3.1.2. Verify equation (3.4).

Specifically, equation (3.4) states that $\prod_{i=1}^{m} \frac{1-q^{n+i}}{1-q^i} = \prod_{i=1}^{m} \frac{1-q^{n-1+i}}{1-q^i} + q^n \prod_{i=1}^{m-1} \frac{1-q^{n+i}}{1-q^i}$

$$\left(\frac{1-q^{n+1}}{1-q^1}\right)\left(\frac{1-q^{n+2}}{1-q^2}\right)\cdots\left(\frac{1-q^{n+m}}{1-q^m}\right) = \left(\frac{1-q^{n-1+1}}{1-q^1}\right)\left(\frac{1-q^{n-1+2}}{1-q^2}\right)\cdots\left(\frac{1-q^{n-1+m}}{1-q^m}\right) +$$

$$q^n \cdot \left(\frac{1-q^{n+1}}{1-q^1}\right)\left(\frac{1-q^{n+2}}{1-q^2}\right)\cdots\left(\frac{1-q^{n+m-1}}{1-q^{m-1}}\right)$$ expand

RHS

$$= \left(\frac{1-q^n}{1-q}\right)\left(\frac{1-q^{n+1}}{1-q^2}\right)\cdots\left(\frac{1-q^{n+m-2}}{1-q^{m-1}}\right)\left(\frac{1-q^{n+m-1}}{1-q^m}\right) +$$

$$q^n\left(\frac{1-q^{n+1}}{1-q}\right)\left(\frac{1-q^{n+2}}{1-q^2}\right)\cdots\left(\frac{1-q^{n+m-1}}{1-q^{m-1}}\right)$$

$$= \frac{(1-q^n)(1-q^{n+1})\cdots(1-q^{n+m-2})(1-q^{n+m-1})}{(1-q)(1-q^2)\cdots(1-q^{m-1})(1-q^m)} + \frac{q^n(1-q^{n+1})(1-q^{n+2})\cdots(1-q^{n+m-1})}{(1-q)(1-q^2)\cdots(1-q^{m-1})}$$

$$= \frac{(1-q^n)(1-q^{n+1})\cdots(1-q^{n+m-2})(1-q^{n+m-1})}{(1-q)(1-q^2)\cdots(1-q^{m-1})(1-q^m)} + \frac{q^n(1-q^{n+1})(1-q^{n+2})\cdots(1-q^{n+m-1})(1-q^m)}{(1-q)(1-q^2)\cdots(1-q^{m-1})(1-q^m)}$$

$$= \frac{(1-q^n)(1-q^{n+1})\cdots(1-q^{n+m-2})(1-q^{n+m-1}) + q^n(1-q^{n+1})(1-q^{n+2})\cdots(1-q^{n+m-1})(1-q^m)}{(1-q)(1-q^2)\cdots(1-q^{m-1})(1-q^m)}$$

So we have established that LHS = ... = RHS :

$$\frac{(1-q^{n+1})(1-q^{n+2})\cdots(1-q^{n+m})}{(1-q^1)(1-q^2)\cdots(1-q^m)} = \frac{(1-q^n)(1-q^{n+1})\cdots(1-q^{n+m-1}) + q^n(1-q^{n+1})(1-q^{n+2})\cdots(1-q^{n+m-1})(1-q^m)}{(1-q)(1-q^2)\cdots(1-q^m)}.$$