University of Glasgow | School of Physics & Astronomy

IGR

LISA Analysis Tools Workshop

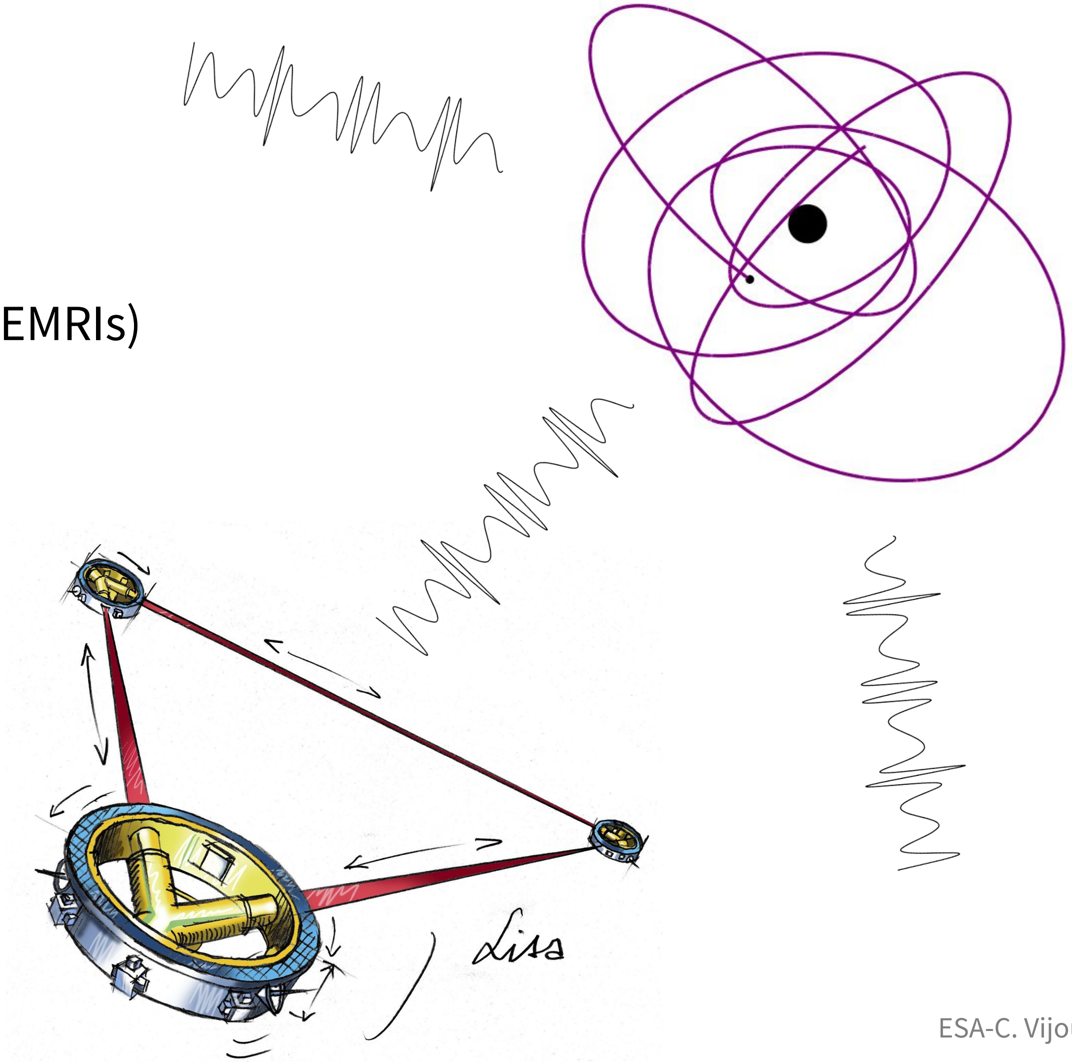
Day 1 - Afternoon Session

EMRI waveforms and the LISA response function

Christian Chapman-Bird
Institute for Gravitational Research, University of Glasgow

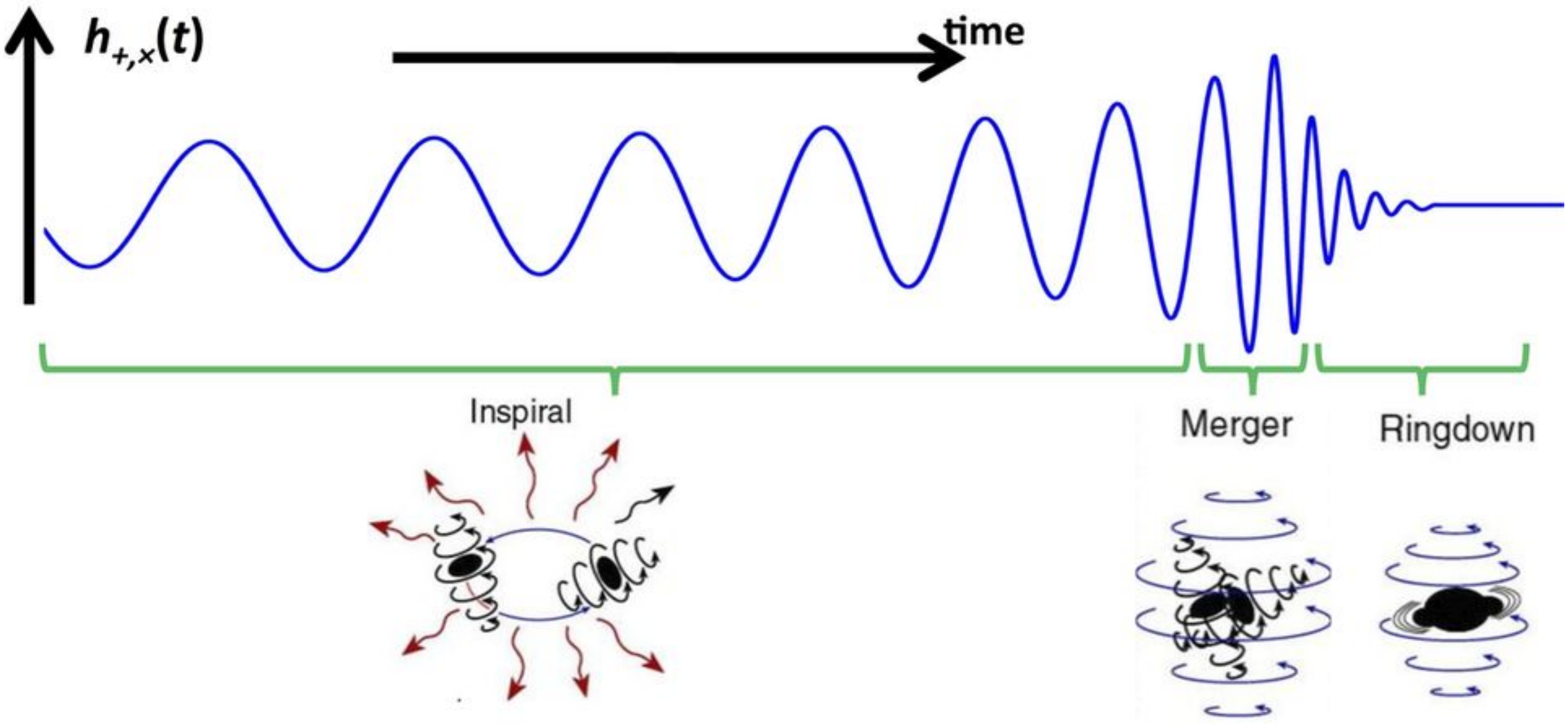
Outline

- Extreme-mass-ratio inspirals (EMRIs)
- FastEMRIWaveforms (**`few`**)
- The LISA response function
- **`fastlisaresponse`**



ESA-C. Vijoux

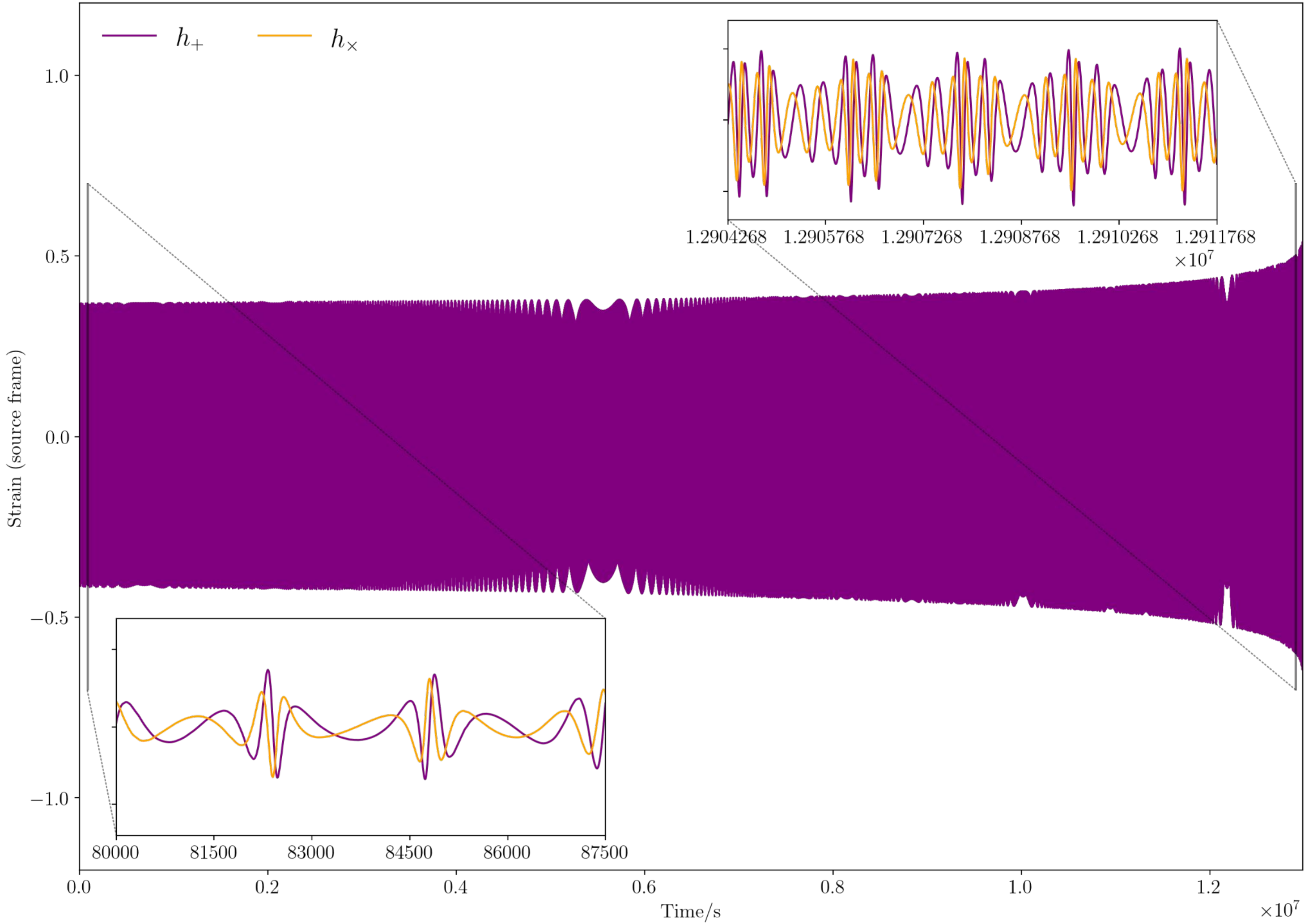
Compact binary coalescences



M. Favata/SXS/K. Thorne

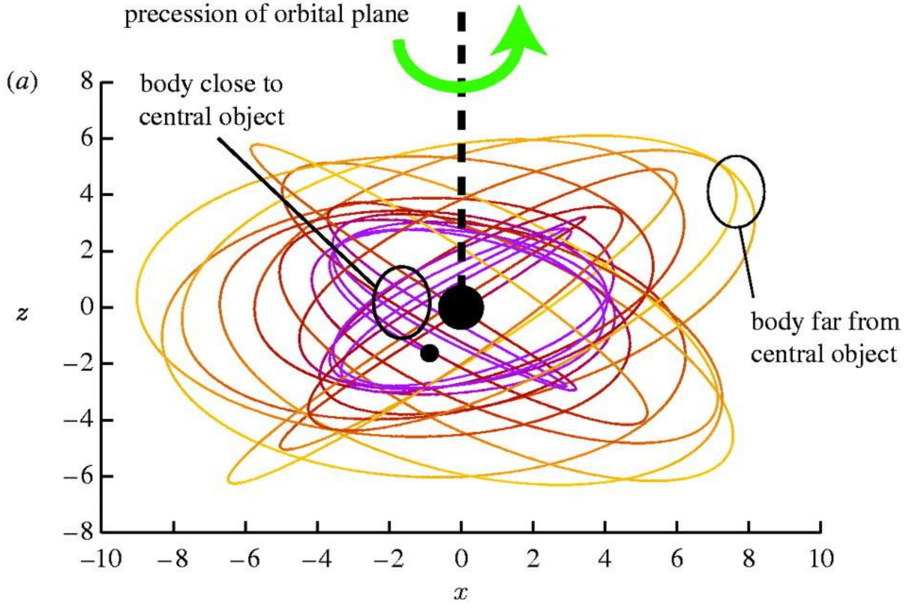
Extreme-mass-ratio inspirals (EMRIs)

- Compact binary coalescence: massive black hole (MBH) + compact object (CO)
- Characterised by mass ratio $q \equiv \mu/M \lesssim 10^{-4}$, two main consequences:
  - CO completes $\mathcal{O}(10^5)$ orbital cycles **in the LISA band** before merger
    - ➢ Highly sensitive to initial conditions
  - Orbits retain significant **eccentricity** and **orbital inclination**
    - ➢ Waveform contains many higher-order modes



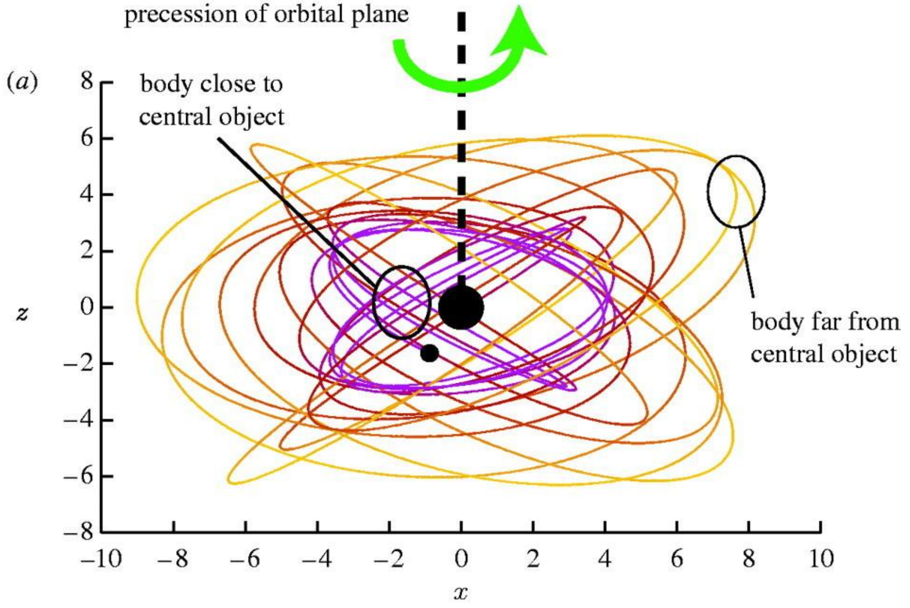
h_+
$h_\times$
1.0
0.5
0.0
−0.5
−1.0
Strain (source frame)
1.2904268
1.2905768
1.2907268
1.2908768
1.2910268
1.2911768
×10^7
80000
81500
83000
84500
86000
87500
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Time/s
×10^7

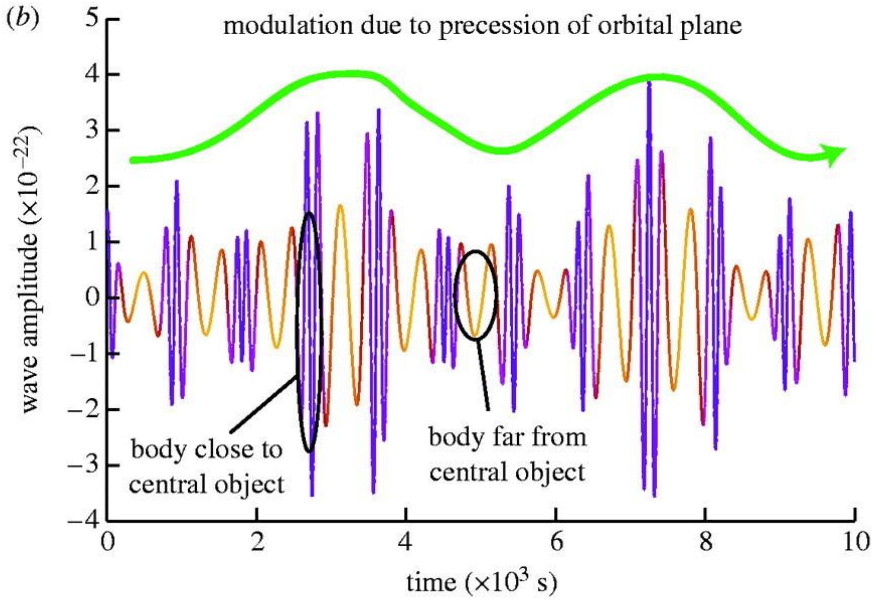
We can identify features in the inspiral trajectory….



Gair (2008) Phil. Trans. R. Soc.

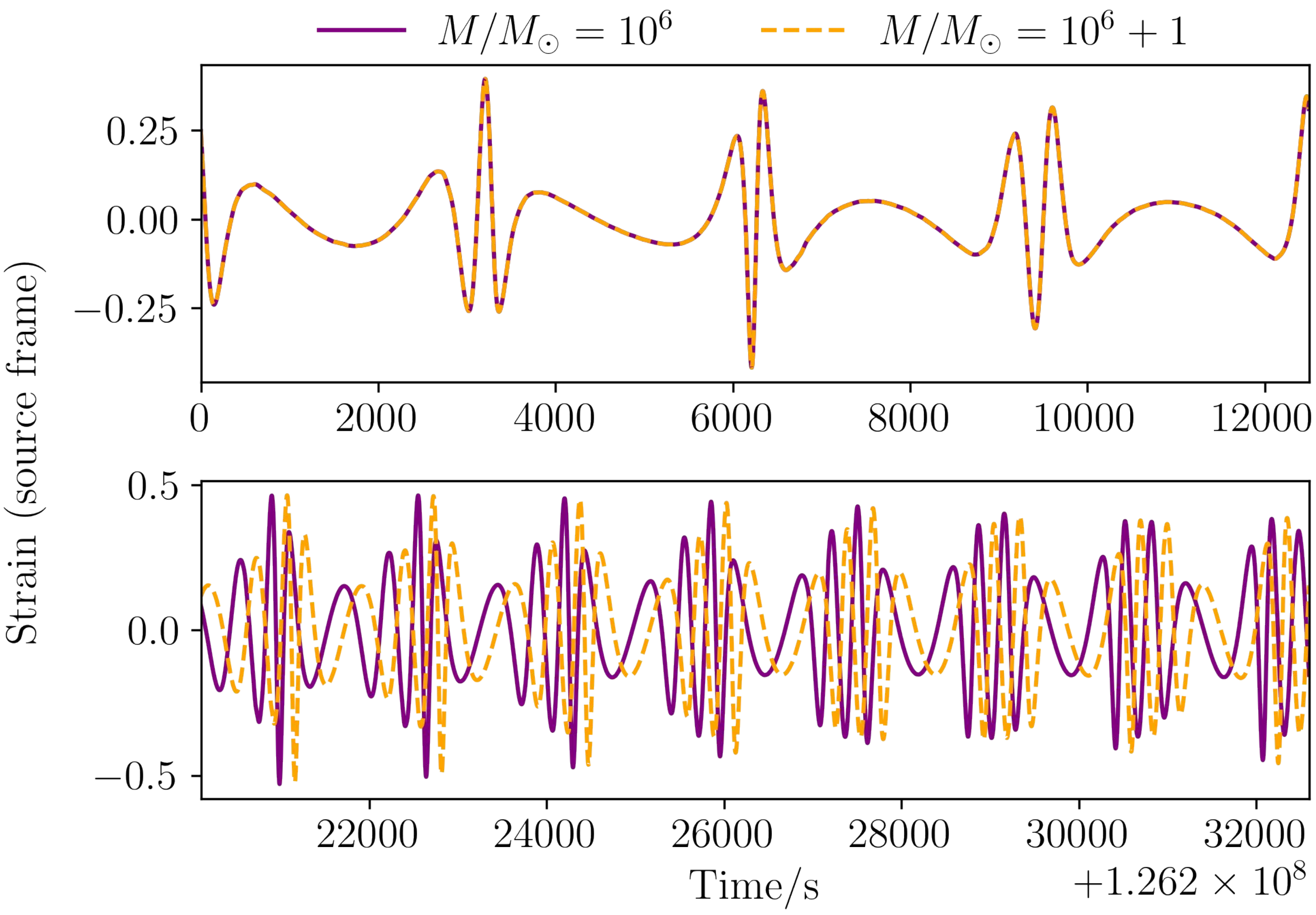
We can identify features in the inspiral trajectory….





… and their impact on the shape of the waveform.

Gair (2008) Phil. Trans. R. Soc.



$M/M_\odot = 10^6$
$M/M_\odot = 10^6 + 1$
Strain (source frame)
0.25
0.00
−0.25
0
2000
4000
6000
8000
10000
12000
0.5
0.0
−0.5
22000
24000
26000
28000
30000
32000
Time/s
$+1.262 \times 10^8$

FastEMRIWaveforms (few)

A part of the Black Hole Perturbation Toolkit
bhptoolkit.org

Nvidia, Dreamstime

$$h(t) = \sum_{lmkn} H_{lmkn}(p(t), e(t), x_I(t), \theta, \phi) e^{-i\Phi_{mkn}(p(t), e(t), x_I(t))}$$

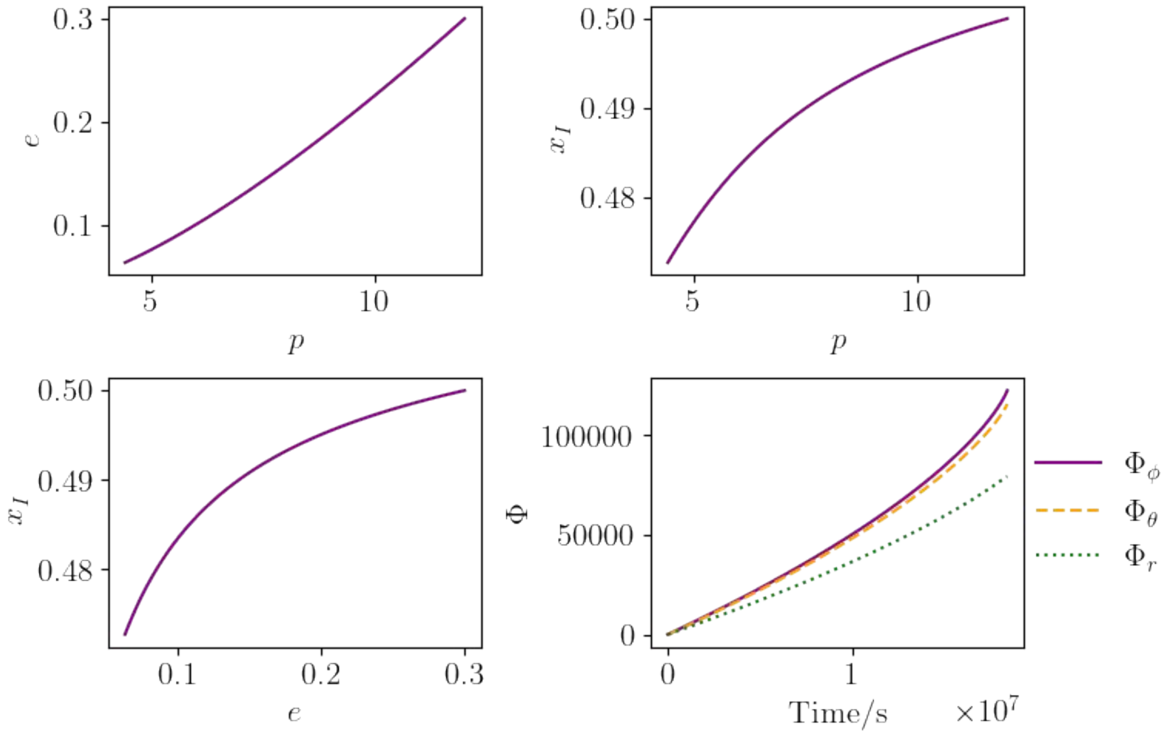
$$(\Phi_{mkn} = m\Phi_\phi + k\Phi_\theta + n\Phi_r)$$

$$h(t) = \sum_{lmkn} H_{lmkn}\left(\boxed{p(t), e(t), x_I(t)}, \theta, \phi\right) e^{\boxed{-i\Phi_{mkn}(p(t), e(t), x_I(t))}}$$

$$(\Phi_{mkn} = m\Phi_\phi + k\Phi_\theta + n\Phi_r)$$

`few.trajectory`

Integrate ODE system

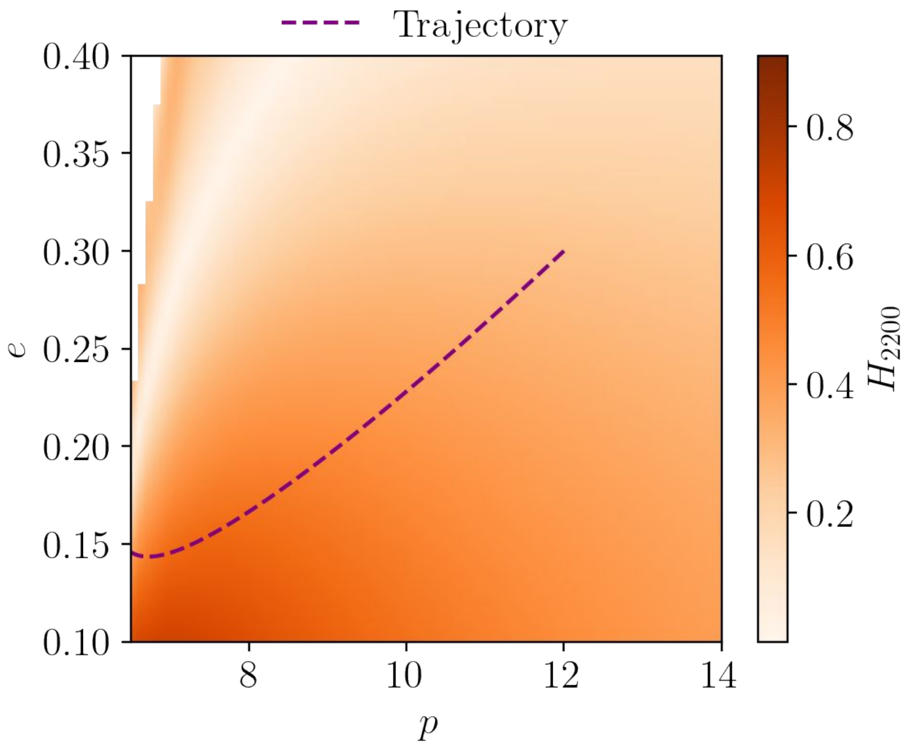


$$h(t) = \sum_{lmkn} \boxed{H_{lmkn}(\boxed{p(t), e(t), x_I(t)}, \theta, \phi)} e^{-\boxed{i\Phi_{mkn}(p(t), e(t), x_I(t))}}$$

$$(\Phi_{mkn} = m\Phi_\phi + k\Phi_\theta + n\Phi_r)$$

`few.trajectory`
Integrate ODE system

`few.amplitude`
Functions of trajectory,
fold in viewing angles



$$h(t) = \boxed{\sum_{lmkn} \boxed{H_{lmkn}(\boxed{p(t), e(t), x_I(t)}, \theta, \phi)} e^{-\boxed{i\Phi_{mkn}(p(t),e(t),x_I(t))}}}$$

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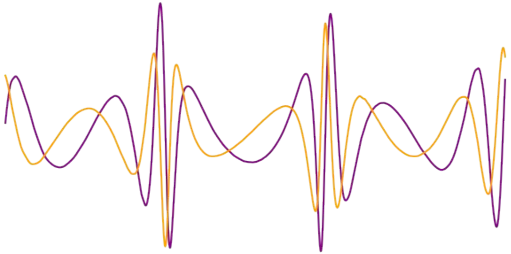
`few.trajectory`
Integrate ODE system

`few.amplitude`
Functions of trajectory, fold in viewing angles

`few.summation`
Sum over all modes to build waveform



$$h(t) = \sum_{lmkn} H_{lmkn}(p(t), e(t), x_I(t), \theta, \phi) e^{-i\Phi_{mkn}(p(t), e(t), x_I(t))}$$

$$(\Phi_{mkn} = m\Phi_\phi + k\Phi_\theta + n\Phi_r)$$

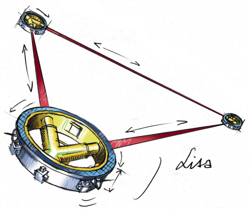
`few.trajectory`
Integrate ODE system

`few.amplitude`
Functions of trajectory, fold in viewing angles

`few.summation`
Sum over all modes to build waveform

`few.waveform`
Joins this all together!

Detector response function



$$h(t) = \boxed{\sum_{lmkn} \boxed{H_{lmkn}\left(\boxed{p(t), e(t), x_I(t)}, \theta, \phi\right)} e^{\boxed{-i\Phi_{mkn}(p(t),e(t),x_I(t))}}}$$

$$(\Phi_{mkn} = m\Phi_\phi + k\Phi_\theta + n\Phi_r)$$

Challenges:

- Sum contains tens of thousands of terms (one per mode)
- For $10\,\mathrm{s}$ sampling cadence, $\mathcal{O}(10^7)$ data points per mode

This is a **computationally expensive operation!**

FastEMRIWaveforms `(few)`

Nvidia, Dreamstime

FastEMRIWaveforms `(few)`

Performant

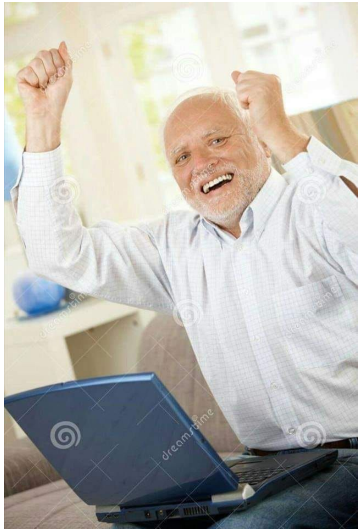
Interpolation, graphics processing unit (GPU) vectorisation

Nvidia, Dreamstime

FastEMRIWaveforms (`few`)

Performant

Interpolation, graphics processing unit (GPU) vectorisation



Easy to use

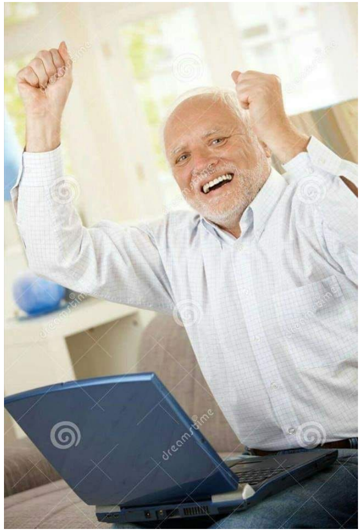
EMRI waveforms out of the box!

Nvidia, Dreamstime

FastEMRIWaveforms `(few)`

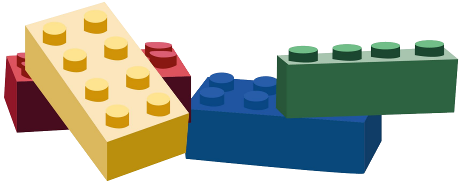
Performant

Interpolation, graphics processing unit (GPU) vectorisation



Easy to use

EMRI waveforms out of the box!



Modular

General framework for building custom EMRI waveforms

Nvidia, Dreamstime

FastEMRIWaveforms `(few)`

Currently supported waveform models:

- **`FastSchwarzschildEccentricFlux`** (Non-spinning black holes)
- **`Pn5AAKWaveform`** (Kludge amplitudes, generic PN inspirals)

Coming soon (names a WIP…):

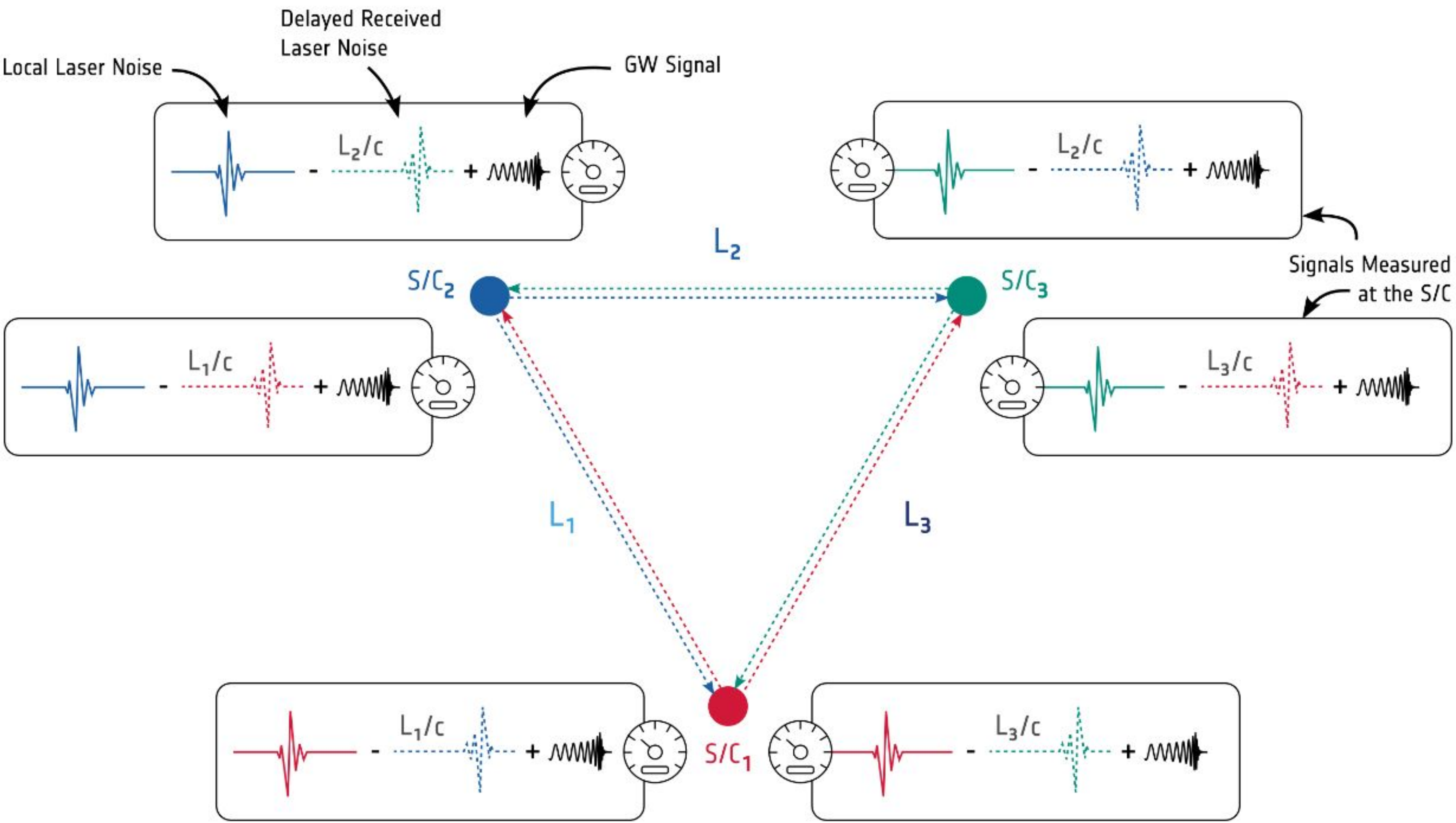
- **`KerrEccentricEquatorialFlux`** (Spinning MBH, equatorial inspirals)
- **`Pn5TrajPn5AdiabaticWaveform`** (Generic PN inspirals and amplitudes)

The LISA response function

LISA data will come in the form of **time delay interferometry (TDI) observables**

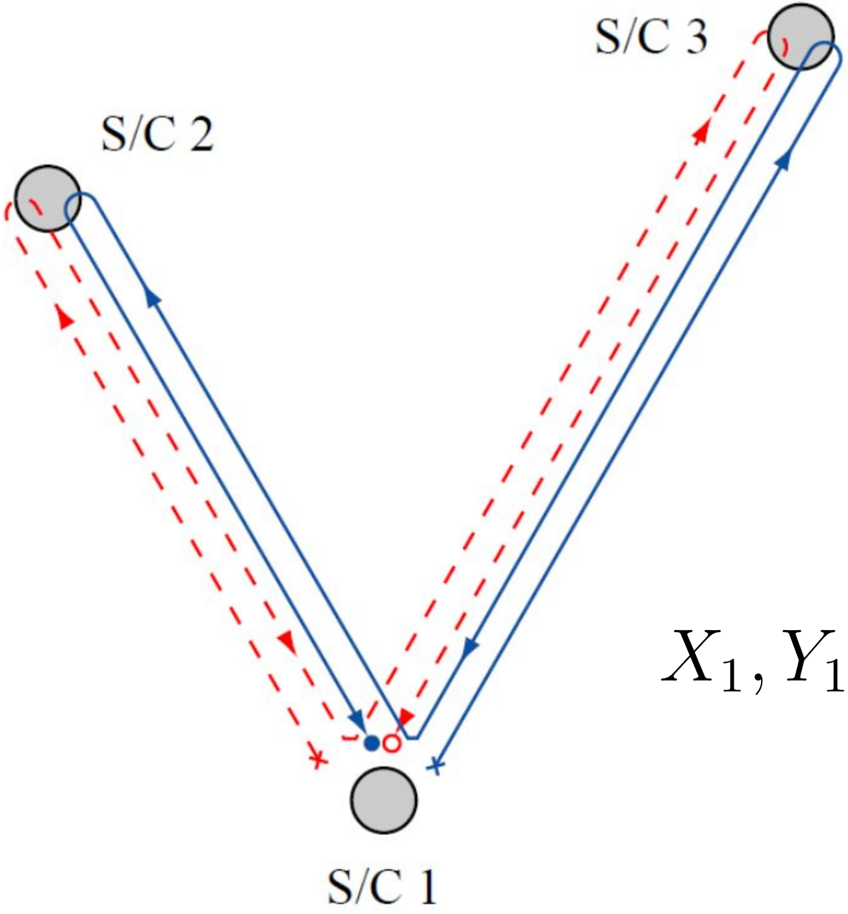
- Linear combinations of data streams delayed in time
- Necessary to suppress **laser frequency noise**
- Delays are multiples of light-travel time between spacecraft
  - Spacecraft orbits **change with time**, so the delays do too

An efficient implementation of the LISA response **essential** for feasible analyses


Delayed Received
Laser Noise
Local Laser Noise
GW Signal
L_2/c
L_2/c
L_2
S/C$_2$
S/C$_3$
Signals Measured
at the S/C
L_1/c
L_3/c
L_1
L_3
L_1/c
L_3/c
S/C$_1$

TDI Combinations

- Construct effective paths from delayed laser link measurements and subtract them
- First-generation TDI combinations account for unequal arms
- Second-generation TDI combinations (constructed from first-gen combinations) account for linear changes in arm-lengths with respect to time

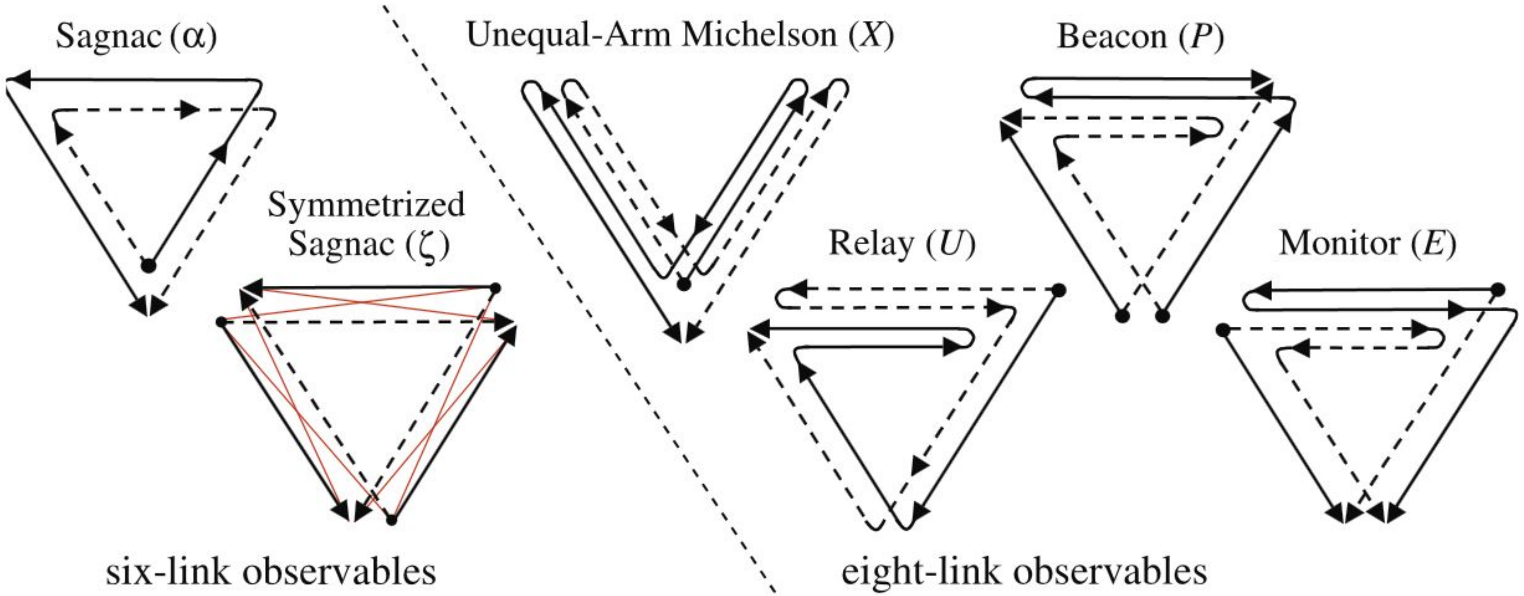


$$X_1, Y_1, Z_1$$

Tinto, Dhurandhar (2015) Liv. Rev. Rel.

Time-delay interferometry

Many combinations are useful…



Vallisneri (2005) Phys. rev. D

`fastlisaresponse` (`lisa-on-gpu`)

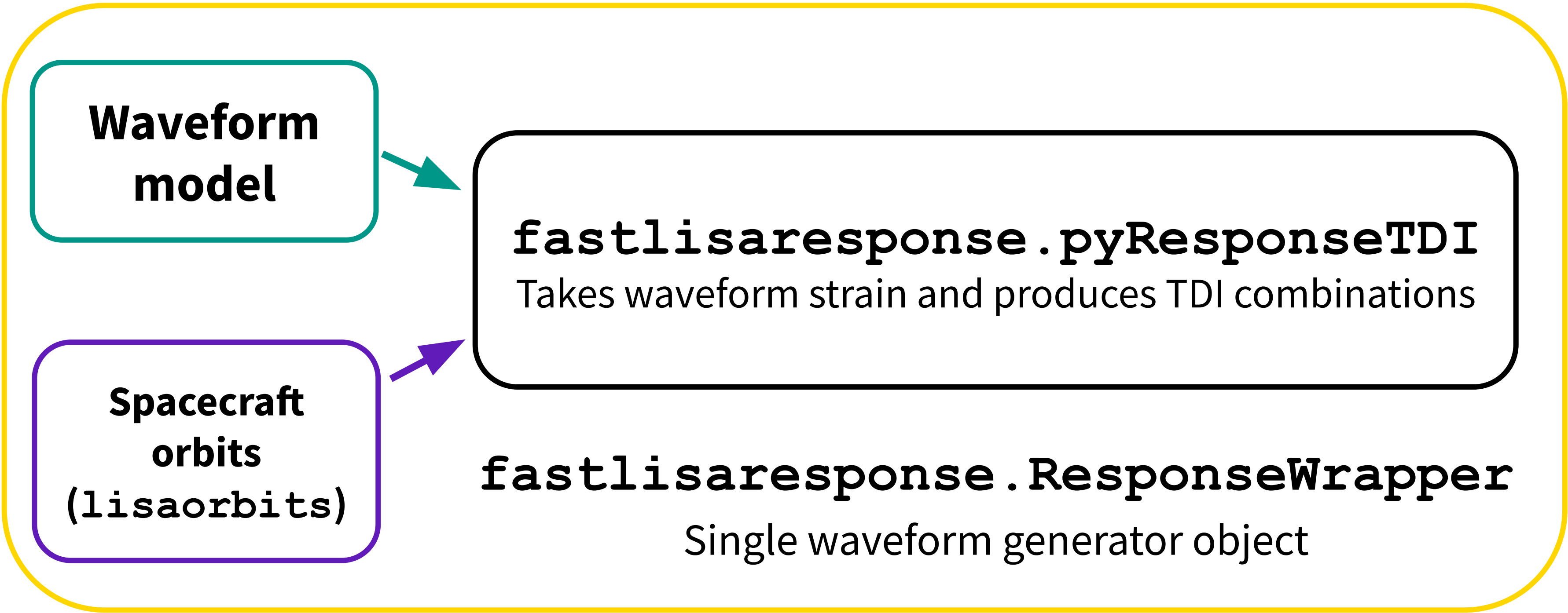
One-stop shop for time-domain LISA response:

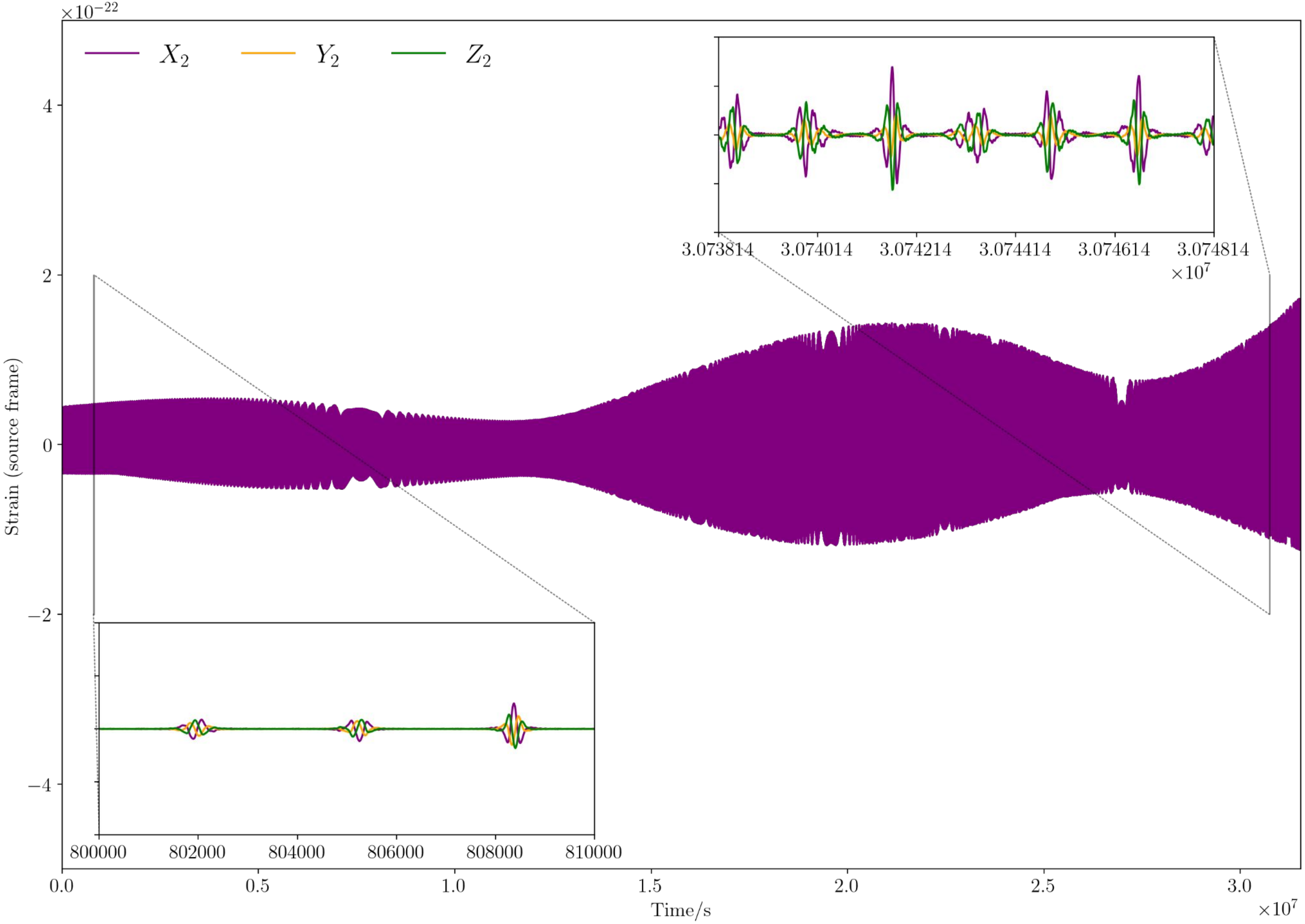
- **GPU vectorisation** for millisecond response evaluation
- Link-based TDI generation: **construct any combination!**
- Commonly-used TDI combinations (XYZ, AET) are included **out-of-the-box**

Interfaces with **`lisaorbits`** via **`lisatools.detector`**:

- Generic orbital interface
- Sample orbit files provided for synthetic data studies

fastlisaresponse (lisa-on-gpu)



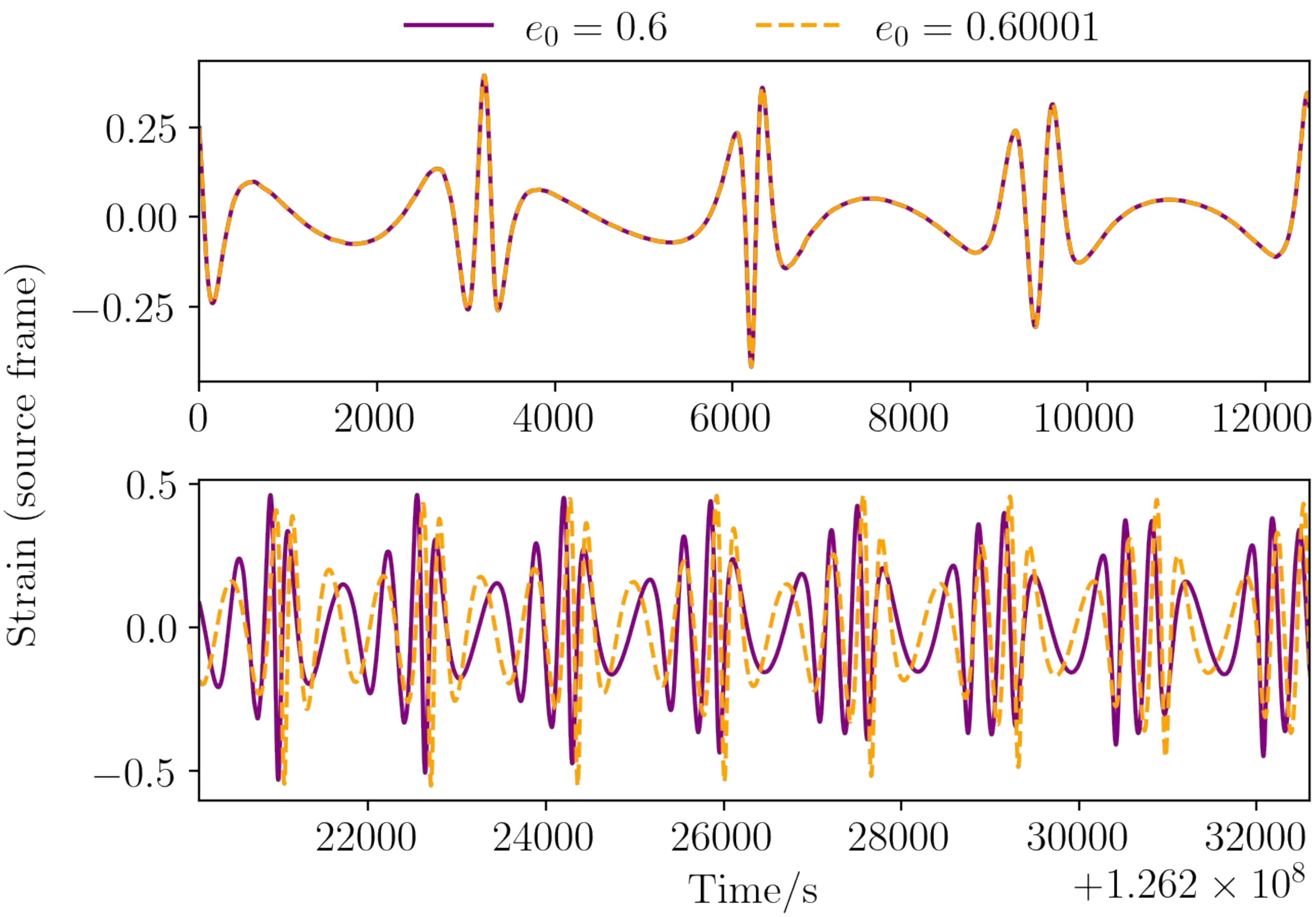


$\times 10^{-22}$
X_2
Y_2
Z_2
4
2
0
−2
−4
Strain (source frame)
3.073814
3.074014
3.074214
3.074414
3.074614
3.074814
$\times 10^{7}$
800000
802000
804000
806000
808000
810000
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Time/s
$\times 10^{7}$

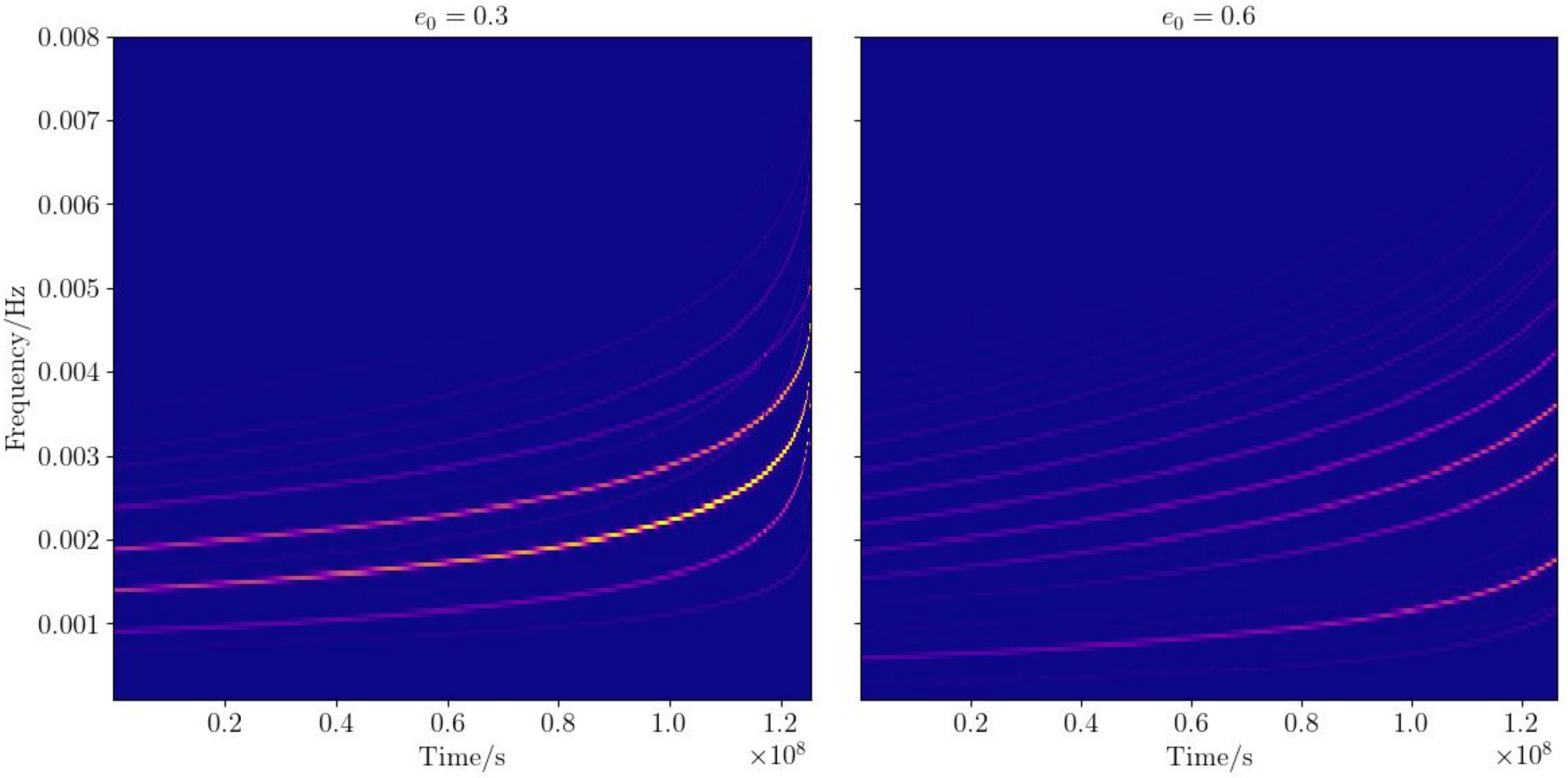
Conclusions

- EMRI waveforms are complicated and computationally expensive
- `few`:
    - **Out-of-the-box** EMRI waveforms
    - Highly-optimised GPU methods: **millisecond waveform generation**
    - Modular: easily implement your own waveform models
- LISA data analysis requires flexible, efficient response implementation
- `fastlisaresponse`
    - Time-domain response, fast via **GPU vectorisation**
    - Support for **generic orbits**
    - Build **any TDI combination**: base support for XYZ/AET

Bonus slides



$e_0 = 0.6$
$e_0 = 0.60001$
Strain (source frame)
0.25
0.00
−0.25
0
2000
4000
6000
8000
10000
12000
0.5
0.0
−0.5
22000
24000
26000
28000
30000
32000
Time/s
$+1.262 \times 10^8$



$e_0 = 0.3$
$e_0 = 0.6$
Frequency/Hz
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0.2
0.4
0.6
0.8
1.0
1.2
Time/s
$\times 10^8$