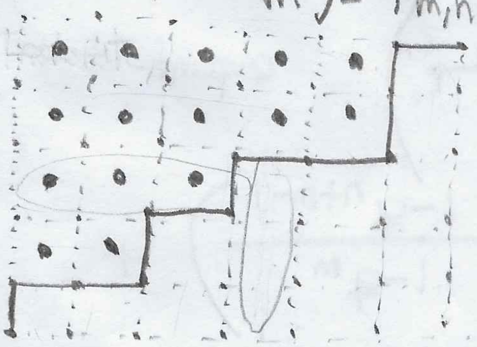
Exercises

3.1.1. Find a one-to-one correspondence (bijection) between partitions into at most m parts, each less than or equal to n, and partitions into at most n parts, each less than or equal to m. This proves combinatorially that

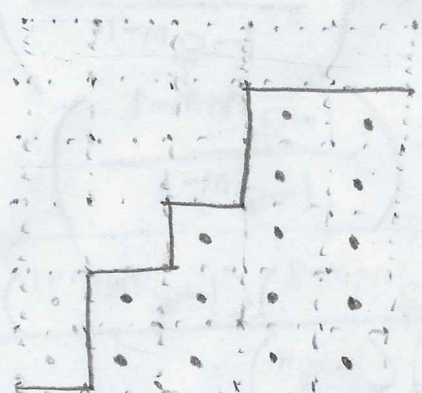
$$\begin{bmatrix} m+n \\ m \end{bmatrix} = \begin{bmatrix} m+n \\ n \end{bmatrix}.$$

So Proposition 3.1 says, the total number of partitions into at most m parts with each part less than or equal to n is equal to $\binom{m+n}{m}$.

Furthermore, $\binom{m+n}{m} = f_{m,n}(1)$, where $f_{m,n}(q) = \begin{bmatrix} m+n \\ m \end{bmatrix}_q = \begin{bmatrix} m+n \\ m \end{bmatrix}$ by notation.



From the book: Lattice paths from (0,0) to (6,5) produce partitions with at most five parts, and each part will be less than or equal to six.



$5+5+3+2 = 15$

$5+3+3+2+1+1 = 15$
(this isn't always the case)

Set $l = m$. // m is the height.
For each part x in our original partition:
For integers i s.t. $x < i \leq n$:
l −= # of parts of length i.

$(n, m) = (6, 3)$.

Bijection: Given a list $p_{old} = [p_1, p_2, \ldots]$ of parts:
Store $p_{new} = p_{old}$
For each part $p_i \in p_{new}$:
Set int $l = m$. // m is the height of the original graph //
For i in (x+1):n :
l −= # of parts of length i in p_{old}.
Set $p_i = l$
Return p_{new}.

$6+3+2$
$\rightarrow 2+2+2+1+0+0$