CPEN 416 / EECE 571O

Lecture 02: Single-qubit systems; introducing PennyLane

Tuesday 09 September 2025

Announcements

- Practice assignment 1 available later today
- Quiz 1 today

Recap from last time

github.com/glassnotes/CPEN-416

Qubits are physical quantum systems with two **basis states**:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Arbitrary states are complex-valued linear combinations

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

where $|\alpha|^2 + |\beta|^2 = 1$ and $\alpha, \beta \in \mathbb{C}$.

Qubit states live in **Hilbert space**.

Recap from last time

Unitary matrices (gates/operations) modify a qubit's state.

A matrix U is unitary if

dagger: conjugate transpose

$$UU^\dagger = U^\dagger U = I$$

They preserve normalization of state vectors (and angles between them - can prove on practice assignment 1).

We saw three examples:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

NOT

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

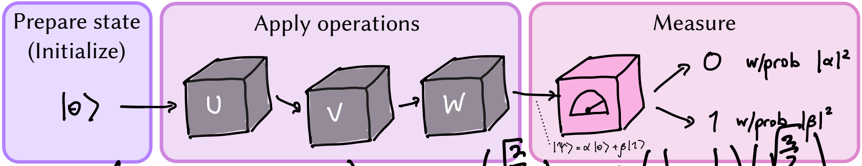
Recap from last time

$$Z|0\rangle = |0\rangle$$

$$H(Z|0\rangle) = H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

After applying gates, we measure and observe the qubit in either state $|0\rangle$ or $|1\rangle$.

Measurement is probabilistic: the outcome probabilities depend on the amplitudes.



$$H\left(\sqrt{\tfrac{2}{3}}|0\rangle + \sqrt{\tfrac{1}{3}}|1\rangle\right) = H\begin{pmatrix}\sqrt{\tfrac{2}{3}} \\ \sqrt{\tfrac{1}{3}}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}\begin{pmatrix}\sqrt{\tfrac{2}{3}} \\ \sqrt{\tfrac{1}{3}}\end{pmatrix}$$

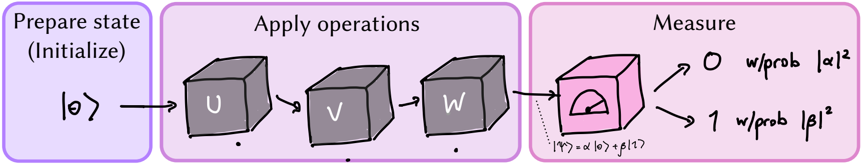
$$= \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2/3} + \sqrt{1/3} \\ \sqrt{2/3} - \sqrt{1/3}\end{pmatrix}$$

Learning outcomes

1. Express quantum computations as quantum circuits, and as quantum functions in PennyLane
2. Represent the state of a single qubit on the Bloch sphere
3. Describe the behaviour of common single-qubit gates

Quantum circuits

This representation is cumbersome:

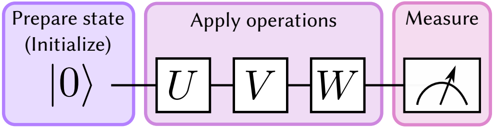


So is the matrix representation:

$$WVU|0\rangle$$

Alternative: quantum circuits!

if not specified, assume |0⟩

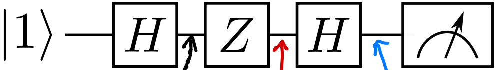


Quantum circuits

linearity: $M(a\vec{v} + b\vec{w}) = M(a\vec{v}) + M(b\vec{w})$

$= a \cdot M\vec{v} + b \cdot M\vec{w}$

Exercise: perform the quantum computation in the circuit below. What is the probability of observing the qubit in state $|0\rangle$ after measurement?



$H\,|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$= |-\rangle$

$Z\left(\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right)$

$= Z\left(\frac{1}{\sqrt{2}}|0\rangle\right) - Z\left(\frac{1}{\sqrt{2}}|1\rangle\right)$

$= \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = |+\rangle$

$H\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$

$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$

$+ \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right)$

$= |0\rangle \Rightarrow \mathrm{Prob}(|0\rangle) = 1$

Programming quantum computers

Everything we've done so far is just matrix-vector multiplication:

```python
def ket_0():
    return np.array([1.0, 0.0])

def apply_ops(ops, state):
    for op in ops:
        state = np.dot(op, state)
    return state

def measure(state, num_samples=100):
    prob_0 = state[0] * state[0].conj()
    prob_1 = state[1] * state[1].conj()

    samples = np.random.choice(
        [0, 1], size=num_samples, p=[prob_0, prob_1]
    )
    return samples

H = (1/np.sqrt(2)) * np.array([[1, 1], [1, -1]])
X = np.array([[0, 1], [1, 0]])
Z = np.array([[1, 0], [0, -1]])

input_state = ket_0()
output_state = apply_ops([H, X, Z], input_state)
results = measure(output_state, num_samples=10)

print(results)
```

```
[1 0 0 1 0 0 0 1 1 0]
```

Sample NumPy

... better to use real quantum software instead.

PennyLane

PennyLane is a Python framework developed by **Xanadu** (a Toronto-based quantum startup).

```python
import pennylane as qml

H = qml.Hamiltonian(...)

dev = qml.device('default.qubit', wires=2)

@qml.qnode(dev)
def quantum_circuit(params):
    qml.RY(params[0], wires=0)
    qml.RY(params[1], wires=1)
    qml.CNOT(wires=[0, 1])
    qml.RY(params[2], wires=0)
    return qml.expval(H)

quantum_circuit([0.1, 0.2, 0.3])
```

GitHub: `https://github.com/PennyLaneAI/PennyLane`
Documentation: `https://pennylane.readthedocs.io/en/stable/`
Demonstrations: `https://pennylane.ai/qml/demonstrations.html`
Discussion Forum: `https://discuss.pennylane.ai/`

Quantum functions

We can express circuits as **quantum functions** in PennyLane. They are similar to normal Python functions, except they

1. apply one or more quantum operations to qubits (0-indexed)
2. must return a quantum measurement

```
import pennylane as qml

def my_quantum_function():
    qml.Hadamard(wires=0) # Apply Hadamard gate to qubit 0
    qml.PauliZ(wires=0)   # Apply Pauli Z gate to qubit 0
    qml.PauliX(wires=0)   # Apply Pauli X gate to qubit 0
    return qml.probs()    # Return measurement probs
```

Q: Why `wires`? A: Lines in a circuit diagram are called wires, and PennyLane can be used for continuous-variable quantum computing, which isn't qubit-based. (Note: keyword argument no longer needed.)

Devices

Quantum functions are executed on **devices**.

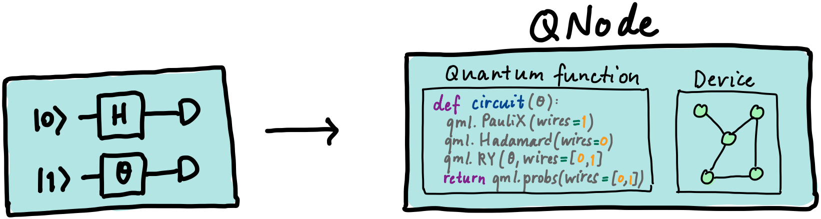
```
# Example 1: 5 qubits, analytic
dev = qml.device("default.qubit", wires=5)

# Example 2: 1 qubit, probabilistic (100 measurements)
dev = qml.device("default.qubit", wires=1, shots=100)
```

Devices can be either *simulators*, or *actual quantum hardware*.

Qubits on a device are initialized in $|0\rangle$.

Quantum nodes

A **QNode (quantum node)** is an object that binds a quantum function to a device, and executes it.

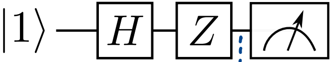



```
# Create a QNode (version 1)
my_qnode = qml.QNode(my_quantum_function, dev)

# Execute the QNode
result = my_qnode()
```

Image: https://pennylane.ai/qml/glossary/quantum_node.html

Executing quantum computations with PennyLane

Exercise: perform the quantum computation below in PennyLane on a device with 1000 shots. What is the probability of observing the qubit in state $|0\rangle$ after measurement?



$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

0
1
2

dev.wires = [0, 1, 2]

```python
import pennylane as qml

dev = qml.device("default.qubit", wires=?, shots=?)

def my_quantum_function():
    # Your gates here
    return qml.probs()

my_qnode = qml.QNode(my_quantum_function, dev)
my_qnode()
```

Run locally, or try our interactive debugger, CircInspect: `circinspect.ece.ubc.ca`

You probably have some questions...

1. Where's the state?
   - Inside the device
2. How do gates get applied?
   - Operations are recorded onto a "tape"
   - The QNode constructs the tape when it is called
   - The tape is then executed on the device.

Single-qubit quantum states

So far we know the following 3 gates:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

But, a general qubit state looks like

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

What about the rest?

1. First, we'll explore a way to visualize arbitrary quantum states
2. Next we'll learn new unitary operations to create them with

Parametrization of qubit states

$|\alpha|^2 = \alpha\alpha^*$

$\alpha = re^{i\phi} \rightarrow |\alpha| = r$

Exercise (part A): Consider the states

$$|\psi_1\rangle = \alpha|0\rangle + \beta|1\rangle, \quad |\psi_2\rangle = \alpha e^{i\phi}|0\rangle + \beta e^{i\phi}|1\rangle$$

How does $e^{i\phi}$ affect the measurement outcome probabilities of $|\psi_2\rangle$ compared to $|\psi_1\rangle$?

$|\psi_1\rangle$: $\text{Prob}(|0\rangle) = |\alpha|^2$

$\text{Prob}(|1\rangle) = |\beta|^2$

$|\psi_2\rangle$: $\text{Prob}(|0\rangle) = (\alpha e^{i\phi})(\alpha^* e^{-i\phi})$

$= \alpha\alpha^*$

$= |\alpha|^2$

$\text{Prob}(|1\rangle) = |\beta|^2$

$|\psi_2\rangle = e^{i\phi}(\alpha|0\rangle + \beta|1\rangle)$

global phase

Parametrization of qubit states

Exercise (part B): How many real numbers are required to *fully specify* a single-qubit state vector?

$\alpha = re^{i\phi}$

$\alpha = a + bi$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \Rightarrow 4$

$\alpha \to r_1 e^{i\phi_1}$, $\beta \to r_2 e^{i\phi_2}$

$= \cancel{e^{i\phi_1}} \left(r_1 \alpha |0\rangle + r_2 e^{i\underbrace{(\phi_2 - \phi_1)}_{\varphi}} |1\rangle \right)$

$r_2 \to \sqrt{(1 - r_1^2)}$

$|\alpha|^2 + |\beta|^2 = 1$

$\downarrow$

$r_1^2 + r_2^2 = 1$

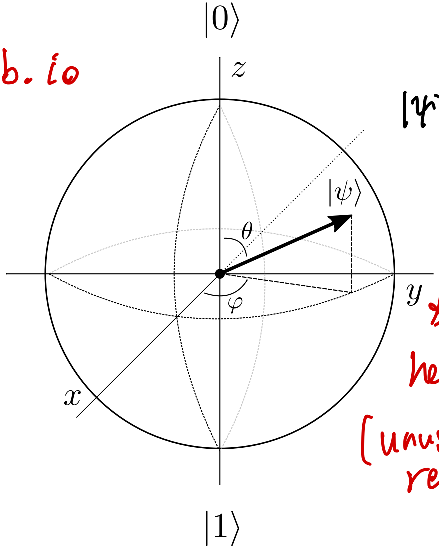
$(\cos\theta)^2 + (\sin\theta)^2 = 1 \Rightarrow$

$$|\psi\rangle = \cos\theta/2\,|0\rangle + \sin\theta/2\, e^{i\varphi}|1\rangle$$

The Bloch sphere

Play with:

bloch.kherb.io



$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

☆ Start from here next time

(unused slides removed)

Next time

Content:

- The theory of projective measurements
- Measuring in different bases

Action items:

1. Work on problems in practice Assignments 0 and 1
2. Explore the PennyLane docs and learn about the different kinds of operations and measurements
3. Visualize combinations of operations on the Bloch sphere

Recommended reading:

- Codebook modules IQC, SQ
- Nielsen & Chuang 4.2, 2.2.3, 2.2.5, 2.2.7