Assignment: ordinary differential equations

1 Intro

This assignment consists of two parts. In part one, you will have to implement four methods for solving simple ODEs numerically. In part two, you will have to solve a small system of ODEs using the classical Runge-Kutta and Runge-Kutta-Fehlberg methods. For both parts, you should make plots, which you should compile into one document and submit with all of the Python codes.

2 Part 1

Write a code that solves the following initial value problem.

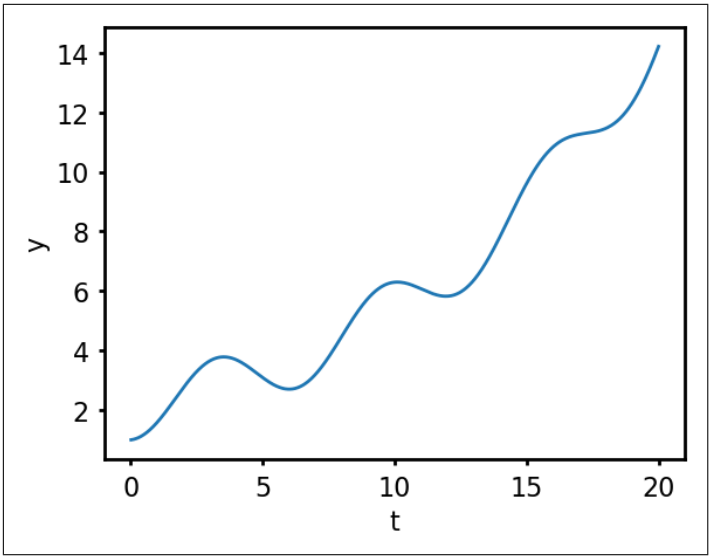
$$y(t=0) = 1 \tag{1}$$

$$\frac{dy}{dt} = -2ty \tag{2}$$

You should use your code to calculate the value of y between $t = 0$ and $t = 3$. ***Solve the problem separately using the following methods:***

- Forward Euler method
- Classical Runge-Kutta method
- Backward Euler method
- Crank-Nicolson method

For Backward Euler and Crank-Nicolson, you should use Newton iteration. You can either write a single code that implements all four methods separately, or four separate codes that implement each method individually. ***Derive the analytic solution to this problem*** (the derivation does not need to be submitted) and then ***make plots that show how these methods perform for different numbers of timesteps.*** You should choose the timestep lengths so that it is clear from the plots how many time steps are needed for the numerical solution to become a good approximation to the analytic solution.



3 Part 2

Write a code that solves the following initial value problem.

$$y(t=0) = 1 \tag{3}$$

$$\frac{dy}{dt} = 0.1y + \sin(t) \tag{4}$$

You should use your code to calculate the value of y between $t = 0$ and $t = 20$ using the Runge-Kutta-Fehlberg method presented in the lecture. The code should take the desired local truncation error, LTE_{tol}, as input and at each timestep determine the timestep length automatically. A perfectly accurate solution should look like the figure (assuming I calculated it correctly).

Solve this problem separately using LTE$_{\text{tol}}$ values of 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, ***and*** 10^{-6}. From your results, you should construct three figures showing y vs t , dt vs t, and the number of timesteps taken vs LTE_{tol}. These figures should include the results for all LTE_{tol} values on the same figure.