ELEC 221 Lecture 21
Modulation and communication systems

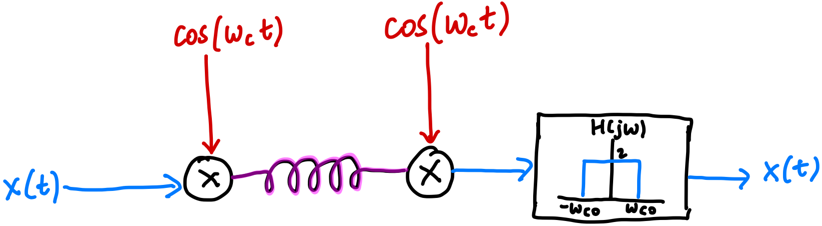
Tuesday 22 November 2022

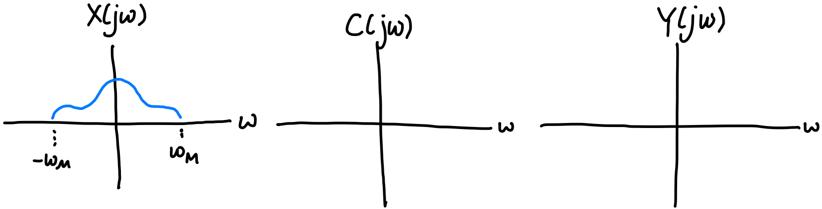
Announcements

- Quiz 9 today
- Midterm grading in progress (nearly done)
- Assignment 6 still under construction (will be short)

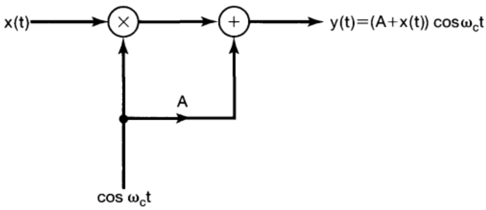
Last time

We did sinusoidal amplitude modulation with a real carrier signal:





We distinguished between synchronous and asynchronous modulation and defined the *modulation index*.



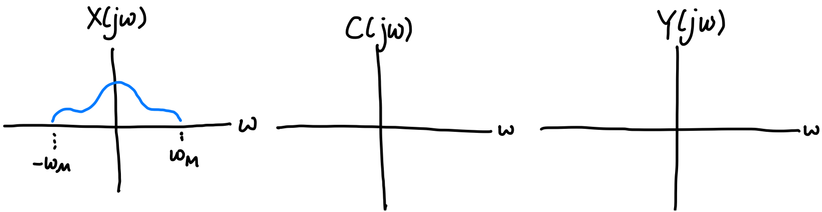
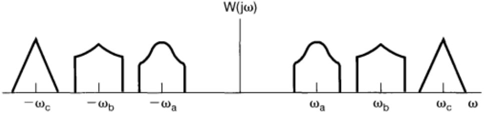


Image credit: Oppenheim 8.3

We learned that multiple signals can be sent over the same channel using *frequency-division multiplexing*.



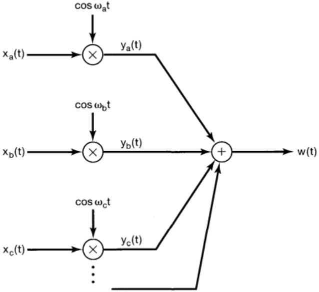


Image credit: Oppenheim 8.3

Last time

We saw how single-sideband modulation can save bandwidth.

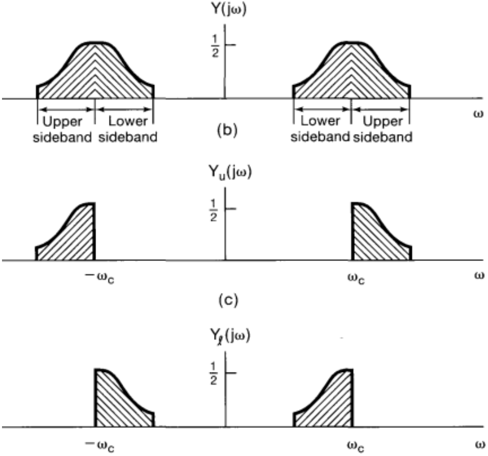


Image credit: Oppenheim 8.4

Today

Partially an "infotainment" lecture.

Learning outcomes:

- perform amplitude modulation on a discrete-time signal
- describe two approaches to frequency modulation
- modulate and demodulate to perform *frequency-shift keying*
- explain (at a high level!) how cell phone communication works

DT sinusoidal AM

Suppose we have carrier signal

$$c[n] = e^{j\omega_c n}$$

Modulation in the time domain

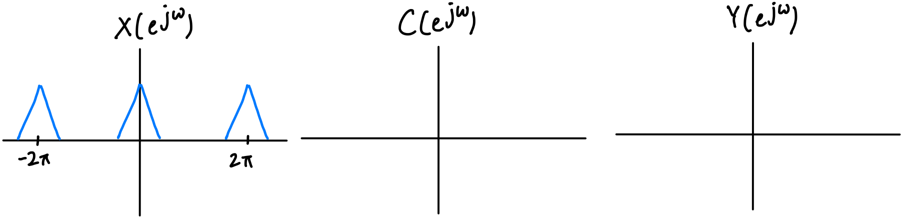
$$y[n] = x[n]c[n]$$

corresponds to convolution in the frequency domain:

$$Y(e^{j\omega}) = \frac{1}{2\pi} \int_{2\pi} X(e^{j\theta}) C(e^{j(\omega-\theta)}) d\theta$$

DT sinusoidal AM

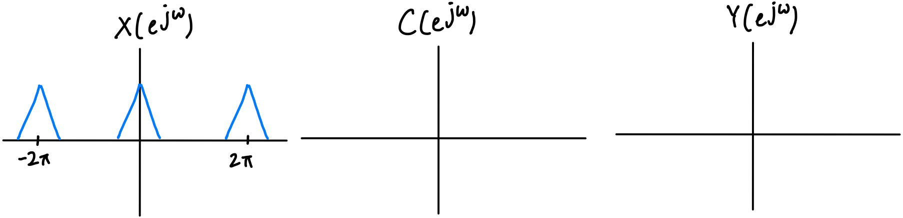
What happens to the spectra?



How about demodulation?

What if we use the carrier signal

$$c[n] = \cos(\omega_c n)$$



Are there any conditions on ω_c?

- $\omega_c > \omega_m$
- $\omega_c < \pi - \omega_m$

Exercise: DT sinusoidal AM

8.16. Suppose $x[n]$ is a real-valued discrete-time signal whose Fourier transform $X(e^{j\omega})$ has the property that

$$X(e^{j\omega}) = 0 \quad \text{for } \frac{\pi}{8} \leq \omega \leq \pi.$$

We use $x[n]$ to modulate a sinusoidal carrier $c[n] = \sin(5\pi/2)n$ to produce

$$y[n] = x[n]c[n].$$

Determine the values of ω in the range $0 \leq \omega \leq \pi$ for which $Y(e^{j\omega})$ is guaranteed to be zero.

Exercise: DT sinusoidal AM

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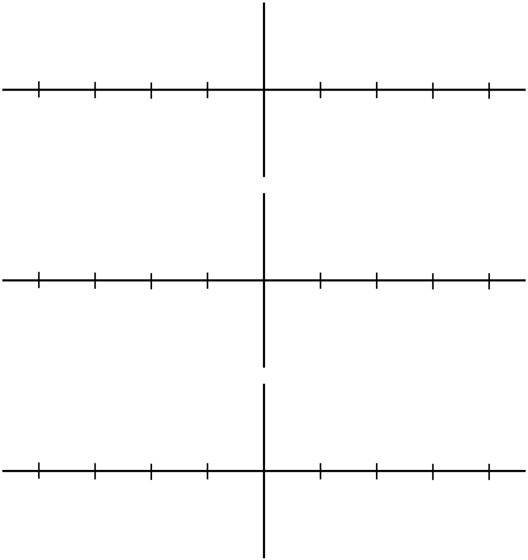
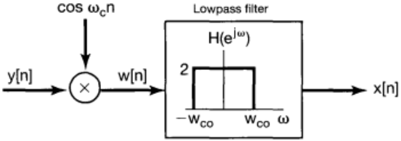
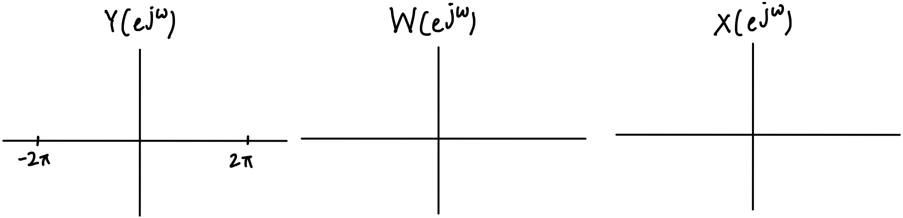
Exercise: DT sinusoidal AM

Solution:

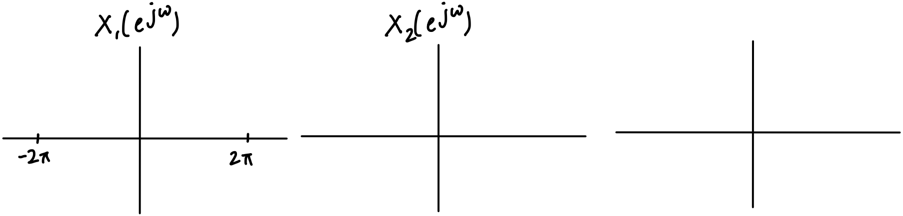


Demodulation





What could go wrong?

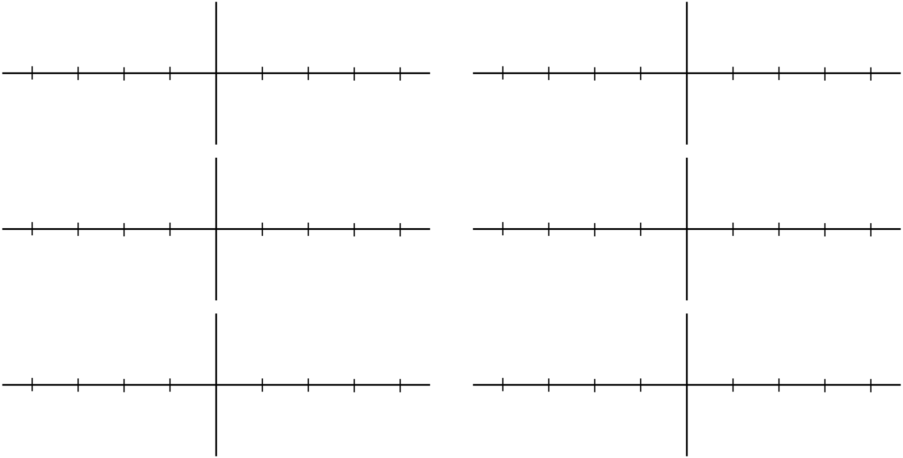


Consider the following for 4 signals, and then the general case of M signals.

8.33. Let us consider the frequency-division multiplexing of discrete-time signals $x_i[n]$, $i = 0, 1, 2, 3$. Furthermore, each $x_i[n]$ potentially occupies the entire frequency band $(-\pi < \omega < \pi)$. The sinusoidal modulation of upsampled versions of each of these signals may be carried out by using either double-sideband techniques or single-sideband techniques.

(a) Suppose each signal $x_i[n]$ is appropriately upsampled and then modulated with $\cos[i(\pi/4)n]$. What is the minimum amount of upsampling that must be carried out on each $x_i[n]$ in order to ensure that the spectrum of the FDM signal does not have any aliasing?

Exercise: DT frequency-division modulation



Frequency modulation

We already saw an example of this:

```
def phase_modulation(signal, time_range, carrier_frequency):
    """Apply phase modulation to a carrier signal.

    Args:
        signal (array[float]): The signal (x(t)) that we modulate with.
        time_range (array[float]): The explicit times over which the signal has
            been sampled (in seconds).
        carrier_frequency (int): The frequency (in Hz) of the cos wave that
            we modulate.

    Returns:
        array[float]: The modulated signal.
    """
    return cos_wave(time_range, carrier_frequency, phase=signal)
```

```
modulating_signal = 3 * (triangle_wave(t_range, 2))
```

```
Audio(phase_modulation(modulating_signal, t_range, 440), rate=sample_rate)
```

Let's take a closer look (but only at a high level).

Two key ways of doing frequency modulation (they are related).

In general, we model frequency modulation as

$$y(t) = A\cos(\theta(t))$$

where $\theta(t)$ depends on a modulating signal $x(t)$.

Last Tuesday you implemented *phase modulation*:

$$y(t) = A\cos(\omega_c t + \theta_c(t)) = A\cos(\omega_c t + \theta_0 + k_p x(t))$$

Another method is to set

$$\frac{d\theta(t)}{dt} = \omega_c + k_f x(t)$$

This is typically called *frequency modulation*.

It is related to phase modulation though:

$$y(t) = A\cos(\theta(t)) = A\cos(\omega_c t + \theta_0 + k_p x(t))$$

$$\frac{d\theta(t)}{dt} = \omega_c + k_p \frac{dx(t)}{dt}$$

Phase modulation with $x(t)$ is equivalent to frequency modulation with $dx(t)/dt$:

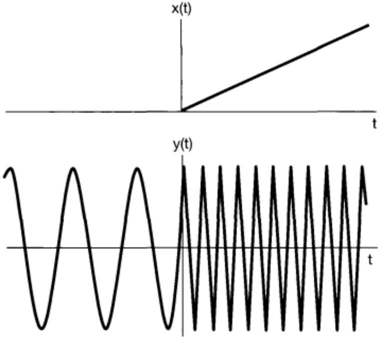


Image credit: Oppenheim 8.7

Frequency modulation with a square wave

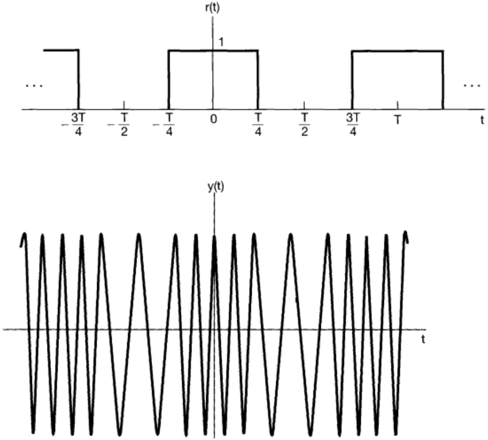


Image credit: Oppenheim 8.7

Frequency modulation with a square wave

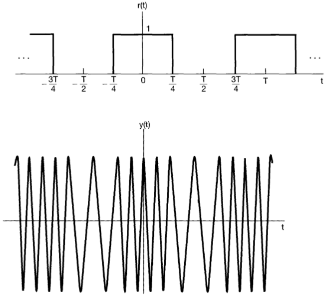


Image credit: Oppenheim 8.7

Suppose the square wave changes the frequency between two values: $\omega_c + \Delta\omega$ and $\omega_c - \Delta\omega$.

We can write

$$\begin{aligned} y(t) = & \quad r(t)\cos((\omega_c + \Delta\omega)t) \\ & + \quad r(t - T/2)\cos((\omega_c - \Delta\omega)t) \end{aligned}$$

What are we doing to these signals?

Frequency modulation with a square wave

This is two AM signals!

Their combined spectrum is the sum of the individual spectra:

$$Y_1(j\omega) = \frac{1}{2}\left[R(j(\omega + (\omega_c + \Delta\omega))) + R(j(\omega - (\omega_c + \Delta\omega)))\right]$$
$$Y_2(j\omega) = \frac{1}{2}\left[R_T(j(\omega + (\omega_c - \Delta\omega))) + R_T(j(\omega - (\omega_c - \Delta\omega)))\right]$$

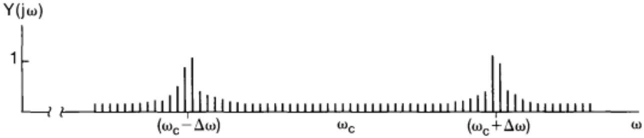


Image credit: Oppenheim 8.7

Main takeways:

- FM is more complicated to analyze than AM
- FM radio uses more bandwidth than AM radio

FM radio is allocated the frequencies between 88-108 MHz.

Channels are allocated 200kHz each; the station frequency is the centre of this band (all FM stations end in odd decimals!).

Image credit: `https://www.mcintoshlabs.com/products/tuners/MR87`

Frequency-shift keying

(Oppenheim 8.39.)

Suppose we wish to send one of two messages:

$$\begin{aligned} m_0(t) &= \cos(\omega_0 t) \\ m_1(t) &= \cos(\omega_1 t) \end{aligned}$$

Send a "burst" of one of these messages over a time interval T.

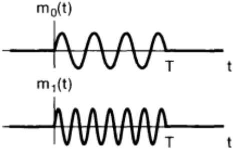


Image credit: Oppenheim P8.39

(Oppenheim 8.39.)

Show that the following system will correctly distinguish between m_0 and m_1:

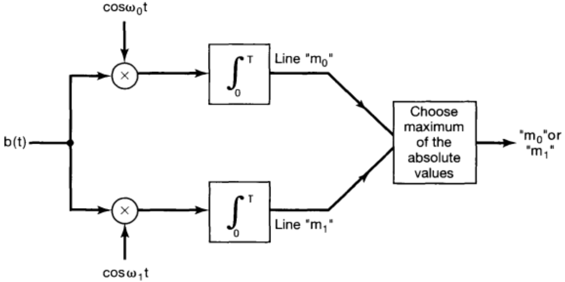


Image credit: Oppenheim P8.39

Exercise: frequency-shift keying

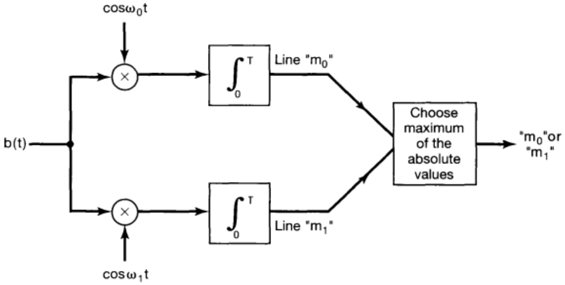
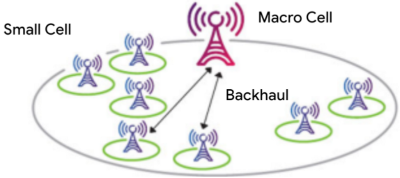
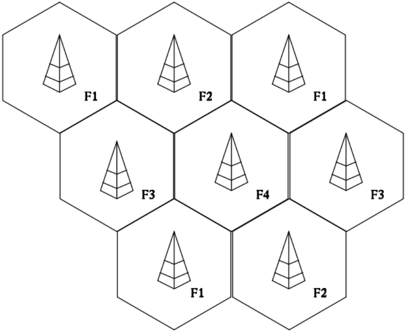


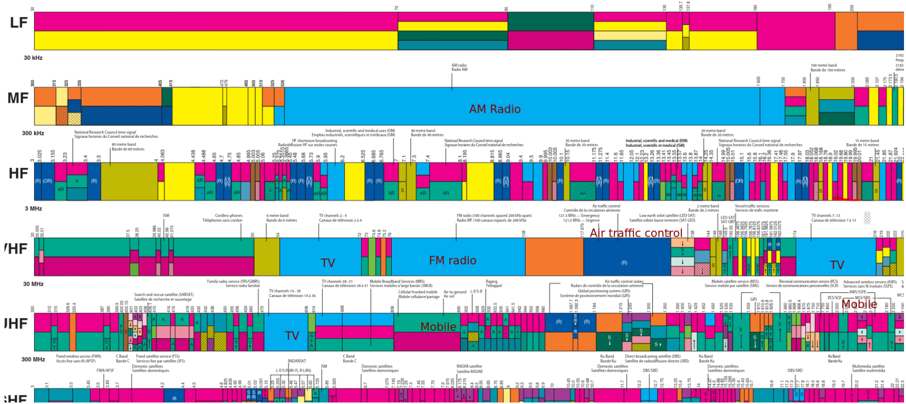
Image credit: Oppenheim P8.39

A cell phone is basically just a really fancy radio.





Infotainment: cell phone communication systems



See the full graphic here: `https://www.ic.gc.ca/eic/site/smt-gst.nsf/vwapj/2018_Canadian_Radio_Spectrum_Chart.PDF/$FILE/2018_Canadian_Radio_Spectrum_Chart.PDF`

Infotainment: cell phone communication systems

3500 MHz band spectrum auction

From: Innovation, Science and Economic Development Canada

Backgrounder

3500 MHz band

The Government of Canada is committed to ensuring that Canadians have access to high-quality wireless services and benefit from ubiquitous coverage and affordable prices. In a globalized world, the deployment of 5th generation—or “5G”—telecommunications standards and technologies is essential to Canada’s economic competitiveness. The deployment of 5G will also help ensure that Canadians continue to benefit from world-class wireless infrastructure.

In June 2018, Innovation, Science and Economic Development Canada (ISED) released *Spectrum Outlook 2018 to 2022*, which included plans to release spectrum that would support 5G services. The Outlook indicated that releasing the 3450–3650 MHz band (referred to as the 3500 MHz band), a key band for 5G, was a high priority. In June 2019, ISED published the *Decision on Revisions to the 3500 MHz Band to Accommodate Flexible Use and Preliminary Decisions on Changes to the 3800 MHz Band* as the first step toward making this band available.

Through the release of the *Policy and Licensing Framework for Spectrum in the 3500 MHz Band*, ISED is setting the rules for the upcoming spectrum auction, the next step toward making this band available for 5G services.

Spectrum to be auctioned

ISED is making 200 MHz of spectrum available in the 3500 MHz band for “flexible use” licensing that allows licensees to choose the type of services they will deploy, such as mobile (5G) or fixed wireless services (e.g. Internet-to-the-home). Through the transition process, existing licensees are eligible

Image credit: `https://www.canada.ca/en/innovation-science-economic-development/news/2020/03/3500-mhz-band-spectrum-auction.html`

Infotainment: cell phone communication systems

Different "generations" of service use different bands.

Higher frequencies enable faster transmission of information, at the cost of working better for shorter distances.

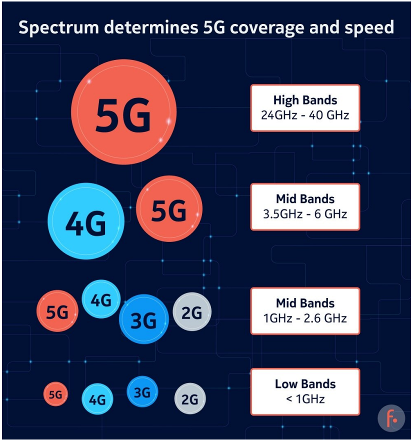


Image credit: `https://www.nokia.com/networks/insights/spectrum-bands-5g-world/`

Learn more:

https://en.wikipedia.org/wiki/Cellular_network

https://www.myamplifiers.com/articles/
lte-frequency-bands-of-us-and-canada-carriers/

https://en.wikipedia.org/wiki/LTE_frequency_bands

https://www.nokia.com/networks/insights/spectrum-bands-5g-world/

Today

Learning outcomes:

- perform amplitude modulation on a discrete-time signal
- describe two approaches to frequency modulation
- modulate and demodulate to perform *frequency-shift keying*
- explain (at a high level!) how a cellphone works

Oppenheim practice problems:

For next time

Content:

- the Laplace transform

Action items:

1. Assignment 6 (computational) available soon

Recommended reading:

- From this class: Oppenheim 8.5-8.9
- For next class: Oppenheim 9.0-9.4