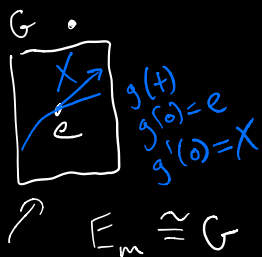
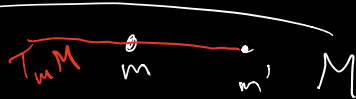
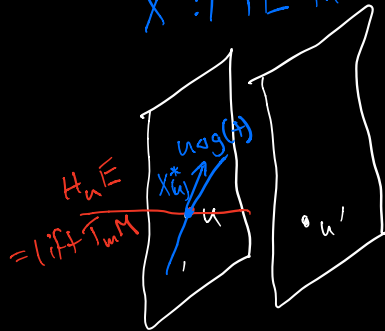


$$E \times_{\mathfrak{g}} \xrightarrow{\text{verticals, ker } T_p} VE \rightarrow TE \rightarrow p^* TM$$

$(u, X) \mapsto X_u^*$



$X^*: \Gamma TE \text{ in fact } \Gamma VE$



$$E \xrightarrow{p} G$$

M

$$\rightarrow R_g: E \rightarrow E$$

$$M = E/G$$

G Lie group
 $\mathfrak{g} \cong T_e G$ Lie alg

$$\Gamma TE := E \xrightarrow{f} TE \text{ s.t.}$$

$$\text{can take } E = \underline{TM}$$

$$TE = \underline{T(TM)}$$

$$E = FM \text{ frames of } TM$$

$$E \times_{\mathfrak{g}} \rightarrow VE \rightarrow TE \rightarrow p^* TM$$

$(u, X) \mapsto (u, X)$
 $(u, X) \mapsto (u, X)$
 $(u, X) \mapsto (u, X)$
 $(u, X) \mapsto (u, X)$

$$E \times_{\mathfrak{g}} \rightarrow E \rightarrow M$$

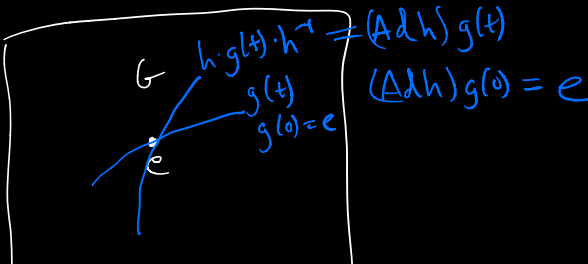
$$E \times_{\mathfrak{g}} \rightarrow VE \rightarrow TE \rightarrow p^* TM$$

$(u, X) \mapsto (u, X)$
 $(u, X) \mapsto (u, X)$
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 $(u, X) \mapsto (u, X)$

$$ad: G \times \mathfrak{g} \rightarrow \mathfrak{g}$$

$$Ad: G \times G \rightarrow G$$

$g, h \mapsto hgh^{-1}$



$$\text{ad} = T_e \text{Ad}$$



Lemma: $G, \sigma, X: \sigma$

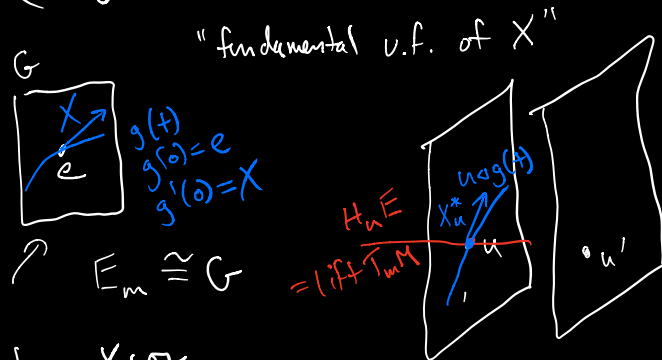
G action on E on the right

X^* : fundamental v.f. of X on TE

$g: G$

$R_g: E \rightarrow E$

$$\longrightarrow (TR_g)X^*(u) = ((\text{ad } g^{-1})X)^* u$$



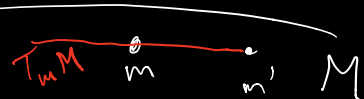
1) choose $X: \sigma$

→ 2) choose $u: E$

3) $X^*_u: T_u E \rightarrow V_u E$

ii

$TR(X)$



NOT 1) choose $X: \sigma$

2) choose $u: E$

3) choose $u': E$, $u' = u \cdot g$

$$TR_g(X^*_u) = X^*_{(u')}$$

$$G \times E \xrightarrow{R} E$$

$$TR(-, -)|_{(e, -)}$$

$$R(e, -) : E \rightarrow E$$

$$\begin{array}{ccccc}
 & E \times \mathfrak{g} & \xleftarrow{\quad \underline{VE} \quad} & TE & p^* TM \\
 \nearrow \quad \downarrow & \downarrow & \xleftarrow{\quad \omega_A \quad} & \downarrow & \downarrow \\
 \rightarrow \quad ad E & \xrightarrow{\quad \downarrow \quad} & TE/G & \xrightarrow{\quad \quad \quad} & TM \\
 \quad \quad \downarrow & & \downarrow & & \downarrow \\
 \quad \quad M & & M & & M
 \end{array}$$

Atiyah sequence
exact

$E \times_{G \times \mathfrak{g}} / \text{ad action of } G \text{ on } \mathfrak{g} \text{ and } G \text{ on } E$

$\omega_A - \omega_{A_1} : \Omega^1(M; ad E)$

$$\Rightarrow \Gamma_{ad E} \longrightarrow \Gamma_{TE/G} \xleftarrow{\quad \quad \quad} \Gamma_{TM} \quad \text{v.f. on } M : \Gamma_{TM}$$

$$F_A(X, Y) := \omega_A(\tilde{X}, \tilde{Y})$$

$$X, Y : \Gamma_{TM}$$

$$\sim := \text{lift}$$

$$F_A \neq 0 \iff F_A = 0$$

$$F_A : \Omega^2(M; ad E)$$

$$F_A(X, Y) : \Gamma_{ad E}$$

$$\begin{array}{ccc}
 E : M & \longrightarrow & BG \\
 & \searrow & \uparrow \\
 & & B_{\nabla} G
 \end{array}$$

$$(x : BG) \times \underline{\Omega^1(M; ad E)}$$