Assignment 1: introduction and visualisation

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1 Intro

This assignment consists of three tasks to be completed. When you have finished the tasks, you should write a **brief** description of what you did and submit it to me **with the codes** by email. My email address is colin.johnstone(at)univie.ac.at. All codes must be written in Python.

2 Task 1

Write a code that can integrate a function, $f(x)$***, numerically between two values of*** x ***using Simpson's rule***, which you should look up on Wikipedia. Specifically, you should calculate the value of

$$\int_{x_1}^{x_2} f(x)dx. \tag{1}$$

The function that you should integrate is

$$f(x) = 2\sin(x) + 1. \tag{2}$$

You should integrate this between x values of 0 and π. By testing with different numbers of bins, i.e. different N, work out how many bins you need to get an accurate result. ***Make a figure showing the result of the integration as a function of the number of bins chosen.***

A simple example code that does an integration using a simpler method is provided with this assignment. This inregrates the function $\sin(x)$ within the same limits using a similar technique that is somewhat simpler than Simpson's rule. You can use this code as the basis of your own, and modify it to complete this task.

3 Task 2

Write a code that can be used to solve simple equations using Newton Iteration. The equations that you should solve are

$$x^3 - x + 1 = 0, \tag{3}$$

$$\cos(x) - 2x = 0. \tag{4}$$

Make a figure showing your approximate solution as a function of the number of iterations used.

4 Task 3

Write a code that uses the bisection method to calculate the speed of an isothermal Parker wind at radius r. The equation for the velocity, v, of an isothermal Parker wind is

$$v \exp\left(-\frac{v^2}{2c_\mathrm{s}^2}\right) = c_\mathrm{s} \left(\frac{r_\mathrm{c}}{r}\right)^2 \exp\left(-\frac{2r_\mathrm{c}}{r} + \frac{3}{2}\right), \tag{5}$$

where c_s is the sound speed given by

$$c_\mathrm{s} = \sqrt{\frac{k_\mathrm{B}T}{\bar{m}}}, \tag{6}$$

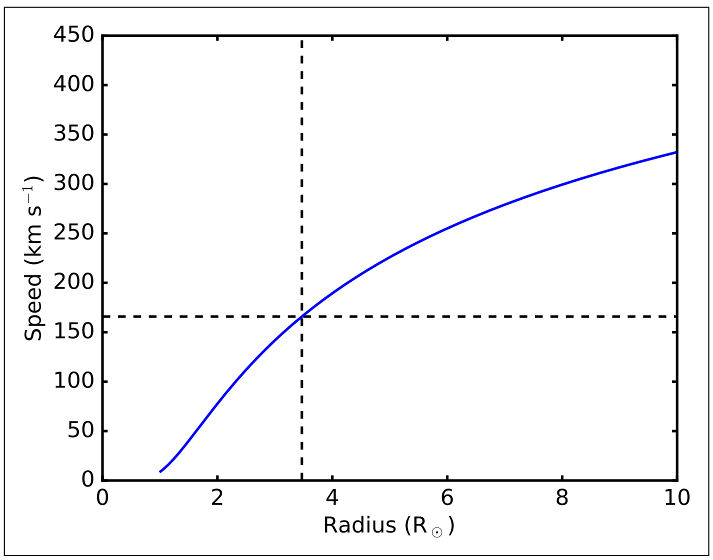
and r_c is the radius of the critical point, given by

$$r_\mathrm{c} = \frac{GM_\star}{2c_\mathrm{s}^2}. \tag{7}$$

For the average molecular mass, $\bar{m}$, assume $0.5M_\mathrm{p}$, where M_p is the mass of a proton (i.e. the wind is mostly just made of H^+ and e^-). For the mass of the star, $M_\star$, assume the solar mass. Remember to be careful to keep all quantities in the same unit system, and make sure to use the correct values of the constants G and k_B for that system.

Use the code to calculate the wind speed in 100 radial bins evenly spaced between radii of 2 $R_\odot$ and 1 AU for winds with temperatures of 2, 4, 6, 8, and 10 MK and make a figure showing v against r for each of these temperatures.

As a hint, you can see in the figure a typical solution for the above equations. The dashed lines show the radius and speed of the critical point, where the wind becomes supersonic. Notice that when $r < r_\mathrm{c}$, the wind is subsonic (i.e. $v < c_\mathrm{s}$), and when $r > r_\mathrm{c}$, the wind is supersonic (i.e. $v > c_\mathrm{s}$). This is useful for helping you decide which initial estimates for the upper and lower bounds to take in the bisection method.



450
400
350
300
250
200
150
100
50
0
Speed (km s^{-1})
0
2
4
6
8
10
Radius (R$_\odot$)