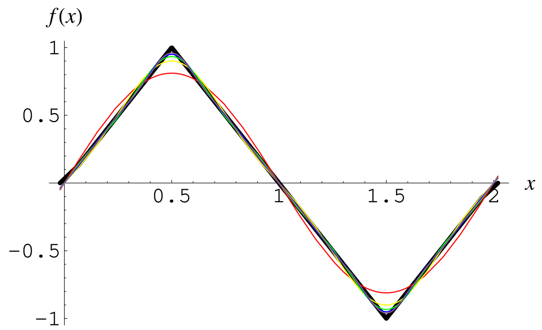
Fourier Series--Triangle Wave



Consider a symmetric triangle wave $T(x)$ of period $2L$. Since the function is odd,

$$a_0 = 0 \tag{1}$$

$$a_n = 0, \tag{2}$$

and

$$b_n = \frac{2}{L}\left\{\int_0^{L/2} \frac{x}{L/2}\sin\left(\frac{n\pi x}{L}\right)dx + \int_{L/2}^{L}\left[1 - \frac{2}{L}\left(x - \tfrac{1}{2}L\right)\right]\sin\left(\frac{n\pi x}{L}\right)dx\right\} \tag{3}$$

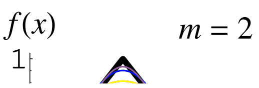
$$= \frac{32}{\pi^2 n^2}\cos\left(\tfrac{1}{4}n\pi\right)\sin^3\left(\tfrac{1}{4}n\pi\right) \tag{4}$$

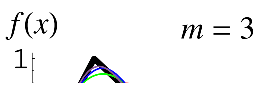
$$= \frac{32}{\pi^2 n^2}\begin{cases} 0 & n = 0, 4, \ldots \\ \frac{1}{4} & n = 1, 5, \ldots \\ 0 & n = 2, 6, \ldots \\ -\frac{1}{4} & n = 3, 7, \ldots \end{cases} \tag{5}$$

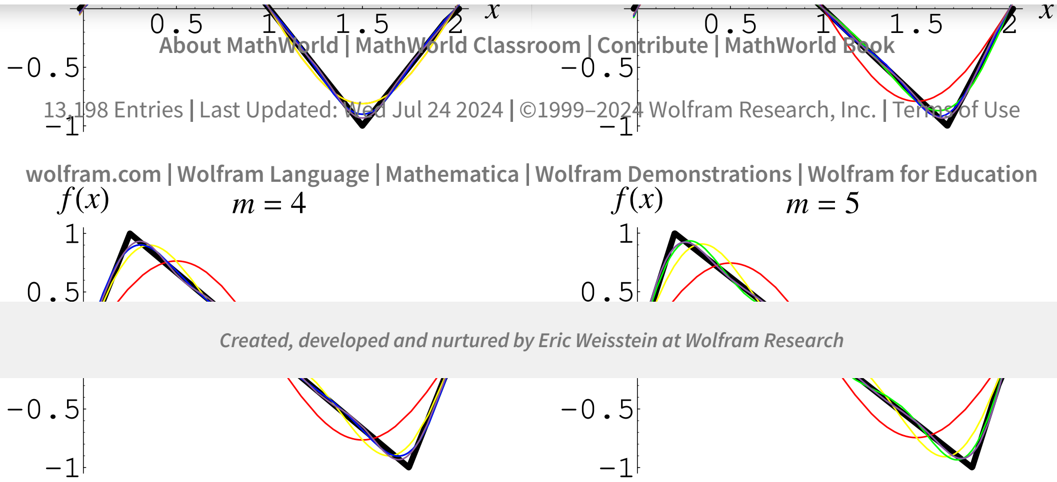
$$= \frac{8}{\pi^2 n^2}\begin{cases} (-1)^{(n-1)/2} & \text{for } n \text{ odd} \\ 0 & \text{for } n \text{ even.} \end{cases} \tag{6}$$

The Fourier series for the triangle wave is therefore

$$f(x) = \frac{8}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{(-1)^{(n-1)/2}}{n^2}\sin\left(\frac{n\pi x}{L}\right). \tag{7}$$







Now consider the asymmetric triangle wave pinned an x-distance which is $(1/m)$th of the distance L. The displacement as a function of x is then

$$f_m(x) = \begin{cases} \frac{m x}{L} & \text{for } 0 \leq x \leq \frac{L}{m} \\ 1 - \frac{m}{(m-1) L}\left(x - \frac{L}{m}\right) & \text{for } \frac{L}{m} \leq x \leq 2L - \frac{L}{m} \\ \frac{m}{L}(x - 2L) & \text{for } 2L - \frac{L}{m} \leq x \leq 2L. \end{cases} \quad (8)$$

The coefficients are therefore

$$a_0 = 0 \quad (9)$$

$$a_n = 0 \quad (10)$$

$$b_n = -\frac{2(-1)^n m^2}{n^2 (m-1) \pi^2} \sin\left[\frac{n(m-1)\pi}{m}\right]. \quad (11)$$

Taking $m = 2$ gives the same Fourier series as before.

SEE ALSO

Fourier Series, Fourier Series--Sawtooth Wave, Fourier Series--Square Wave, Triangle Wave

EXPLORE WITH WOLFRAM|ALPHA

WolframAlpha {{2,3},{4,5}}^(-1)

More things to try: {{2,3},{4,5}}^(-1) | div (grad f) | local minimum calculator

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