edge("e", "a", 5)
print_graph()
print ("This graph: ", graph)
```

```
vertex a : (b, 1)
vertex a : (c, 6)
vertex a : (d, 7)
vertex b : (c, 2)
vertex c : (d, 3)
vertex d : (e, 4)
vertex e : (a, 5)
This graph:  {'a': [['b', 1], ['c', 6], ['d', 7]], 'b': [['c', 2]], 'c': [['d', 3]], 'd':
[['e', 4]], 'e': [['a', 5]]}
```

3. Comparsion and Implementation

Adjacency matrix or Adjacency list

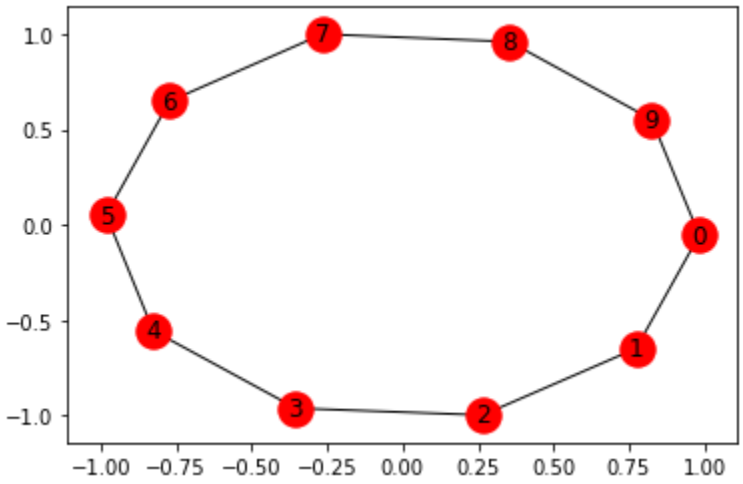
The graph may be sparse or dense
for sparse graph: m = $O(n)$
for dense graph: m = $\Omega(n)$
Adjacency matrix
1. Space required : $\theta(n^2)$
2. Time for going through all edges : $\theta(n^2)$
3. time for finding an edge exists: $\theta(1)$
Adjacency lists
1. Space required: $\theta(n+m)$
2. Time for going through all edges : $\theta(n+m)$
3. Time for finding if an edge exists : $\theta(max(Adj(n)))$
|V| = n, |E| = m
To conclude, Adjacency matrix is better for dense graphs and Adjacency list is better for sparse graphs

The common way to store network data is adjacency list, because most of graphs are sparse.

Considering facebook networking case, if the storage using adjacency matrix will be **NOT** efficient becasue the vertices are facebook users (~ 2.45 billion users). For example, I have ~ 2000 friend.
If I want to find my connection using adjacency matrix,
then I need to traverse all facebook users to find who are connected to me,
this takes a long time and waste memory,
Please be noticed, the considertions are cases in general.

In [87]:
```
## Visualization adjacency matrix
import networkx as nx
G = nx.cycle_graph(10)
A = nx.adjacency_matrix(G)
print(A.todense())
nx.draw_networkx(G)
```

```
[[0 1 0 0 0 0 0 0 0 1]
 [1 0 1 0 0 0 0 0 0 0]
 [0 1 0 1 0 0 0 0 0 0]
 [0 0 1 0 1 0 0 0 0 0]
 [0 0 0 1 0 1 0 0 0 0]
 [0 0 0 0 1 0 1 0 0 0]
 [0 0 0 0 0 1 0 1 0 0]
 [0 0 0 0 0 0 1 0 1 0]
 [0 0 0 0 0 0 0 1 0 1]
 [1 0 0 0 0 0 0 0 1 0]]
```



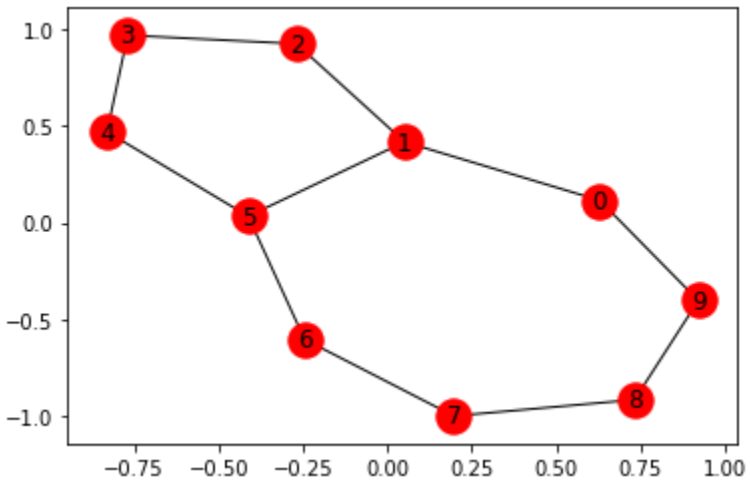
In [2]:
```
import matplotlib.pyplot as plt
nx.draw_networkx(G)
plt.show()
```

```
C:\Users\gladies\Anaconda3\lib\site-packages\networkx\drawing\nx_pylab.py:563: MatplotlibDeprecationWarning:
The iterable function was deprecated in Matplotlib 3.1 and will be removed in 3.3. Use np.iterable instead.
  if not cb.iterable(width):
C:\Users\gladies\Anaconda3\lib\site-packages\networkx\drawing\nx_pylab.py:611: MatplotlibDeprecationWarning:
The is_numlike function was deprecated in Matplotlib 3.0 and will be removed in 3.2. Use isinstance(..., numbers.Number) instead.
  if cb.is_numlike(alpha):
```

```
<Figure size 640x480 with 1 Axes>
```

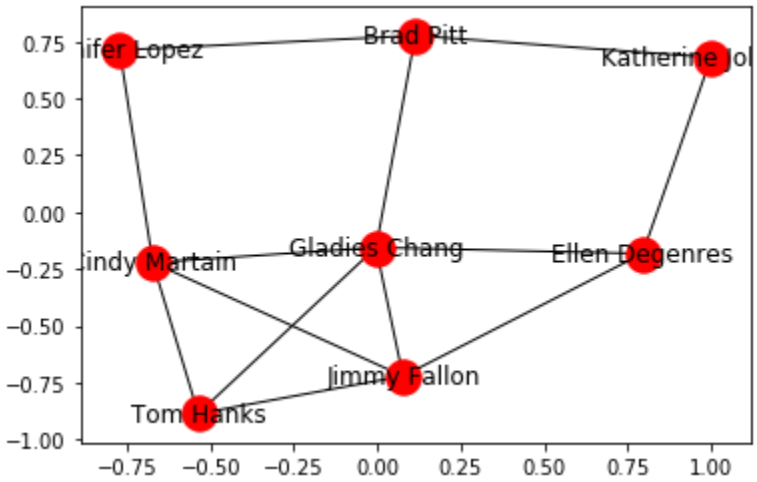
In [3]:

```
G.add_edge(1,5)
nx.draw_networkx(G)
plt.show()
```



In [4]:

```
## Visualization adjacency list -this is "my network"
import networkx as nx
G_symmetric = nx.Graph()
G_symmetric.add_edge('Cindy Martain','Jennifer Lopez')
G_symmetric.add_edge('Cindy Martain','Jimmy Fallon')
G_symmetric.add_edge('Cindy Martain','Tom Hanks')
G_symmetric.add_edge('Cindy Martain','Gladies Chang')
G_symmetric.add_edge('Ellen Degenres','Katherine Johnson')
G_symmetric.add_edge('Ellen Degenres','Katherine Johnson')
G_symmetric.add_edge('Brad Pitt','Katherine Johnson')
G_symmetric.add_edge('Katherine Johnson','Brad Pitt')
G_symmetric.add_edge('Ellen Degenres','Jimmy Fallon')
G_symmetric.add_edge('Jimmy Fallon','Gladies Chang')
G_symmetric.add_edge('Brad Pitt','Gladies Chang')
G_symmetric.add_edge('Brad Pitt','Jennifer Lopez')
G_symmetric.add_edge('Jimmy Fallon','Tom Hanks')
G_symmetric.add_edge('Tom Hanks','Gladies Chang')
G_symmetric.add_edge('Ellen Degenres','Gladies Chang')
nx.draw_networkx(G_symmetric)
```



In [10]:

```
## I check facebook dataset from website  - this is facebook network -
## I refer to datacamp site to compare with "my network"
import networkx as nx
G_fb = nx.read_edgelist("facebook_combined.txt", create_using = nx.Graph(), nodetype=i
```

In [11]:

```
print(nx.info(G_fb))
```

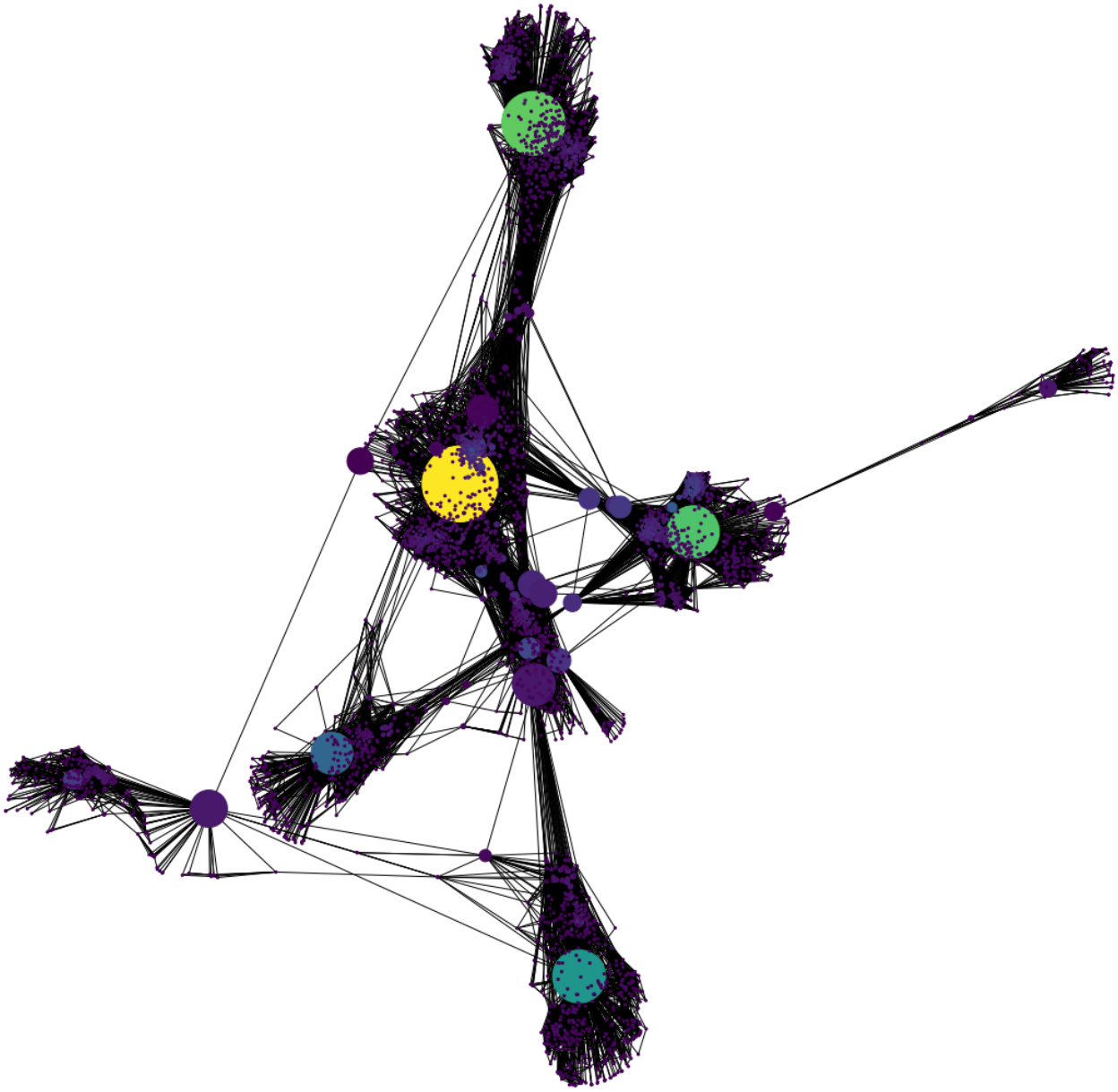
```
Name:
Type: Graph
Number of nodes: 4039
Number of edges: 88234
Average degree:  43.6910
```

In [12]:

```
from matplotlib import pyplot as plt
pos = nx.spring_layout(G_fb)
betCent = nx.betweenness_centrality(G_fb, normalized=True, endpoints=True)
node_color = [20000.0 * G_fb.degree(v) for v in G_fb]
node_size =  [v * 10000 for v in betCent.values()]
plt.figure(figsize=(20,20))
nx.draw_networkx(G_fb, pos=pos, with_labels=False,
                 node_color=node_color,
                 node_size=node_size )
plt.axis('off')
```

Out[12]:

```
(-0.9657380995161217,
 1.0946510252363366,
 -0.8358074340622605,
 0.8237482241812333)
```



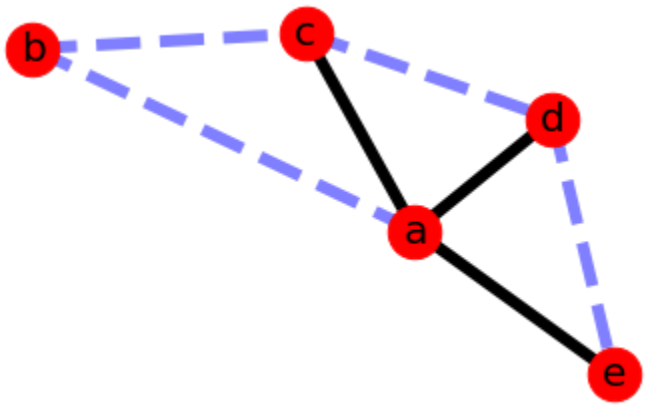
4. Python implementation to of depth-first-search (DFS) on

graphs

(5 points) Read an adjacency list/matrix representation of graph and convert it into a DiGraph object in networkx. You can find more network data here.

In [29]:

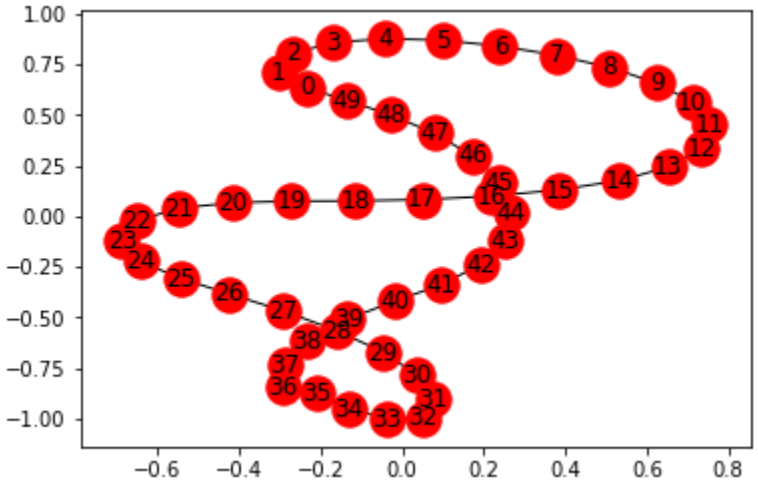
```
import networkx as nx
import matplotlib.pyplot as plt
G=nx.Graph()
G.add_edge("a", "b", weight = 1)
G.add_edge("a", "c", weight = 6)
G.add_edge("a", "d", weight = 7)
G.add_edge("b", "c", weight = 2)
G.add_edge("c", "d", weight = 3)
G.add_edge("d", "e", weight = 4)
G.add_edge("e", "a", weight = 5)
elarge = [(u, v) for (u, v, d) in G.edges(data=True) if d['weight'] > 4]
esmall = [(u, v) for (u, v, d) in G.edges(data=True) if d['weight'] <= 4]
# labels
pos = nx.spring_layout(G)  # positions for all nodes
# nodes
nx.draw_networkx_nodes(G, pos, node_size=700)
# edges
nx.draw_networkx_edges(G, pos, edgelist=elarge,
                       width=6)
nx.draw_networkx_edges(G, pos, edgelist=esmall,
                       width=6, alpha=0.5, edge_color='b', style='dashed')
nx.draw_networkx_labels(G, pos, font_size=20, font_family='sans-serif')
plt.axis('off')
plt.show()
```



In [45]:

```
## Visualization adjacency matrix
import networkx as nx
G = nx.cycle_graph(50)
A = nx.adjacency_matrix(G)
print(A.todense())
nx.draw_networkx(G)
```

```
[[0 1 0 ... 0 0 1]
 [1 0 1 ... 0 0 0]
 [0 1 0 ... 0 0 0]
 ...
 [0 0 0 ... 0 1 0]
 [0 0 0 ... 1 0 1]
 [1 0 0 ... 0 1 0]]
```



(70 points) Implement DFS and perform it on G from an arbitrary node s and allow users to make queries on the reachability from s to anywhere in G. You can return two list/arrays from your DFS(G,s) calls: visited: is a list with n (number of nodes) elements (Boolean type). EdgeTo: a list with n elements that shows the previous n Hint: Use of Stack is recommended.

In [81]:

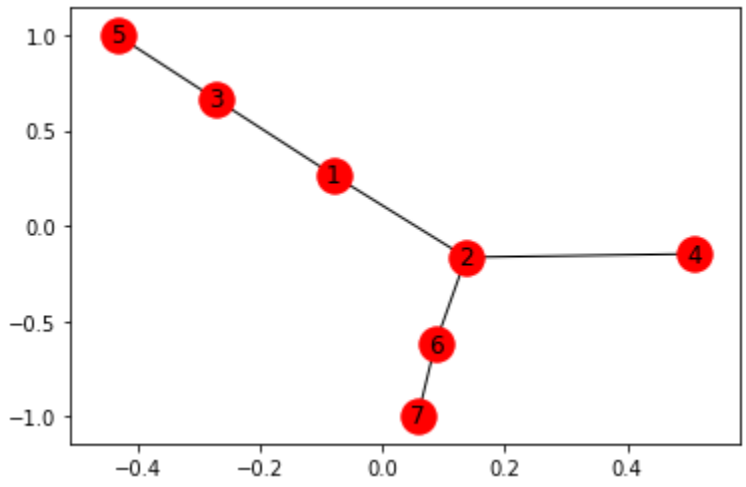
```
graph = {
    '1' : ['2','9'],
    '2' : ['1'],
    '3' : ['4','5','6','9'],
    '4' : ['3'],
    '5' : ['3','8'],
    '6' : ['3','7'],
    '7' : ['6','9'],
    '8' : ['5','7'],
    '9' : ['1','3','7']
}

def dfs(graph, vertex):
    visited = [vertex]
    stack = [vertex]
    while stack:
        vertex = stack[-1]
        if vertex not in visited:
            visited.append(vertex)
        pop_stack = True
        for next in graph[vertex]:
            if next not in visited:
                stack.extend(next)
                pop_stack = False
                break
        if pop_stack:
            stack.pop()
    return visited

print (dfs(graph, '1'))
```

```
['1', '2', '9', '3', '4', '5', '8', '7', '6']
```

(15) Build a DFS tree using EdgeTo and visualize it in the graph (different edge width and color). See this as a reference for drawing graphs.

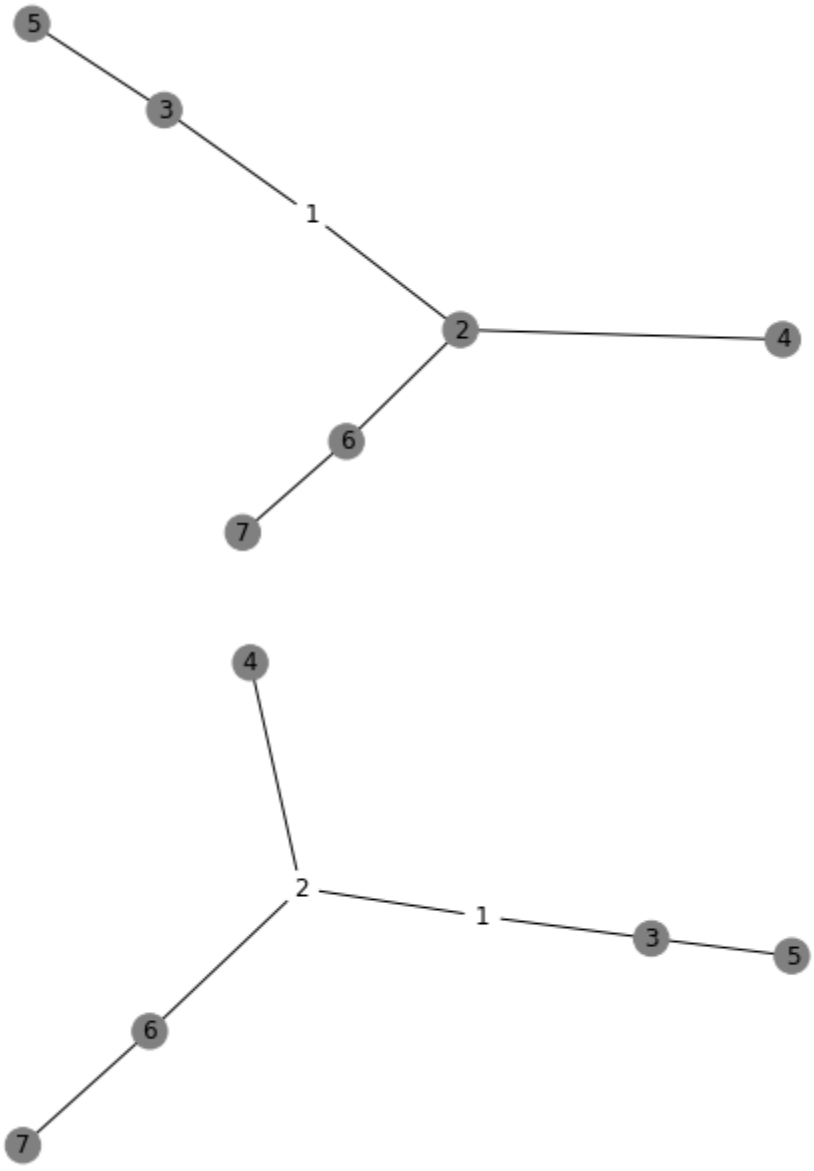
In [55]:

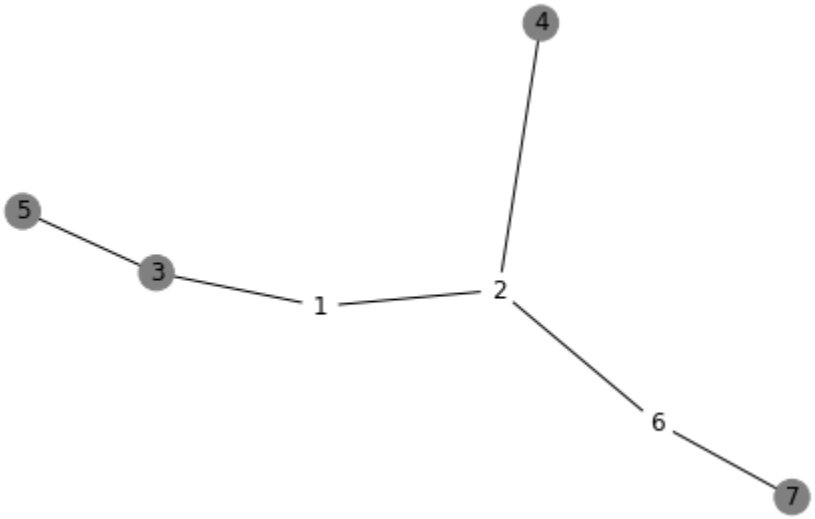
```
import networkx as nx
import matplotlib.pyplot as plt
import numpy as np
import seaborn as sns
gf=nx.Graph()

gf.add_node(1)
gf.add_node(2)
gf.add_node(3)
gf.add_node(4)
gf.add_node(5)
gf.add_node(6)
gf.add_node(7)
gf.add_edge(1,2)
gf.add_edge(1,3)
gf.add_edge(2,6)
gf.add_edge(2,4)
gf.add_edge(3,5)
gf.add_edge(6,7)
nx.draw_networkx(gf)
plt.show()
```

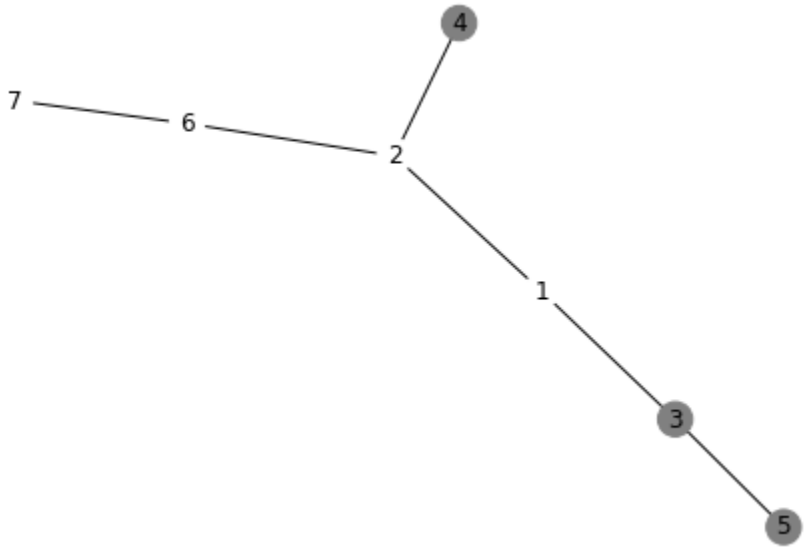


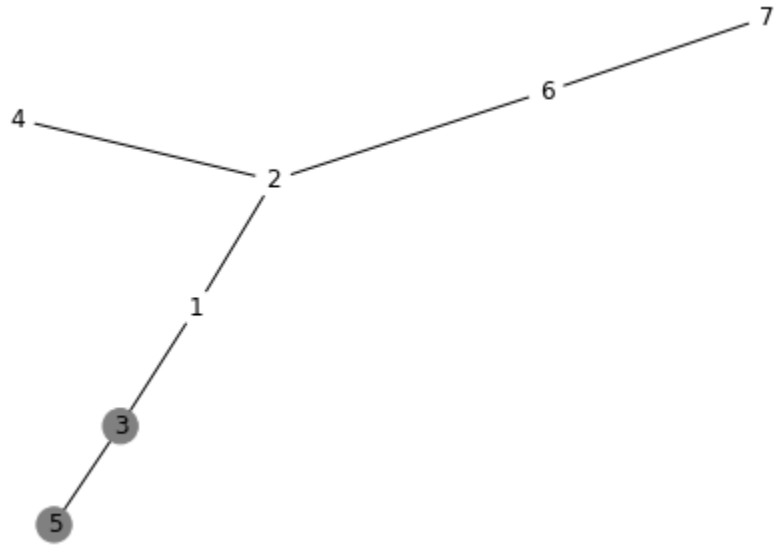
In [86]:

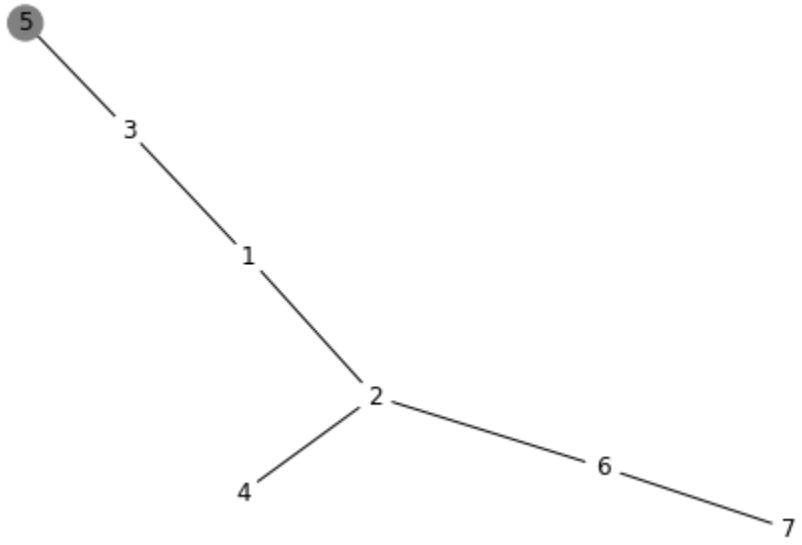
```
visited=[]
color_map=['gray']*(gf.number_of_nodes()) #Coloring them yellow.
def dfs(x):
    if(x not in visited):
        color_map[x-1]='white'
        nx.draw(gf,node_color=color_map,with_labels=True)
        plt.show()
        visited.append(x)
        all_n=nx.neighbors(gf,x)
        for x_tov in all_n:
            dfs(x_tov)
dfs(1)
```

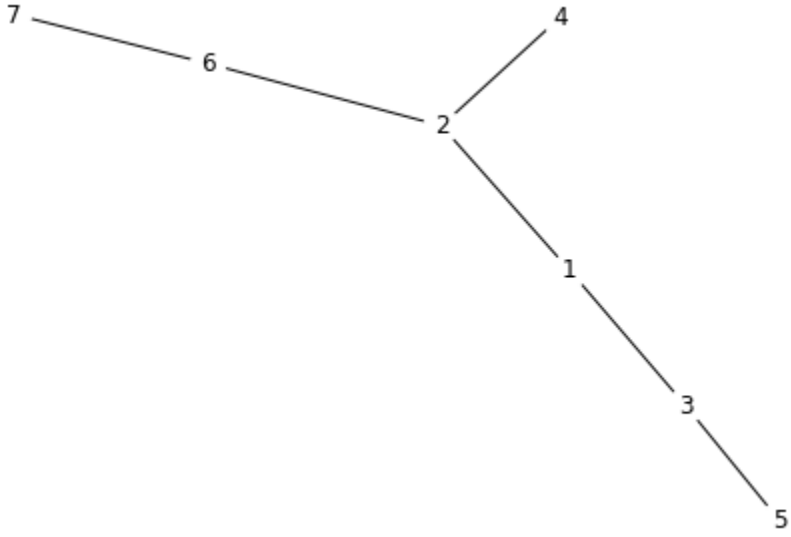



4
5
3
1
2
6
7


4
7
6
2
1
3
5


7
6
4
2
1
3
5





(10) Compare the use of loop or recursion for coding DFS. Which one do you recommend and why

In [85]:

```python
## recursive
graph = {
    '1' : ['2','9'],
    '2' : ['1'],
    '3' : ['4','5','6','9'],
    '4' : ['3'],
    '5' : ['3','8'],
    '6' : ['3','7'],
    '7' : ['6','9'],
    '8' : ['5','7'],
    '9' : ['1','3','7']
}
def dfs(graph, vertex, visited):
    if vertex not in visited:
        visited.append(vertex)
        for n in graph[vertex]:
            dfs(graph,n, visited)
    return visited

visited = dfs(graph,'1', [])
print(visited)
```

```
['1', '2', '9', '3', '4', '5', '8', '7', '6']
```

```
I would like to recommend recursive one, because the code is more succinct and easy to read.
```