ELEC 221 Lecture 04
CT convolution integral; the impulse response and system properties

Tuesday 17 September 2024

Announcements

- Assignment 1 due Thursday 23:59 (final question moved to Assignment 2)
- Thursday class on Zoom (link in Canvas)
- Friday office hour cancelled this week

Start with Quiz 2.

Learning outcomes:

- Define the CT unit impulse and step functions
- Define the convolution integral and use it to compute the output of a system
- Use the convolution sum/integral to show that complex exponentials are eigenfunctions of LTI systems

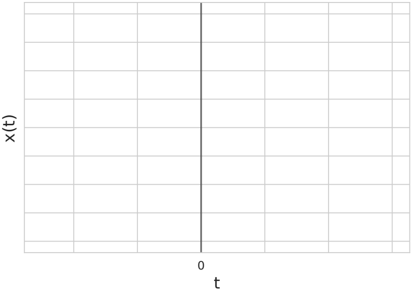
The convolution sum

We expressed signals as weighted sums of impulses

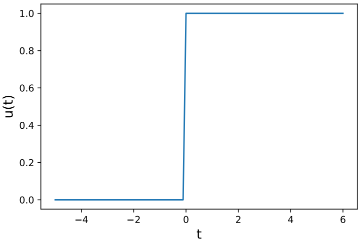
If we know what an LTI system does to a unit impulse (i.e., the impulse response $h[n]$), we know what it does to any other signal:

This is the **convolution sum**.
Today we will see the **convolution integral** in continuous time.

The CT unit impulse



The CT unit step



Just like in DT, the unit impulse and step are related:

The CT analogue of convolution sum is the **convolution integral**.

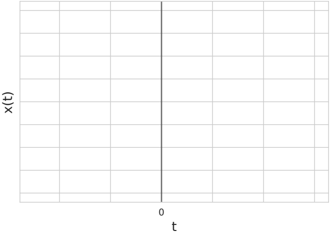
where $h(t)$ is the **CT impulse response**.

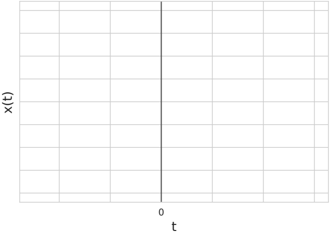
It has the same properties (commutative, associative, distributive).

Example: convolution

(Oppenheim Ex. 2.6 Var.) Consider system with impulse response

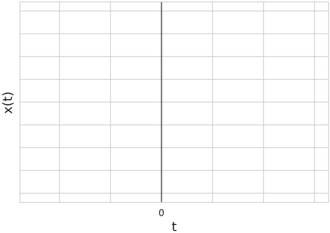
What is the output of the system for the input signal

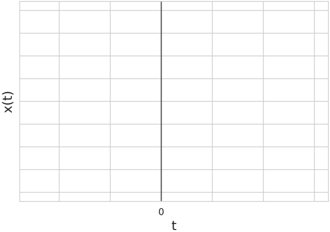




Example: convolution

Example: convolution

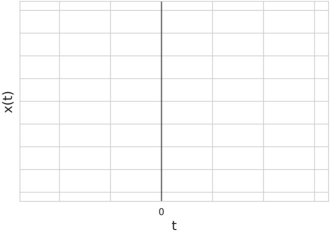


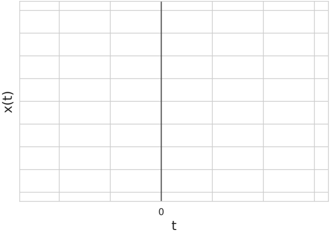


Exercise: convolution

(Oppenheim 2.8) Consider system with impulse response

What is the output of the system for the input signal

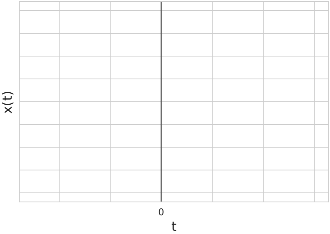


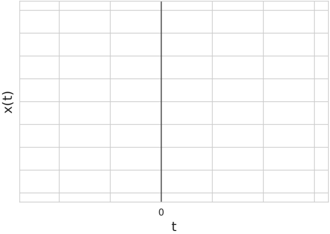


Exercise: convolution

Direct integration:

Visual intuition:





To reiterate: the convolution sum

and convolution integral

show that as long as we know how a system responds to a unit impulse, we can determine its response to any other signal.

The impulse response also allows us to reason about key system properties.

A system is memoryless if the output depends only on the input at the same time. This implies $h[n] = 0$ for $n \neq 0$, meaning

(And analogous for CT case)

If a system is invertible, it has an inverse system.
Suppose impulse response of a system is $h(t)$. Then

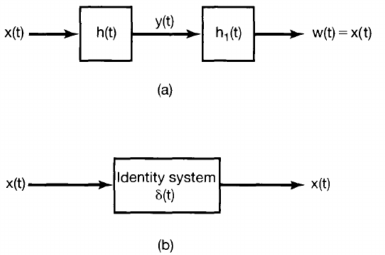


Figure 2.26 Concept of an inverse system for continuous-time LTI systems. The system with impulse response $h_1(t)$ is the inverse of the system with impulse response $h(t)$ if $h(t) * h_1(t) = \delta(t)$.

(And analagous for DT case. We will see this later in the course.)

Image: Oppenheim, Fig 2.26

Impulse response and stability

Suppose $x(t)$ is boudned, $|x(t)| \leq B$. If the system is stable, the output should be bounded.

(And analogous for DT case)

Impulse response and stability

As long as

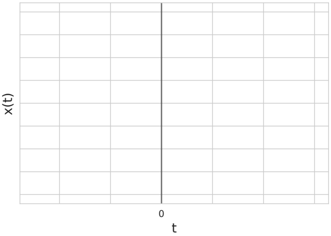
is bounded (i.e., $h(t)$ is absolutely integrable), the system is stable.

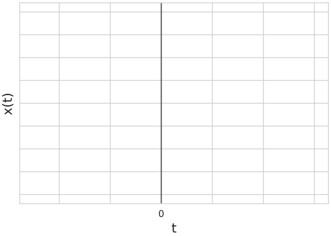
(And analogous for DT case)

Example/exercise: stability

Consider systems A and B with impulse responses

Are they stable?





Example/exercise: stability

Impulse response and causality

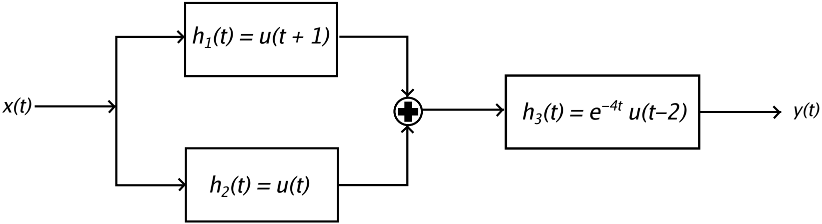
Recall definition of causal signal and consider the convolution sum:

What properties does $h[n]$ need to have for system to be causal?

(Analogous holds for CT systems)

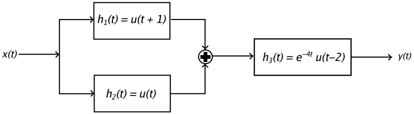
Final exercise

Consider the following combination of systems:

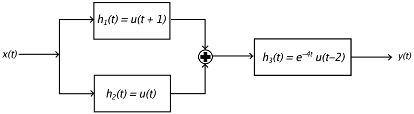


Is this system causal and/or stable?

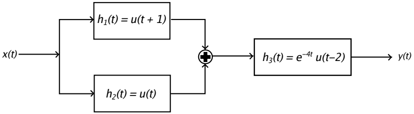
Final exercise



Final exercise



Final exercise

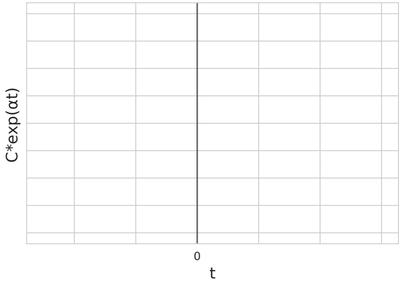


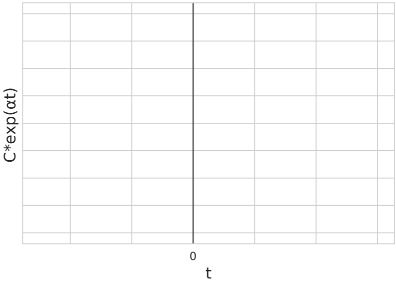
Review: complex exponential functions

Most general form:

where α can be real or complex.

Case: both C and α are real-valued.





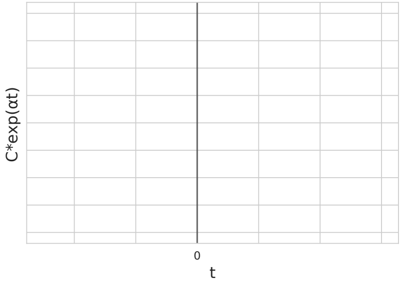
Case: α is complex. Recall can write any complex z in two ways:

Euler's relation allows us to write

As a result,

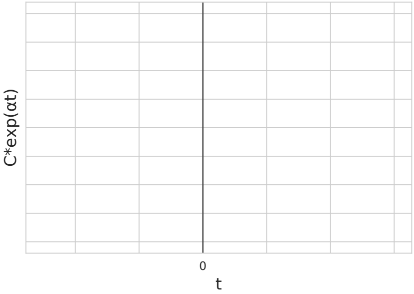
Review: complex exponential functions

Case: α is complex, $\alpha = j\omega$. Then,



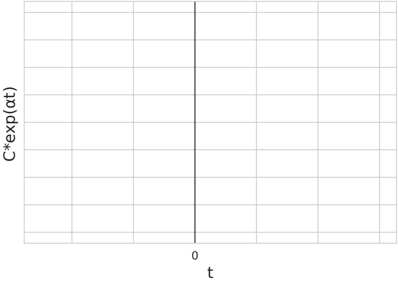
Review: complex exponential functions

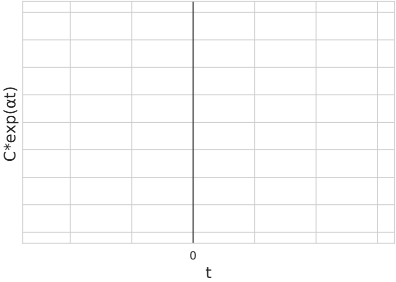
$x(t) = Ce^{j\omega t}$ is periodic:



Review: complex exponential functions

Case: α is a complex number $\alpha = r + j\omega$. Then,





LTI systems and complex exponential functions

Recall the convolution integral:

What happens when $x(t)$ is a complex exponential signal?

Write $x(t) = e^{st}$. Then:

LTI systems and complex exponential functions

To summarize:

Complex exponentials are **eigenfunctions** of LTI systems.

$H(s)$ is the **system function** (*frequency response*) of an LTI system.

...so what?

Recall the "L" in "LTI system" stands for linear...

If all the $x_i(t)$ are complex exponential functions and

then

...so what?

The response of LTI systems to superpositions of complex exponential signals is a superposition **of those same signals**.

How can we express arbitrary signals using complex exponentials?

Recap

Today's learning outcomes were:

- Define the CT unit impulse and step functions
- Define the convolution integral and use it to compute the output of a system
- Use the convolution sum/integral to show that complex exponentials are eigenfunctions of LTI systems

For next time

Content:

- CT Fourier series representation and properties
- Dirichlet conditions, and the Gibbs phenomenon
- Power and energy of signals and Parseval's relation

Action items:

1. Assignment 1 due Thursday 23:59

Recommended reading:

- From today's class: Oppenheim 1.4, 2.2-2.3
- practice problems: 2.8-2.12, 2.14-16, 2.22, 2.28, 2.29
- For next class: Oppenheim 1.3, 3.0-3.5