Técnicas para calcular distribuciones de creencias honestas

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Incertidumbre

Sorpresa

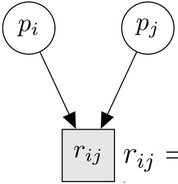
Sorpresa
El punto débil de las creencias

¿Cómo estimar habilidades?

Arpad Elo

Modelo Elo

Rendimiento aleatorio oculto (p)
centrado en la habilidad estimada (s)

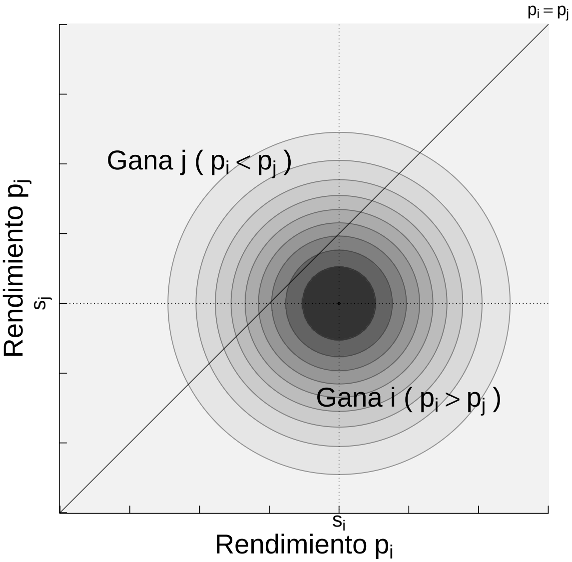


$p \sim N(s, \beta^2)$

Resultado observado (r)

$r_{ij} = \mathbb{I}(p_i > p_j)$

Probabilidad de ganar (gráfica)



Estimación Elo

$$s_i^{\text{new}} = s_i^{\text{old}} + K\Delta$$

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$$\Delta = \underbrace{(2\, r_{ij} - 1)}_{\substack{\textbf{Signo} \\ \text{del resultado}}} \underbrace{(1 - P(r_{ij} | s_i, s_j))}_{\substack{\textbf{Sorpresa} \\ \text{del resultado}}}$$

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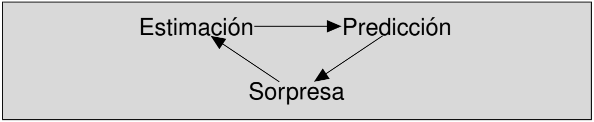
Modelo de solución



Hoy
Estimación de habilidad en la industria del video juego

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Inferencia Bayesiana

¿Por qué inferencia Bayesiana?

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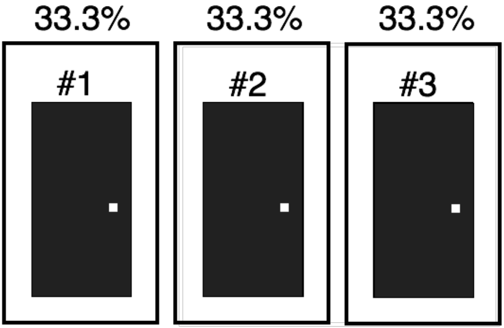
Permite computar creencias óptimas
dadas restricciones: modelos y datos

Books: Jaynes. Probability theor, the logic of science – Bishop. Pattern recognition and machine learning

Honestidad: principio de máxima incertidumbre



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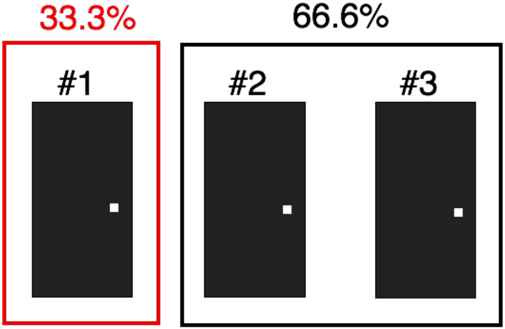

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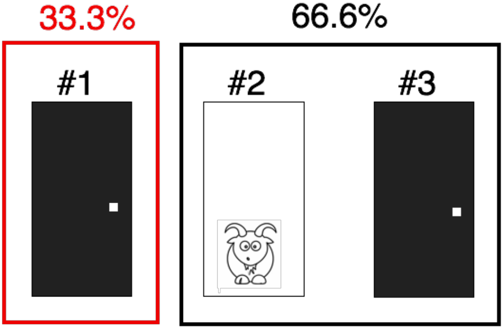

Entropía: la sorpresa fuente de información

Información extraida de la sorpresa,

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$$h(x) = -\log P(x)$$

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Shannon, 1948. A mathematical theory of communication – Jaynes, 1957. Information theory and statistical mechanics

Razonamiento para actualizar creencias

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Principios deseable

- Que permita representar las creencias con valores reales

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Teorema de Cox, 1946
Las reglas de la probabilidad de Laplace-Jeffreys

Cox, R (1946). Probability, frequency and reasonable expectation.

Regla de Bernoulli

Estados igualmente posibles: $\{(1,1),(1,2),\dots,(6,6)\}$

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$$p(A) = \frac{\text{Cantidad de estados en que } A \text{ es verdadera}}{\text{Cantidad de estados totales}}$$

La reglas de la probabilidad

$$
\begin{array}{c|c|c|c|c}
\multicolumn{1}{c}{} & \multicolumn{1}{c}{} & \multicolumn{1}{c}{y_j} & \multicolumn{1}{c}{} & \\ \cline{2-4}
 & & & & \\ \cline{2-4}
x_i & & n_{ij} & & f_i \\ \cline{2-4}
 & & & & \\ \cline{2-4}
\multicolumn{1}{c}{} & \multicolumn{1}{c}{} & \multicolumn{1}{c}{c_j} & \multicolumn{1}{c}{} & N
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 & & y_j & & \\
\hline
 & & & & \\
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\hline
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La reglas de la probabilidad

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x_i		n_{ij}		f_i
		c_j		N

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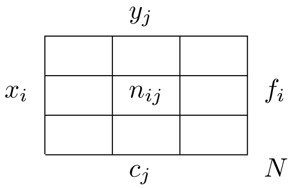
La reglas de la probabilidad



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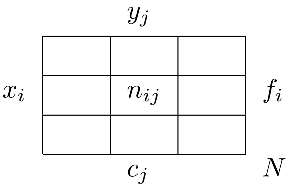

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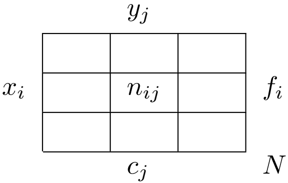

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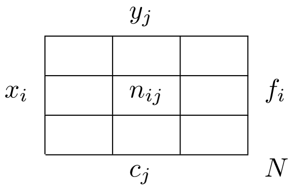

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La reglas de la probabilidad

$$\text{Marginal}_i = \sum_j \text{Conjunta}_{ij} \qquad \text{Condicional}_{j|i} = \frac{\text{Conjunta}_{ij}}{\text{Marginal}_i}$$

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Regla de la suma

$$P(X) = \sum_Y P(X,Y)$$

Cualquier distribución marginal puede ser obtenida integrando la distribución conjunta

Regla del producto

$$P(X,Y) = P(Y|X)P(X)$$

Cualquier distribución conjunta puede ser expresada como el producto de distribuciones condicionales uni-dimensionles.

Teorema de Bayes

$$P(Y|X) = \frac{p(X|Y)p(Y)}{p(X)}$$

Teorema de Bayes

$$P(\text{Creencia} \mid \text{Datos}) = \frac{P(\text{Datos} \mid \text{Creencia})P(\text{Creencia})}{P(\text{Datos})}$$

Teorema de Bayes

$$\underbrace{P(\text{Creencia} \mid \text{Datos})}_{\text{Posteriorii}} = \frac{\overbrace{P(\text{Datos} \mid \text{Creencia})}^{\text{Verosimilitud}}\,\overbrace{P(\text{Creencia})}^{\text{Priorii}}}{\underbrace{P(\text{Datos})}_{\text{Evidencia}}}$$

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$$\underbrace{P(\text{Creencia} \mid \text{Datos, Modelo})}_{\text{Posteriorii}} = \frac{\overbrace{P(\text{Datos} \mid \text{Creencia, Modelo})}^{\text{Verosimilitud}}\overbrace{P(\text{Creencia} \mid \text{Modelo})}^{\text{Priorii}}}{\underbrace{P(\text{Datos} \mid \text{Modelo})}_{\text{Evidencia}}}$$

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El **modelo** es lo que permite relacionar
los **datos** con nuestras **creencias**!

El filtro de las creencias

$$\overbrace{P(C|D,M)}^{\text{Posteriorii}} = \frac{\overbrace{P(D|C,M)}^{\text{Verosimilitud}}\overbrace{P(C,M)}^{\text{Priorii}}}{\underbrace{P(D|M)}_{\text{Evidencia}}}$$

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Las creencias inverosímiles, se anula.
Las creencias verosímiles, resisten.

La sorpresa:
El filtro de las creencias

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Verosimilitud: el jardín de los caminos que se bifurcan

Datos: ● ○ ● Creencias: ○○○○, ●○○○, ●●○○, ●●●○, ●●●●

Modelo: Data $\sim$ Binomial(n, p)

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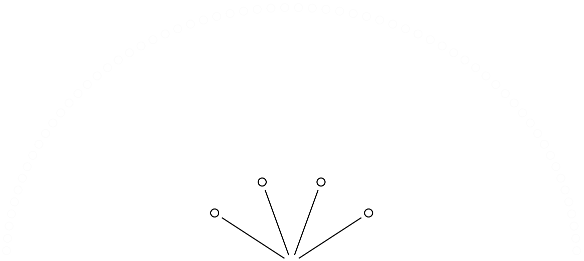

Caminos dado M y $C =$ ○○○○ (Primer marcador)

Verosimilitud: el jardín de los caminos que se bifurcan

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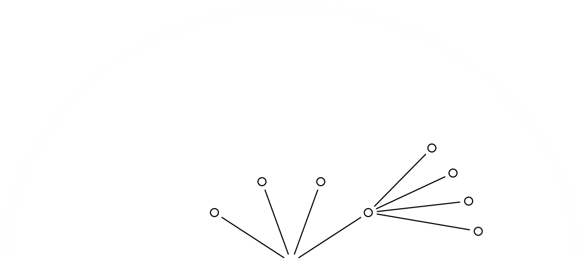

Caminos dado M y $C =$ ○○○○ (Segundo marcador)

Verosimilitud: el jardín de los caminos que se bifurcan

Datos: ● ○ ● Creencias: ○○○○, ●○○○, ●●○○, ●●●○, ●●●●

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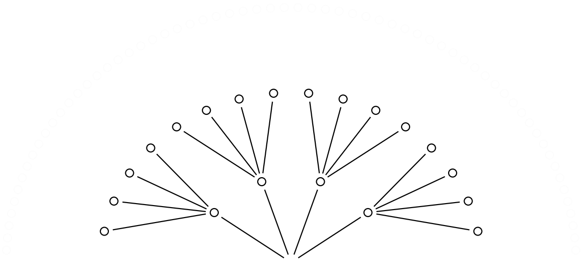

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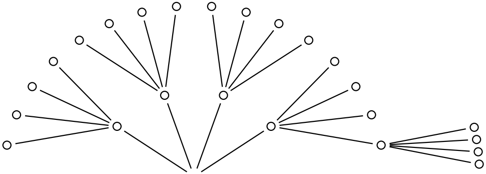

Caminos dado M y $C =$ ○○○○ (Tercer marcador)

Verosimilitud: el jardín de los caminos que se bifurcan

Datos: ● ○ ● Creencias: ○○○○, ●○○○, ●●○○, ●●●○, ●●●●

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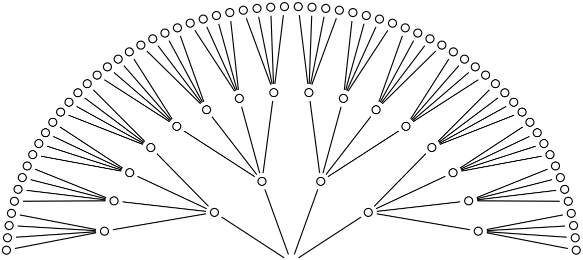

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Verosimilitud: el jardín de los caminos que se bifurcan

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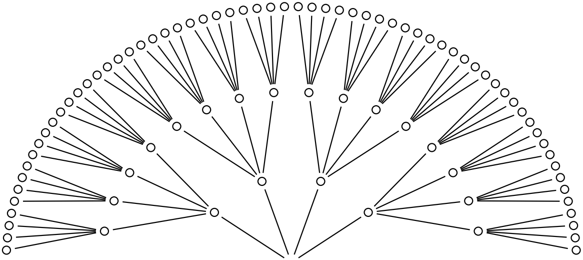


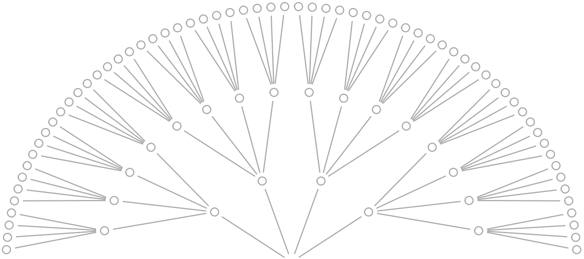
Caminos dado M y $C =$ ○○○○

Creencia	Caminos que conducen ●○●
○○○○	

Verosimilitud: el jardín de los caminos que se bifurcan

Datos: ● ○ ●   Creencias: ○○○○, ●○○○, ●●○○, ●●●○, ●●●●

Modelo: Data $\sim$ Binomial(n, p)



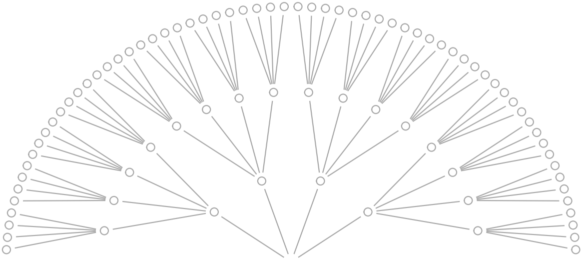
Caminos dado M y $C =$ ○○○○

Creencia	Caminos que conducen ● ○ ●
○○○○	$0 \times 4 \times 0 = 0$

Verosimilitud: el jardín de los caminos que se bifurcan

Datos: ● ○ ●   Creencias: ○○○○, ●○○○, ●●○○, ●●●○, ●●●●

Modelo: Data $\sim$ Binomial(n, p)



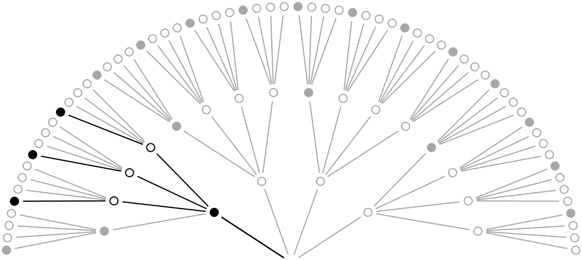
Caminos dado M y $C =$ ○○○○

Creencia	Caminos que conducen ● ○ ●	Verosimilitud	Priori	Posteriori $\propto$
○○○○	$0 \times 4 \times 0 = 0$	$\frac{0\times4\times0}{4\times4\times4} = \frac{0}{64}$	$1/5$	$\frac{0}{64}\,\frac{1}{5}$

Verosimilitud: el jardín de los caminos que se bifurcan

Datos: ● ○ ● Creencias: ○○○○, ●○○○, ●●○○, ●●●○, ●●●●

Modelo: Data $\sim$ Binomial(n, p)



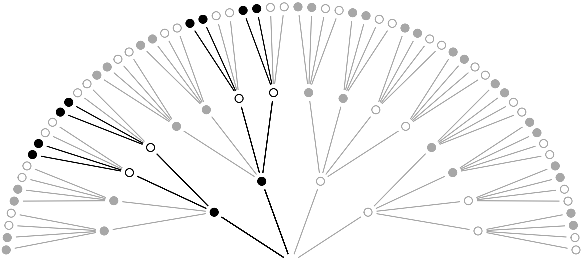
Caminos dado M y $C =$ ●○○○

Creencia	Caminos que conducen ● ○ ●	Verosimilitud	Priori	Posteriori $\propto$
○○○○	$0 \times 4 \times 0 = 0$	$\frac{0\times4\times0}{4\times4\times4} = \frac{0}{64}$	$1/5$	$\frac{0}{64}\,\frac{1}{5}$
●○○○	$1 \times 3 \times 1 = 3$	$3/64$	$1/5$	$\frac{3}{64}\,\frac{1}{5}$

Verosimilitud: el jardín de los caminos que se bifurcan

Datos: ● ○ ●   Creencias: ○○○○, ●○○○, ●●○○, ●●●○, ●●●●

Modelo: Data $\sim$ Binomial(n, p)



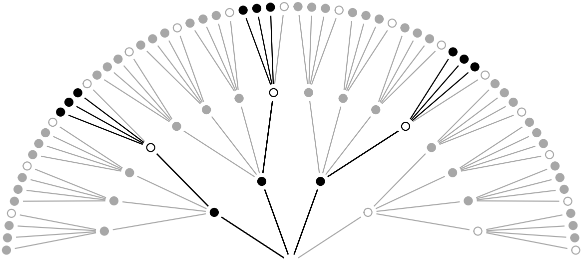
Caminos dado M y $C =$ ●●○○

Creencia	Caminos que conducen ● ○ ●	Verosimilitud	Priori	Posteriori $\propto$
○○○○	$0 \times 4 \times 0 = 0$	$\frac{0 \times 4 \times 0}{4 \times 4 \times 4} = \frac{0}{64}$	$1/5$	$\frac{0}{64}\,\frac{1}{5}$
●○○○	$1 \times 3 \times 1 = 3$	$3/64$	$1/5$	$\frac{3}{64}\,\frac{1}{5}$
●●○○	$2 \times 2 \times 2 = 8$	$8/64$	$1/5$	$\frac{8}{64}\,\frac{1}{5}$

Verosimilitud: el jardín de los caminos que se bifurcan

Datos: ● ○ ●   Creencias: ○○○○, ●○○○, ●●○○, ●●●○, ●●●●

Modelo: Data $\sim$ Binomial(n, p)



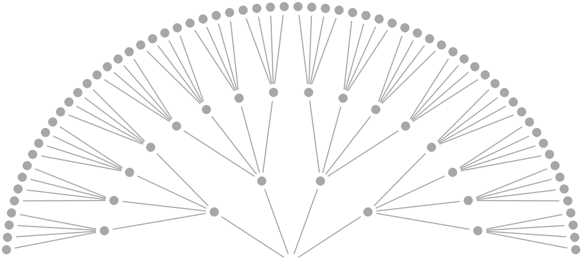
Caminos dado M y $C =$ ●●●○

Creencia	Caminos que conducen ● ○ ●	Verosimilitud	Priori	Posteriori $\propto$
○○○○	$0 \times 4 \times 0 = 0$	$\frac{0\times4\times0}{4\times4\times4} = \frac{0}{64}$	$1/5$	$\frac{0}{64}\,\frac{1}{5}$
●○○○	$1 \times 3 \times 1 = 3$	$3/64$	$1/5$	$\frac{3}{64}\,\frac{1}{5}$
●●○○	$2 \times 2 \times 2 = 8$	$8/64$	$1/5$	$\frac{8}{64}\,\frac{1}{5}$
●●●○	$3 \times 1 \times 3 = 9$	$9/64$	$1/5$	$\frac{9}{64}\,\frac{1}{5}$

Verosimilitud: el jardín de los caminos que se bifurcan

Datos: ● ○ ●   Creencias: ○○○○, ●○○○, ●●○○, ●●●○, ●●●●

Modelo: Data $\sim$ Binomial(n, p)



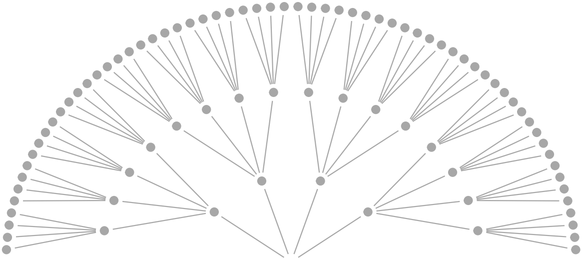
Caminos dado M y $C =$ ●●●●

Creencia	Caminos que conducen ● ○ ●	Verosimilitud	Priori	Posteriori $\propto$
○○○○	$0 \times 4 \times 0 = 0$	$\frac{0\times4\times0}{4\times4\times4} = \frac{0}{64}$	$1/5$	$\frac{0}{64}\,\frac{1}{5}$
●○○○	$1 \times 3 \times 1 = 3$	$3/64$	$1/5$	$\frac{3}{64}\,\frac{1}{5}$
●●○○	$2 \times 2 \times 2 = 8$	$8/64$	$1/5$	$\frac{8}{64}\,\frac{1}{5}$
●●●○	$3 \times 1 \times 3 = 9$	$9/64$	$1/5$	$\frac{9}{64}\,\frac{1}{5}$
●●●●	$4 \times 0 \times 4 = 0$	$0/64$	$1/5$	$\frac{0}{64}\,\frac{1}{5}$

Verosimilitud: el jardín de los caminos que se bifurcan

Datos: ● ○ ●   Creencias: ○○○○, ●○○○, ●●○○, ●●●○, ●●●●

Modelo: Data $\sim$ Binomial(n, p)



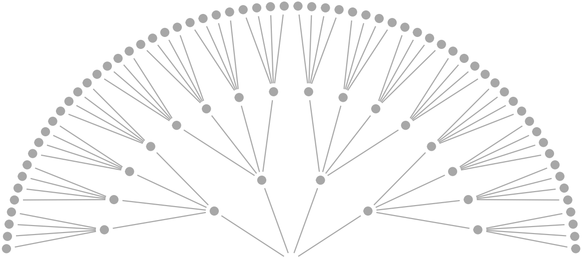
Caminos dado M y $C =$ ●●●●

Creencia	Caminos que conducen ● ○ ●	Verosimilitud	Priori	Posteriori $\propto$
○○○○	$0 \times 4 \times 0 = 0$	$\frac{0\times4\times0}{4\times4\times4} = \frac{0}{64}$	$1/5$	$\frac{0}{64}\,\frac{1}{5}$
●○○○	$1 \times 3 \times 1 = 3$	$3/64$	$1/5$	$\frac{3}{64}\,\frac{1}{5}$
●●○○	$2 \times 2 \times 2 = 8$	$8/64$	$1/5$	$\frac{8}{64}\,\frac{1}{5}$
●●●○	$3 \times 1 \times 3 = 9$	$9/64$	$1/5$	$\frac{9}{64}\,\frac{1}{5}$
●●●●	$4 \times 0 \times 4 = 0$	$0/64$	$1/5$	$\frac{0}{64}\,\frac{1}{5}$
				$P(D\|M)$

Verosimilitud: el jardín de los caminos que se bifurcan

Datos: ● ○ ● Creencias: ○○○○, ●○○○, ●●○○, ●●●○, ●●●●

Modelo: Data $\sim$ Binomial(n, p)



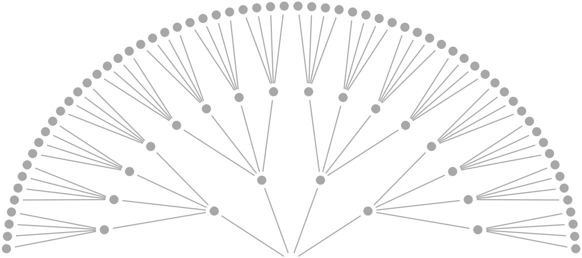
Caminos dado M y $C =$ ●●●●

Creencia	Caminos que conducen ● ○ ●	Verosimilitud	Priori	Posteriori $\propto$
○○○○	$0 \times 4 \times 0 = 0$	$\frac{0\times4\times0}{4\times4\times4} = \frac{0}{64}$	$1/5$	$\frac{0}{64}\,\frac{1}{5}$
●○○○	$1 \times 3 \times 1 = 3$	$3/64$	$1/5$	$\frac{3}{64}\,\frac{1}{5}$
●●○○	$2 \times 2 \times 2 = 8$	$8/64$	$1/5$	$\frac{8}{64}\,\frac{1}{5}$
●●●○	$3 \times 1 \times 3 = 9$	$9/64$	$1/5$	$\frac{9}{64}\,\frac{1}{5}$
●●●●	$4 \times 0 \times 4 = 0$	$0/64$	$1/5$	$\frac{0}{64}\,\frac{1}{5}$
				$\frac{3+8+9}{64\cdot5}$

Verosimilitud: el jardín de los caminos que se bifurcan

Datos: ● ○ ●   Creencias: ○○○○, ●○○○, ●●○○, ●●●○, ●●●●

Modelo: Data $\sim$ Binomial(n, p)



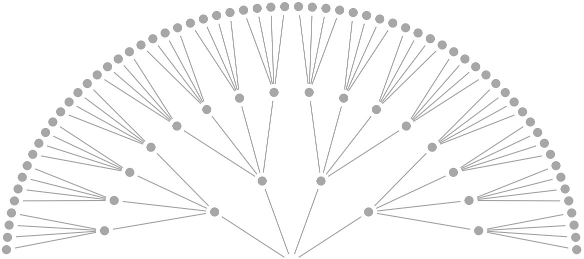
Caminos dado M y $C =$ ●●●●

Creencia	Caminos que conducen ● ○ ●	Verosimilitud	Priori	Posteriori $\propto$	Posteriori
○○○○	$0 \times 4 \times 0 = 0$	$\frac{0\times4\times0}{4\times4\times4} = \frac{0}{64}$	$1/5$	$\frac{0}{64}\,\frac{1}{5}$	$\frac{0}{64}\,\frac{1}{5}\,\frac{64\cdot5}{3+8+9}$
●○○○	$1 \times 3 \times 1 = 3$	$3/64$	$1/5$	$\frac{3}{64}\,\frac{1}{5}$	
●●○○	$2 \times 2 \times 2 = 8$	$8/64$	$1/5$	$\frac{8}{64}\,\frac{1}{5}$	
●●●○	$3 \times 1 \times 3 = 9$	$9/64$	$1/5$	$\frac{9}{64}\,\frac{1}{5}$	
●●●●	$4 \times 0 \times 4 = 0$	$0/64$	$1/5$	$\frac{0}{64}\,\frac{1}{5}$	
				$\frac{3+8+9}{64\cdot5}$	

Verosimilitud: el jardín de los caminos que se bifurcan

Datos: ● ○ ●   Creencias: ○○○○, ●○○○, ●●○○, ●●●○, ●●●●

Modelo: Data $\sim$ Binomial(n,p)



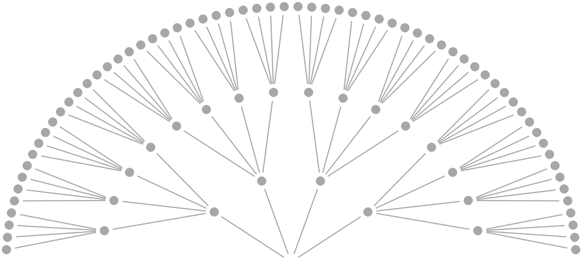
Caminos dado M y $C =$ ●●●●

Creencia	Caminos que conducen ● ○ ●	Verosimilitud	Priori	Posteriori $\propto$	Posteriori
○○○○	$0 \times 4 \times 0 = 0$	$\frac{0\times4\times0}{4\times4\times4} = \frac{0}{64}$	$1/5$	$\frac{0}{64}\,\frac{1}{5}$	$\frac{0}{3+8+9} = 0.00$
●○○○	$1 \times 3 \times 1 = 3$	$3/64$	$1/5$	$\frac{3}{64}\,\frac{1}{5}$	
●●○○	$2 \times 2 \times 2 = 8$	$8/64$	$1/5$	$\frac{8}{64}\,\frac{1}{5}$	
●●●○	$3 \times 1 \times 3 = 9$	$9/64$	$1/5$	$\frac{9}{64}\,\frac{1}{5}$	
●●●●	$4 \times 0 \times 4 = 0$	$0/64$	$1/5$	$\frac{0}{64}\,\frac{1}{5}$	
				$\frac{3+8+9}{64\cdot5}$	

Verosimilitud: el jardín de los caminos que se bifurcan

Datos: ● ○ ● Creencias: ○○○○, ●○○○, ●●○○, ●●●○, ●●●●

Modelo: Data $\sim$ Binomial(n, p)



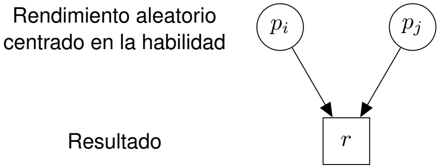
Caminos dado M y $C =$ ●●●●

Creencia	Caminos que conducen ● ○ ●	Verosimilitud	Priori	Posteriori $\propto$	Posteriori
○○○○	$0 \times 4 \times 0 = 0$	$\frac{0\times4\times0}{4\times4\times4} = \frac{0}{64}$	$1/5$	$\frac{0}{64}\,\frac{1}{5}$	$\frac{0}{3+8+9} = 0.00$
●○○○	$1 \times 3 \times 1 = 3$	$3/64$	$1/5$	$\frac{3}{64}\,\frac{1}{5}$	$\frac{3}{3+8+9} = 0.15$
●●○○	$2 \times 2 \times 2 = 8$	$8/64$	$1/5$	$\frac{8}{64}\,\frac{1}{5}$	$\frac{8}{3+8+9} = 0.40$
●●●○	$3 \times 1 \times 3 = 9$	$9/64$	$1/5$	$\frac{9}{64}\,\frac{1}{5}$	$\frac{9}{3+8+9} = 0.45$
●●●●	$4 \times 0 \times 4 = 0$	$0/64$	$1/5$	$\frac{0}{64}\,\frac{1}{5}$	$\frac{0}{3+8+9} = 0.00$
				$\frac{3+8+9}{64\cdot5}$	

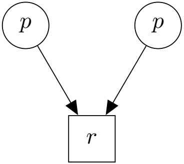

DE LOS LABERINTOS
SE SALE SÓLO
POR ARRIBA
PITTY

Modelos gráficos

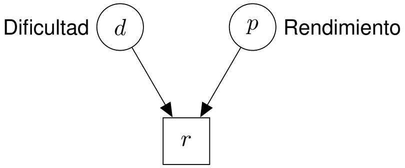
Modelos gráficos



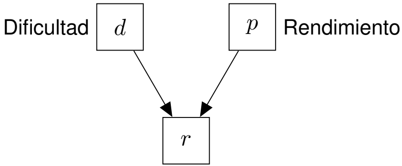
Modelos gráficos



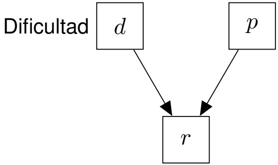
Modelos gráficos



Modelos gráficos



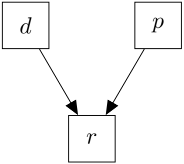
Modelos gráficos



p^0	p^1
0,70	0,30

Modelos gráficos

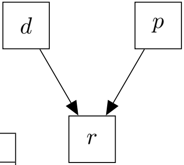
d^0	d^1
0,15	0,85



p^0	p^1
0,70	0,30

Modelos gráficos

d^0	d^1
0,15	0,85

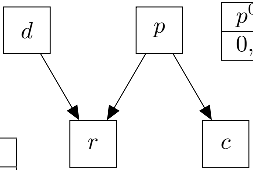


p^0	p^1
0,70	0,30

	r^{-1}	r^0	r^1
(p^0, d^0)	0,30	0,40	0,30
(p^0, d^1)	0,70	0,25	0,05
(p^1, d^0)	0,02	0,08	0,90
(p^1, d^1)	0,20	0,30	0,50

Modelos gráficos

d^0	d^1
0,15	0,85



p^0	p^1
0,70	0,30

Clasificador (Elo / TrueSkill)

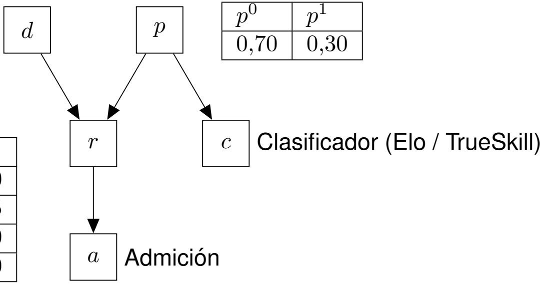
	r^{-1}	r^0	r^1
(p^0, d^0)	0,30	0,40	0,30
(p^0, d^1)	0,70	0,25	0,05
(p^1, d^0)	0,02	0,08	0,90
(p^1, d^1)	0,20	0,30	0,50

Modelos gráficos

d^0	d^1
0,15	0,85

p^0	p^1
0,70	0,30

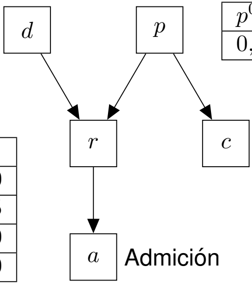
	r^{-1}	r^0	r^1
(p^0, d^0)	0,30	0,40	0,30
(p^0, d^1)	0,70	0,25	0,05
(p^1, d^0)	0,02	0,08	0,90
(p^1, d^1)	0,20	0,30	0,50



Modelos gráficos

d^0	d^1
0,15	0,85

p^0	p^1
0,70	0,30



	r^{-1}	r^0	r^1
(p^0, d^0)	0,30	0,40	0,30
(p^0, d^1)	0,70	0,25	0,05
(p^1, d^0)	0,02	0,08	0,90
(p^1, d^1)	0,20	0,30	0,50

	c^0	c^1
p^0	0,95	0,05
p^1	0,2	0,8

Modelos gráficos

d^0	d^1
0,15	0,85

p^0	p^1
0,70	0,30

$$
\begin{tikzcd}
d \arrow[dr] & & p \arrow[dl] \arrow[dr] & \\
& r \arrow[d] & & c \\
& a & &
\end{tikzcd}
$$

	c^0	c^1
p^0	0,95	0,05
p^1	0,2	0,8

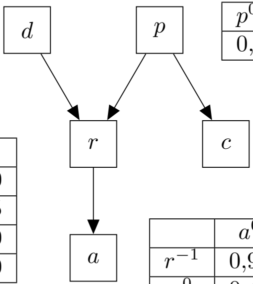
	r^{-1}	r^0	r^1
(p^0, d^0)	0,30	0,40	0,30
(p^0, d^1)	0,70	0,25	0,05
(p^1, d^0)	0,02	0,08	0,90
(p^1, d^1)	0,20	0,30	0,50

	a^0	a^1
r^{-1}	0,99	0,01
r^0	0,40	0,60
r^1	0,10	0,90

Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85



$P(p)$:

p^0	p^1
0,70	0,30

$P(r|d,p)$

	r^{-1}	r^0	r^1
(p^0, d^0)	0,30	0,40	0,30
(p^0, d^1)	0,70	0,25	0,05
(p^1, d^0)	0,02	0,08	0,90
(p^1, d^1)	0,20	0,30	0,50

$P(c|p)$:

	c^0	c^1
p^0	0,95	0,05
p^1	0,2	0,8

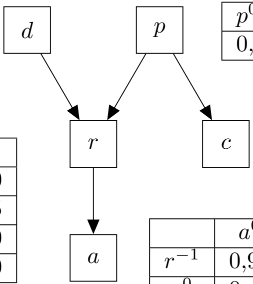
$P(a|r)$:

	a^0	a^1
r^{-1}	0,99	0,01
r^0	0,40	0,60
r^1	0,10	0,90

Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85



p^0	p^1
0,70	0,30

: $P(p)$

$P(r|d,p)$

	r^{-1}	r^0	r^1
(p^0, d^0)	0,30	0,40	0,30
(p^0, d^1)	0,70	0,25	0,05
(p^1, d^0)	0,02	0,08	0,90
(p^1, d^1)	0,20	0,30	0,50

	c^0	c^1
p^0	0,95	0,05
p^1	0,2	0,8

: $P(c|p)$

	a^0	a^1
r^{-1}	0,99	0,01
r^0	0,40	0,60
r^1	0,10	0,90

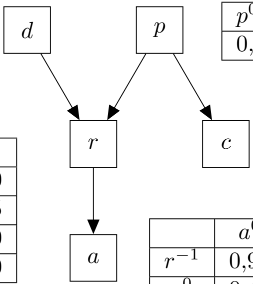
: $P(a|r)$

Los factores son útiles para definir distribuciones de probabilidad en espacios de alta dimensión.

Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85



: $P(p)$

p^0	p^1
0,70	0,30

$P(r|d,p)$

	r^{-1}	r^0	r^1
(p^0, d^0)	0,30	0,40	0,30
(p^0, d^1)	0,70	0,25	0,05
(p^1, d^0)	0,02	0,08	0,90
(p^1, d^1)	0,20	0,30	0,50

: $P(c|p)$

	c^0	c^1
p^0	0,95	0,05
p^1	0,2	0,8

: $P(a|r)$

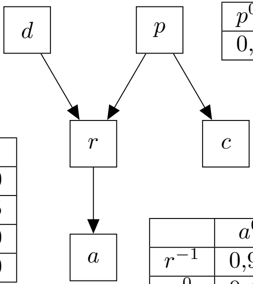
	a^0	a^1
r^{-1}	0,99	0,01
r^0	0,40	0,60
r^1	0,10	0,90

$$P(d, p, r, c, a) =$$

Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85



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0,70	0,30

: $P(p)$

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(p^1, d^1)	0,20	0,30	0,50

	c^0	c^1
p^0	0,95	0,05
p^1	0,2	0,8

: $P(c|p)$

	a^0	a^1
r^{-1}	0,99	0,01
r^0	0,40	0,60
r^1	0,10	0,90

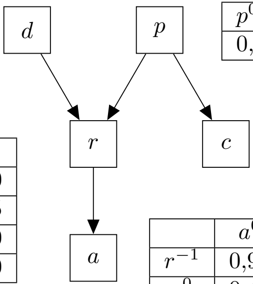
: $P(a|r)$

$$P(d,p,r,c,a) = P(d)P(p)P(r|d,p)P(c|p)P(a|r)$$

Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85



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	a^0	a^1
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r^1	0,10	0,90

: $P(a|r)$

$$P(d^1, p^1, r^1, c^1, a^1) =$$

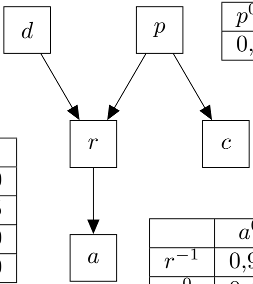
Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85

$: P(p)$

p^0	p^1
0,70	0,30



$P(r|d,p)$

	r^{-1}	r^0	r^1
(p^0, d^0)	0,30	0,40	0,30
(p^0, d^1)	0,70	0,25	0,05
(p^1, d^0)	0,02	0,08	0,90
(p^1, d^1)	0,20	0,30	0,50

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	c^0	c^1
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$: P(a|r)$

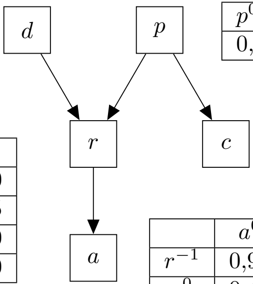
	a^0	a^1
r^{-1}	0,99	0,01
r^0	0,40	0,60
r^1	0,10	0,90

$$P(d^1, p^1, r^1, c^1, a^1) = 0.85 \cdot 0.30 \cdot 0.50 \cdot 0.80 \cdot 0.90$$

Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85



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: $P(p)$

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p^0	0,95	0,05
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	a^0	a^1
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r^1	0,10	0,90

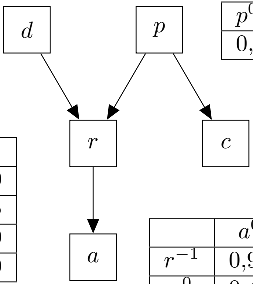
: $P(a|r)$

$$P(d^1, p^1, r^1, c^1, a^1) = 0.85 \cdot 0.30 \cdot 0.50 \cdot 0.80 \cdot 0.90 \approx 0.09$$

Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85



$P(p)$:

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0,70	0,30

$P(r|d,p)$

	r^{-1}	r^0	r^1
(p^0, d^0)	0,30	0,40	0,30
(p^0, d^1)	0,70	0,25	0,05
(p^1, d^0)	0,02	0,08	0,90
(p^1, d^1)	0,20	0,30	0,50

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	c^0	c^1
p^0	0,95	0,05
p^1	0,2	0,8

$P(a|r)$:

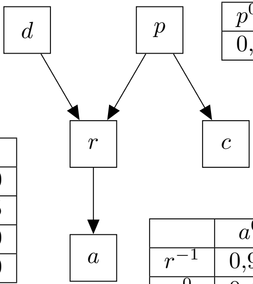
	a^0	a^1
r^{-1}	0,99	0,01
r^0	0,40	0,60
r^1	0,10	0,90

$P(r^1) =$

Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85



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: $P(p)$

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r^1	0,10	0,90

: $P(a|r)$

$$P(r^1) = \sum_{i,j,l,m} P(d^i, p^j, r^1, c^l, a^m)$$

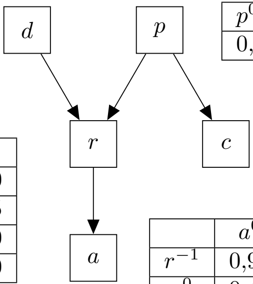
Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85

p^0	p^1
0,70	0,30

: $P(p)$



$P(r|d,p)$

	r^{-1}	r^0	r^1
(p^0, d^0)	0,30	0,40	0,30
(p^0, d^1)	0,70	0,25	0,05
(p^1, d^0)	0,02	0,08	0,90
(p^1, d^1)	0,20	0,30	0,50

	c^0	c^1
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	a^0	a^1
r^{-1}	0,99	0,01
r^0	0,40	0,60
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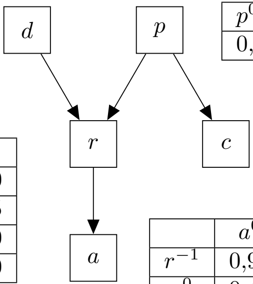
: $P(a|r)$

$$P(r^1) = \sum_{i,j,l,m} P(d^i)P(p^j)P(r^1|d^i,p^j)P(c^l|p^j)P(a^m|r^k)$$

Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85



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: $P(p)$

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r^1	0,10	0,90

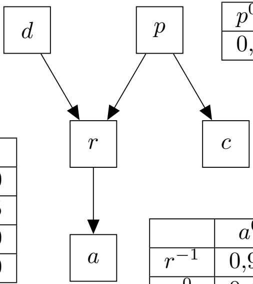
: $P(a|r)$

$$P(r^1) = \left(\sum_m P(a^m|r^1)\right)\left(\sum_{i,j,l} P(d^i)P(p^j)P(r^1|d^i,p^j)P(c^l|p^j)\right)$$

Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85



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0,70	0,30

: $P(p)$

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	a^0	a^1
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r^0	0,40	0,60
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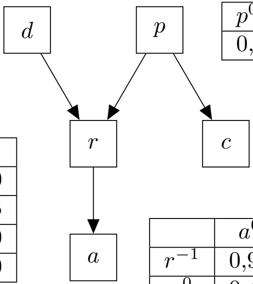
: $P(a|r)$

$$P(r^1) = \left(\sum_m P(a^m|r^1) \right) \left(\sum_{i,j} P(d^i) P(p^j) P(r^1|d^i, p^j) \Big(\sum_l P(c^l|p^j) \Big) \right)$$

Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85



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$$P(r^1) = \left(\cancel{\sum_m P(a^m|r^1)}\right)\left(\sum_{i,j} P(d^i)P(p^j)P(r^1|d^i,p^j)\Big(\sum_l P(c^l|p^j)\Big)\right)$$

Modelos gráficos

$P(d)$:

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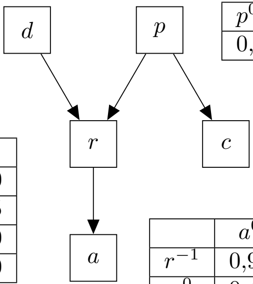

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Modelos gráficos

$P(d)$:

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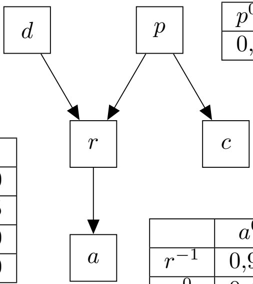

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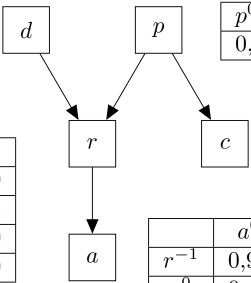
$$P(r^1) = \sum_{i,j} P(d^i)P(p^j)P(r^1|d^i,p^j)$$

Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85



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r^1	0,10	0,90

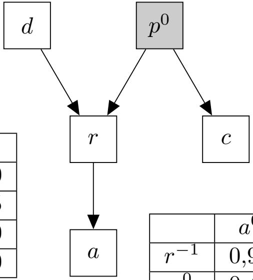
: $P(a|r)$

$$P(r^1) = \sum_{i,j} P(d^i)P(p^j)P(r^1|d^i, p^j) \approx 0.18$$

Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85



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r^{-1}	0,99	0,01
r^0	0,40	0,60
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: $P(a|r)$

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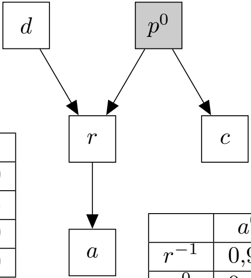
$$P(r^1|p^0) =$$

Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85



$P(r|d,p)$

	r^{-1}	r^0	r^1
(p^0, d^0)	0,30	0,40	0,30
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: $P(a|r)$

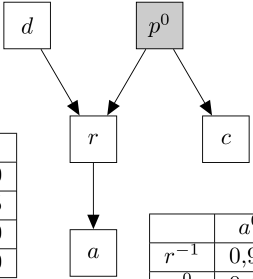
$$P(r^1) = \sum_{i,j} P(d^i)P(p^j)P(r^1|d^i,p^j) \approx 0.18$$

$$P(r^1|p^0) = \frac{P(r^1,p^0)}{P(p^0)}$$

Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85



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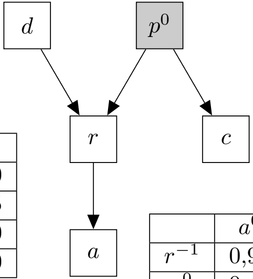
$$P(r^1|p^0) = \frac{1}{P(p^0)} \sum_{i,l,m} P(d^i)P(p^0)P(r^1|d^i,p^0)P(c^l|p^0)P(a^m|r^k)$$

Modelos gráficos

$P(d)$:

d^0	d^1
0,15	0,85



$P(r|d,p)$

	r^{-1}	r^0	r^1
(p^0,d^0)	0,30	0,40	0,30
(p^0,d^1)	0,70	0,25	0,05
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Modelos gráficos

$P(d)$:

d^0	d^1
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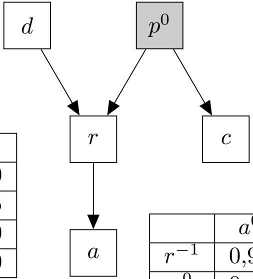

$P(r|d,p)$

	r^{-1}	r^0	r^1
(p^0, d^0)	0,30	0,40	0,30
(p^0, d^1)	0,70	0,25	0,05
(p^1, d^0)	0,02	0,08	0,90
(p^1, d^1)	0,20	0,30	0,50

	c^0	c^1
p^0	0,95	0,05
p^1	0,2	0,8

: $P(c|p)$

	a^0	a^1
r^{-1}	0,99	0,01
r^0	0,40	0,60
r^1	0,10	0,90

: $P(a|r)$

$$P(r^1) = \sum_{i,j} P(d^i)P(p^j)P(r^1|d^i,p^j) \approx 0.18$$

$$P(r^1|p^0) = \frac{1}{P(p^0)} \sum_i P(d^i)P(p^0)P(r^1|d^i,p^0) \approx 0.13$$

Flujos de inferencia

Flujos de inferencia

	$V \notin$ Observable	$V \in$ Observable
$X \to V \to Y$		
$X \leftarrow V \leftarrow Y$		
$X \leftarrow V \to Y$		
$X \to V \leftarrow Y$		

Flujos de inferencia

	$V \notin$ Observable	$V \in$ Observable
$X \rightarrow V \rightarrow Y$	Sí	
$X \leftarrow V \leftarrow Y$		
$X \leftarrow V \rightarrow Y$		
$X \rightarrow V \leftarrow Y$		

Flujos de inferencia

	$V \notin$ Observable	$V \in$ Observable
$X \to V \to Y$	Sí	
$X \leftarrow V \leftarrow Y$	Sí	
$X \leftarrow V \to Y$		
$X \to V \leftarrow Y$		

Flujos de inferencia

	$V \notin$ Observable	$V \in$ Observable
$X \rightarrow V \rightarrow Y$	Sí	
$X \leftarrow V \leftarrow Y$	Sí	
$X \leftarrow V \rightarrow Y$	Sí	
$X \rightarrow V \leftarrow Y$		

Flujos de inferencia

	$V \notin$ Observable	$V \in$ Observable
$X \to V \to Y$	Sí	
$X \leftarrow V \leftarrow Y$	Sí	
$X \leftarrow V \to Y$	Sí	
$X \to V \leftarrow Y$	No	

Flujos de inferencia

	$V \notin$ Observable	$V \in$ Observable
$X \rightarrow V \rightarrow Y$	Sí	No
$X \leftarrow V \leftarrow Y$	Sí	
$X \leftarrow V \rightarrow Y$	Sí	
$X \rightarrow V \leftarrow Y$	No	

Flujos de inferencia

	$V \notin$ Observable	$V \in$ Observable
$X \rightarrow V \rightarrow Y$	Sí	No
$X \leftarrow V \leftarrow Y$	Sí	No
$X \leftarrow V \rightarrow Y$	Sí	
$X \rightarrow V \leftarrow Y$	No	

Flujos de inferencia

	$V \notin$ Observable	$V \in$ Observable
$X \to V \to Y$	Sí	No
$X \leftarrow V \leftarrow Y$	Sí	No
$X \leftarrow V \to Y$	Sí	No
$X \to V \leftarrow Y$	No	

Flujos de inferencia

	$V \notin$ Observable	$V \in$ Observable
$X \rightarrow V \rightarrow Y$	Sí	No
$X \leftarrow V \leftarrow Y$	Sí	No
$X \leftarrow V \rightarrow Y$	Sí	No
$X \rightarrow V \leftarrow Y$	No	Sí

Flujos de inferencia

	$V \notin$ Observable	$V \in$ Observable
$X \to V \to Y$	Sí	No
$X \leftarrow V \leftarrow Y$	Sí	No
$X \leftarrow V \to Y$	Sí	No
$X \to V \leftarrow Y$	No	$\underbrace{\text{Sí}}_{\text{O algún descendiente observable}}$

Flujos de inferencia

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$X \rightarrow V \rightarrow Y$	Sí	No
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Un flujo de inferencia permenece abierto si:

- Toda consecuencia común (o alguno de sus descendientes) es observable
- Ningúna otra variable es observable

Flujos de inferencia

	$V \notin$ Observable	$V \in$ Observable
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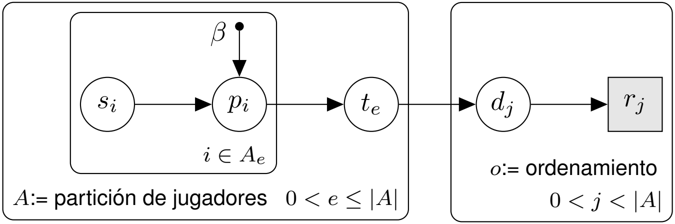
Pearl, J. 2018. The Book of Why: The New Science of Cause and Effect

TrueSkill

TrueSkill



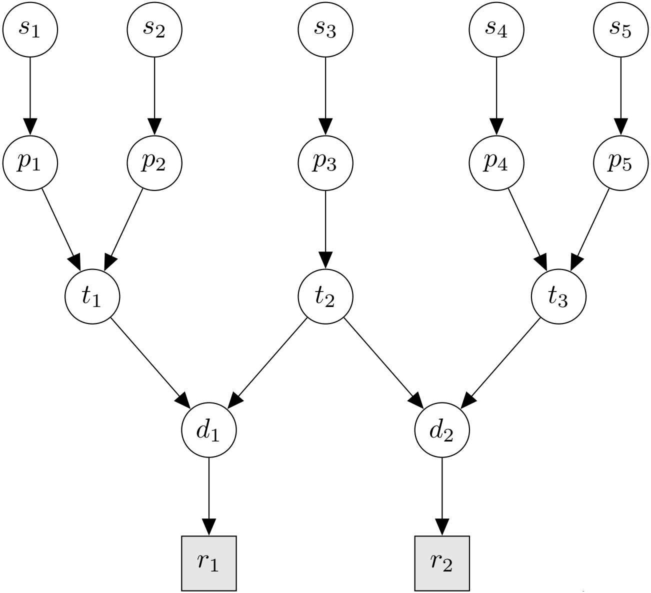
$$s_i \sim N(\mu_i, \sigma_i^2)$$

$$p_i \sim N(s_i, \beta^2)$$

$$t_e = \sum_{i \in A_e} p_i$$

$$d_j = t_{o_j} - t_{o_{j+1}}$$

$$r_j = d_j > 0$$



$$s_i \sim N(\mu_i, \sigma_i^2)$$

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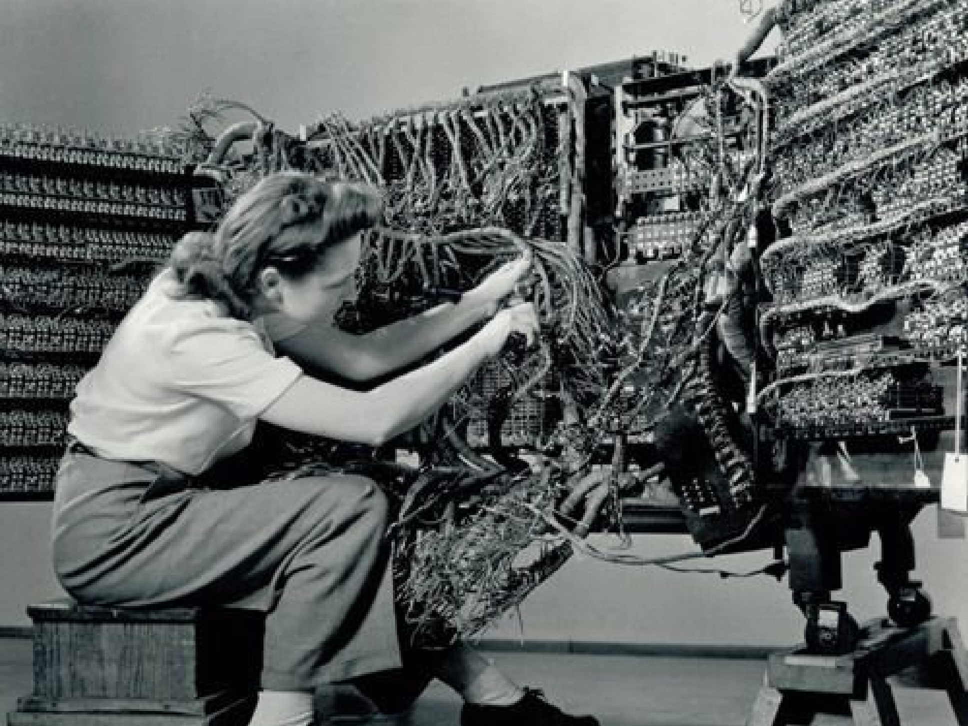

Factor graph

$$g(x_1, \ldots, x_n) = \prod f_j(X)$$

Factor graph

$$g(x_1, x_2, x_3, x_4, x_5) = f_A(x_1) f_B(x_2) f_C(x_1, x_2, x_3) f_D(x_3, x_4) f_E(x_3, x_5)$$

Factor graph

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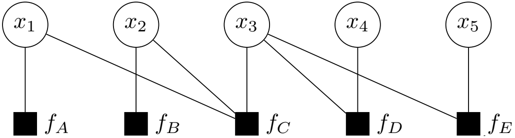

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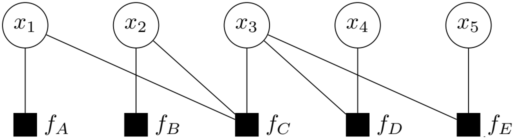
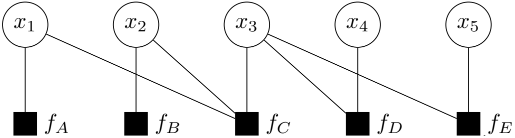


Un factor graph sin ciclos codifica:

- La factorización de una función $g(x_1, \ldots, x_n)$

Factor graph

$$g(x_1, x_2, x_3, x_4, x_5) = f_A(x_1) f_B(x_2) f_C(x_1, x_2, x_3) f_D(x_3, x_4) f_E(x_3, x_5)$$



Un factor graph sin ciclos codifica:

- La factorización de una función $g(x_1, \ldots, x_n)$
- **Las operaciones para computar sus marginales** $g_i(x_i)$

Ejemplo

$$g_1(x_1) = \sum_{x_2} \sum_{x_3} \sum_{x_4} \sum_{x_5} f_A(x_1) f_B(x_2) f_C(x_1, x_2, x_3) f_D(x_3, x_4) f_E(x_3, x_5)$$

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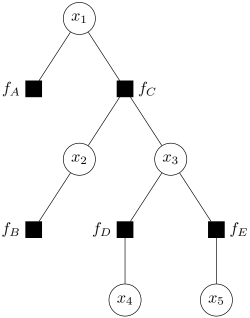
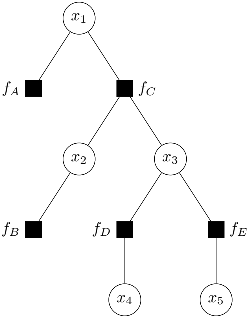
$$g_1(x_1) = \sum_{x_2} \sum_{x_3} \sum_{x_4} \sum_{x_5} f_A(x_1) f_B(x_2) f_C(x_1, x_2, x_3) f_D(x_3, x_4) f_E(x_3, x_5)$$



Ejemplo

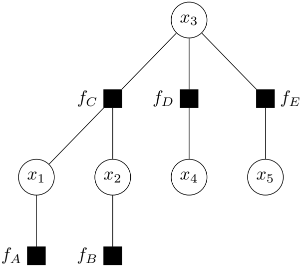
$$g_1(x_1) = \sum_{x_2} \sum_{x_3} \sum_{x_4} \sum_{x_5} f_A(x_1) f_B(x_2) f_C(x_1, x_2, x_3) f_D(x_3, x_4) f_E(x_3, x_5)$$



$$g_1(x_1) = f_A(x_1) \left(\sum_{x_2, x_3} f_B(x_2) f_C(x_1, x_2, x_3) \left(\sum_{x_4} f_D(x_3, x_4) \right) \left(\sum_{x_5} f_E(x_3, x_5) \right) \right)$$

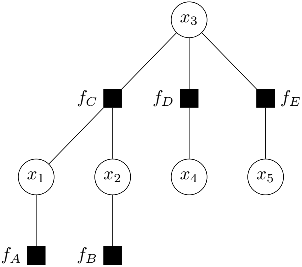
Ejemplo

$$g_3(x_3) = \sum_{x_1} \sum_{x_2} \sum_{x_4} \sum_{x_5} f_A(x_1) f_B(x_2) f_C(x_1, x_2, x_3) f_D(x_3, x_4) f_E(x_3, x_5)$$



Ejemplo

$$g_3(x_3) = \sum_{x_1}\sum_{x_2}\sum_{x_4}\sum_{x_5} f_A(x_1) f_B(x_2) f_C(x_1,x_2,x_3) f_D(x_3,x_4) f_E(x_3,x_5)$$



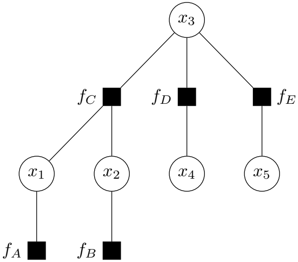
$$g_3(x_3) = \left(\sum_{x_1,x_2} f_A(x_1) f_B(x_2) f_C(x_1,x_2,x_3)\right)\left(\sum_{x_4} f_D(x_3,x_4)\right)\left(\sum_{x_5} f_E(x_3,x_5)\right)$$

Sum-product algorithm

$m_{x\to f}(x)$ **:** Mensaje enviado por el nodo variable x al nodo factor f

$m_{f\to x}(x)$ **:** Mensaje enviado por un nodo factor f a un nodo variable x.

$n(v)$ **:** Conjunto de nodos vecinos a v.



$$g_i(x_i) = \prod_{h\in n(x_i)} m_{h\to x_i}$$

Sum-product algorithm

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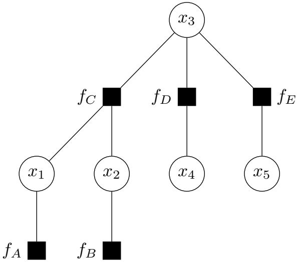

$$g_i(x_i) = \prod_{h \in n(x_i)} m_{h \to x_i}$$

$$m_{x \to f}(x) = \prod_{h \in n(x) \setminus \{f\}} m_{h \to x}(x) \qquad (1)$$

Sum-product algorithm

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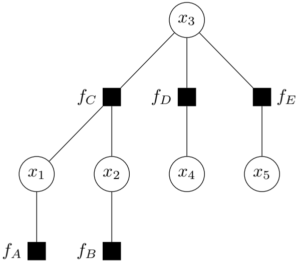

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$$m_{f\to x}(x) = \sum_{X\setminus\{x\}} \left(f(X) \prod_{h\in n(f)\setminus\{x\}} m_{h\to f}(h) \right) \qquad (2)$$

Sum-product algorithm

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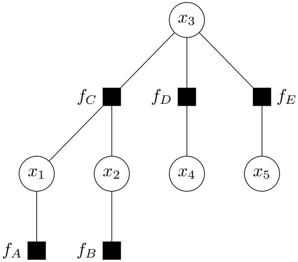

$$g_i(x_i) = \prod_{h \in n(x_i)} m_{h \to x_i}$$

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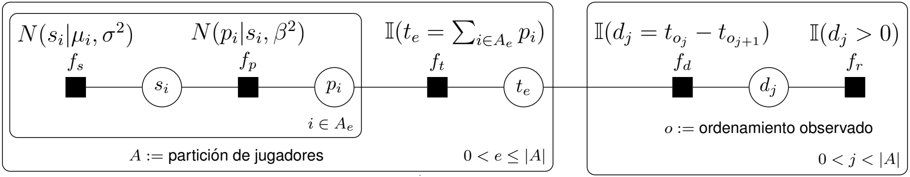
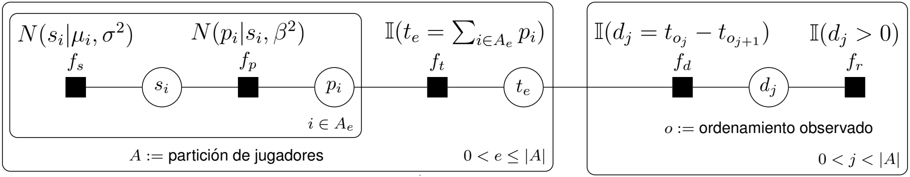
Kschischang FR, Frey BJ, Loeliger HA. Factor graphs and the sum-product algorithm. 2001

TrueSkill


$N(s_i|\mu_i,\sigma^2)$
$N(p_i|s_i,\beta^2)$
$\mathbb{I}(t_e = \sum_{i\in A_e} p_i)$
$\mathbb{I}(d_j = t_{o_j} - t_{o_{j+1}})$
$\mathbb{I}(d_j > 0)$
f_s
f_p
f_t
f_d
f_r
s_i
p_i
t_e
d_j
$i \in A_e$
$A :=$ partición de jugadores
$0 < e \leq |A|$
$o :=$ ordenamiento observado
$0 < j < |A|$

TrueSkill


$N(s_i|\mu_i, \sigma^2)$
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f_s
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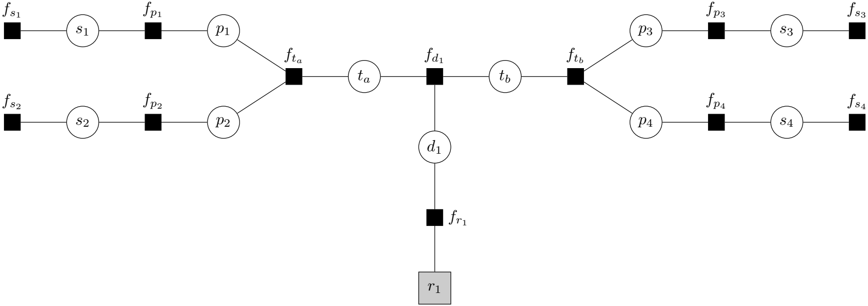
f_{s_1}
s_1
f_{p_1}
p_1
f_{s_2}
s_2
f_{p_2}
p_2
f_{t_a}
t_a
f_{d_1}
t_b
f_{t_b}
p_3
f_{p_3}
s_3
f_{s_3}
p_4
f_{p_4}
s_4
f_{s_4}
d_1
f_{r_1}
r_1

Propiedades

$$N(x|\mu, \sigma^2) = N(\mu|x, \sigma^2) = N(-\mu|-x, \sigma^2) = N(-x|-\mu, \sigma^2) \tag{3}$$

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$$\frac{\partial}{\partial x}\Phi(x|\mu,\sigma^2) = N(x|\mu,\sigma^2) \tag{5}$$

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$$\iint_{-\infty}^{\infty} \mathbb{I}(x = h(y,z))\, f(x)\, g(y)\, dx\, dy = \int_{-\infty}^{\infty} f(h(y,z))\, g(y)\, dy \tag{6}$$

Propiedades

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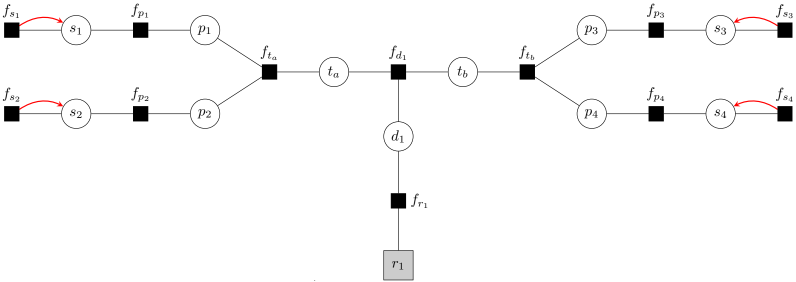
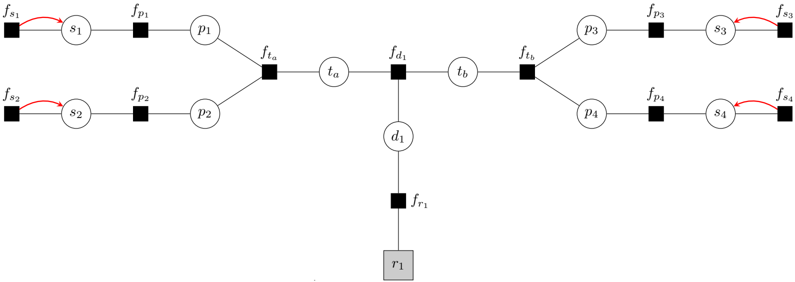
$$\int_{-\infty}^{\infty} N(x|\mu_x,\sigma_x^2) N(x|\mu_y,\sigma_y^2)\, dx \overset{*}{=} \int_{-\infty}^{\infty} \underbrace{N(\mu_x|\mu_y,\sigma_x^2+\sigma_y^2)}_{\text{constante}} \underbrace{N(x|\mu_*,\sigma_*^2)dx}_{\text{integra } 1} \tag{7}$$

$$m_{f_s \rightarrow s}(s)$$



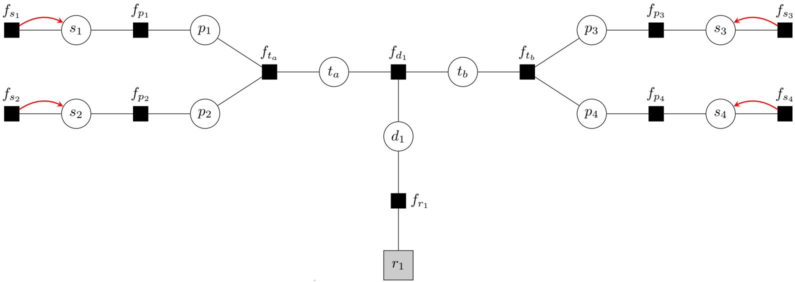
$$m_{f_{s_i} \rightarrow s_i}(s_i) =$$

$$m_{f_s \to s}(s)$$



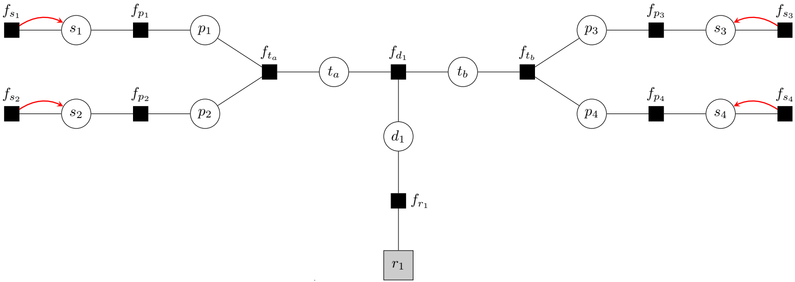
$$m_{f_{s_i} \to s_i}(s_i) = \int f_{s_i}(\boldsymbol{x}) \prod_{h \in n(f_{s_i}) \setminus \{s_i\}} m_{h \to f_{s_i}}(h) d\boldsymbol{x}_{\setminus \{s_i\}}$$

$$m_{f_s \to s}(s)$$



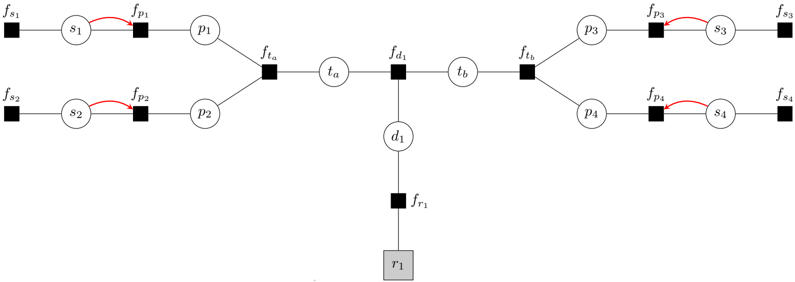
$$m_{f_{s_i} \to s_i}(s_i) = \int N(s_i | \mu_i, \sigma_i^2) d\boldsymbol{x}_{\setminus \{s_i\}}$$

$$m_{f_s \to s}(s)$$



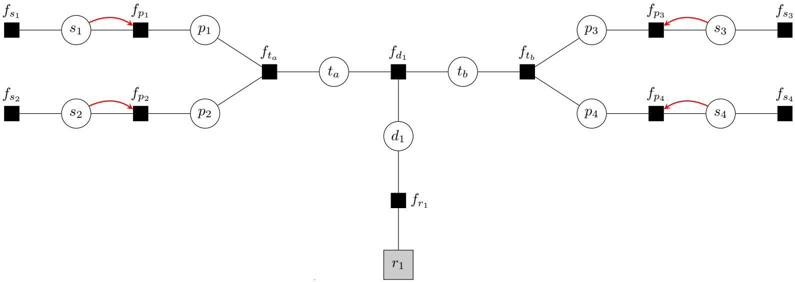
$$m_{f_{s_i} \to s_i}(s_i) = \int N(s_i|\mu_i, \sigma_i^2) d\boldsymbol{x}_{\setminus\{s_i\}} = N(s_i|\mu_i, \sigma_i^2)$$

$$m_{s \to f_p}(s)$$



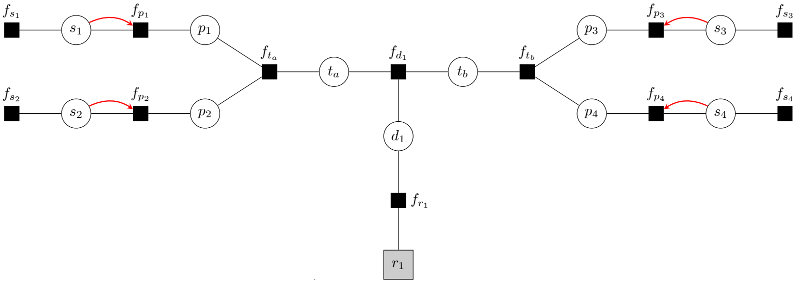
$$m_{s_i \to f_{p_i}}(s_i) =$$

$$m_{s\to f_p}(s)$$



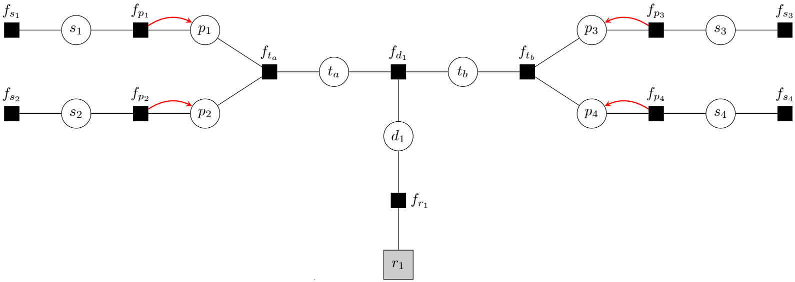
$$m_{s_i\to f_{p_i}}(s_i) = \prod_{h\in n(s_i)\setminus\{f_{p_i}\}} m_{h\to s_i}(s_i)$$

$$m_{s \to f_p}(s)$$



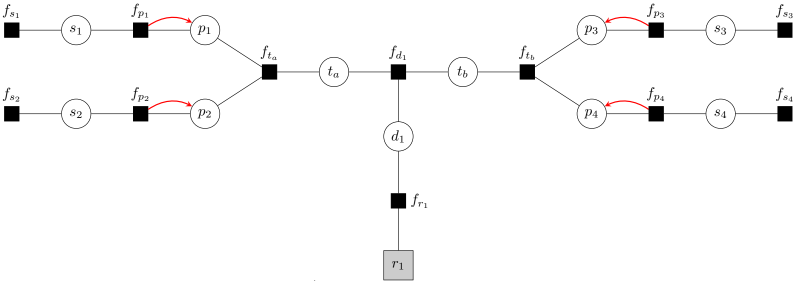
$$m_{s_i \to f_{p_i}}(s_i) = N(s_i | \mu_i, \sigma_i^2)$$

$$m_{f_p \rightarrow p}(p)$$



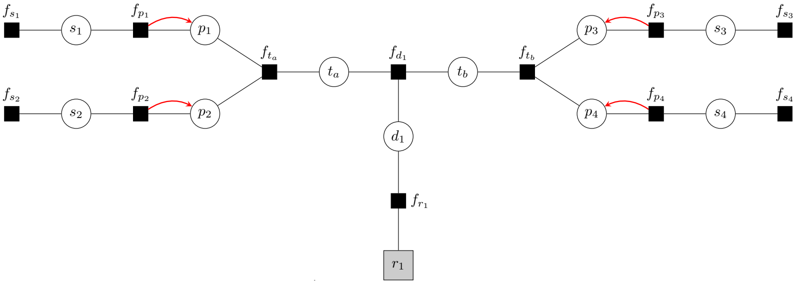
$$m_{f_{p_i} \rightarrow p_i}(p_i) =$$

$$m_{f_p \to p}(p)$$



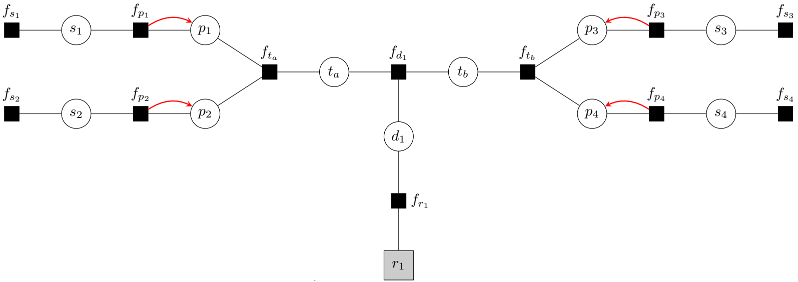
$$m_{f_{p_i} \to p_i}(p_i) = \int f_{p_i}(\boldsymbol{x}) \Big(\prod_{h \in n(f_{p_i}) \setminus \{p_i\}} m_{h \to f_{p_i}}(h) \Big) d\boldsymbol{x}_{\setminus \{p_i\}}$$

$$m_{f_p \to p}(p)$$



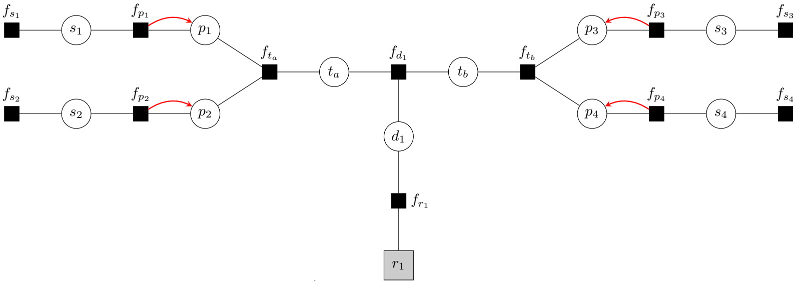
$$m_{f_{p_i} \to p_i}(p_i) = \int N(p_i | s_i, \beta^2)\, N(s_i | \mu_i, \sigma_i^2)\, ds_i$$

$$m_{f_p \to p}(p)$$



$$m_{f_{p_i} \to p_i}(p_i) = \int N(p_i|s_i, \beta^2)\, N(s_i|\mu_i, \sigma_i^2)\, ds_i = \int N(s_i|p_i, \beta^2)\, N(s_i|\mu_i, \sigma_i^2)\, ds_i$$

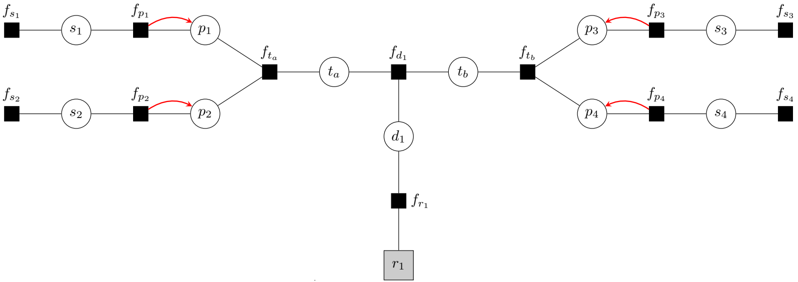
$$m_{f_p \to p}(p)$$



$$m_{f_{p_i} \to p_i}(p_i) = \int N(p_i|s_i,\beta^2)\, N(s_i|\mu_i,\sigma_i^2)\, ds_i = \int N(s_i|p_i,\beta^2)\, N(s_i|\mu_i,\sigma_i^2)\, ds_i$$

$$= \int \underbrace{N(p_i|\mu_i,\beta^2+\sigma_i^2)}_{\text{const.}}\, \underbrace{N(s_i|\mu_*,\sigma_*^2)\, ds_i}_{1}$$

$$m_{f_p \to p}(p)$$

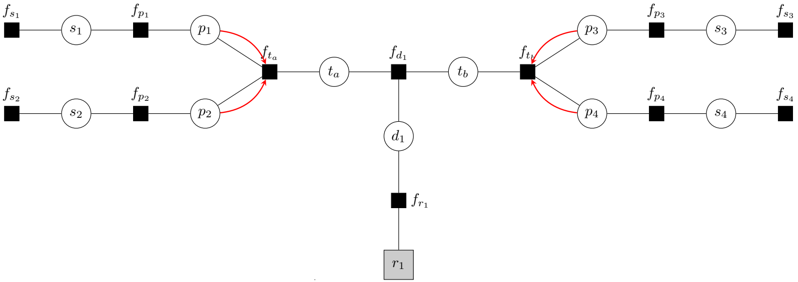


$$m_{f_{p_i} \to p_i}(p_i) = \int N(p_i|s_i,\beta^2)\, N(s_i|\mu_i,\sigma_i^2)\, ds_i = \int N(s_i|p_i,\beta^2)\, N(s_i|\mu_i,\sigma_i^2)\, ds_i$$

$$= \int \underbrace{N(p_i|\mu_i,\beta^2+\sigma_i^2)}_{\text{const.}}\, \underbrace{N(s_i|\mu_*,\sigma_*^2)\, ds_i}_{1}$$

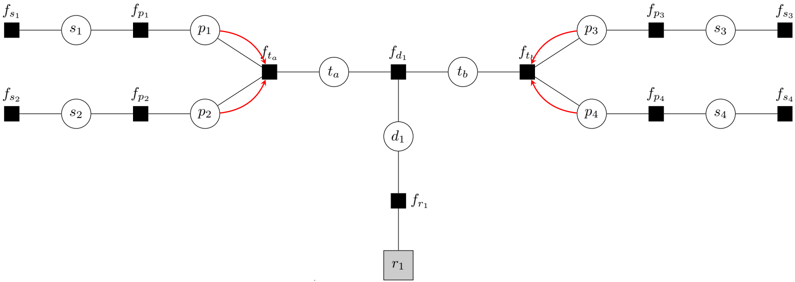
$$= N(p_i|\mu_i,\beta^2+\sigma_i^2)$$

$$m_{p \rightarrow f_t}(p)$$



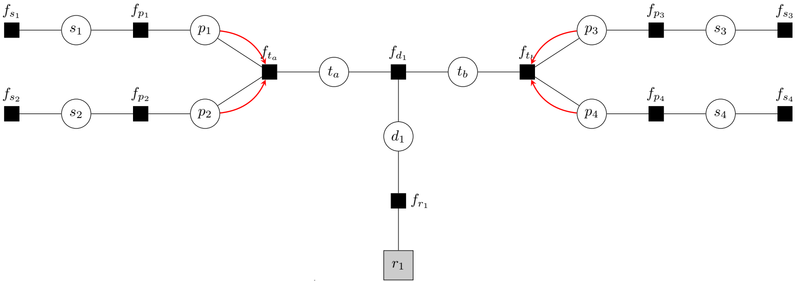
$$m_{p_i \rightarrow f_{t_e}}(p_i) =$$

$$m_{p \to f_t}(p)$$



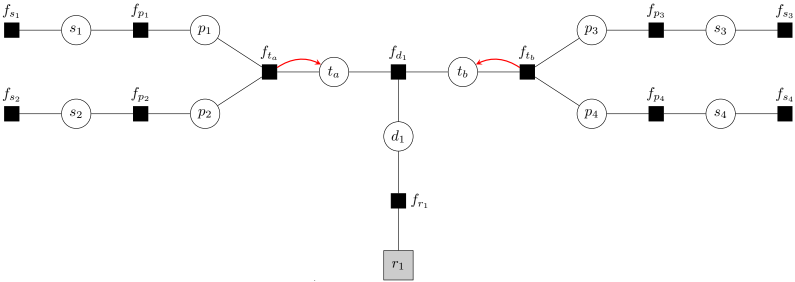
$$m_{p_i \to f_{t_e}}(p_i) = \prod_{h \in n(p_i) \setminus \{f_{t_e}\}} m_{h \to p_i}(p_i)$$

$$m_{p\to f_t}(p)$$



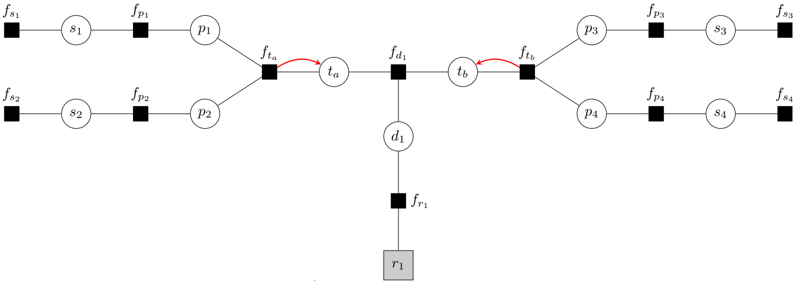
$$m_{p_i\to f_{t_e}}(p_i) = \prod_{i\in e} N(p_i|\mu_i, \beta^2+\sigma_i^2)$$

$$m_{f_t \to t}(t)$$



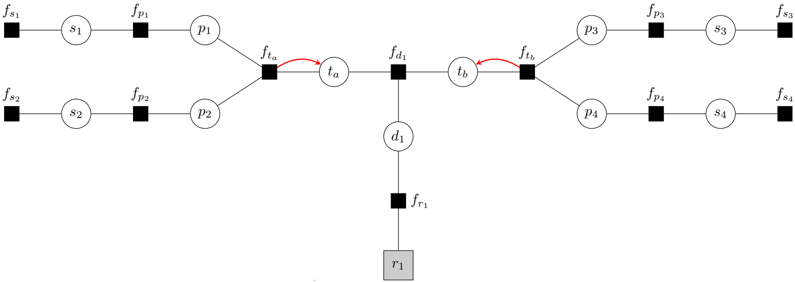
$$m_{f_{t_e} \to t_e}(t_e) =$$

$$m_{f_t \to t}(t)$$



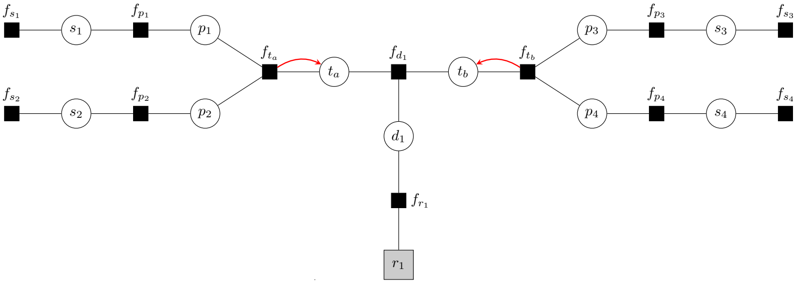
$$m_{f_{t_e} \to t_e}(t_e) = \int f_{t_e}(\boldsymbol{x}) \Big(\prod_{h \in n(f_{t_e}) \setminus \{t_e\}} m_{h \to f_{t_e}}(h) \Big) d\boldsymbol{x}_{\setminus \{t_e\}}$$

$$m_{f_t \to t}(t)$$



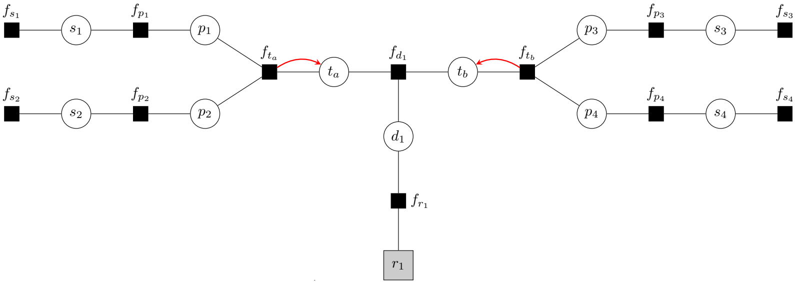
$$m_{f_{t_e} \to t_e}(t_e) = \iint \mathbb{I}(t_e = p_i + p_j) N(p_i | \mu_i, \beta^2 + \sigma_i^2) N(p_j | \mu_j, \beta^2 + \sigma_j^2) dp_i dp_j$$

$$m_{f_t \to t}(t)$$



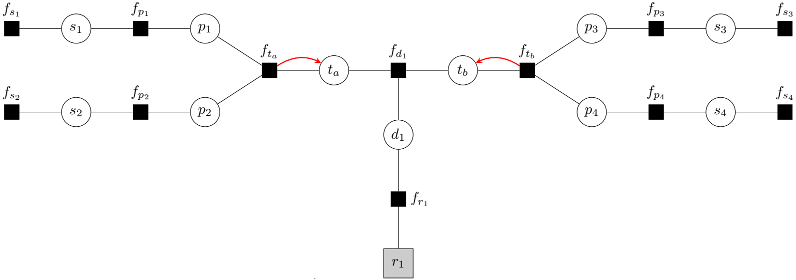
$$\begin{aligned} m_{f_{t_e} \to t_e}(t_e) &= \iint \mathbb{I}(t_e = p_i + p_j) N(p_i|\mu_i, \beta^2 + \sigma_i^2) N(p_j|\mu_j, \beta^2 + \sigma_j^2) dp_i dp_j \\ &= \int N(p_i|\mu_i, \beta^2 + \sigma_i^2) N(t_e - p_i|\mu_j, \beta^2 + \sigma_j^2) dp_i \end{aligned}$$

$$m_{f_t \to t}(t)$$



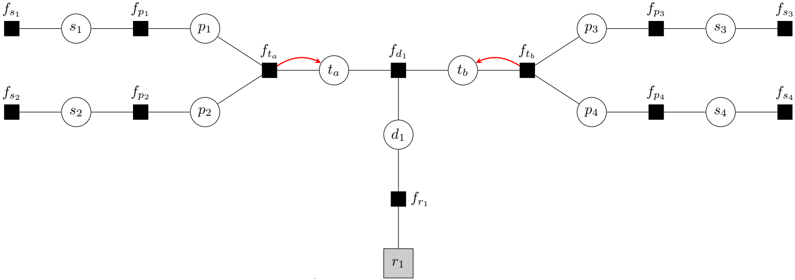
$$m_{f_{t_e} \to t_e}(t_e) = \iint \mathbb{I}(t_e = p_i + p_j) N(p_i | \mu_i, \beta^2 + \sigma_i^2) N(p_j | \mu_j, \beta^2 + \sigma_j^2) dp_i dp_j$$
$$= \int N(p_i | \mu_i, \beta^2 + \sigma_i^2) N(p_i | t_e - \mu_j, \beta^2 + \sigma_j^2) dp_i$$

$$m_{f_t \to t}(t)$$



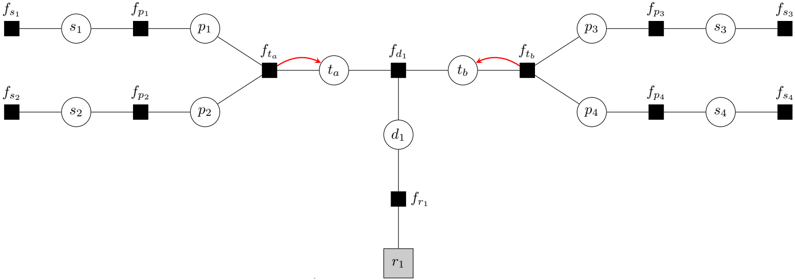
$$\begin{aligned} m_{f_{t_e} \to t_e}(t_e) &= \iint \mathbb{I}(t_e = p_i + p_j) N(p_i|\mu_i, \beta^2 + \sigma_i^2) N(p_j|\mu_j, \beta^2 + \sigma_j^2) dp_i dp_j \\ &= \int N(p_i|\mu_i, \beta^2 + \sigma_i^2) N(p_i|t_e - \mu_j, \beta^2 + \sigma_j^2) dp_i \\ &= \int \underbrace{N(t_e|\mu_i + \mu_j, 2\beta^2 + \sigma_i^2 + \sigma_j^2)}_{\text{const.}} \underbrace{N(p_i|\mu_*, \sigma_*^2) dp_i}_{1} \end{aligned}$$

$$m_{f_t \to t}(t)$$



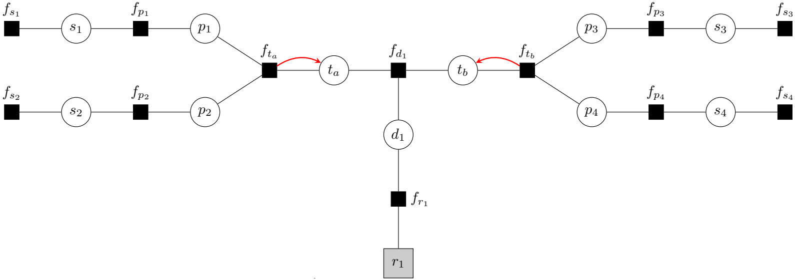
$$\begin{aligned} m_{f_{t_e} \to t_e}(t_e) &= \iint \mathbb{I}(t_e = p_i + p_j) N(p_i|\mu_i, \beta^2 + \sigma_i^2) N(p_j|\mu_j, \beta^2 + \sigma_j^2) dp_i dp_j \\ &= \int N(p_i|\mu_i, \beta^2 + \sigma_i^2) N(p_i|t_e - \mu_j, \beta^2 + \sigma_j^2) dp_i \\ &= N(t_e|\mu_i + \mu_j, 2\beta^2 + \sigma_i^2 + \sigma_j^2) \end{aligned}$$

$$m_{f_t \to t}(t)$$



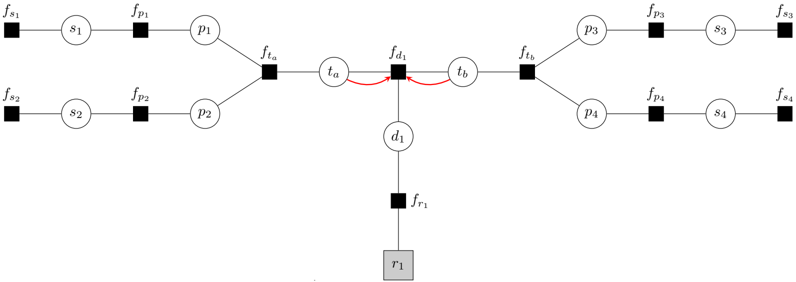
$$\begin{aligned} m_{f_{t_e} \to t_e}(t_e) &= \iint \mathbb{I}(t_e = p_i + p_j) N(p_i|\mu_i, \beta^2 + \sigma_i^2) N(p_j|\mu_j, \beta^2 + \sigma_j^2) dp_i dp_j \\ &= \int N(p_i|\mu_i, \beta^2 + \sigma_i^2) N(p_i|t_e - \mu_j, \beta^2 + \sigma_j^2) dp_i \\ &= N(t_e| \sum_{i \in A_e} \mu_i, \sum_{i \in A_e} \beta^2 + \sigma_i^2) \end{aligned}$$

$$m_{f_t \to t}(t)$$



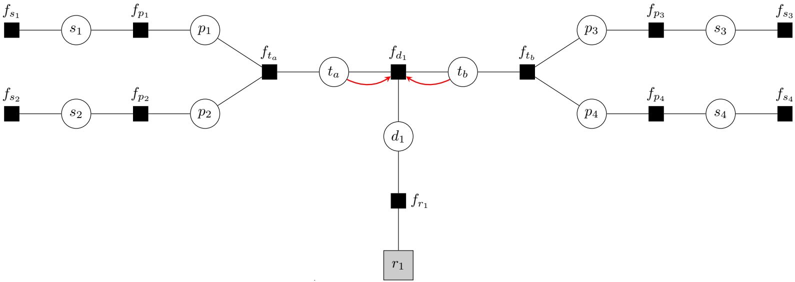
$$\begin{aligned}
m_{f_{t_e} \to t_e}(t_e) &= \iint \mathbb{I}(t_e = p_i + p_j) N(p_i|\mu_i, \beta^2 + \sigma_i^2) N(p_j|\mu_j, \beta^2 + \sigma_j^2) dp_i dp_j \\
&= \int N(p_i|\mu_i, \beta^2 + \sigma_i^2) N(p_i|t_e - \mu_j, \beta^2 + \sigma_j^2) dp_i \\
&= N(t_e| \sum_{i \in A_e} \mu_i, \sum_{i \in A_e} \beta^2 + \sigma_i^2) = N(t_e|\mu_e, \sigma_e^2)
\end{aligned}$$

$$m_{t \rightarrow f_d}(t)$$



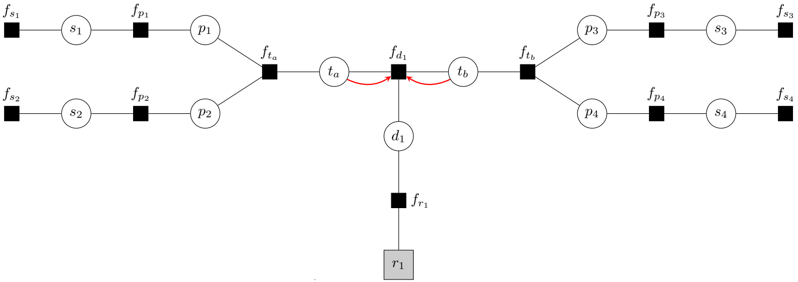
$$m_{t_e \rightarrow f_d}(t_e) =$$

$$m_{t \to f_d}(t)$$



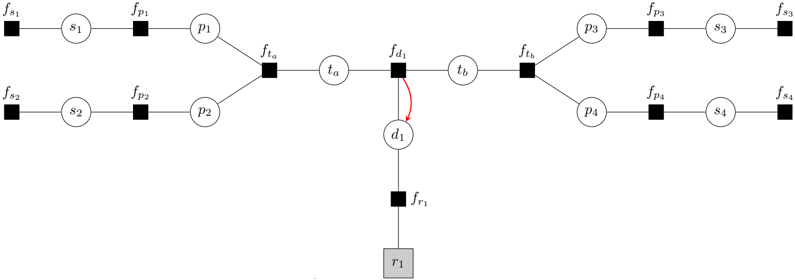
$$m_{t_e \to f_d}(t_e) = \prod_{h \in n(t_e) \setminus \{f_d\}} m_{h \to t_e}(t_e)$$

$$m_{t \to f_d}(t)$$



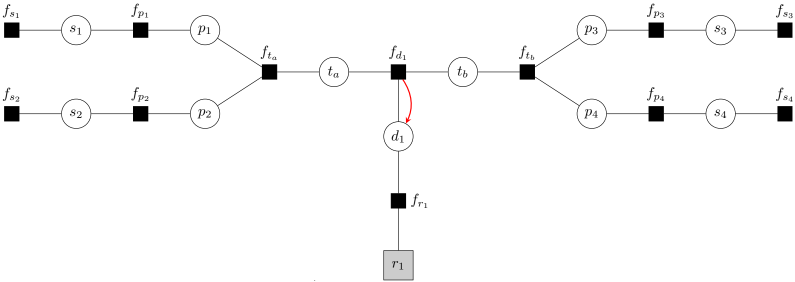
$$m_{t_e \to f_d}(t_e) = N(t_e|\mu_e, \sigma_e^2)$$

$$m_{f_d \to d}(d)$$



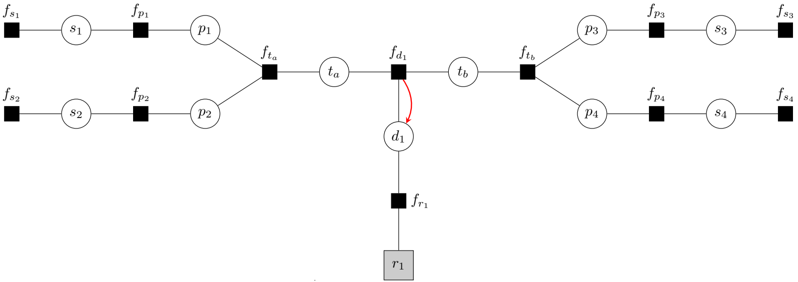
$$m_{f_d \to d}(d) =$$

$$m_{f_d \to d}(d)$$



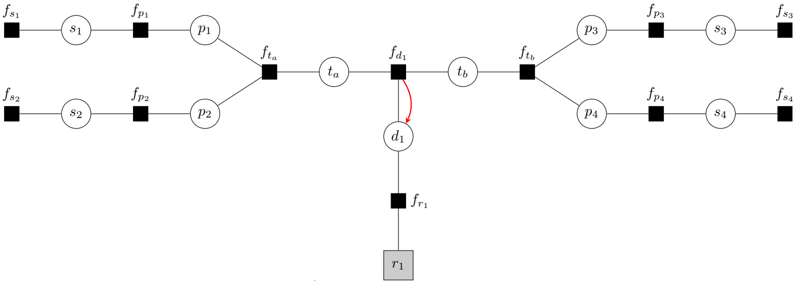
$$m_{f_d \to d}(d) = \int f_d(\boldsymbol{x}) \Big(\prod_{h \in n(f_d) \setminus \{d\}} m_{h \to f_d}(h) \Big) d\boldsymbol{x}_{\setminus \{d\}}$$

$$m_{f_d \to d}(d)$$



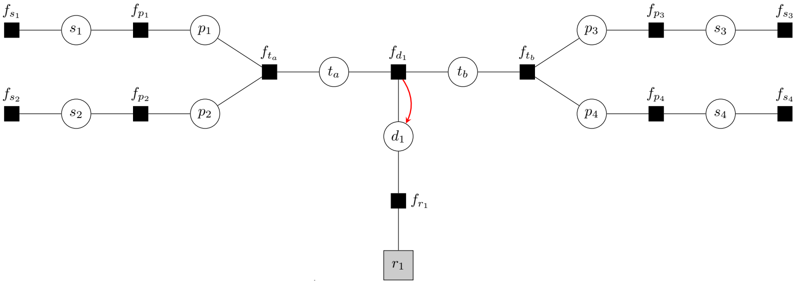
$$m_{f_d \to d}(d) = \iint \mathbb{I}(d = t_a - t_b) N(t_a|\mu_a, \sigma_a^2) N(t_b|\mu_b, \sigma_b^2)\, dt_a dt_b$$

$$m_{f_d \to d}(d)$$



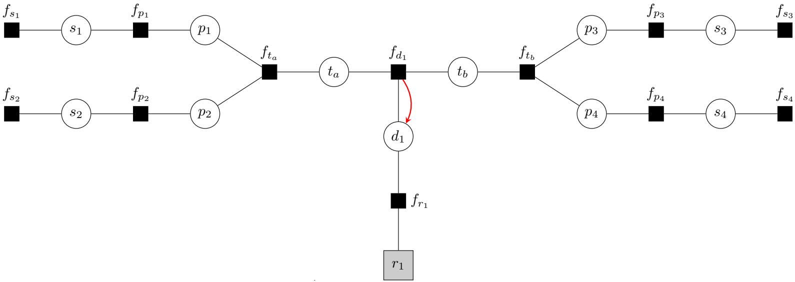
$$\begin{aligned} m_{f_d \to d}(d) &= \iint \mathbb{I}(d = t_a - t_b) N(t_a|\mu_a, \sigma_a^2) N(t_b|\mu_b, \sigma_b^2)\, dt_a dt_b \\ &= \int N(d + t_b|\mu_a, \sigma_a^2) N(t_b|\mu_b, \sigma_b^2)\, dt_b \end{aligned}$$

$$m_{f_d \to d}(d)$$



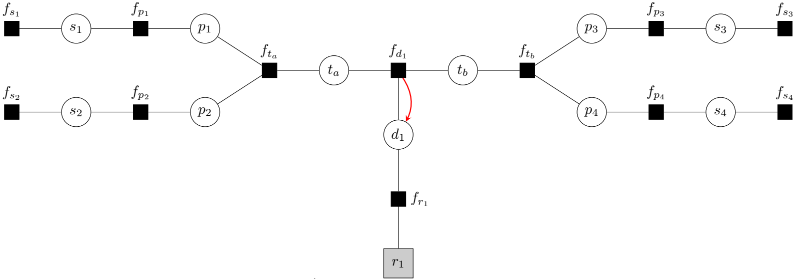
$$\begin{aligned} m_{f_d \to d}(d) &= \iint \mathbb{I}(d = t_a - t_b) N(t_a|\mu_a, \sigma_a^2) N(t_b|\mu_b, \sigma_b^2)\, dt_a dt_b \\ &= \int N(t_b|\mu_a - d, \sigma_a^2) N(t_b|\mu_b, \sigma_b^2)\, dt_b \end{aligned}$$

$$m_{f_d \to d}(d)$$



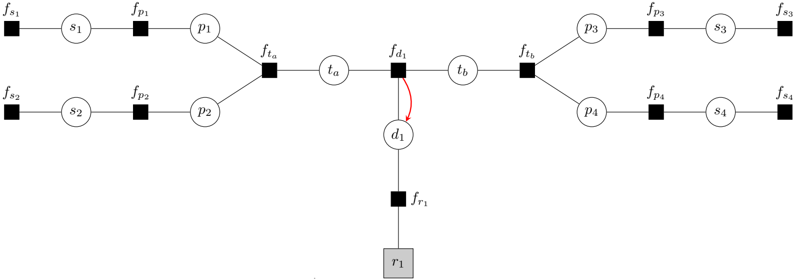
$$\begin{aligned} m_{f_d \to d}(d) &= \iint \mathbb{I}(d = t_a - t_b) N(t_a|\mu_a, \sigma_a^2) N(t_b|\mu_b, \sigma_b^2)\, dt_a dt_b \\ &= \int N(t_b|\mu_a - d, \sigma_a^2) N(t_b|\mu_b, \sigma_b^2)\, dt_b \\ &= \int \underbrace{N(d|\mu_a - \mu_b, \sigma_a^2 + \sigma_b^2)}_{\text{const.}} \underbrace{N(t_b|\mu_*, \sigma_*^2)\, dt_b}_{1} \end{aligned}$$

$$m_{f_d \to d}(d)$$



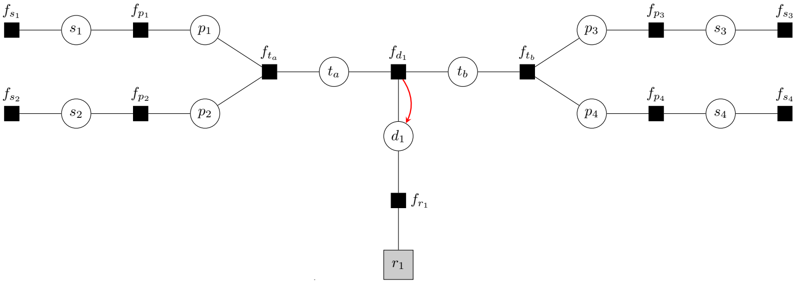
$$\begin{aligned} m_{f_d \to d}(d) &= \iint \mathbb{I}(d = t_a - t_b) N(t_a|\mu_a, \sigma_a^2) N(t_b|\mu_b, \sigma_b^2)\, dt_a dt_b \\ &= \int N(t_b|\mu_a - d, \sigma_a^2) N(t_b|\mu_b, \sigma_b^2)\, dt_b \\ &= N(d|\, \mu_a - \mu_b, \sigma_a^2 + \sigma_b^2) \end{aligned}$$

$$m_{f_d \to d}(d)$$



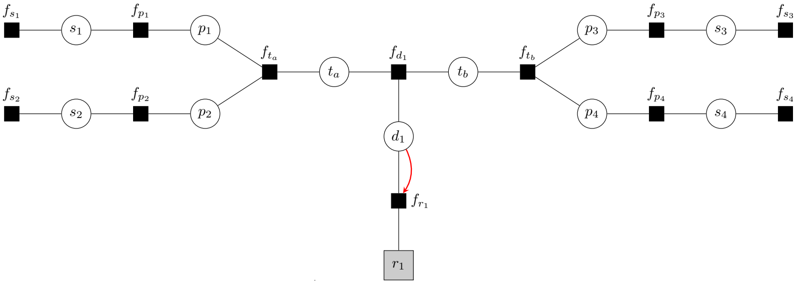
$$\begin{aligned} m_{f_d \to d}(d) &= \iint \mathbb{I}(d = t_a - t_b) N(t_a|\mu_a, \sigma_a^2) N(t_b|\mu_b, \sigma_b^2)\, dt_a dt_b \\ &= \int N(t_b|\mu_a - d, \sigma_a^2) N(t_b|\mu_b, \sigma_b^2)\, dt_b \\ &= N(d|\underbrace{\mu_a - \mu_b}_{\text{Diferencia esperada } \delta}, \underbrace{\sigma_a^2 + \sigma_b^2}_{\text{Varianza total } \vartheta}) \end{aligned}$$

$$m_{f_d \to d}(d)$$



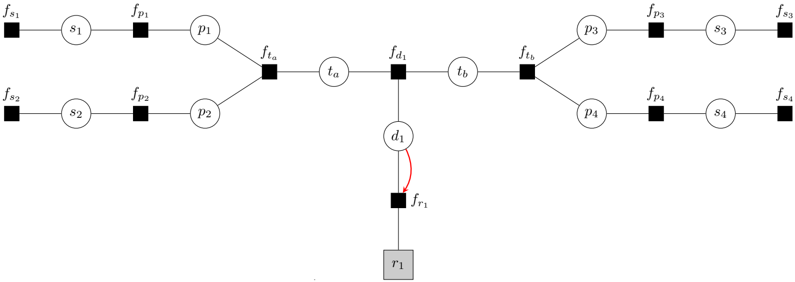
$$\begin{aligned} m_{f_d \to d}(d) &= \iint \mathbb{I}(d = t_a - t_b) N(t_a|\mu_a, \sigma_a^2) N(t_b|\mu_b, \sigma_b^2)\, dt_a dt_b \\ &= \int N(t_b|\mu_a - d, \sigma_a^2) N(t_b|\mu_b, \sigma_b^2)\, dt_b \\ &= N(d|\underbrace{\mu_a - \mu_b}_{\text{Diferencia esperada } \delta}, \underbrace{\sigma_a^2 + \sigma_b^2}_{\text{Varianza total } \vartheta}) = N(d|\delta, \vartheta^2) \end{aligned}$$

$$m_{d \to f_r}(d)$$



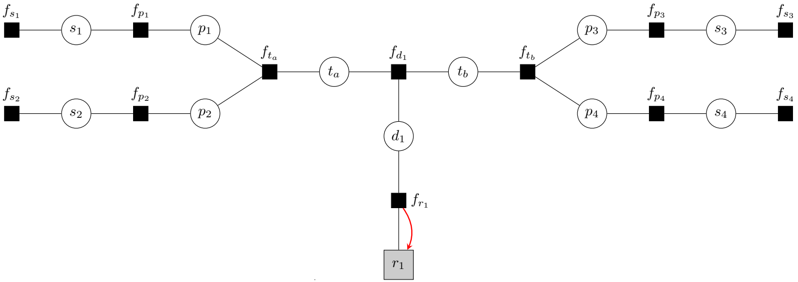
$$m_{t_e \to f_d}(t_e) =$$

$$m_{d \to f_r}(d)$$



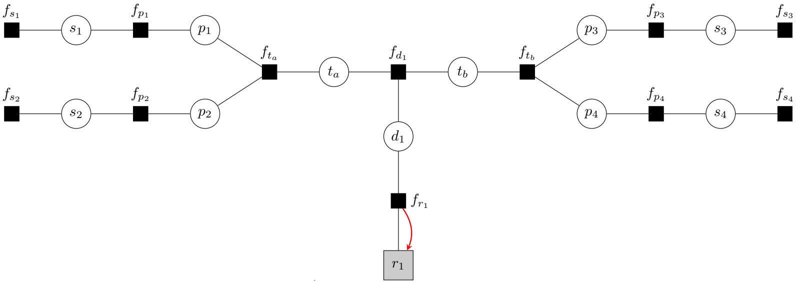
$$m_{t_e \to f_d}(t_e) = N(d|\delta, \vartheta^2)$$

$$m_{f_r \to r}(r)$$



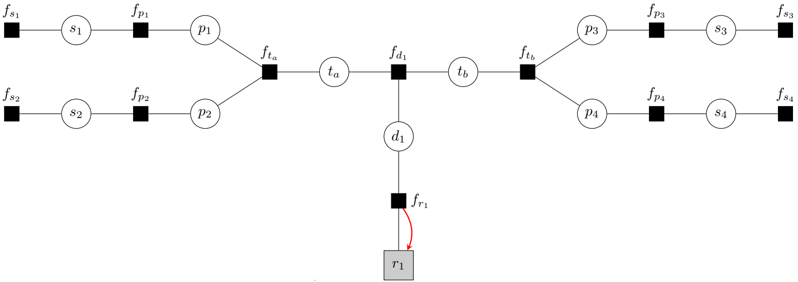
$$m_{f_r \to r}(r) =$$

$$m_{f_r \to r}(r)$$



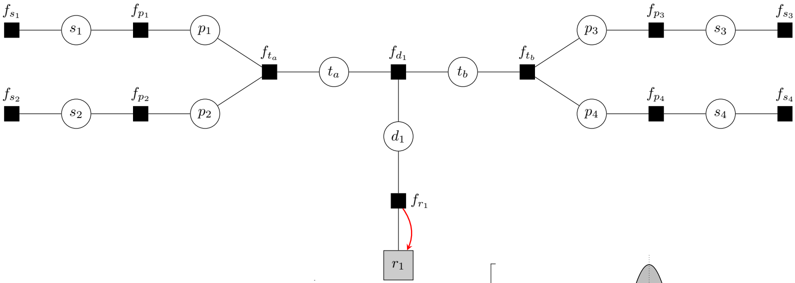
$$m_{f_r \to r}(r) = \int \mathbb{I}(d > 0) N(d|\delta, \vartheta^2)\, dd$$

$$m_{f_r \to r}(r)$$

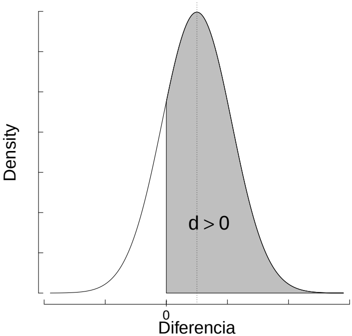


$$\begin{aligned} m_{f_r \to r}(r) &= \int \mathbb{I}(d > 0) N(d|\delta, \vartheta^2)\, dd \\ &= \int_0^\infty N(d|\delta, \vartheta^2)\, dd \end{aligned}$$

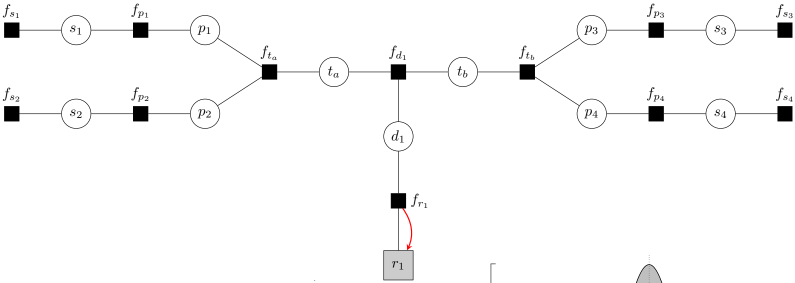
$$m_{f_r \to r}(r)$$



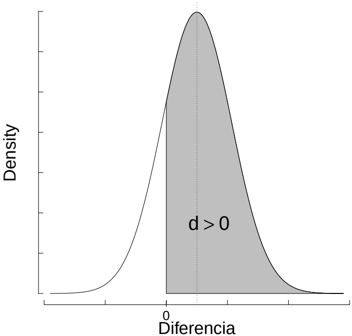
$$\begin{aligned} m_{f_r \to r}(r) &= \int \mathbb{I}(d > 0) N(d|\delta, \vartheta^2)\, dd \\ &= \int_0^{\infty} N(d|\delta, \vartheta^2)\, dd \end{aligned}$$



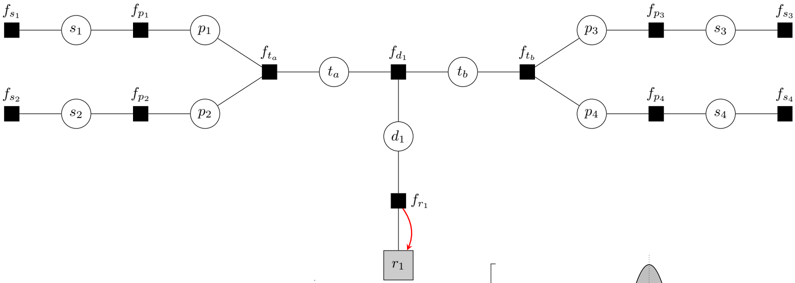
$$m_{f_r \to r}(r)$$



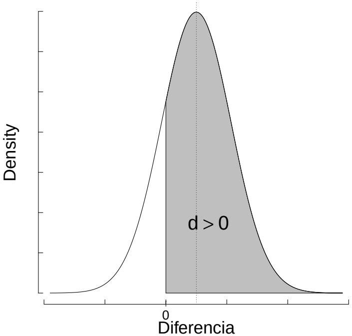
$$\begin{aligned} m_{f_r \to r}(r) &= \int \mathbb{I}(d > 0) N(d|\delta, \vartheta^2)\, dd \\ &= \int_0^\infty N(d|\delta, \vartheta^2)\, dd \\ &= 1 - \Phi(0|\delta, \vartheta^2) \end{aligned}$$



$$m_{f_r \rightarrow r}(r)$$



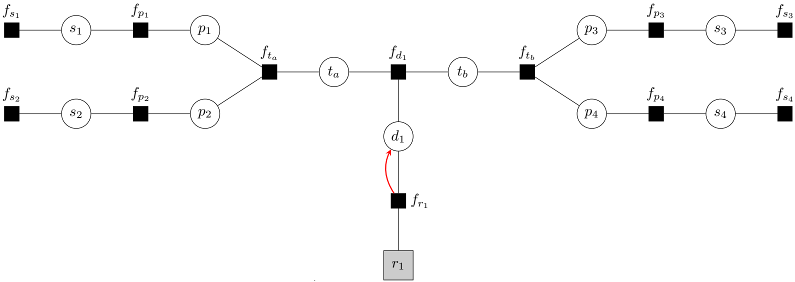
$$\begin{aligned} m_{f_r \rightarrow r}(r) &= \int \mathbb{I}(d > 0) N(d|\delta, \vartheta^2)\, dd \\ &= \int_0^{\infty} N(d|\delta, \vartheta^2)\, dd \\ &= 1 - \Phi(0|\delta, \vartheta^2) = \Phi\left(\frac{\delta}{\vartheta}\right) \end{aligned}$$



Evidencia

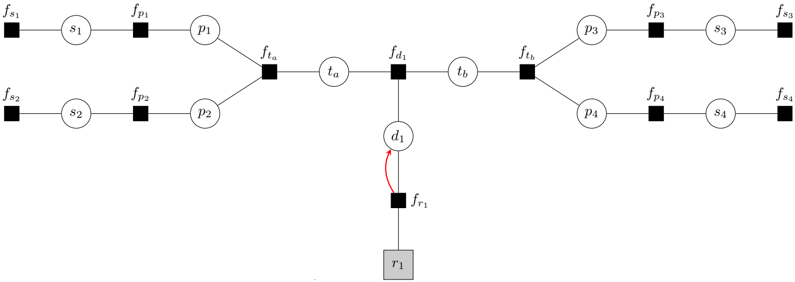
$$P(r|s,A) = \Phi\left(\frac{\delta}{\vartheta}\right) \tag{8}$$

$$m_{f_r \to d}(d)$$



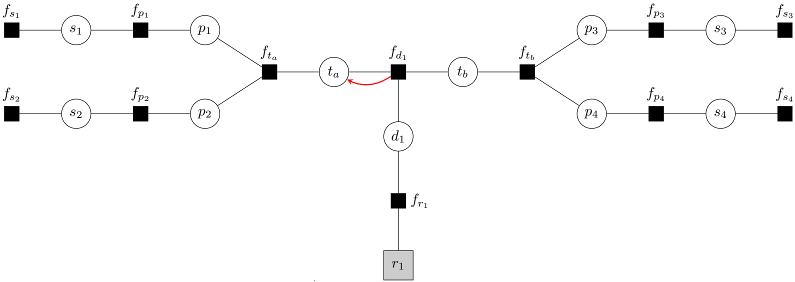
$$m_{f_r \to d}(d) =$$

$$m_{f_r \to d}(d)$$



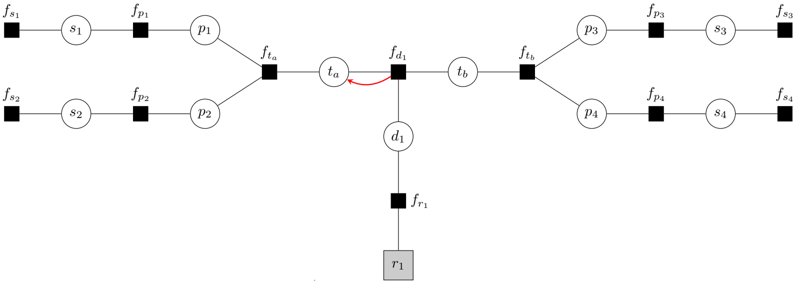
$$m_{f_r \to d}(d) = \mathbb{I}(d > 0)$$

$$m_{f_d \to t_a}(t_a)$$



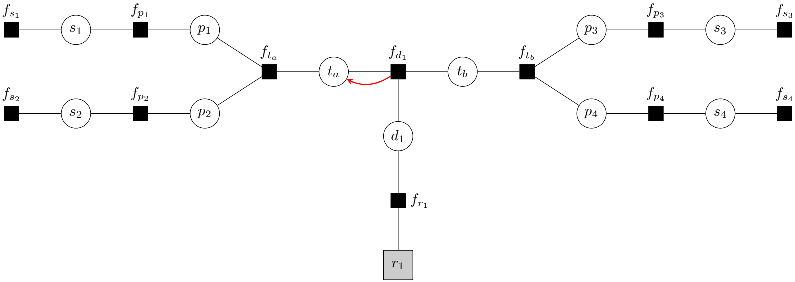
$$m_{f_d \to t_a}(t_a) =$$

$$m_{f_d \to t_a}(t_a)$$



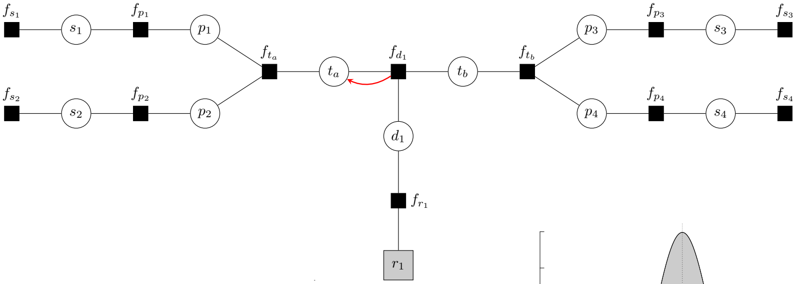
$$m_{f_d \to t_a}(t_a) = \iint \mathbb{I}(d = t_a - t_b)\mathbb{I}(d > 0) N(t_b|\mu_b, \sigma_b^2)\, dt_b\, dd$$

$$m_{f_d \to t_a}(t_a)$$

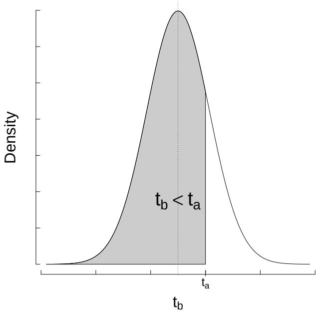


$$m_{f_d \to t_a}(t_a) = \int \mathbb{I}(t_a > t_b) N(t_b | \mu_b, \sigma_b^2) \, dt_b \, dd$$

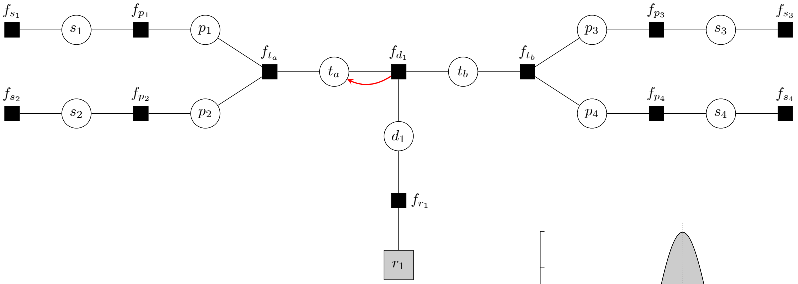
$$m_{f_d \to t_a}(t_a)$$



$$m_{f_d \to t_a}(t_a) = \int \mathbb{I}(t_a > t_b) N(t_b|\mu_b, \sigma_b^2)\, dt_b\, dd$$

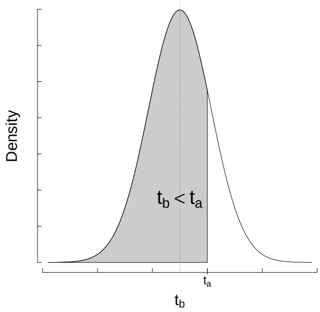


$$m_{f_d \to t_a}(t_a)$$

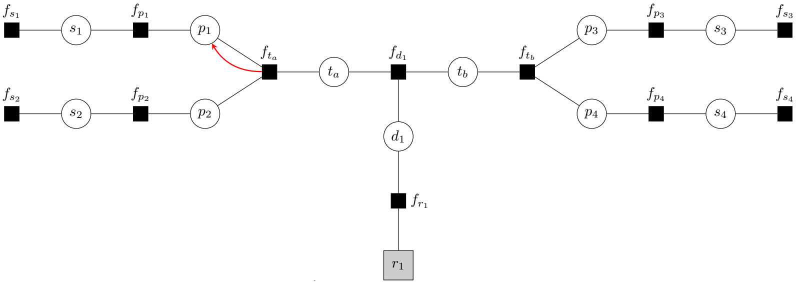


$$m_{f_d \to t_a}(t_a) = \int \mathbb{I}(t_a > t_b) N(t_b|\mu_b, \sigma_b^2)\, dt_b\, dd$$

$$= \Phi(t_a|\mu_b, \sigma_b^2)$$

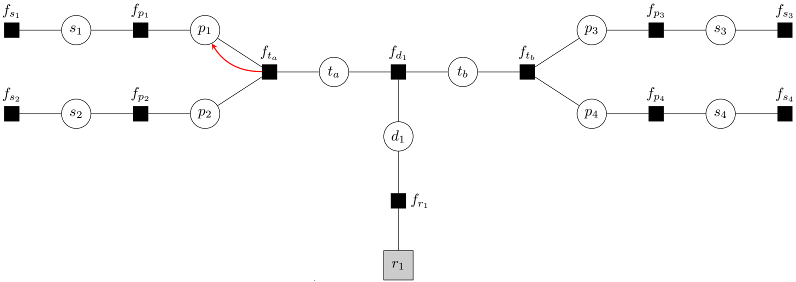


$$m_{f_t \to p}(p)$$



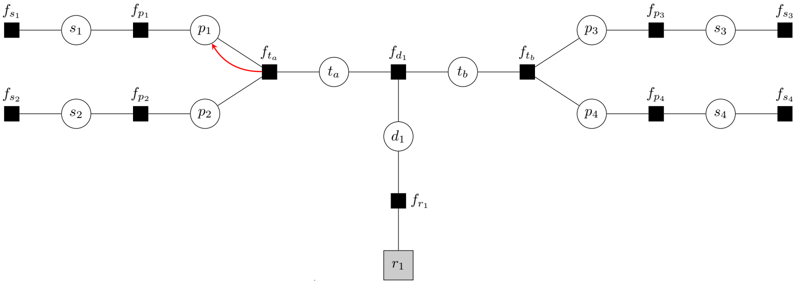
$$m_{f_{t_a} \to p_1}(p_1) =$$

$$m_{f_t \to p}(p)$$



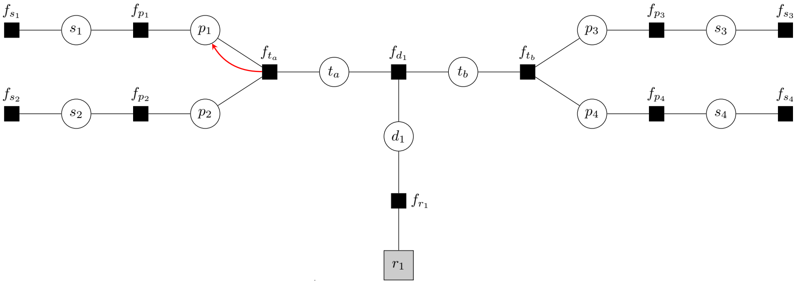
$$m_{f_{t_a} \to p_1}(p_1) = \iint \mathbb{I}(t_a = p_1 + p_2) N(p_2|\mu_2, \sigma_2^2 + \beta^2)\, \Phi(t_a|\mu_b, \sigma_b^2)\, dt_a\, dp_2$$

$$m_{f_t \to p}(p)$$



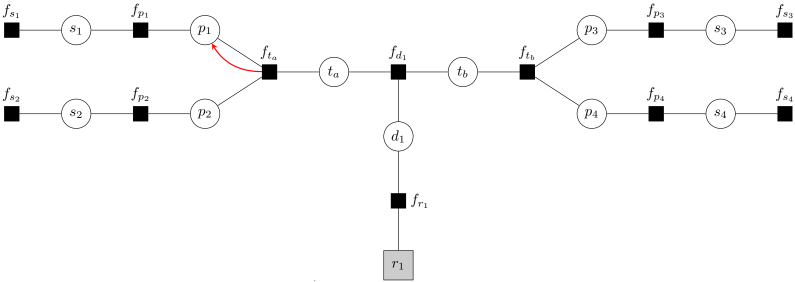
$$m_{f_{t_a} \to p_1}(p_1) = \int N(p_2|\mu_2, \sigma_2^2 + \beta^2)\, \Phi(p_1 + p_2|\mu_b, \sigma_b^2)\, dp_2$$

$$m_{f_t \to p}(p)$$



$$m_{f_{t_a} \to p_1}(p_1) = \int N(p_2|\mu_2, \sigma_2^2 + \beta^2)\, \Phi(p_1|\mu_b - p_2, \sigma_b^2)\, dp_2$$

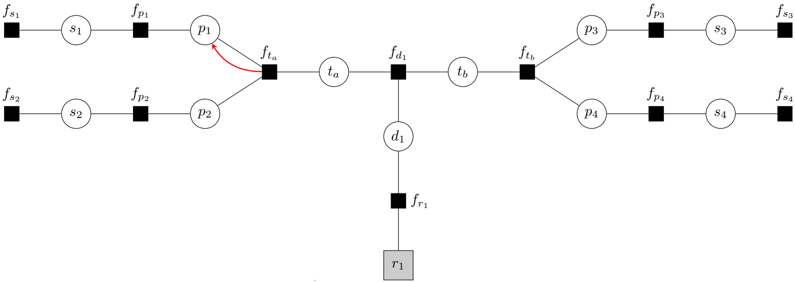
$$m_{f_t \to p}(p)$$



$$m_{f_{t_a} \to p_1}(p_1) = \int N(p_2|\mu_2, \sigma_2^2 + \beta^2)\, \Phi(p_1|\mu_b - p_2, \sigma_b^2)\, dp_2$$

$$\frac{\partial}{\partial p_1} m_{f_{t_a} \to p_1}(p_1) = \frac{\partial}{\partial p_1} \int N(p_2|\mu_2, \sigma_2^2 + \beta^2)\, \Phi(p_1|\mu_b - p_2, \sigma_b^2)\, dp_2$$

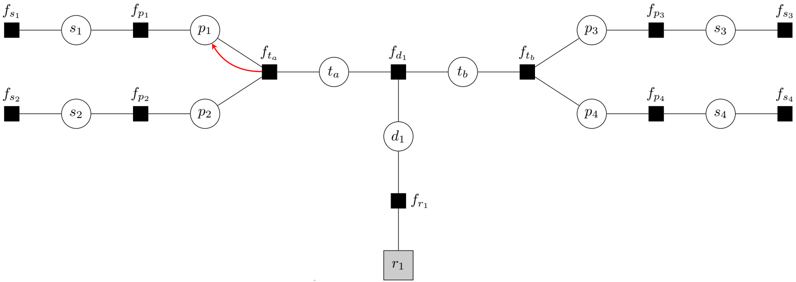
$$m_{f_t \to p}(p)$$



$$m_{f_{t_a} \to p_1}(p_1) = \int N(p_2|\mu_2, \sigma_2^2 + \beta^2)\, \Phi(p_1|\mu_b - p_2, \sigma_b^2)\, dp_2$$

$$\frac{\partial}{\partial p_1} m_{f_{t_a} \to p_1}(p_1) = \int N(p_2|\mu_2, \sigma_2^2 + \beta^2)\, \frac{\partial}{\partial p_1} \Phi(p_1|\mu_b - p_2, \sigma_b^2)\, dp_2$$

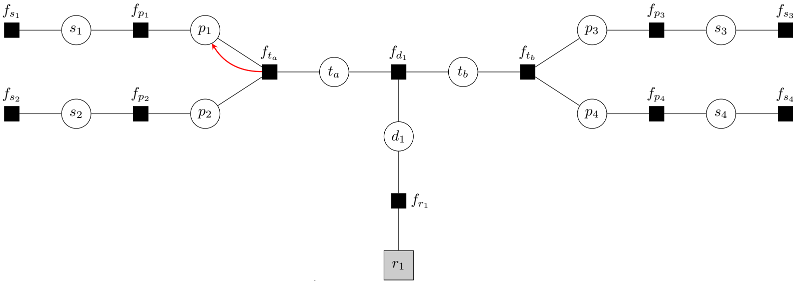
$$m_{f_t \to p}(p)$$



$$m_{f_{t_a} \to p_1}(p_1) = \int N(p_2|\mu_2, \sigma_2^2 + \beta^2)\, \Phi(p_1|\mu_b - p_2, \sigma_b^2)\, dp_2$$

$$\frac{\partial}{\partial p_1} m_{f_{t_a} \to p_1}(p_1) = \int N(p_2|\mu_2, \sigma_2^2 + \beta^2)\, N(p_1|\mu_b - p_2, \sigma_b^2)\, dp_2$$

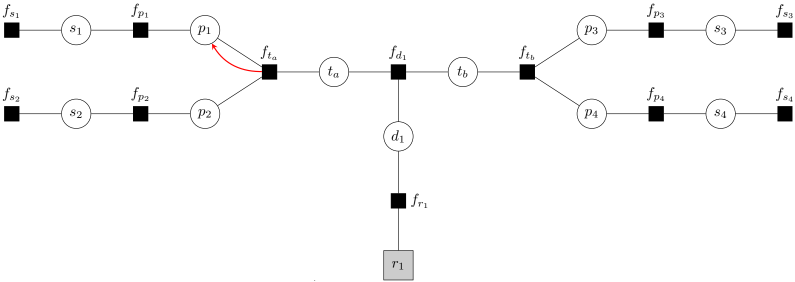
$$m_{f_t \to p}(p)$$



$$m_{f_{t_a} \to p_1}(p_1) = \int N(p_2|\mu_2, \sigma_2^2 + \beta^2)\, \Phi(p_1|\mu_b - p_2, \sigma_b^2)\, dp_2$$

$$\frac{\partial}{\partial p_1} m_{f_{t_a} \to p_1}(p_1) = \int N(p_2|\mu_2, \sigma_2^2 + \beta^2)\, N(p_2|\mu_b - p_1, \sigma_b^2)\, dp_2$$

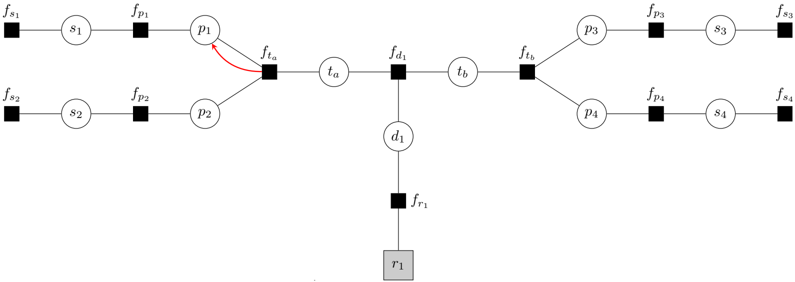
$$m_{f_t \to p}(p)$$



$$m_{f_{t_a} \to p_1}(p_1) = \int N(p_2|\mu_2, \sigma_2^2 + \beta^2)\, \Phi(p_1|\mu_b - p_2, \sigma_b^2)\, dp_2$$

$$\frac{\partial}{\partial p_1} m_{f_{t_a} \to p_1}(p_1) = \int \underbrace{N(\mu_2|\mu_b - p_1, \sigma_b^2 + \sigma_2^2 + \beta^2)}_{\text{const.}} \underbrace{N(p_2|\mu_*, \sigma_*^2) dp_2}_{1}$$

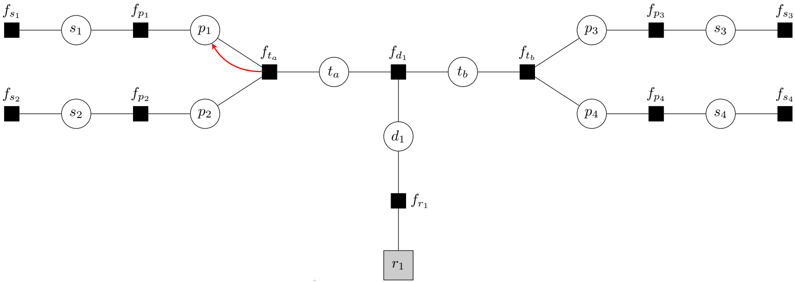
$$m_{f_t \to p}(p)$$



$$m_{f_{t_a} \to p_1}(p_1) = \int N(p_2 | \mu_2, \sigma_2^2 + \beta^2)\, \Phi(p_1 | \mu_b - p_2, \sigma_b^2)\, dp_2$$

$$\frac{\partial}{\partial p_1} m_{f_{t_a} \to p_1}(p_1) = N(p_1 | \mu_b - \mu_2, \sigma_b^2 + \sigma_2^2 + \beta^2)$$

$$m_{f_t \to p}(p)$$

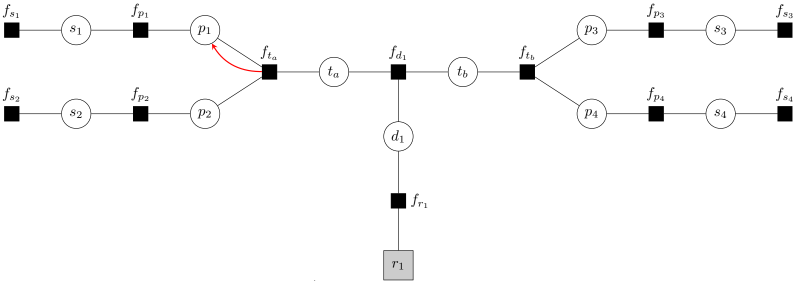


$$m_{f_{t_a} \to p_1}(p_1) = \int N(p_2|\mu_2, \sigma_2^2 + \beta^2)\, \Phi(p_1|\mu_b - p_2, \sigma_b^2)\, dp_2$$

$$\frac{\partial}{\partial p_1} m_{f_{t_a} \to p_1}(p_1) = N(p_1|\mu_b - \mu_2, \sigma_b^2 + \sigma_2^2 + \beta^2)$$

$$m_{f_{t_a} \to p_1}(p_1) =$$

$$m_{f_t \to p}(p)$$

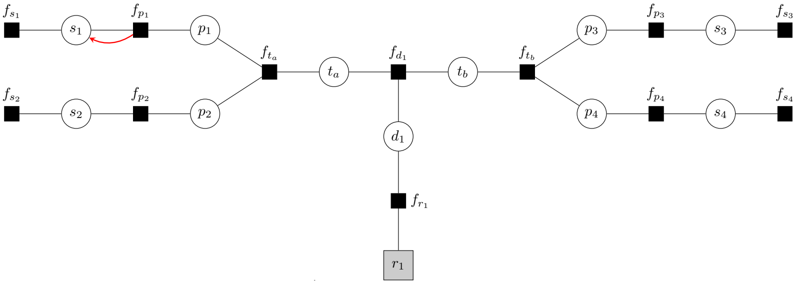


$$m_{f_{t_a} \to p_1}(p_1) = \int N(p_2|\mu_2, \sigma_2^2 + \beta^2)\, \Phi(p_1|\mu_b - p_2, \sigma_b^2)\, dp_2$$

$$\frac{\partial}{\partial p_1} m_{f_{t_a} \to p_1}(p_1) = N(p_1|\mu_b - \mu_2, \sigma_b^2 + \sigma_2^2 + \beta^2)$$

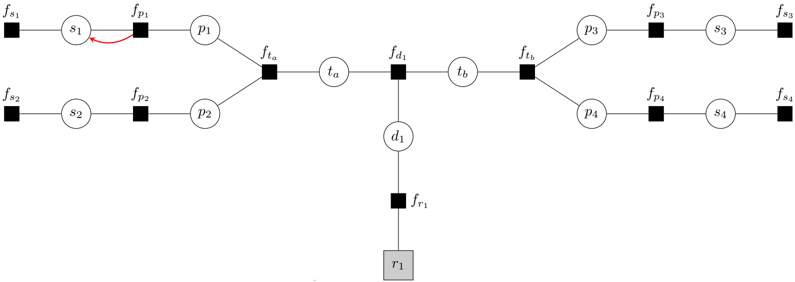
$$m_{f_{t_a} \to p_1}(p_1) = \Phi(p_1|\mu_b - \mu_2, \sigma_b^2 + \sigma_2^2 + \beta^2)$$

$$m_{f_p \to s}(s)$$



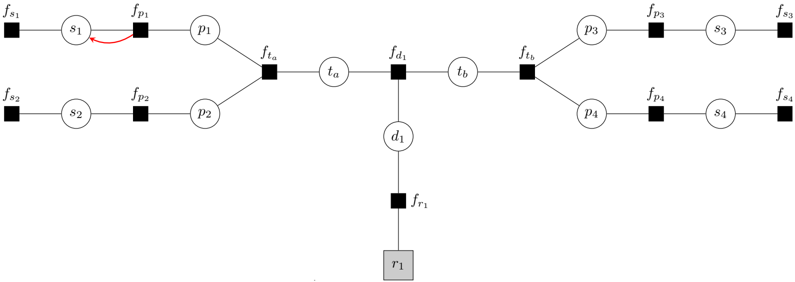
$$m_{f_{p_1} \to s_1}(s_1) =$$

$$m_{f_p \to s}(s)$$



$$m_{f_{p_1} \to s_1}(s_1) = \int N(p_1 | s_1, \beta^2)\, \Phi(p_1 | \mu_b - \mu_2, \sigma_b^2 + \sigma_2^2 + \beta^2)\, dp_1$$

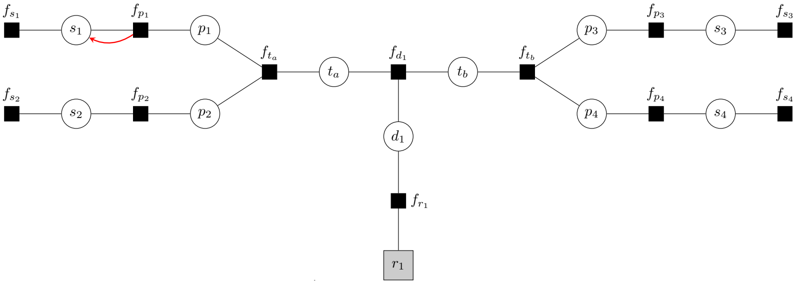
$$m_{f_p \to s}(s)$$



$$m_{f_{p_1} \to s_1}(s_1) = \int N(p_1 | s_1, \beta^2)\, \Phi(p_1 | \mu_b - \mu_2, \sigma_b^2 + \sigma_2^2 + \beta^2)\, dp_1$$

$$\frac{\partial}{\partial \mu_2} m_{f_{s_1} \to s_1}(s_1) =$$

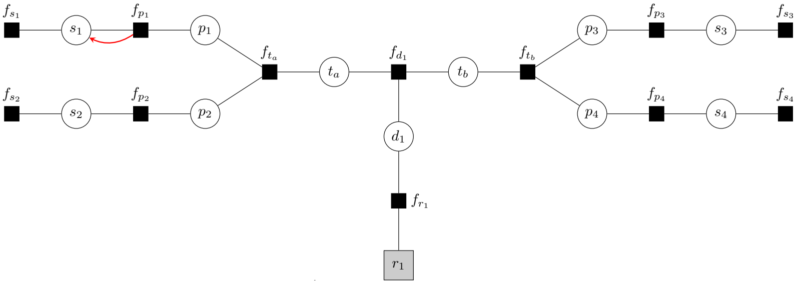
$$m_{f_p \to s}(s)$$



$$m_{f_{p_1} \to s_1}(s_1) = \int N(p_1|s_1, \beta^2)\, \Phi(p_1|\mu_b - \mu_2, \sigma_b^2 + \sigma_2^2 + \beta^2)\, dp_1$$

$$\frac{\partial}{\partial \mu_2} m_{f_{s_1} \to s_1}(s_1) = \int N(p_1|s_1, \beta^2)\, N(p_1|\mu_b - \mu_2, \sigma_b^2 + \sigma_2^2 + \beta^2)\, dp_1$$

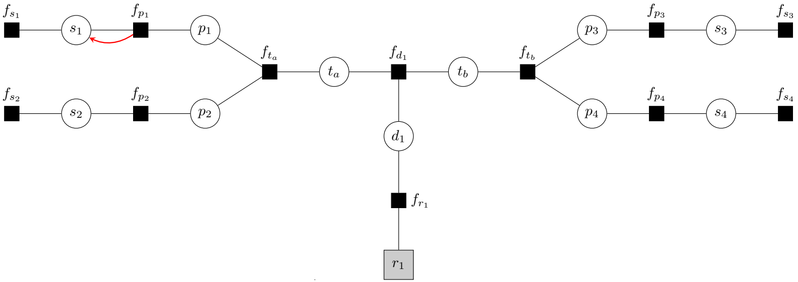
$$m_{f_p \to s}(s)$$



$$m_{f_{p_1} \to s_1}(s_1) = \int N(p_1 | s_1, \beta^2)\, \Phi(p_1 | \mu_b - \mu_2, \sigma_b^2 + \sigma_2^2 + \beta^2)\, dp_1$$

$$\frac{\partial}{\partial \mu_2} m_{f_{s_1} \to s_1}(s_1) = N(s_1 | \mu_b - \mu_2, \sigma_b^2 + \sigma_2^2 + 2\beta^2)$$

$$m_{f_p \to s}(s)$$

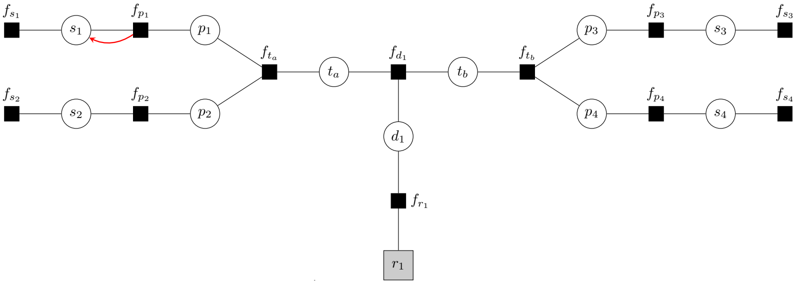


$$m_{f_{p_1} \to s_1}(s_1) = \int N(p_1|s_1, \beta^2)\, \Phi(p_1|\mu_b - \mu_2, \sigma_b^2 + \sigma_2^2 + \beta^2)\, dp_1$$

$$\frac{\partial}{\partial \mu_2} m_{f_{s_1} \to s_1}(s_1) = N(s_1|\mu_b - \mu_2, \sigma_b^2 + \sigma_2^2 + 2\beta^2)$$

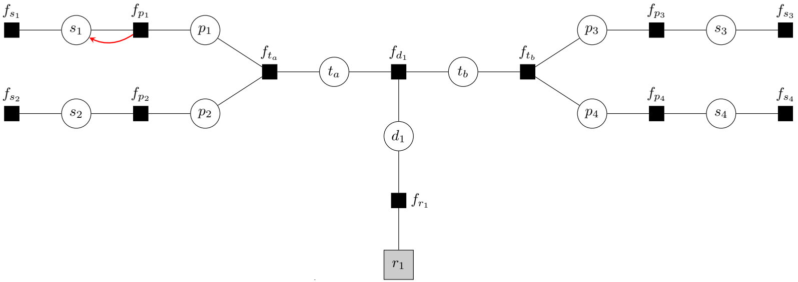
$$m_{f_{p_1} \to s_1}(s_1) = \Phi(s_1|\mu_b - \mu_2, \sigma_b^2 + \sigma_2^2 + 2\beta^2)$$

$$m_{f_p \to s}(s)$$



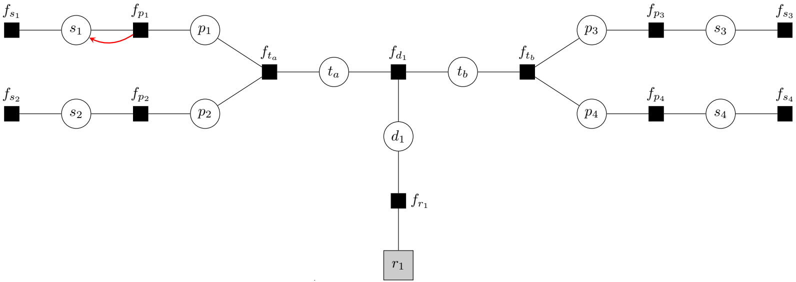
$$m_{f_{p_1} \to s_1}(s_1) = \Phi(s_1 | \mu_b - \mu_2, \sigma_b^2 + \sigma_2^2 + 2\beta^2)$$

$$m_{f_p \to s}(s)$$



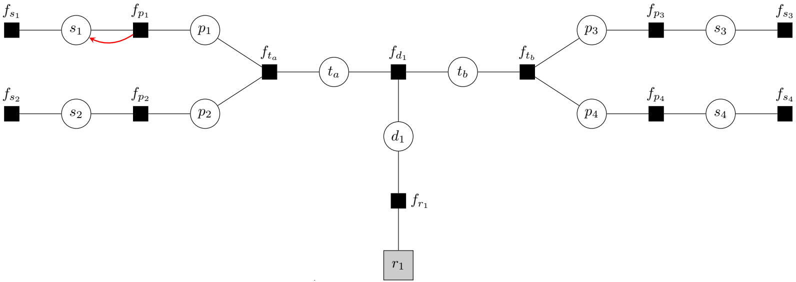
$$m_{f_{p_1} \to s_1}(s_1) = \Phi(s_1 | \mu_b - \underbrace{\mu_2}_{\mu_a - \mu_1}, \sigma_b^2 + \underbrace{\sigma_2^2 + 2\beta^2}_{\sigma_a^2 - \sigma_1^2})$$

$$m_{f_p \to s}(s)$$



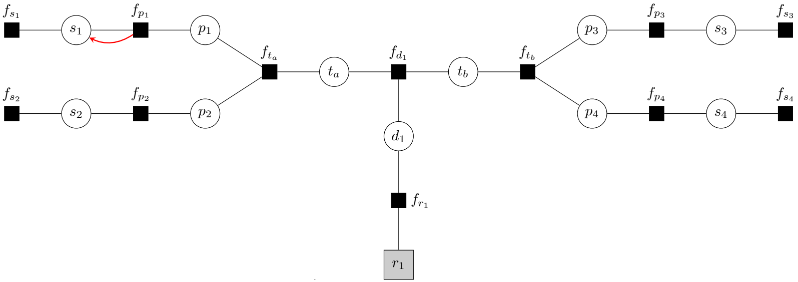
$$m_{f_{p_1} \to s_1}(s_1) = \Phi(s_1 | \mu_b - \mu_a + \mu_1, \sigma_b^2 + \sigma_a^2 - \sigma_1^2)$$

$$m_{f_p \to s}(s)$$



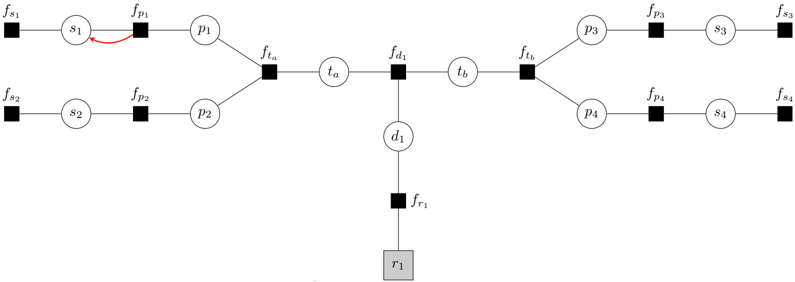
$$m_{f_{p_1} \to s_1}(s_1) = \Phi(s_1 | \underbrace{\mu_b - \mu_a}_{-\delta} + \mu_1, \underbrace{\sigma_b^2 + \sigma_a^2}_{\vartheta^2} - \sigma_1^2)$$

$$m_{f_p \to s}(s)$$



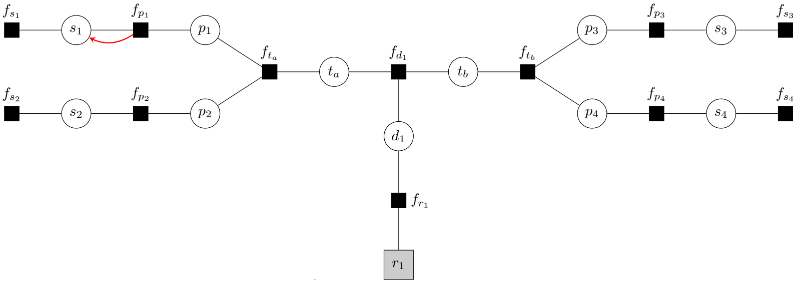
$$m_{f_{p_1} \to s_1}(s_1) = \Phi(s_1 | -\delta + \mu_1, \vartheta^2 - \sigma_1^2)$$

$$m_{f_p \to s}(s)$$



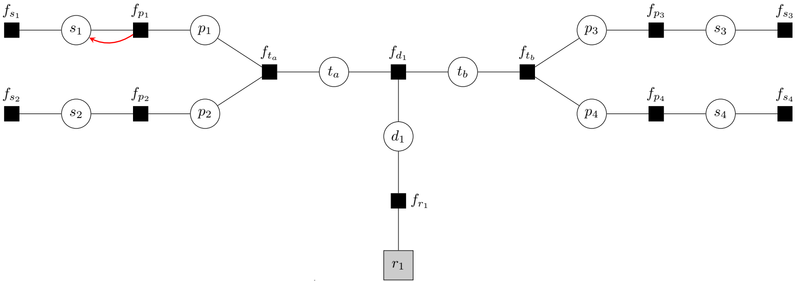
$$m_{f_{p_1} \to s_1}(s_1) = \Phi(0| - \delta + \mu_1 - s_1, \vartheta^2 - \sigma_1^2)$$

$$m_{f_p \to s}(s)$$



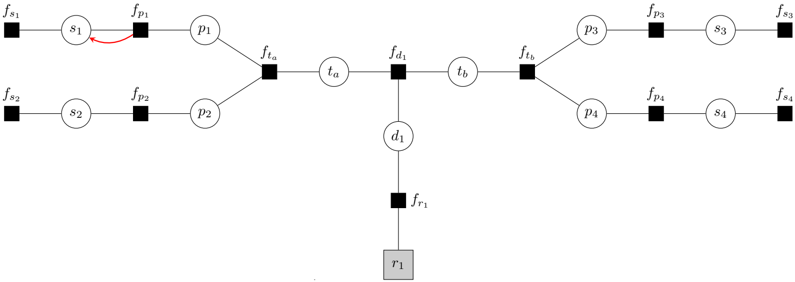
$$\begin{aligned} m_{f_{p_1} \to s_1}(s_1) &= \Phi(0| -\delta + \mu_1 - s_1, \vartheta^2 - \sigma_1^2) \\ &= 1 - \Phi(0|\, \delta - \mu_1 + s_1\, , \vartheta^2 - \sigma_1^2) \end{aligned}$$

$$m_{f_p \to s}(s)$$



$$
\begin{aligned}
m_{f_{p_1} \to s_1}(s_1) &= \Phi(0 | -\delta + \mu_1 - s_1, \vartheta^2 - \sigma_1^2) \\
&= 1 - \Phi(0 | \underbrace{\delta - \mu_1 + s_1}_{\text{Diferencia esperada parametrizada en } s}, \underbrace{\vartheta^2 - \sigma_1^2}_{\text{Sin incertidumbre respecto de } s})
\end{aligned}
$$

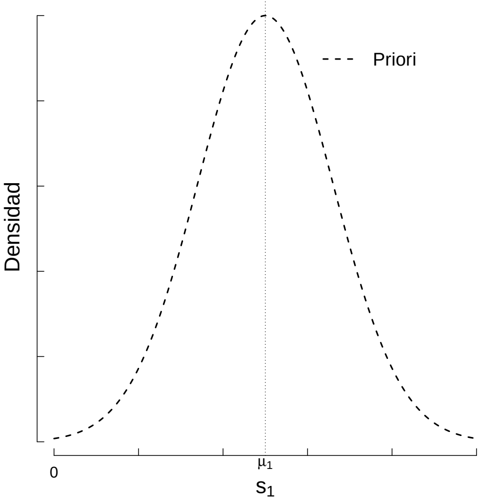
$$m_{f_p \to s}(s)$$



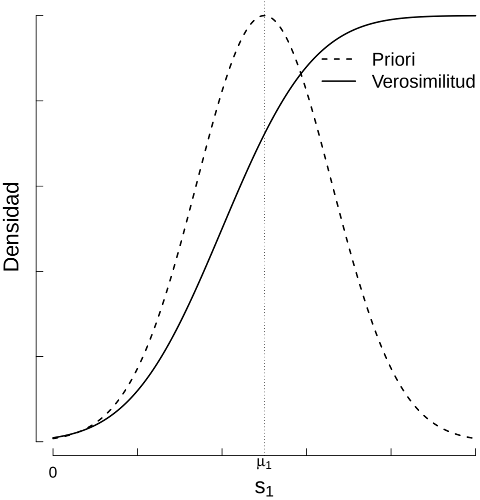
$$\begin{aligned} m_{f_{p_1} \to s_1}(s_1) &= \Phi(0| - \delta + \mu_1 - s_1, \vartheta^2 - \sigma_1^2) \\ &= 1 - \Phi(0| \underbrace{\delta - \mu_1 + s_1}_{\text{Diferencia esperada parametrizada en } s} , \underbrace{\vartheta^2 - \sigma_1^2}_{\text{Sin incertidumbre respecto de } s}) \\ &= \text{Probabilidad de ganar si la verdadera habilidad fuera } s_1 \end{aligned}$$

$$\overbrace{P(s_1 \mid r, \text{Modelo})}^{\text{Posteriori}} \propto \overbrace{N(s_1 \,|\, \mu_1, \sigma_1^2)}^{\text{Priori}} \; \overbrace{1 - \Phi(0 \,|\delta - \mu_1 + s_1, \vartheta^2 - \sigma_1^2)}^{\text{Verosimilitud}} \quad \text{Caso ganador}$$

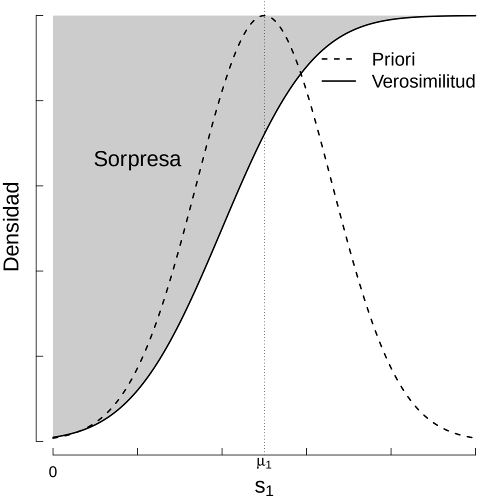
$$\overbrace{P(s_1 \mid r, \text{Modelo})}^{\text{Posteriori}} \propto \overbrace{N(s_1 \,|\, \mu_1, \sigma_1^2)}^{\text{Priori}} \; \overbrace{1 - \Phi(0 \,|\delta - \mu_1 + s_1, \vartheta^2 - \sigma_1^2)}^{\text{Verosimilitud}} \quad \text{Caso ganador}$$



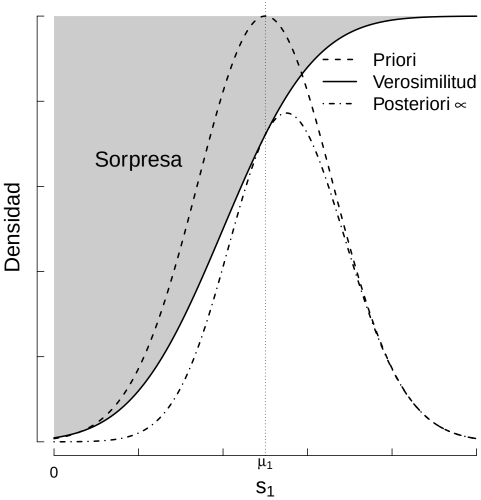
$$\overbrace{P(s_1 \mid r, \text{Modelo})}^{\text{Posteriori}} \propto \overbrace{N(s_1 \,|\, \mu_1, \sigma_1^2)}^{\text{Priori}} \; \overbrace{1 - \Phi(0 \,|\delta - \mu_1 + s_1, \vartheta^2 - \sigma_1^2)}^{\text{Verosimilitud}} \quad \text{Caso ganador}$$



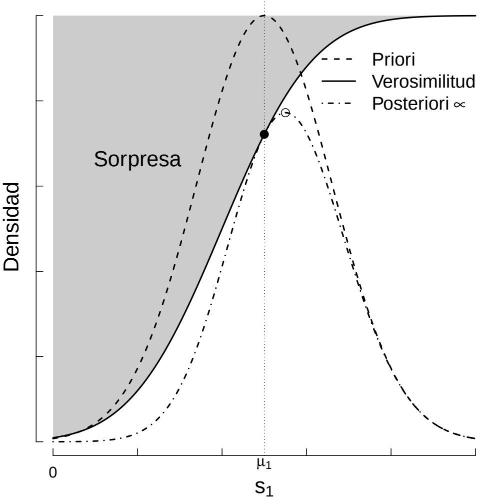
$$\overbrace{P(s_1 \mid r, \text{Modelo})}^{\text{Posteriori}} \propto \overbrace{N(s_1 \,|\, \mu_1, \sigma_1^2)}^{\text{Priori}} \; \overbrace{1 - \Phi(0 \,|\delta - \mu_1 + s_1, \vartheta^2 - \sigma_1^2)}^{\text{Verosimilitud}} \quad \text{Caso ganador}$$



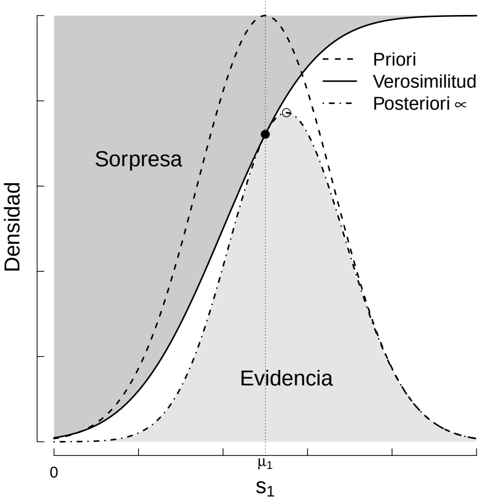
$$\overbrace{P(s_1 \mid r, \text{Modelo})}^{\text{Posteriori}} \propto \overbrace{N(s_1 \,|\, \mu_1, \sigma_1^2)}^{\text{Priori}} \; \overbrace{1 - \Phi(0 \,|\delta - \mu_1 + s_1, \vartheta^2 - \sigma_1^2)}^{\text{Verosimilitud}} \quad \text{Caso ganador}$$



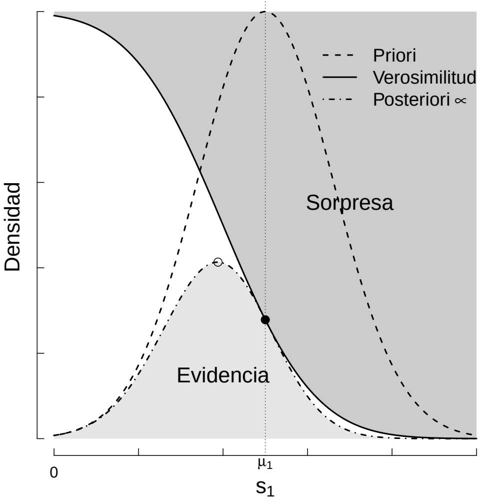
$$\overbrace{P(s_1 \mid r, \text{Modelo})}^{\text{Posteriori}} \propto \overbrace{N(s_1 \,|\, \mu_1, \sigma_1^2)}^{\text{Priori}} \ \overbrace{1 - \Phi(0 \,|\delta - \mu_1 + s_1, \vartheta^2 - \sigma_1^2)}^{\text{Verosimilitud}} \quad \text{Caso ganador}$$



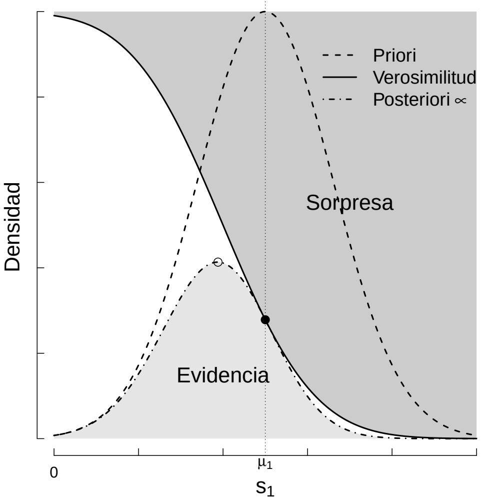
$$\overbrace{P(s_1 \mid r, \text{Modelo})}^{\text{Posteriori}} \propto \overbrace{N(s_1 \,|\, \mu_1, \sigma_1^2)}^{\text{Priori}} \; \overbrace{1 - \Phi(0 \,|\delta - \mu_1 + s_1, \vartheta^2 - \sigma_1^2)}^{\text{Verosimilitud}} \quad \text{Caso ganador}$$



$$\overbrace{P(s_3 \mid r, \text{Modelo})}^{\text{Posteriori}} \propto \overbrace{N(s_3 \,|\, \mu_3, \sigma_3^2)}^{\text{Priori}} \ \overbrace{\Phi(0 \,|\delta + \mu_3 - s_3, \vartheta^2 - \sigma_3^2)}^{\text{Verosimilitud}} \quad \text{Caso perdedor}$$



$$\overbrace{P(s_3 \mid r, \text{Modelo})}^{\text{Posteriori}} \propto \overbrace{N(s_3 \,|\, \mu_3, \sigma_3^2)}^{\text{Priori}} \, \overbrace{\Phi(0 \,|\delta + \mu_3 - s_3, \vartheta^2 - \sigma_3^2)}^{\text{Verosimilitud}} \quad \text{Caso perdedor}$$



Todos los detalles en: Landfried. TrueSkill: Technical Report. 2019

Selección de modelo

¿Y nuestras creencias respecto de modelos alternativos?

Selección de modelo

$$P(\mathsf{M}|\mathsf{D}) = \frac{P(\mathsf{D}|\mathsf{M})P(\mathsf{M})}{P(\mathsf{D})}$$

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Selección de modelo

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Lo único que necesitamos es la **evidencia**!

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Lo único que necesitamos es la **evidencia**!

Sobre comparación de modelos ver: Kass & Raftery. Bayes factors. 1995.

Evidencia

$$P(\mathsf{C}|\mathsf{D},\mathsf{M}) = \frac{P(\mathsf{D}|\mathsf{C},\mathsf{M})P(\mathsf{C}|\mathsf{M})}{\underbrace{P(\mathsf{D}|\mathsf{M})}_{\text{Evidencia}}}$$

Evidencia

$$P(\mathsf{C}|\mathsf{D},\mathsf{M}) = \frac{P(\mathsf{D}|\mathsf{C},\mathsf{M})P(\mathsf{C}|\mathsf{M})}{\underbrace{P(\mathsf{D}|\mathsf{M})}_{\text{Evidencia}}}$$

$$P(\mathsf{D}|\mathsf{M}) = \sum_{C} P(\mathsf{D}|\mathsf{C},\mathsf{M})\, P(\mathsf{C}|\mathsf{M})$$

Evidencia

$$P(\mathsf{C}|\mathsf{D},\mathsf{M}) = \frac{P(\mathsf{D}|\mathsf{C},\mathsf{M})P(\mathsf{C}|\mathsf{M})}{\underbrace{P(\mathsf{D}|\mathsf{M})}_{\text{Evidencia}}}$$

$$P(\mathsf{D}|\mathsf{M}) = \sum_{C} \underbrace{P(\mathsf{D}|\mathsf{C},\mathsf{M})}_{\substack{\text{Predicción} \\ \text{de D dado C}}} \underbrace{P(\mathsf{C}|\mathsf{M})}_{\substack{\text{Creencia de} \\ \text{C a Priori}}}$$

Evidencia

$$P(\mathsf{C}|\mathsf{D},\mathsf{M}) = \frac{P(\mathsf{D}|\mathsf{C},\mathsf{M})P(\mathsf{C}|\mathsf{M})}{\underbrace{P(\mathsf{D}|\mathsf{M})}_{\text{Evidencia}}}$$

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Evidencia

$$P(\mathsf{C}|\mathsf{D},\mathsf{M}) = \frac{P(\mathsf{D}|\mathsf{C},\mathsf{M})P(\mathsf{C}|\mathsf{M})}{\underbrace{P(\mathsf{D}|\mathsf{M})}_{\text{Evidencia}}}$$

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Preferimos modelos con la menor **sorpresa** en la evidencia!

Evidencia

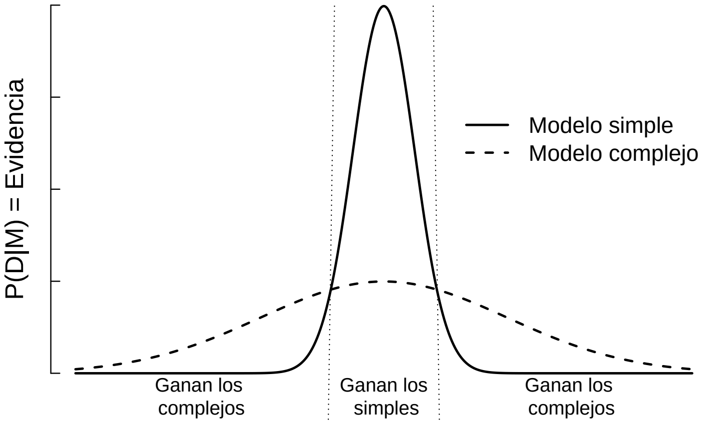
$$P(\mathsf{C}|\mathsf{D},\mathsf{M}) = \frac{P(\mathsf{D}|\mathsf{C},\mathsf{M})P(\mathsf{C}|\mathsf{M})}{\underbrace{P(\mathsf{D}|\mathsf{M})}_{\text{Evidencia}}}$$

$$P(\mathsf{D}|\mathsf{M}) = \underbrace{\sum_{C} \underbrace{P(\mathsf{D}|\mathsf{C},\mathsf{M})}_{\substack{\text{Predicción} \\ \text{de D dado C}}} \underbrace{P(\mathsf{C}|\mathsf{M})}_{\substack{\text{Creencia de} \\ \text{C a Priori}}}}_{\substack{\text{Predicción de la datos observados} \\ \text{pesando todas las creencias a priori}}}$$

Preferimos modelos con la menor **sorpresa** en la evidencia!

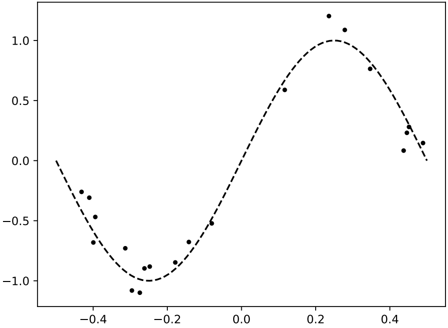
$$\boxed{P(M_q|D) > P(M_r|D) \iff P(D|M_q) > P(D|M_r)}$$

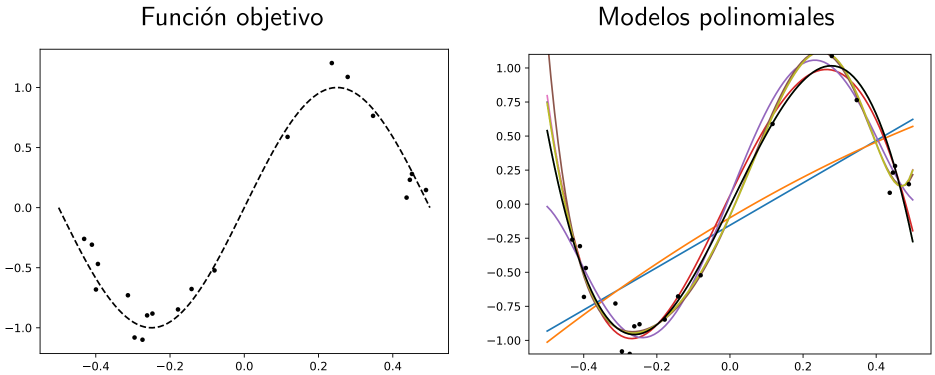
Evidencia



Balance natural entre complejidad y predicción

Función objetivo




Función objetivo
1.0
0.5
0.0
−0.5
−1.0
−0.4
−0.2
0.0
0.2
0.4
Modelos polinomiales
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−0.4
−0.2
0.0
0.2
0.4

Evidencia vs Verosimilitud