Assignment 4 - Marcov Chain Monte Carlo

Christian Wiskott

University of Vienna — November 26, 2020

Algorithm

In order to find the numeric values for the mean and the standard deviation of the given data set "Dataset17.txt", the Metropolis-Hastings algorithm was employed by following the guidelines of the assignment sheet. The code was provided in conjunction with this report.

First, the initial values for the mean (μ) and the standard deviation (σ) were computed based on the true values. These were obtained by using the proprietary numpy-functions, while adding a random fraction of the respective value in order to obtain different starting values each run. The following code shows the calculations.

```
Calculates the initial values for μ and σ

 # Random plus or minus operators
 op1 = np.random.choice([1, -1])
 op2 = np.random.choice([1, -1])

 # Initial values with added random fraction
 mu = np.mean(data) + op1 * (np.mean(data) * np.random.rand())
 sigma = np.std(data) + op2 * (np.std(data) * np.random.rand())
```

After checking whether σ is greater than 0 and computing the likelihood of the initial values, new proposal values were computed by adding random noise sampled from a Gaussian distribution with its mean being the respective parameter. Following the recommendation of the assignment sheet the standard deviation was chosen as 0.1. The calculation of the new proposal values was implemented using the *scipy.stats* function *norm*:

```
New proposal values for μ and σ

 mu_prop = norm(mu, 0.1).rvs()
 sigma_prop = norm(sigma, 0.1).rvs()
```

After, again checking the validity of the σ-value, the new likelihood was computed and compared to the old value following the assignment instructions. Depending on the respective likelihood, the proposal was either accepted and its values were used for the next iteration, or the proposal was rejected and new values were computed.

Results

Fig. 1 shows the results of the calculation for N=3000. The two plots in the upper row show the convergence of the results to the true values of the data set. The bottom-left plot shows the convergence to the true value-pair (μ, σ), starting from the initial values. As can be seen by the random walk, the algorithm seems to move toward the true solution before starting to fluctuate around the true values and converging given some (non-trivial) error. The last plot shows the normalized histogram of the data as well as the sought-for probability density function, using the values of the final proposal for the model. The resulting PDF fits the data very well.
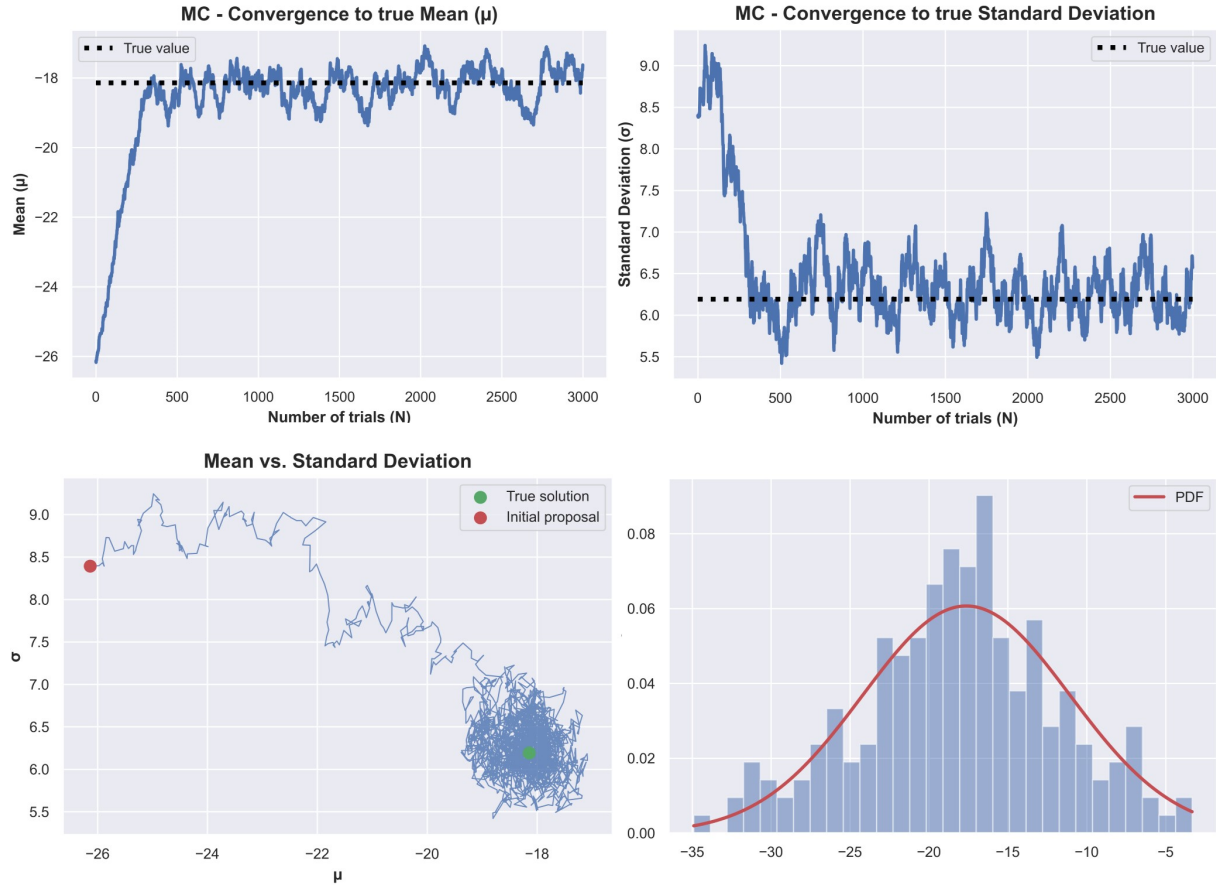

Figure 1: Results of the Metropolis-Hastings algorithm for N=3000. Shows the convergence behaviour of the results to the true values for mean and standard deviation of the data set. The random walk in the lower left plot shows the convergence to the true values as well. Bottom-right: Normalized histogram with the corresponding pdf-function using the values of the final proposal.