An Investigation into the Learnings of Social Constructivism

My interest in social constructivism (which began with our discussion of Lakatos and how mathematical proofs are not fully understood without regarding their historical context) aims to determine the extent of objectivity in math, the purpose of social constructivism, and its applications. In his article "Aphorism and Critical Mathematics Education" which delineates the problems within scientific rationality (rational constructivism), Skovsmose writes that modern authors' (including Lakatos' and Wittgenstein's) works on social constructivism are "internalist" and ignore the critical social function of mathematics[1]. His conception of social constructivism is that it should not just account for math and math knowledge naturalistically[2]; it needs to support students' education not only in the construction of 'proper' mathematical knowledge but also in challenging the socially acceptable paradigm of optimism in science which stems from the internalist conception of mathematics (rational constructivism)[3].

Furthermore, I believe that it would be helpful to define social constructivism – in its most basic terms (as defined by the subjectivist Ernest) mathematics is a social construct, and we cannot ignore the role which linguistics, interpersonal processes (which turn subjective discoveries into widely-accepted objective knowledge) and social objectivity play in mathematical knowledge[4]. Specifically, Ernest extends the importance

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[1] Skovsmose, *Aphorism and Critical Mathematics Education,* p. 7.

[2] Ibid., p. 7.

[3] Ibid., p. 7.

[4] Sriraman, *Preface to Part II Ernest's Reflections on Theories of Learning,* p. 36.

of Lakatos' philosophical position – that is, math is fallible and socially constructed – to the realm of education by arguing that teachers and students should engage in more dialogue[5]. An interesting topic of note also in this article is the subject of Ramanujan who seemingly came up with profound mathematical knowledge with no socially constructed basis.

Lakatos takes several crucial steps to address the problem of mathematical certainty; the dogmatists would continue to assert that we can achieve some probabilistic degree of certainty in any theorem, but this claim seems to be a fallback on the notion of redefining certainty as exemplified in Lakatos' book. There's also another article about this topic (written by Paul Ernest) which strikes me as important because it addresses the concept of familiarity mentioned in class. From Ernest's perspective, mathematical calculations like those we see in calculus basically require invariance (and hence a false sense of certainty) in order to achieve their social goals[6] – mathematical objects such as numbers, functions and even group theory have been so static and we have become so familiar with them over the centuries that they only seem to be certain.

In reality, we see from Ernest (as a key figure in social constructivism) that mathematics smoothens uncertainty and appropriates concepts which are troubling and may seem contradictory via the method of proof and the way mathematics continues to be taught in schools[7]. Furthermore, a lot of mathematical problems become nonsensical when we look at them closely. We talked about this in class extensively, with the picture

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[5] Sriraman, p. 36.

[6] Ernest, *The problem of certainty in mathematics,* p. 391.

[7] Ibid., p. 391.

frame and tunnels and other figures which reduce the definition of polyhedra to absurdity. The boundaries of mathematical certainty are always fuzzy – we as human beings have to engage in continuous dialogue in order to achieve something close to certainty (as the ancient philosophers Plato and Aristotle already knew). We concluded that mathematical truth is not absolute but is only necessarily true because the entire foundation of certain branches of math such as set theory rely on hidden assumptions including the vast nature of infinity. As Ernest says, we don't have any proof-based warrant for the axioms and beginning postulates, we can never achieve full rigorousness within the boundary of professionalism and many theories such as Gödel's incompleteness theorem demonstrate most of math is unprovable (proof and proof-analysis are limited); the arrival of intuitionism is where finite certainty breaks down in math[8]. As you said, looking at your bag and saying it is not black is not the same as finding a black raven, but logically they are equivalent which does not make much sense – we could just as easily substitute the word "raven" for any other imaginary object and come to similar unmeaningful results.

The topic of logical positivism is addressed extensively in *Proofs and Refutations* – Lakatos acknowledges that some problems about a theory can only be approached after it has been formalized, yet this formalization is analogous to death in the biological analysis of bodies[9]. Furthermore, the dogmatist/skeptic ideologies contrasted in Lakatos (that we can attain or cannot attain truth) are exemplified in various means to demonstrate that the history of metamathematics invalidates modern mathematical dogmatism[10].

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[8] Ernest, p. 389.

[9] Lakatos, *Proofs and Refutations,* p. 3.

[10] Ibid., p. 4-5.

In fact, the topic of the principle of the excluded third (which is discussed by Brouwer) is an example of a seemingly intuition-based and logical statement which can be refuted. Consider the notion of a drift – that is, given the union Y of a converging sequence of counting reals $c_1(Y)$, $c_2(Y)$, … (where if all elements of the sequence < the kernel $c(Y)$ the drift is left-winged and if they're all > $c(Y)$ the drift is right-winged)[11] we can show that if Y is right-winged (where counting-numbers are all rational and serve as indices for untested mathematical conjectures) that we cannot judge for certain whether or not the sequence converges to a rational number and so cannot judge the truth of the conjecture[12]. What Brouwer is basically saying is that the principle of the excluded third when expressed on the number line implies that either classical mathematics is false or that we need to reevaluate the either-or assumption which it makes. Brouwer addresses the crisis in mathematics as an acknowledgement that even the most basic assumptions we make are socially ingrained. Furthermore, computer-based proof generators remove the suspension of disbelief which is required for proofs to carry the warrant of truth.

The question addressed by Lakatos – is there a relation between the number of vertices V, the number of edges E and the number of faces F of polyhedra that is analogous to the trivial relation $V = E$?[13] – is crucial because the methodology used to arrive at conjectures comes into scrutiny. It is noted that after trial and error[14] they notice that $V - E + F = 2$ (the characteristic polynomial as mathematicians would say). The

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[11] Brouwer, *Consciousness, philosophy, and mathematics,* p. 93.

[12] Ibid., p. 94.

[13] Lakatos, p. 6.

[14] Ibid., p. 6.

question of course becomes, by which trial and error did this occur? It's noted that Pólya among others believed that you have to guess a mathematical theorem before you prove it[15]; that is, the result precedes the investigation. How can we account for results such as the Descartes-Euler conjecture or Ramanujan's profound theorems (which lack a socially constructed basis)? As Lakatos demonstrates, we account for them after the fact which means that mutual familiarity with concepts plays a crucial role. Throughout his fictional story (which involves a dialogue in an imaginary classroom between teachers and students) we uncover concept-contraction (from Delta) and concept expansion (from Alpha) in the discussion of the picture-frame[16]. The dialogue devolves into a debate over what constitutes a genuine polyhedron; concept-stretching, monster-barring and local/ global counterexamples become the focus of discussion.

In fact, Poincare in 1908 stated specifically that logic (and not just its slumber) produces monsters. To paraphrase, he says that for half a century now they've seen a host of bizarre functions arise which serve no purpose and make the most general claims; these functions are made explicitly to demonstrate the shortcomings of the reasonings of the old mathematicians[17]. Specifically, in the context of Lakatos we learn about proof-analysis without proof – an example of this would be developing the notion that all polyhedra have at least 17 edges. These are just *ad hoc* conjectures; they are made up on the spot arbitrarily and their only purpose and meaning is to use the internal, intrinsic

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[15] Ibid., p. 10.

[16] Lakatos, p. 23.

[17] Ibid., p. 25.

structure of the theorem and method of proof-analysis to justify a theorem[18]; any number

of misconceptions on part of the inventor can be made explicit and thus, by granting

generous charity to the progenitor of the proof we can justify anything.

Furthermore, the debate over history continues in Lakatos; Sigma argues that the

only thing making inductive conjectures possible is historical accident[19]; real chaos

obliterates any preconceived notion of what, for instance a polyhedron is. Because there

is no inductive basis for naïve conjectures as Teacher says but rather trial and error

through conjectures and refutations[20], Euler's theorem was thus based on a very biased

pool of polyhedra. This of course is analogous to the scientific method wherein

hypotheses are formed and we test and refute them. But even Lakatos' deductive

guessing has to start on some kind of basis, and it is clear that this basis is social. The

entirety of Euler's polyhedral theorem seemingly rests on the laurels of its historical

context and Euler's credibility. Lakatos alludes to ancient informal logic (this is the logic

of construction or thought-experiment); philosophers such as Descartes, Kant and

Poincaré extolled the virtues of content-rich informal logic. The later changes in logical

theory come from changes in logical practice[21]. What this means is that originally, broad-

sweeping claims such as Euler's were viewed as very good and powerful by merit of their

ability to understand the world. They were assumed to be true on little actual merit but

merely as the simplest explanation of an occurrence.
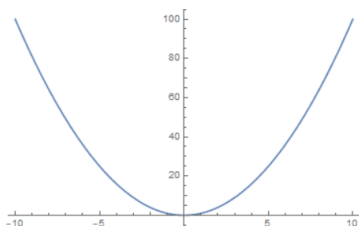
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[18] Ibid., p. 53.

[19] Ibid., p. 77.

[20] Lakatos, p. 78.

[21] Ibid., p. 87.

More specifically, the Descartes-Euler conjecture is subjectively true for its time; as Lakatos describes, their conception of polyhedron included all sorts of convex polyhedral and even concave ones[22]. As we can see, mathematics evolves over time. The mathematician Reuben Hersh discussed why social constructivism provides a more accurate view of mathematics as opposed to the traditional Platonist view of math (in which pure entities exist in a different dimension or realm independent from our physically or psychologically perceived reality). The flaw in Platonism basically amounts to, how can we have knowledge of such a realm even if it exists[23]? As in Lakatos, we do not have a concrete sense of familiarity with what a polyhedron truly is.

For the social subjectivist Hersh, an object is constructed by the mathematical community and then takes on a livelihood of its own. For Ernest on the other hand, functions like x^2 could become odd at any time because oddness $(f(-x) \neq f(x))$ is only true for the present socially defined state of mathematics[24]. Of course, the cyclic subgroups of $Z/6Z$ all have even order as well – we have $\langle 0 \rangle = \{0\}$, $\langle 1 \rangle = \{1,2,3,4,5,6 \bmod 6 = 0\}$, $\langle 2 \rangle = \{2,4,0\}$ and $\langle 3 \rangle = \{3,0\}$. Clearly, each order (1, 6, 3 and 2) is not equal to 5. Yet Ernest's point is equally valid in my opinion because it alludes to the fact that subgroups (subsets which are also groups) and their generators are defined socially by consensus of the mathematical community. We could just as easily say that the cyclic subgroup $1 = \{1,2,3,4,5\}$ excluding 0 because 6 mod 6 is

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[22] Ibid., p. 89.

[23] Gold, *Social Constructivism as a Philosophy of Mathematics,* p. 376.

[24] Ibid., p. 377.

a separate entity from 0 and is not logically equivalent. It seems obvious to me that mathematics is a human invention designed to find patterns in the natural world but is not entirely perfect. Intuition depends on our historical interpretation of concepts.

Furthermore, Gold concedes that mathematics is *discovered* by humans[25]. I disagree with this, believing that it is entirely a human invention and that the rules of logic change over time. Consider the classical logical principle wherein if P is false and Q is true, then the statement $P \Rightarrow Q$ is always true. Certainly this is true based on an overwhelming majority consensus; I once spoke to Krusemeyer regarding this matter and he told me that I would be disagreeing with the entire mathematical community if I did not believe this statement. Given P false, Q true we can say that P implies Q. But what if when P is true, Q turns out to be false? That is, Q being true depends explicitly on the truth value of P. In that case, the truth value of P does matter and $P \Rightarrow Q$ is not always true whenever Q is true. Similar questions arise for another statement (P false, Q false implies $P \Rightarrow Q$).

Essentially, there are several leaps of faith that one has to make when studying logic. For instance, another topic in set theory would be induction. Certainly, numerical induction is valid yet still requires us to assume that because it is true for the base case that it is true for all n. Experimental induction as in the case of Lakatos' polyhedra proves to be nigh impossible and demonstrates that our vague historical conception of polyhedra is rooted in its applicability to the world rather than in its truth. As I discussed

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[25] Ibid., p. 380.

earlier, the extent to which we are willing to draw inferences from a handful of socially-biased examples is highly dependent on historical circumstances.
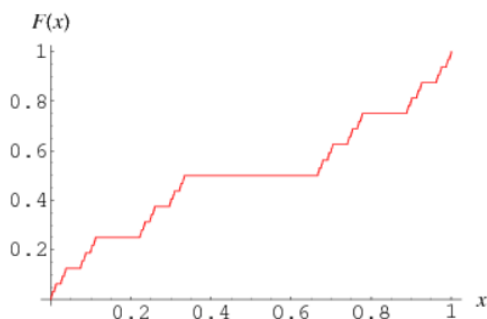
For one final example, consider the Cantor function (graphically representing the Cantor set constructed by recursively removing middle thirds; flat portions on the function represent removals). It's clear that somewhere within the remaining dust we achieve non-absolute continuity[26]; the function's derivative is always 0 but it continues to increase and so demonstrates our limited knowledge of integrals against Cantorian infinity. In order to maintain depth and progression in mathematics, it is crucial that we do not let ideas be certain. Math as a human invention is an excellent but fallible, socially-constructed system for understanding the world.

[26] Weisstein, Eric W. "Cantor Function." From MathWorld.

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What the Paper Does Well

1. The paper explores some interesting social constructivist themes. 2. It's great that you've done some independent research and found some relevant sources. 3. At the level of sentence and paragraph, the writing is generally clear. Moreover, within the span of a single paragraph, it's usually clear what point the paper is trying to make.

Ways to Improve the Paper

1. The paper has no clear overall structure. It needs something like an introduction and a section structure that is made clear by the end of the introductory remarks. 2. The paper doesn't have a clear thesis. I *think* I know what the thesis is at least approximately: namely, the social constructivist view you attribute to Ernest. But this thesis is pretty vague, and you don't explicitly endorse it as your own view. 3. As it is currently written, the paper is really sprawling, with some parts that I think help to give evidence in favor of a social constructivist thesis, and a number of parts that don't help. In the comments I've indicated good examples of places where I think you need to expand and go into more detail, and other places where you could just cut whole paragraphs without loss. Remember that a paper like this is meant to be a reasoned defense of a claim. This should guide what you expand, and what you cut. 4. I don't think I really understand the role Lakatos is meant to play in this paper. At the beginning, he's presented more as a foe of social constructivism for being too internalist (and I don't know what that means, by the way). Later on, he's presented as a source of evidence for social

constructivism, or a proponent of it. It would help to be clear about what role you think Lakatos is playing in your argument (vs. Ernest's argument, for instance).

Grade: B/B+. Good work!