# Assignment: hydrodynamics 2

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## 1 Intro

The second two parts of this assignment are about the construction of a simple 1D hydrodynamic code and the Sod shock tube test.

Imagine you have a 1D spatial domain extending from $x = 0$ to $x = 1$. Within this domain, you have a uniform fluid with $\rho = 0.125$, $u = 0$, and $P = 0.1$. However, in the region between $x = 0.35$ and $x = 0.65$, you have $\rho = 1.0$, $u = 0$, and $P = 1.0$.

What should happen is that the fluid, at the boundaries on either side of the region of enhanced density and pressure, should expand into the other regions in a manner similar to what you can see in the normal Sod shock tube test (see Wikipedia for details). In order to model this, you need to solve the equations of hydrodynamics, given by

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left(\rho u\right) = 0 \tag{1}$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial}{\partial x}\left(\rho u^2\right) = -\frac{\partial}{\partial x} P \tag{2}$$

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial}{\partial x}\left(\epsilon u\right) = -\frac{\partial}{\partial x}\left(P u\right) \tag{3}$$

where $\epsilon = P/(\gamma - 1) + 0.5\rho u^2$ and $\gamma = 5/3$. Assume for your grid, 1000 grid cells.

## 2 Part 3 (9 marks)

1. Firstly, write a code that solves this problem using the two step Lax-Wendroff method for the advection and the central difference for the source terms. *Show and describe what happens when you evolve the system forward in time.*

2. Secondly, write a code that does the same thing, but using the upwind scheme instead. *Show and describe what happens when you evolve the system forward in time.*

In both cases, you can simply hold the quantities at the cells on the edges of the grid constant and stop your simulations before the disturbances reach the edges of the grid. You will need to choose the time step to be small enough that the calculations are stable, or you can calculate the time step properly by calculating how long it takes sound waves to cross each grid point. When you make plots for the protocol (the thing to hand in), present plots of the density, velocity, and temperature (assuming a particle mass equal to that of atomic hydrogen; $P = \rho/m_H k_B T$) at two or more time steps. Hint: in both cases, the simulations should be symmetric about the point $x = 0.5$; if they are not, then there is something wrong with the code (and this is especially easy to get wrong for the upwind scheme).

# 3 Part 4 (1 mark)

You have now seen and implemented the Lax-Wendroff method in three different forms. Firstly, you have seen it as a finite difference scheme that uses central differencing for spatial derivatives and adds just enough artificial diffusion to keep the scheme stable. Secondly, you have seen it as a flux conserving scheme where the quantities in grid cells are assumed to vary linearly in space and the slopes are calculated using a downwind assumption. Finally, you have seen it as a two step algorithm. For the final mark, *show analytically that these three schemes are identical for the problem of advecting an arbitrary function $q_i^n$, assuming a constant and uniform advection velocity.* You can hand in this part in written form (i.e. hand written if you want) or in pdf form (from latex for example).