

# Wavelet-Project Handout

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## 1 Introduction

During the Conference on Advances in Communication and Control Systems 2013, a paper named "Digital Image Watermarking Using 3 levels Discrete Wavelet Transform" initiated a new technique of image watermarking based on the decomposition in the space of three-level wavelet. The principle is quite intuitive : we select two images, the first one (I1) is the picture we want to protect and the other one (I2) acts like a certificate (watermark) that we want to embed in I1 without too adversely affecting its quality. In order to do so, we decompose I1 and I2 in three parts thanks to a three-level wavelet, we retrieve the third part of each (LL3 (I1) and WLL3 (I2) ) and we affect to LL3 a linear combination of LL3 and WLL3 (the coefficients which intervene in this operation are denoted  $k$  and  $q$ ). We then recompose the new image I1'. Regarding the choice of  $k$  and  $q$ , we should obtain an image more and less similar to the original image I1. We can retrieve the watermark from I1', knowing only the coefficient  $k$  and having the original image I1. We can then prove the paternity of an image.

## 2 Novelty of the methods

There exist two general techniques to create watermarked images: the spatial watermarking modifies directly the value of its pixels in order to embed the watermark. However, this method has many fragilities. Another way is to modify the frequency component of the picture to be protected. Various methods have been implemented using DCT, DWT, or a mix of DCT-DWT. The proposed method has the advantage of using a 3-level wavelet decomposition in the Haar basis and an image as a watermark. The technique seems to be similar to the paper 'Image watermarking using 3-Level Discrete wavelet transform' except that the basis used is daubecheis.

### 3 Extension: Comparison of the methods with other methods

Because the implementation of the method is very similar to the functions we have studied in the previous lecture, we will consider extending the result to additional methods:

- Least Significant Bit insertion (LSBI), a spatial watermarking where each pixel of the original and watermarked undergoes a bit shift and replaces the value of the pixel of the original image by the addition of the two transformed pixels.
- A combination of DCT and DWT (DCT-DWT).
- A watermarked using DWT but with pixel-wise marking (DWT-PWM)

We particularly assess the imperceptibility of the embedding method by computing the Mean Square Error (MSE) as well as the Peak Signal-to-Noise Ratio (PSNR). The formula of the two measures are given below:

$$MSE = \frac{\sum_{i=0}^{N-1} \sum_{j=0}^{M-1} I_{reconstructed}(i,j) - I_{original}(i,j)}{N * M}$$

Where  $N$  and  $M$  are, respectively, the length and the width of the images.

$$PSNR = 20 * \log\left(\frac{255}{\sqrt{MSE}}\right)$$

We first focus on the impact of the combination of coefficients in the first methods with various watermark and target images (presented in their respected files).

We have the following results:

(i,j) respectively the index of the target image and the watermark	MSE /PNSR	k	q
(1,1)	6758.01/9.83	0.2	0.009
	2588.73/13.99	0.5	0.009
	614.13/20.248	0.75	0.009
	206.05/24.99	0.85	0.009
	83.82/28.89	0.90	0.009
	16.13/36.05	0.95	0.009
	1.2494/47.16	0.99	0.009
	2.9818/43.38	1.0	0.009
	755.28/19.34	1.25	0.009
	2871.045/13.55	1.5	0.009
	11192.92/7.6413	2.0	0.009
(1,2)	6849.64/9.77	0.2	0.009
	2645.25/13.90	0.5	0.009
	641.39/20.05	0.75	0.009
	221.61/24.67	0.85	0.009
	93.53/28.42	0.90	0.009
	19.99/35.12	0.95	0.009
	0.429/ 51.80	0.99	0.009
	0.991/48.16	1.0	0.009
	724.03/19.53	1.25	0.009
	2810.54/13.64	1.5	0.009
	11073.90/7.68	2.0	0.009
(1,3)	18.40/35.48	0.95	0.009
	1.864/45.42	0.99	0.009
	3.184/43.10	1.0	0.009
	745.16/19.40	1.25	0.009
(2,1)	13.92/36.69	0.95	0.009
	1.436/46.55	0.99	0.009
	2.9818/43.38	1.0	0.009
	648.41/20.01	1.25	0.009
(2,2)	17.25/35.76	0.95	0.009
	0.5103/51.05	0.99	0.009
	0.9913/48.16	1.0	0.009
	619.82/20.20	1.25	0.009
(3,3)	26.81/33.84	0.95	0.009
	2.054/45.00	0.99	0.009
	3.1843/43.10	1.0	0.009
	983.09/18.20	1.25	0.009

The results obtained are slightly different from the result of the paper ( we have a local maximum at 0.99 instead of 0.85.

We compare this results with the three other methods and we obtained the following results:

PNSR/MSE				
(i,j)	three level discrete wavelet	LSBI	DCT-DWT	DWT-PWM
(1,1)	1.2494/ 47.16	38272.49/ 2.3019	5.7974/ 40.49	14.062/ 36.65
(1,2)	0.4292/ 51.80	47801.81/ 1.336	5.797/ 40.49	14.062/ 36.65
(1,3)	1.864/ 45.4242	22715.56/ 4.567	3.4873/ 42.70	14.06/ 36.65
(2,1)	1.436/ 46.55	37682.51/ 2.369	5.797/ 40.49	14.06/ 36.65
(2,2)	0.510/ 51.05	46177.75/ 1.486	5.797/ 40.49	14.06/ 36.65
(2,3)	2.1838/ 44.73	22695.63/ 4.571	3.487/ 42.70	14.06/ 36.65

We see that most the case the three level discrete wavelet methods is much more imperceptible than the the others methods. Especially the mixed method DCT-DWT doesn't improve the imperceptibility of the image.

We then focus on the robustness of the recovered watermark, firstly just without transformation:

We add to the previous measure, an indicator that wasn't mentioned in the article which is the correlation factor which is given by the following formula:

$$\rho = \frac{\sum_i \sum_j w(i,j)w'(i,j)}{\sum_i \sum_j (w(i,j))^2}$$
 where  $w(i,j)$  and  $w'(i,j)$  are respectfully the coefficients of the watermarked image and the extracted watermarked image.

MSE/PNSR/correlation factor of the extracted watermark				
(i,j)	three level discrete wavelet	LSBI	DCT-DWT	DWT-PWM
(1,1)	32442.71/3.019/0.9973	11644.44/7.46/0.9053	37450/2.3962/0.0649	10407.04/7.95/0.9513
(1,2)	10556.2/7.8957/0.9999	4144.16/11.95/0.8774	12004/7.33/0.1435	38207.6/2.30/0.5554
(1,3)	34918.20/2.70/0.9873	12362.21/7.20/0.9246	38675.34/2.25/0.2004	40132.61/2.09/0.9462
(2,1)	32432.38/3.02/0.9964	13026.20/6.98/0.8921	37118.18/2.43/0.1139	10371.93/7.97/0.8953
(2,2)	10562.95/7.89/0.9999	4456.13/11.64/0.8718	11640.21/7.47/0.2242	38158.04/2.31/0.5563
(2,3)	34960.26/2.69/0.9801	14070.57/6.64/0.9100	39953.31/2.11/0.093	40196.12/2.09/0.4913
(3,1)	32475.13/3.01/0.9898	12679.84/7.1/0.8898	37104.08/2.44/0.1165	10371.93/7.97/0.8953

We deduce that the MSE and the PNSR measure are not relevant to describe the robustness of the method to recover the embedded watermark. In particular, the PNSR measure is less prominent for the three-level discrete method than the LSBI method, while the correlation factor is significantly more important for the former. We will then keep the correlation factor to assess the robustness of the method. We remark that the three-level discrete remain the method with the better performance for robustness and the imperceptibility of the embedding. We also remark on the low quality of the disembedded process for the DCT-DWT and the DWT-PWM method. While the DWT-PWM method is sensible to the threshold determining if a pixel is much closer to black or white (much of the watermark images have different shade of gray, which make difficult to apply this method because it presupposes that the watermark has only black and white pixels), the DCT-DWT process uses random number for embedding and disembedding the watermark which impacts the reconstruction process.

We are looking to expand this result by testing the robustness of the methods on three main transformations:

- An addition of a normal noise of a variance  $(\sigma)^2$
- A cropping at the center in a window size depending of a factor  $f$
- A rotation of an angle  $\theta$

We give the following result:

correlation factor of the extracted watermark (1,1) undergoing a normal noise				
$\sigma$	three level discrete wavelet	LSBI	DCT-DWT	DWT-PWM
0.5	0.9970	0.8732	0.0649	0.6943
1	0.9963	0.8731	0.0649	0.7246
5	0.9702	0.8727	0.0813	0.6833
10	0.8964	0.8714	0.1573	0.7103
20	0.7065	0.8655	0.1584	0.7075

We observe a slightly degradation of the robustness of the three-level discrete wavelet with respect to the standard variation. The other methods don't seem to be affected by the addition of noise. In reality, their results is far from what was expected .

correlation factor of the extracted watermark (1,1) undergoing a crop of factor $f$				
$f$	three level discrete wavelet	LSBI	DCT-DWT	DWT-PWM
2	0.0177	0.8424	0.2073	0.7945
3	0.2530	0.8507	0.1984	0.8146
4	0.2650	0.8249	0.2108	0.8274

We observe as well a considerable degradation of the correlation factor with respect to the factor  $f$  of reduction.

correlation factor of the extracted watermark (1,1) undergoing a rotation $\theta$				
$\theta$	three level discrete wavelet	LSBI	DCT-DWT	DWT-PWM
0	0.9973	0.9053	0.0649	0.8948
0.05	0.9446	0.8885	0.0649	0.8927
0.10	0.8186	0.8776	0.0947	0.8915
0.2	0.5588	0.8722	0.0993	0.8877
0.5	0.2142	0.8630	0.1545	0.8675

In the same way, we observe a diminution of the robustness of the recovering for the three level discrete wavelet method when we apply a rotation to the watermarked image.

## 4 Conclusion

We have reproduced and extended the results presented in this paper by interesting ourselves on three other alternatives to embed a watermark (representing by a picture) in an image. For each method, we have assessed two major criteria that a good watermarking must satisfy: the imperceptibility of the watermark image and the robustness of the extraction method with respect to transformation of the watermarked image. We have observed that the three-level discrete wavelet technique gave better results for imperceptibility and the embedding didn't affect the quality of the watermark. We then apply three major image transformations which are : the rotation  $\theta$  of the image invariant in its center, the cropping delimited by a window whose size depends on a factor  $f$  and the addition of a Gaussian noise by varying its standard deviation  $\sigma$ . We witness the decrease of this technique when we applied these transformations. More precisely this fall was significant for the rotation and the cropping. The results of the other techniques could give the impression of stability but when we see it qualitatively, we figure out that the reconstruction was far from what we expected. We conclude that no methods are invariant to the transformation. The three-level method remains the technique which is the most efficient.