Conjecture 8: The number of descending plane partitions with largest part less than or equal to r and for which r appears as a part exactly $k-1$ times is equal to the number of $r \times r$ alternating sign matrices with a 1 in the kth column of the first row (page 24).

6.1.2 Prove the following special case of Conjecture 10: The number of permutations $\sigma \in S_n$ for which $\sigma(1) = k$ and $I(\sigma) = p$ is equal to the number of descending plane partitions in $B(n,n,n)$ with exactly $k-1$ parts of size n, no special parts (the entry in position (i,j) must be strictly greater than $j-i$), and a total of p parts.

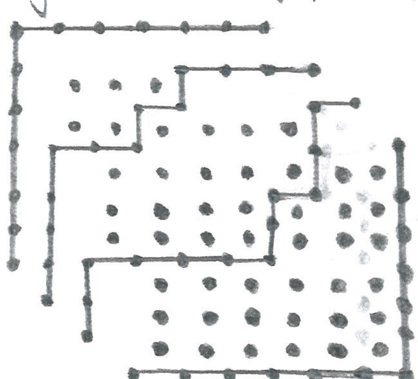
Let's say we are given a modified version of the descending plane partition from page 195 such that it no longer has special parts and it has exactly 7-1 parts of size 7:

7	6	6	6	6	6
	5	5	4	4	
		3	2		

This corresponds to the sequence $(1,5,2,2,1,1)$, called the inversion word, which is generated as $(a_1, a_2, \ldots, a_{n-1})$ where $n-1$ is the length of the top row and a_i = the number of parts in the descending plane partition with value $n-i+1$,

so $(1,5,2,2,1,1)$ = (number of 7s, number of 6s, ..., number of 2s).

Visualizing this descending plane partition as a set of non-intersecting lattice paths, it is



(0,0)

clear that along any path, the number of horizontal steps is always $\leq$ the height above 0 at that point.

This is only true by the fact that there are no special parts, iff the entry (shown by horizontal steps) in position (i,j) on the partition is $> j-i$.

This $\Rightarrow$ for $(a_1, a_2, \ldots, a_{n-1})$, $a_i \leq n-i$ always and so the sequence is by definition an inversion word for which, for $\sigma \in S_n$, a_i counts the number of inversions (i,j) with $\sigma(j) = i$, $i = 1, 2, \ldots, n-1$ and $a_1 + \cdots + a_{n-1} = I(\sigma)$, the number of inversions in σ.

Specifically, a_i is the number of elements to the left of i in the permutation $\sigma \in S_n$ which are $> i$.

In our example, the inversion word $(1,5,\underline{2},2,1,1)$ corresponds to the permutation $7153462 \in S_7$ because we have 1 element to the left of 1 which >1, 5 elements to the left of 2 which is >2, $\underline{2}$ elements to the left of 3 which are >3, etc. Here, $\sigma(1) = 7$ and $I(\sigma) = 12$.