Time Spent: 2 hours

Collaborators and Resources:

Problem 3

For Problem 3, instead of matching n men with n women we're matching celebrities to professional dancers. So I wanted to simplify the problem - instead of trying to find the exact preferences of each celebrity (which we don't have enough information to do because we can't actually run the Gale-Shapley algorithm) I could just find which matchings they are going to prefer (where preferences are determined entirely by whether the celebrities get their optimal dancers or not) based on the previous results we have proved in class . . .

Claim 1. Every celebrity in C has the same preference ordering as Nikki (M_1, M_2 and M_3)

Proof. This problem has the exact same characteristics as the standard stable marriage problem except for the lack of proposals; if we were to run Gale-Shapley (if the n celebrities in C proposed to the n professional dancers in P), we would be able to find one of the three stable matchings.

Specifically, we would find a stable matching in which the celebrities get their best valid (in a stable matching) partner. This is because the very first celebrity c to be rejected by p = best(c) is rejected when c' proposes to p and p prefers c'. Since c is the first to be rejected, c' hasn't been rejected before and so p = best(c'). What we end up with is an unstable matching - (c, p) is in a stable matching by definition of best valid partner and so (c', p) is an instability for that matching because c' prefers p while p prefers c'.

Essentially the whole point of this proof is to show that G-S always returns S* = {(c, best(c)) | c ∈ C}, and as a result S* is among the three stable matchings indicated. Furthermore, this is guaranteed to be a stable matching and if the n professional dancers were the proposers instead we would find a second matching S_2* = {(p, best(p)) | p ∈ P} = {(c, worst(c)) | c ∈ C}. Thus, we know that in one of the stable matchings each celebrity gets their best valid (out of all stable matchings) pick and in another, each celebrity gets their worst valid option.

So if Nikki prefers M_1 because it pairs her with her best valid dancer, all other celebrities are going to prefer M_1 because in it they will all get their best valid dancer. Similarly, if Nikki prefers M_3 the least then all celebrities will prefer M_3 the least as well because they get their worst option - as a result they will all choose M_2 as their middle preference, leading us to the result that every celebrity prefers M_1 to M_2 to M_3 ... $\square$