

ELEC 221 Lecture 19

Amplitude modulation

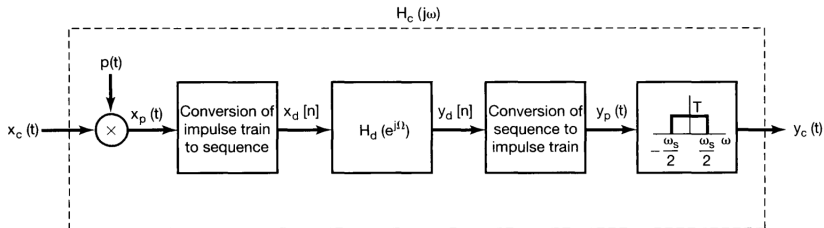
Tuesday 19 November 2024

Announcements

- Quiz 8 today
- Assignment 4 due Saturday 23:59
- Tutorial assignment 4 due Monday 23:59
- Friday office hour cancelled
- Class next Tuesday (26 Nov) on Zoom

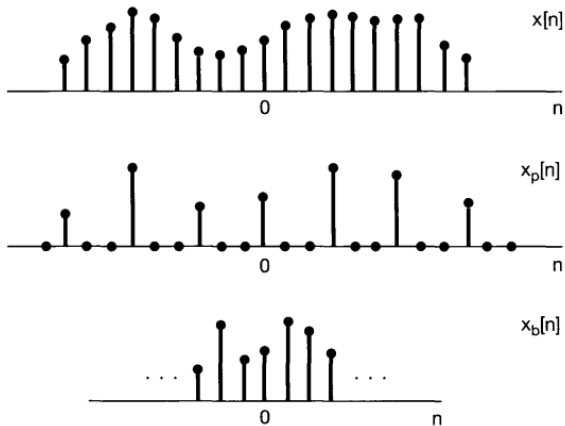
Last time

We performed CT signal processing by sampling, converting to DT, processing, then converting back.



Last time

We sampled DT signals (decimation). If original signal was CT, this is called *downsampling*.



Last time

Decimation spreads out the spectrum.

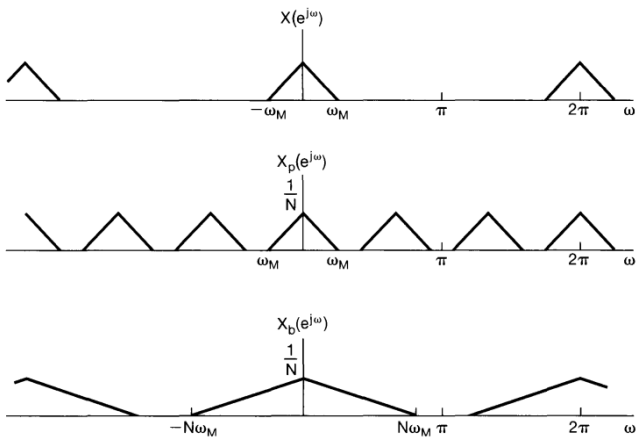
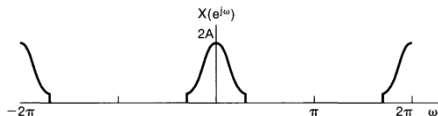
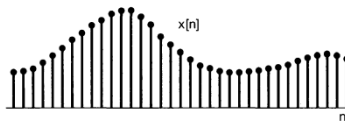
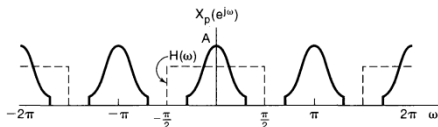
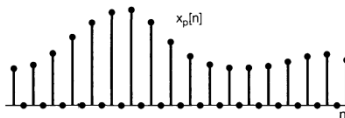
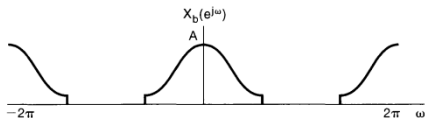
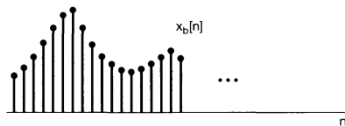


Image credit: Oppenheim 7.35

Last time

The opposite of decimation is interpolation (upsampling).



Today

First: finish example from last time.

Learning outcomes:

- perform sinusoidal amplitude modulation (AM) and demodulation in CT and DT
- differentiate between synchronous and asynchronous modulation techniques, and identify pros/cons of each method
- carry out frequency-division multiplexing

Example: down/upsampling

Oppenheim problem 7.19,

7.19. Consider the system shown in Figure P7.19, with input $x[n]$ and the corresponding output $y[n]$. The zero-insertion system inserts two points with zero amplitude between each of the sequence values in $x[n]$. The decimation is defined by

$$y[n] = w[5n],$$

where $w[n]$ is the input sequence for the decimation system. If the input is of the form

$$x[n] = \frac{\sin \omega_1 n}{\pi n},$$

determine the output $y[n]$ for the following values of ω_1 :

- (a) $\omega_1 \leq \frac{3\pi}{5}$
- (b) $\omega_1 > \frac{3\pi}{5}$

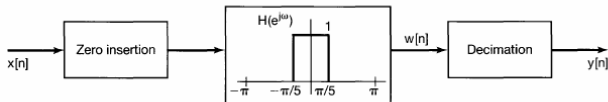


Figure P7.19

Example: down/upsampling

First case: $\omega_1 \leq \frac{3\pi}{5}$

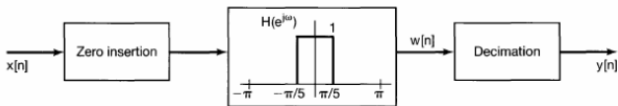


Figure P7.19

Example: down/upsampling

Second case: $\omega_1 > \frac{3\pi}{5}$

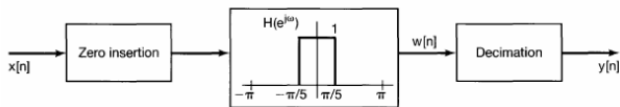


Figure P7.19

Impulse train sampling

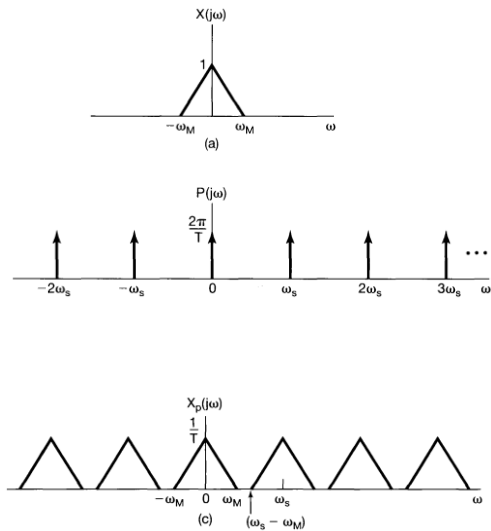
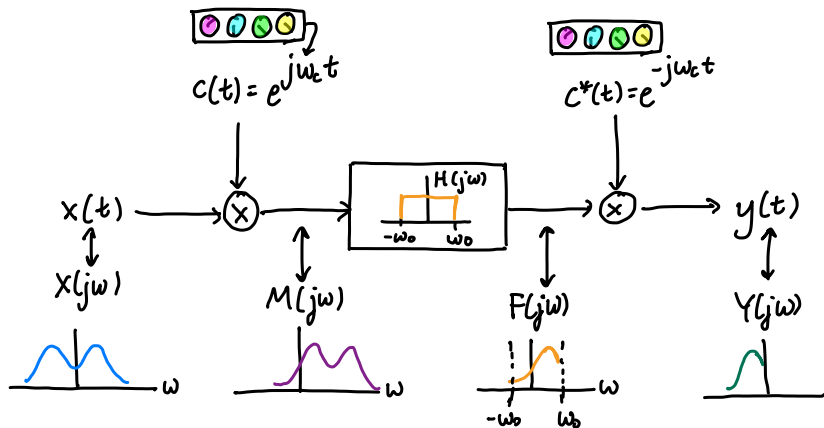


Image credit: Oppenheim 7.1

Frequency-selective filtering with variable centre frequency

Recall from Assignment 3:

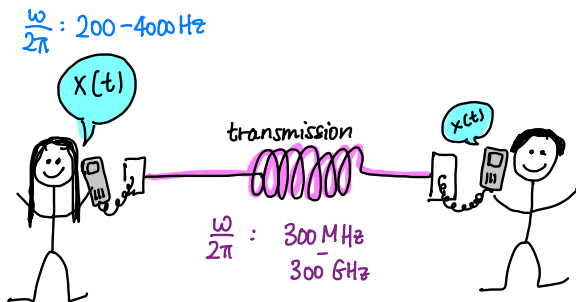
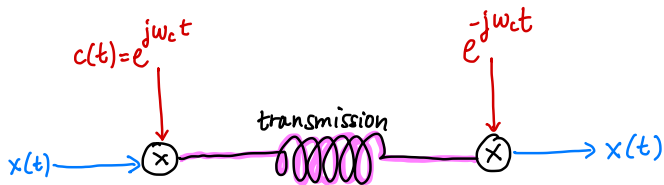


Multiplication property

Both results come from the multiplication property:

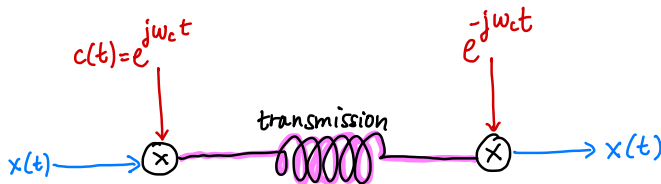
This is also known as the *modulation property*.

Motivation: communication systems



Modulation

The process of embedding an information-bearing signal into a second signal. (Extracting the signal: demodulation)



Two main types (we will only discuss AM):

- amplitude modulation (AM)
- frequency modulation (FM)

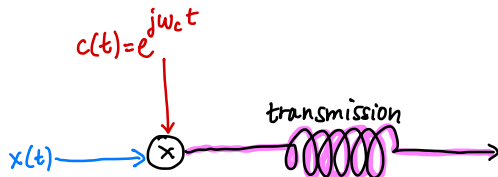
We focus on two types of **carrier signal**, $c(t)$:

- complex exponential signal

- sinusoidal signal

Complex exponential amplitude modulation

We've already seen what happens with the first one.



In practice:

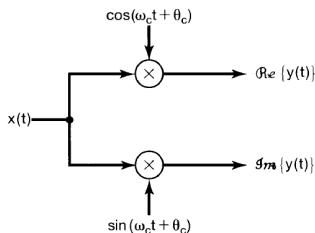
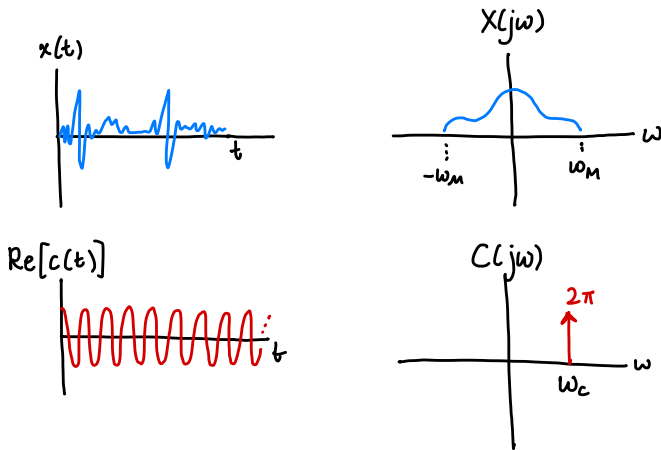


Figure 8.2 Implementation of amplitude modulation with a complex exponential carrier $c(t) = e^{j(\omega_c t + \theta_c)}$.

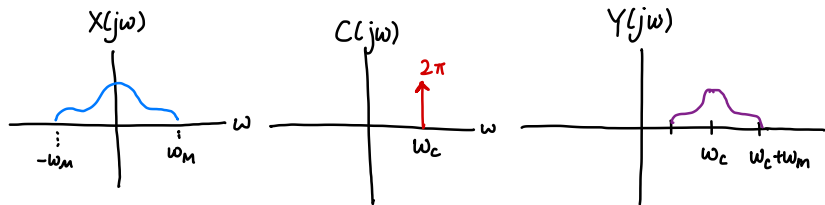
Complex exponential amplitude modulation

Consider the Fourier spectrum of both signals:

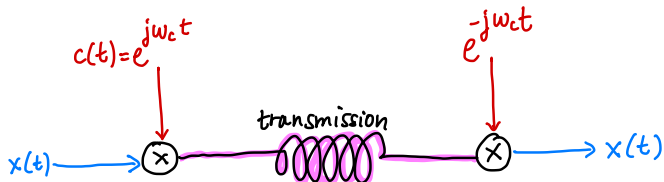


Complex exponential amplitude modulation

Multiplication by $c(t)$ shifts spectrum to different frequency range.

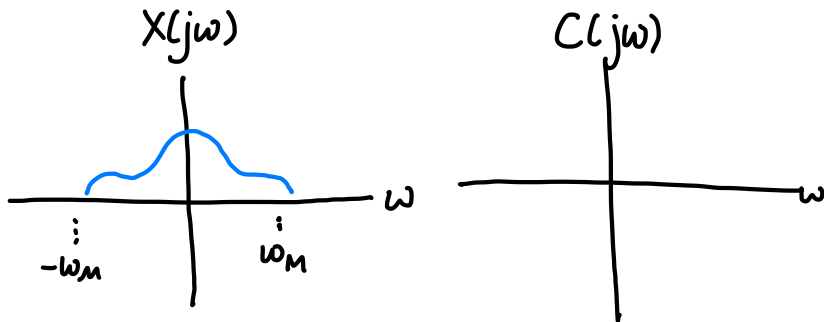


Demodulation is straightforward.

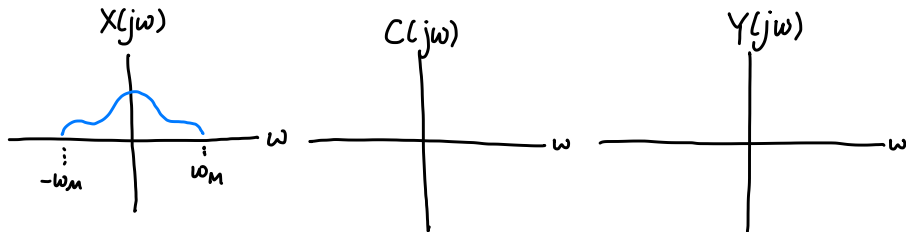


Sinusoidal amplitude modulation

What if we use a sinusoidal signal instead?

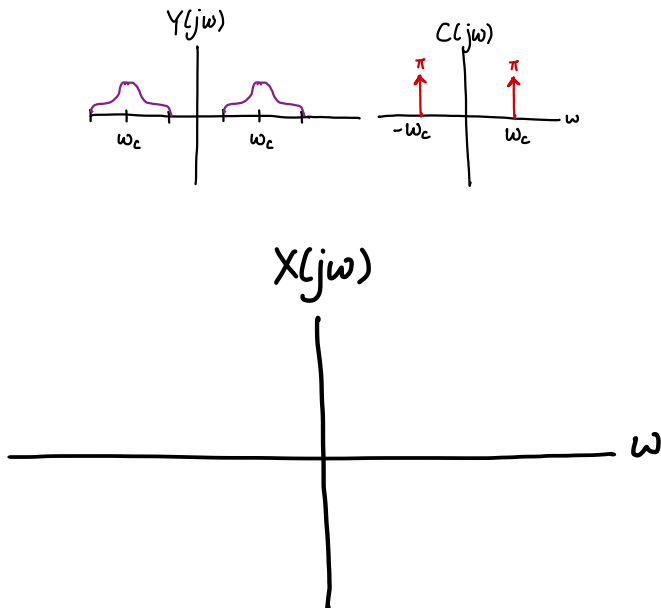


Sinusoidal amplitude modulation

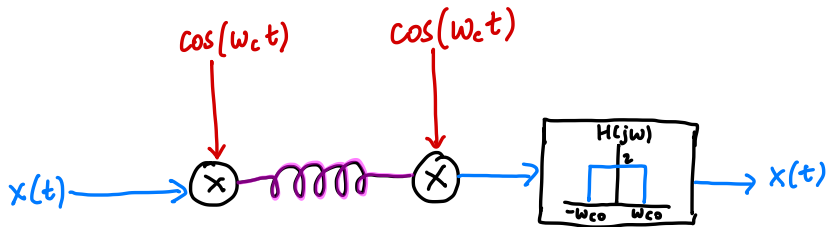


- Any foreseeable problems with this?
- How do we recover the signal?

Sinusoidal amplitude modulation



Sinusoidal amplitude modulation



Let's check the math:

Exercise: sinusoidal amplitude modulation

- 8.3.** Let $x(t)$ be a real-valued signal for which $X(j\omega) = 0$ when $|\omega| > 2,000\pi$. Amplitude modulation is performed to produce the signal

$$g(t) = x(t) \sin(2,000\pi t).$$

A proposed demodulation technique is illustrated in Figure P8.3 where $g(t)$ is the input, $y(t)$ is the output, and the ideal lowpass filter has cutoff frequency $2,000\pi$ and passband gain of 2. Determine $y(t)$.

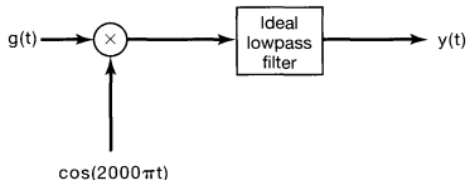


Figure P8.3

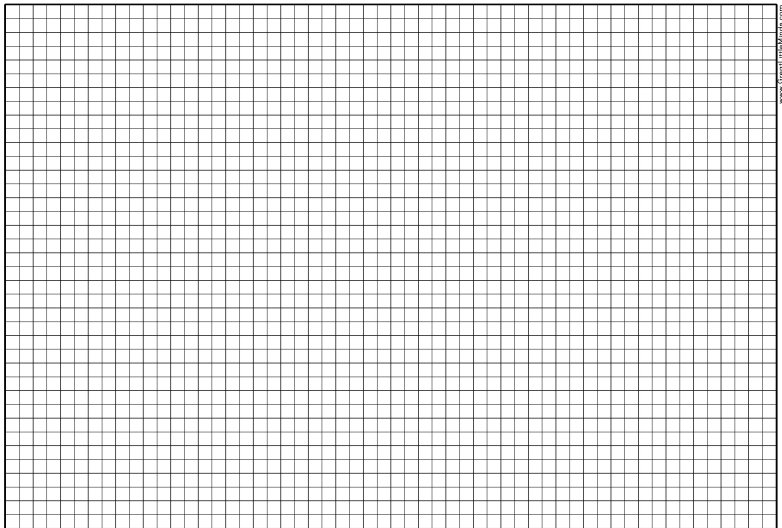
Exercise: sinusoidal amplitude modulation

Solution option 1: mathematical

Evaluate $w(t) = g(t) \cos(2000\pi t)$ (input to filter):

Exercise: sinusoidal amplitude modulation

Solution option 2: analyze the spectra



Synchronous demodulation

More generally, must consider phases in both modulating and demodulating signals:

Output after the lowpass filter is

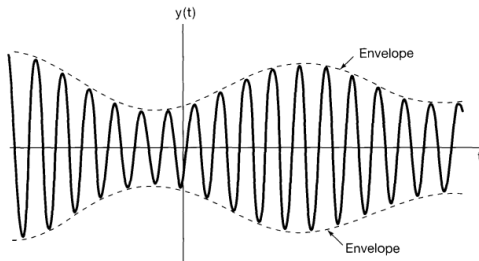
Synchronous demodulation: $\phi_c = \theta_c$. What could go wrong?

Asynchronous demodulation

Suppose the following is true:

- $x(t)$ is positive
- ω_c is much larger than ω_m

The transmitted signal will look something like this:



Asynchronous demodulation

Design a system to track the envelope (we won't go into details).

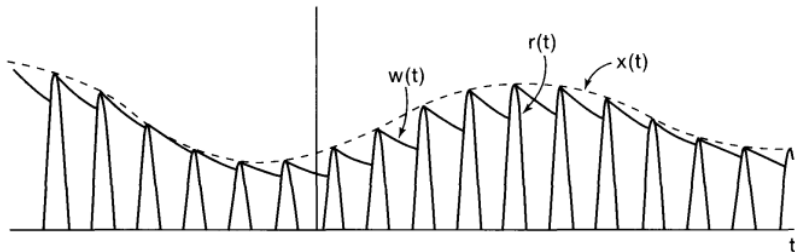


Image credit: Oppenheim 8.2

Asynchronous demodulation

If $x(t)$ is not positive, choose A sufficiently large and add to signal:

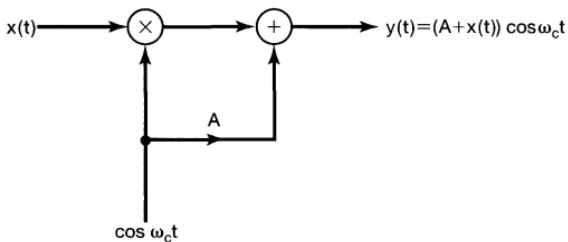
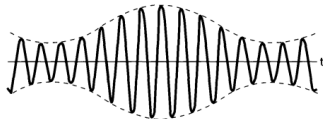


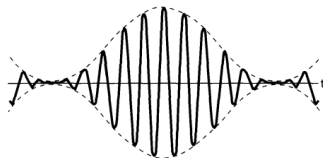
Image credit: Oppenheim 8.2

Asynchronous demodulation

Suppose $|x(t)| \leq K$. Must have $A > K$. Then $m = K/A$ is known as the *modulation index*.



(a)

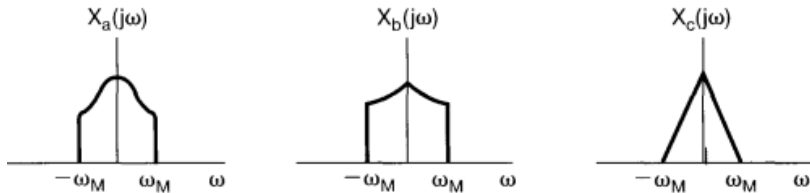


(b)

You will explore this in Assignment 4.

Frequency-division multiplexing (FDM)

Suppose we have more than one signal we wish to transmit:



Modulation can help us send them at the same time!

Image credit: Oppenheim 8.3

Frequency-division multiplexing (FDM)

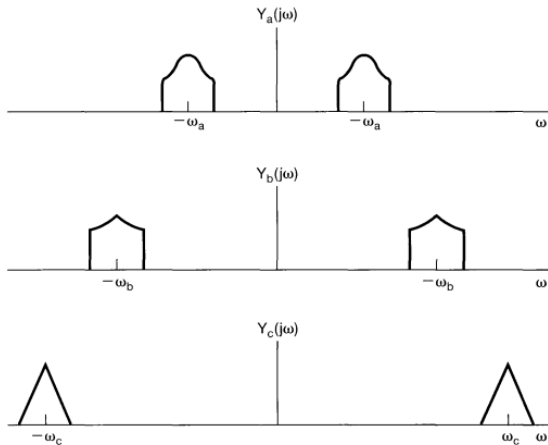
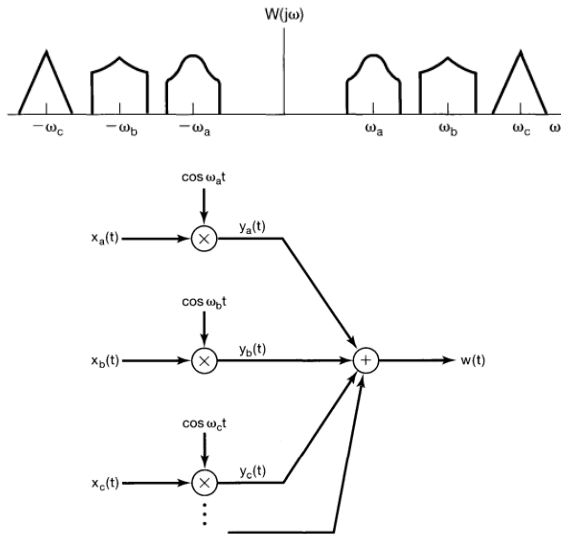


Image credit: Oppenheim 8.3

Frequency-division multiplexing (FDM)

This is called *frequency-division multiplexing*.



Frequency-division multiplexing (FDM)

How to separate out the channels?

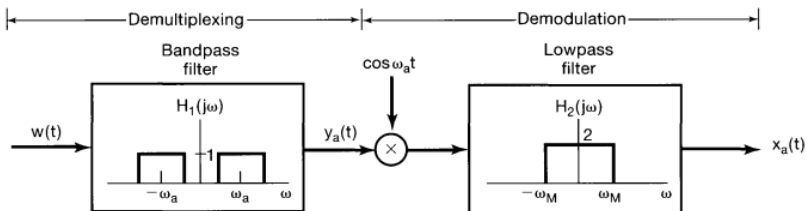
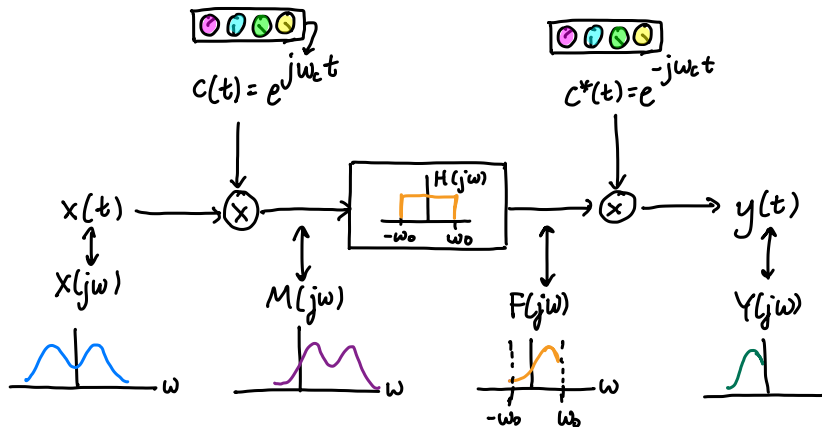


Image credit: Oppenheim 8.3

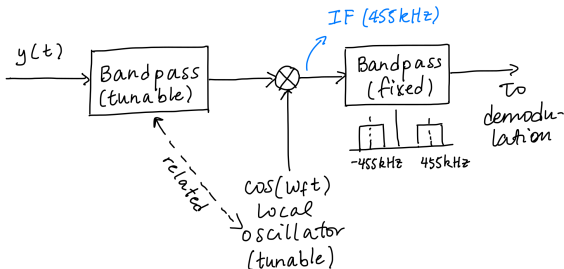
Example: frequency-selective filtering with variable centre frequency



Real-world example: radio



“Superheterodyne receiver” (Tutorial assignment 5!)



For next time

Content:

- single-sideband modulation
- pulse amplitude modulation and time-division multiplexing
- cellphone communication systems

Action items:

1. Assignment 4 due Saturday 23:59
2. Tutorial assignment 4 due Monday 23:59

Recommended reading:

- From this class: Oppenheim 8.0-8.3
- Suggested problems: 8.1, 8.2, 8.4-8.6, 8.8, 8.21-8.23, 8.40
- For next class: Oppenheim 8.4-8.6