# Assignment - Hydrodynamics 3

Christian Wiskott

University of Vienna — January 4, 2021

## Part 5

The numeric solution to the diffusion problem was evaluated using two different methods: the explicit *FTCS* and the implicit *BTCS*, over a grid with 100 points between x=0 and x=10 using the initial conditions:

$$q(x,0) = \begin{cases} 1.0 & \text{if } 3.5 \leq \text{x} \leq 6.5 \\ 0.1 & \text{otherwise} \end{cases}$$

with D = 1.0. The timestep was computed via:

$$\Delta t = c\frac{\Delta x^2}{2D} \tag{1}$$

with $0 \leq c \leq 1$.

The results were stored in a M $x$ N - matrix, where M is the number of grid points and N is the number of timesteps over the course of the calculation. Thus, a column of the matrix represents one single timestep and its corresponding row elements i.e. the results for the grid points $q(x_i, t)$.
The code was written such that, once executed, the results are plotted for all considered methods, in order to get a valid comparison of the performance for the chosen parameters.

1. The analysis of the *FTCS* method shows, that the results become increasingly unstable for increasing $\Delta t$, similar to the *Central Difference* method of the first part of this assignment. The first row of fig. 1 shows the behaviour using the respective $\Delta t$. With $c = 0.5$ the results look very similar compared to the BTCS method as no obvious fluctuations are visible. Using c=1 leads to visible instabilities of the results. It is quite obvious that the chosen $\Delta t$ is too large and thus the results becoming inaccurate. The considerable influence of $\Delta t$ is obvious with c=1.5 as the results diverge quickly and show huge fluctuations.

2. The *BTCS* method provides much more stable results compared to *FTCS*. As $\Delta t$ increases, the results appear to be virtually unaffected. Only minor differences could be identified and are only visible when looking at the results numerically, not visually. Due to the unconditional stability of this method, the results don't show the same unstable behaviour as $\Delta t$ is increased beyond c=1.

   As stated in the assignment sheet, the values at the boundaries where kept constant and show the expected *unrealistic* values.

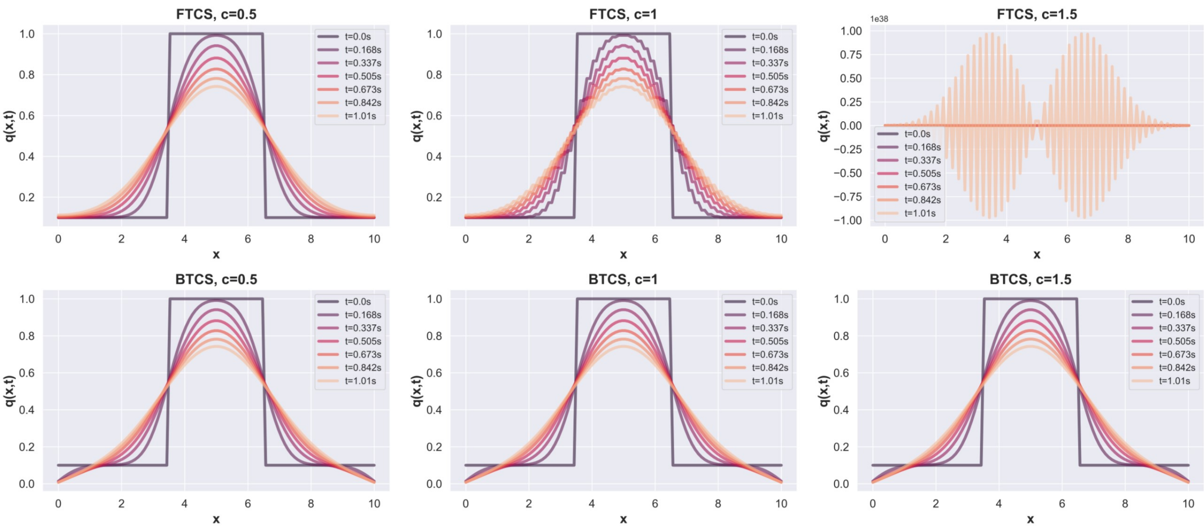


Figure 1: Comparison of the results provided by the two chosen methods. Each row corresponds to the results of the given method using three different Courant numbers (0.5, 1, 1.5) and thus different $\Delta t$'s. The first column was done using $c = 0.5$, the second with $c = 0.1$ and the third with $c = 1.5$. This serves the purpose of visually comparing the stability achieved by each method at each $\Delta t$. As $\Delta t$ increases, the *FTCS* method exhibits increasingly unstable results and completely diverges with c>1. The results of the *BTCS* method indicate much higher stability, as the results seem to be virtually unaffected by the changing $\Delta t$. The boundary values exhibit the expected unrealistic values and where kept constant during the computation.