Poster on the Standard Model

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Dictionary between Physics and Math

Physics	Math
Symmetry	Group ($U(1)$ for EM, $SU(2)$ for weak, $SU(3)$ for strong, Poincaré for translation/rotation/relativistic boosts)
Matter field	Section of an associated vector bundle
Gauge transformation	Bundle automorphism
Gauge field/potential	Connection one-form
Field strength	Curvature of connection
Lagrangian	Map from sections of bundles to real-valued function on base
Interaction	Product of distinct sections in the Lagrangian

(adapted from http://www.disconzi.net/Notes_links_media/more/sheridan_math_thesis_final_05-20.pdf)

Standard model lagrangian

$$
\begin{aligned}
\mathcal{L} = &-\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{8}tr\left(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right) - \frac{1}{2}tr\left(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}\right) && \text{(U(1), SU(2), and SU(3) gauge terms)} \\
&+ (\bar{\nu}_L, \bar{e}_L)\tilde{\sigma}^\mu iD_\mu \begin{pmatrix}\nu_L \\ e_L\end{pmatrix} + \bar{e}_R\sigma^\mu iD_\mu e_R + \bar{\nu}_R\sigma^\mu iD_\mu \nu_R + \text{(h.c.)} && \text{(lepton dynamical term)} \\
&- \frac{\sqrt{2}}{\nu}\left[(\bar{\nu}_L, \bar{e}_L)\phi M^e e_R + \bar{e}_R\bar{M}^e\bar{\phi}\begin{pmatrix}\nu_L \\ e_L\end{pmatrix}\right] && \text{(electron, muon, tauon mass term)} \\
&- \frac{\sqrt{2}}{\nu}\left[(-\bar{e}_L, \bar{\nu}_L)\phi^* M^\nu \nu_R + \bar{\nu}_R\bar{M}^\nu\phi^T\begin{pmatrix}-e_L \\ \nu_L\end{pmatrix}\right] && \text{(neutrino mass term)} \\
&+ \left(\bar{u}_L, \bar{d}_L\right)\tilde{\sigma}^\mu iD_\mu \begin{pmatrix}u_L \\ d_L\end{pmatrix} + \bar{u}_R\sigma^\mu iD_\mu u_R + \bar{d}_R\sigma^\mu iD_\mu d_R + \text{(h.c.)} && \text{(quark dynamical term)} \\
&- \frac{\sqrt{2}}{\nu}\left[\left(\bar{u}_L, \bar{d}_L\right)\phi M^d d_R + \bar{d}_R\bar{M}^d\bar{\phi}\begin{pmatrix}u_L \\ d_L\end{pmatrix}\right] && \text{(down, strange, bottom mass term)} \\
&- \frac{\sqrt{2}}{\nu}\left[\left(-\bar{d}_L, \bar{u}_L\right)\phi^* M^u u_R + \bar{u}_R\bar{M}^u\phi^T\begin{pmatrix}-d_L \\ u_L\end{pmatrix}\right] && \text{(up, charm, top mass term)} \\
&+ \overline{(D_\mu\phi)}D^\mu\phi - \frac{m_h^2\left[\bar{\phi}\phi - \frac{\nu^2}{2}\right]^2}{2\nu^2} && \text{(Higgs dynamical and mass term)}
\end{aligned}
$$

(from https://github.com/SodiumIodide/Standard-Model-Lagrangian)

Lagrangians are the preferred condensed representation of what a theory is. Physics has (or works to create) algorithms to progress from a lagrangian to specific formulas for how a system evolves over time.

Terms like $B_{\mu\nu}B^{\mu\nu}$ are the square of the curvature. Curvature has values in a Lie algebra, so you also take the trace or something similar, to get a function on spacetime.

Terms like $\overline{e}_R\sigma^\mu iD_\mu e_R$ are products of two "e"s and a "D", where the "D" includes the connection (covariant derivative). So such terms are an interaction among three things: two leptons and a force particle.