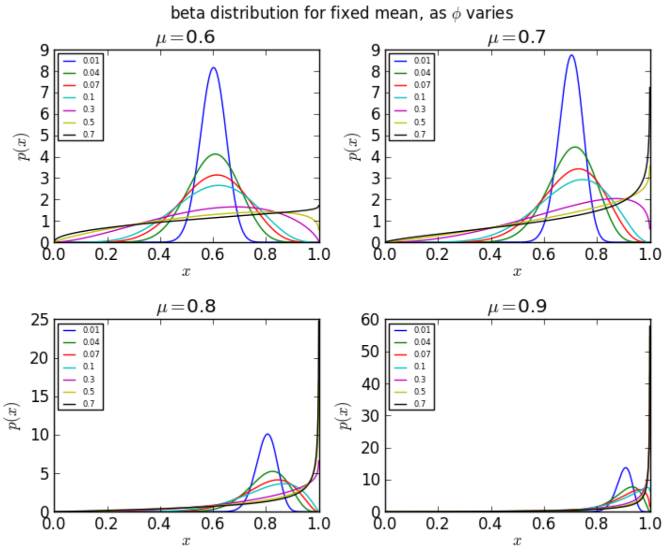

Figure 1:

1 5-28 notes

1.1 New parameterization

I found a parameterization of the beta distribution, where you specify the *mode* and a dispersion parameter. If the dispersion parameter is between 0 and 1, then the distribution is guaranteed to be unimodal.

1.2 Simple model used for analysis

I tried to analyze the cause of bias using the simplest model below:

This is basically the part of the model that predicts a.

$$
\begin{aligned}
f^a(x) &= \frac{1}{1+e^{-x}} && (1)\\
\mu_i^a &= f^a(\mu_{pop}^a + Bx_i) && (2)\\
a_i &\sim Beta(\mu_i^a, \phi^a) && (3)\\
B &\sim U(-\infty, \infty) && (4)
\end{aligned}
$$

1.3 Plots

1. First question is whether we should have a linear model for the mean or mode of the beta distribution that a comes from. That is, should μ_i^a be the mean or mode of a beta distribution? In Figure 1, in each subplot, I plot several beta distributions. All of them have the same *mean*, but different ϕ's.

2. In Figure 2, in each subplot, I plot several beta distributions. All of them have the same *mode*, but different ϕ. It seems better to have a linear model for the mode of beta distributions

3. Let's try to get a grasp on the ϕ parameter. In Figure 3, in each subplot, I fix ϕ and plot a beta distribution with various modes, with that ϕ.

4. Now, goal is to see if this beta regression produces biased MLE estimates of B. In Figure 4, I set B=1.0. Below, for each (μ_{pop}^a, ϕ) combination, I do the following:

   (a) let X_i be number between -1 and 1. The $X_1, \ldots X_n$ are chosen to be equally spaced between -1 and 1. $n = 100$.

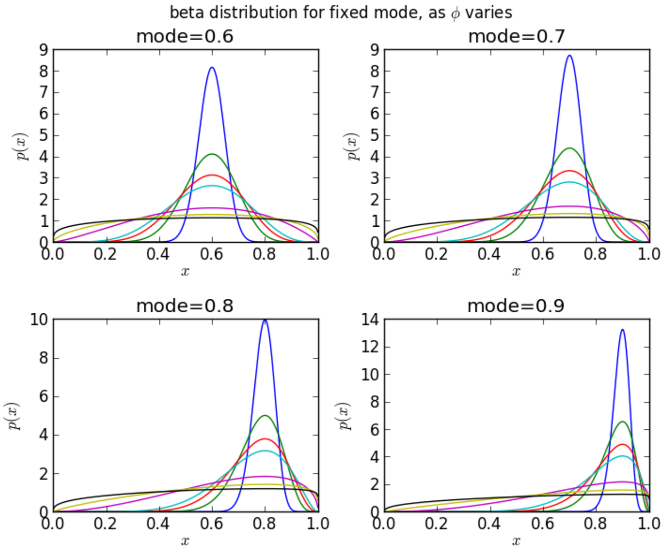

Figure 2:

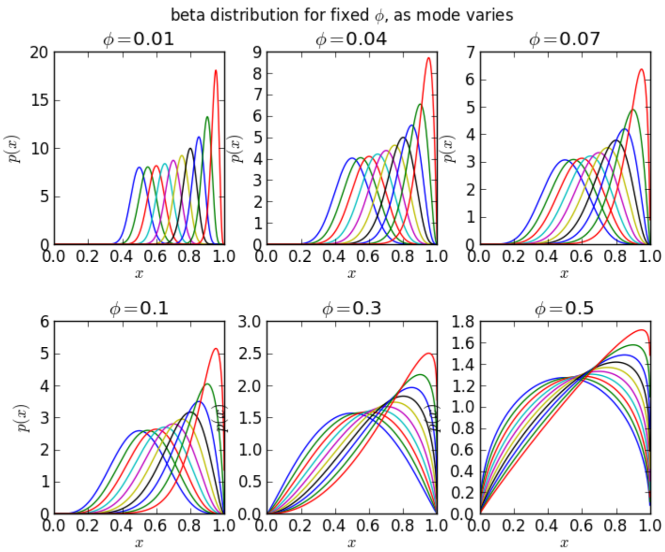

Figure 3:

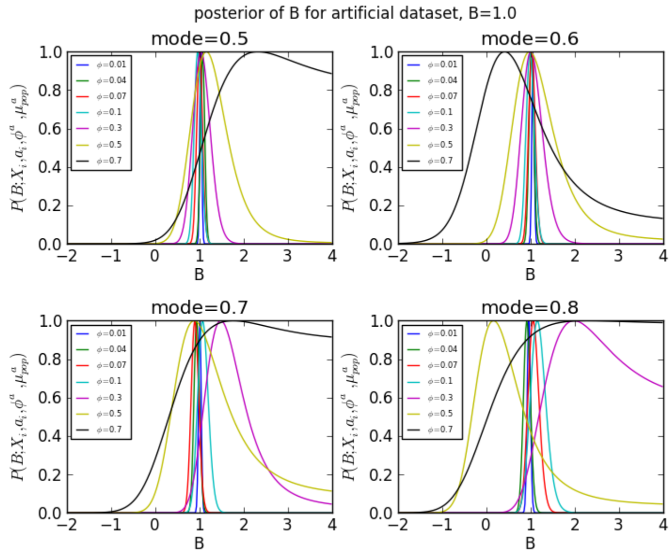

Figure 4:

(b) for each X_i, generate a_i according to the model, with randomness.(μ^a_{pop} and ϕ^a) and B are all specified, so this is well defined.

(c) Plot the distribution of $P(B; X_i, a_i, \phi, \mu^a_{pop})$

These plots are mildly informative. Small ϕ's get rid of bias. It's not clear if higher ϕ leads to bias in one direction.

5. Try to figure out the cause of the bias. In Figure 5, I fix x. I plot P(x;mode,ϕ), assuming $x \sim Beta(mode, \phi)$. Note that for high values of, x the mode of $P(x; mode, \phi)$ does not occur at x. I do this for 4 values of x.

6. From previous plot, we know that if x is close to 1, and we assume x Beta(mode,ϕ), the MLE estimate of mode will be biased. Let's see how this causes the estimate of B to be biased. In Figure 6, I basically repeat the previous plots where I simulated data and tried to infer B. However, here there is only 1 datapoint, $x_0 = 1$, and I let $a_0 = \mu^a_0$.

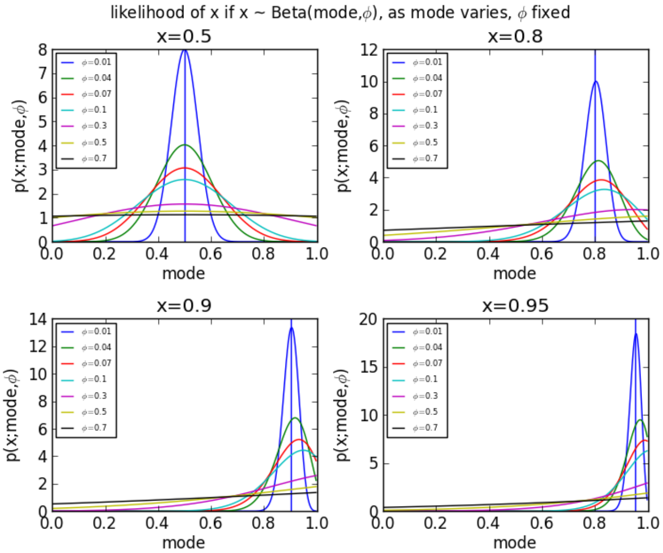

Figure 5:

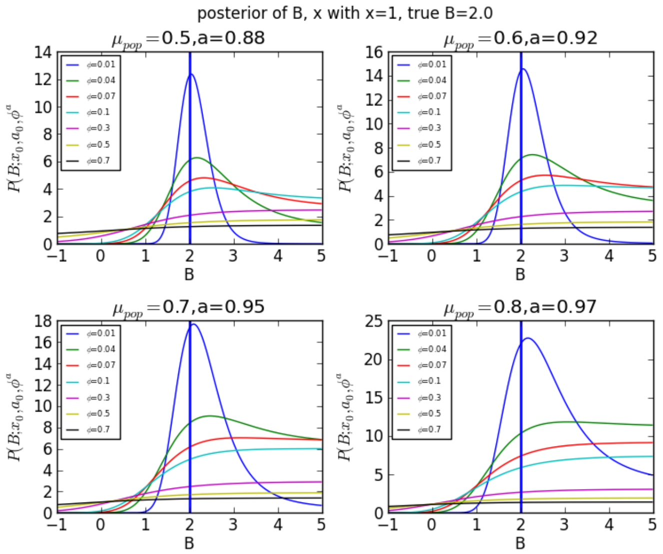

Figure 6: