3.1.3. Given that $f(q,m)=(1-q^{m-1})f(q,m-2)$,
Let $m=1$. $f(q,m)=1-\left[{1 \atop 1}\right]=1-1=0$ in that case.
$f(q,m)=(1-q^{m-1})\cdot f(q,m-2)=(1-q^{m-1})\cdot 0=0$ if we accept the inductive hypothesis,
so $\underline{f(q,m)=0 \text{ if } m \text{ is odd.}}$

Let $m=2$. $f(q,2)=1-\left[{2 \atop 1}\right]+\left[{2 \atop 2}\right]=1-\left[{2 \atop 1}\right]+1=-\left[{2 \atop 1}\right]+2$

$-\left[{2 \atop 1}\right]=-\left(\left[{2-1 \atop 1}\right]+q^{2-1}\left[{2-1 \atop 1-1}\right]\right)=-\left[{1 \atop 1}\right]-q\left[{1 \atop 0}\right]=-1-q$, so
$f(q,2)=-1-q+2=(1-q)$.

$f(q,m)=(1-q^{m-1})\cdot f(q,m-2)=(1-q^{m-1})\cdot\left((1-q)(1-q^3)\cdots(1-q^{m-2-1})\right)$
by the inductive hypothesis, so
$f(q,m)=(1-q)(1-q^3)\cdots(1-q^{m-1})$, if m is even.

3.1.4. Let k be odd. Slow that $\dfrac{1-\alpha^{k-j}}{1-\alpha^j}=-\alpha^{-j}$.

Use this to prove that $\left[{k-1 \atop j}\right]_\alpha=(-1)^j\alpha^{-j(j+1)/2}$,

and therefore $f(\alpha,k-1)=\sum_{j=0}^{k-1}\alpha^{-j(j+1)/2}$.

Let k be odd. $\alpha \doteq e^{2\pi i h/k}$ because this is the general primitive kth root of unity.

$$\frac{1-\alpha^{k-j}}{1-\alpha^j}=\frac{1-(e^{2\pi i h/k})^{k-j}}{1-(e^{2\pi i h/k})^j}=\frac{1-e^{2\pi i h-2\pi i h j/k}}{1-e^{2\pi i h j/k}}$$

$$=\frac{1-e^{-2\pi i h j/k}e^{2\pi i h}}{1-e^{2\pi i h j/k}}=\frac{-e^{-2\pi i h j/k}\cdot(e^{2\pi i h}-e^{2\pi i h j/k})}{1-e^{2\pi i h j/k}}$$

$$=\frac{-e^{-2\pi i h j/k}(1-e^{2\pi i h j/k})}{(1-e^{2\pi i h j/k})}$$ because the primitive 1st root of unity is always 1

$$=-(e^{2\pi i h/k})^{-j}=\underline{-\alpha^{-j}}$$