Assigment: diffusion (hydrodynamics 3)

1 Part 5

The final part of this assignment is about solving the diffusion problem using the explicit and implicit methods discussed in the lectures. As in the fist part of this assigment, imagine that you have a spatial domain extending from $x = 0$ to $x = 10$. Consider the quantity q, which as before is the density of some conserved quantity, which at $t = 0$ has a value of 1.0 between $x = 3.5$ and $x = 6.5$ and 0.1 everywhere else. The evolution of q is described by

$$\frac{\partial q}{\partial t} = D \frac{\partial^2 q}{\partial x^2} \tag{1}$$

where $D = 1.0$.

In the lectures, two numerical schemes for solving this problem were described, both using central differencing for the spatial derivatives and the forward/backward Euler for the time derivative. For the FTCS scheme, you will need to use a courant number less than unity for stability purposes, but not for the BTCS scheme.

Your task is to write codes to evolve q from $t = 0$ to $t = 1.0$ using the FTCS and BTCS methods discussed in the lecture. Make plots showing the results of both this calculation using both schemes. Do the calculations separately for courant numbers of 0.5, 1.0, and 1.5, to show how the two schemes depend on the timestep length.

Note that for the BTCS scheme, you might need to take measures to keep the boundary values realistic. For simplicity, you can just keep the boundary values constant and not worry if the solution is slightly unrealistic at the boundaries. Typically you would want to focus on getting a good treatment of the boundaries, but that is not necessary for this assignment.