

# **ELEC 221 Lecture 20**

## **Amplitude modulation**

Thursday 17 November 2022

- Quiz 9 on Tuesday

# Recap

We have seen convolution and multiplication properties in a few different contexts now:

The latter is sometimes known as the *modulation property*.

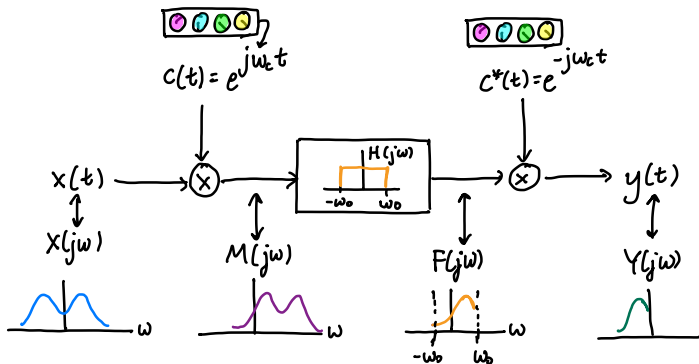
We have used modulation in a few different contexts already.

On Tuesday we did some simple amplitude modulation:

```
def amplitude_modulation(signal, time_range, carrier_frequency):  
    """Apply sinusoidal amplitude modulation.  
  
    Args:  
        signal (array[float]): The modulating signal.  
        time_range (array[float]): The explicit times over which the signal has  
            been sampled (in seconds).  
        carrier_frequency (int): The frequency (in Hz) of the carrier signal.  
  
    Returns:  
        array[float]: The modulated signal.  
    """  
    return signal * cos_wave(time_range, carrier_frequency)
```

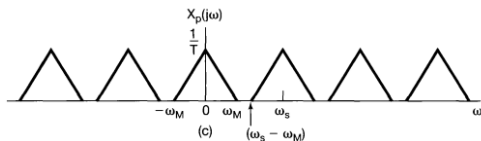
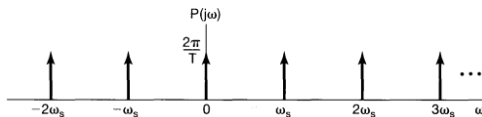
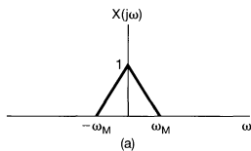
## Recap

In lecture 10 (and homework 4) we explored how modulation could be leveraged for frequency-selective filtering.



# Recap

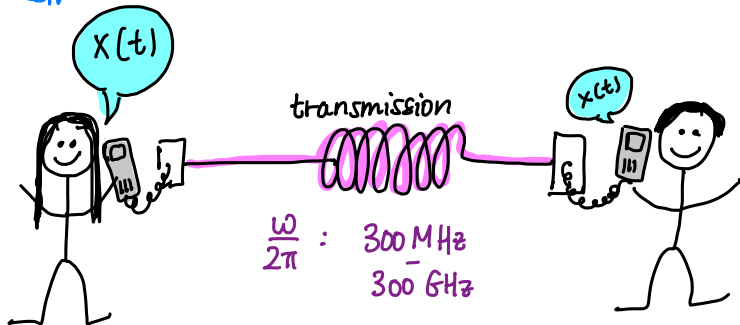
We modeled sampling as multiplication by a periodic impulse train.



## Recap

We briefly discussed the application of phone signal transmission.

$$\frac{\omega}{2\pi} : 200 - 4000 \text{ Hz}$$



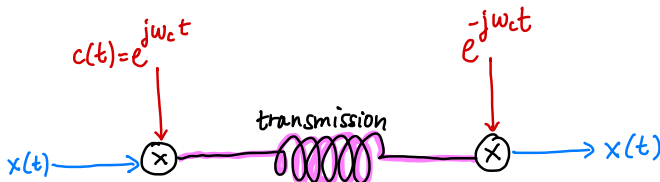
Learning outcomes:

- perform sinusoidal amplitude modulation (AM) and demodulation
- describe the process of frequency-domain multiplexing
- differentiate between double- and single-sideband modulation



# Modulation

The process of embedding an information-bearing signal into a second signal. (Extracting the signal: demodulation)



We will discuss two types:

- amplitude modulation (AM) (today)
- frequency modulation (FM) (next time)

Visualization: [https://global.oup.com/us/companion.websites/fdscontent/uscompanion/us/static/companion.websites/9780199922963/images/AM\\_FM.gif](https://global.oup.com/us/companion.websites/fdscontent/uscompanion/us/static/companion.websites/9780199922963/images/AM_FM.gif)

# Sinusoidal amplitude modulation

We focus on two types of carrier signals:

- complex exponential signal

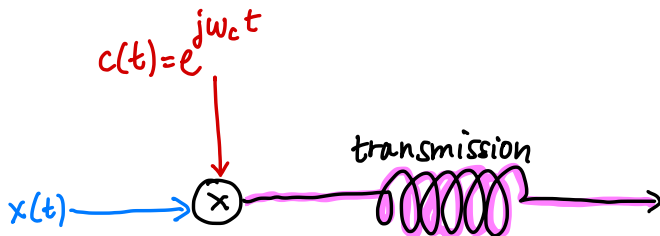
- sinusoidal signal

## Complex exponential amplitude modulation

We've already seen what happens with the first one.

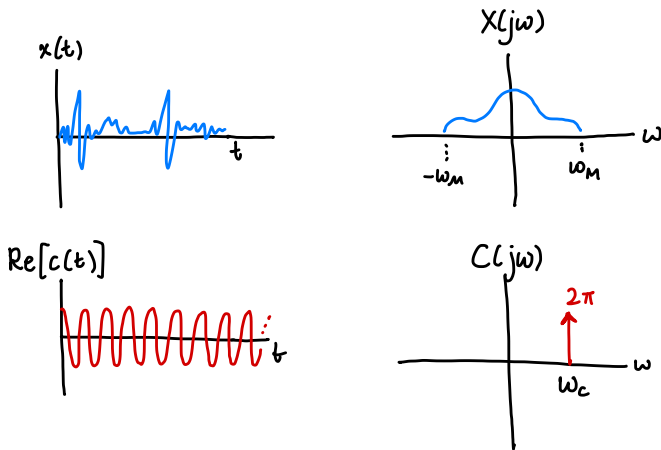
Suppose

(set  $\theta_c = 0$ , we will deal with it later).



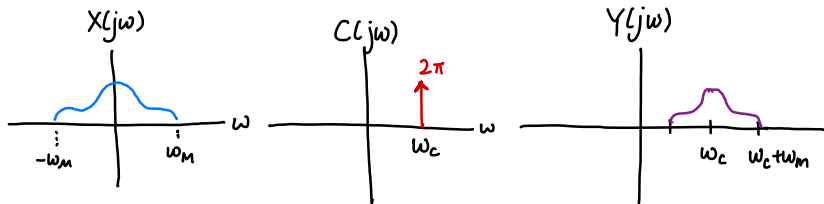
# Complex exponential amplitude modulation

Consider the Fourier spectrum of both signals:

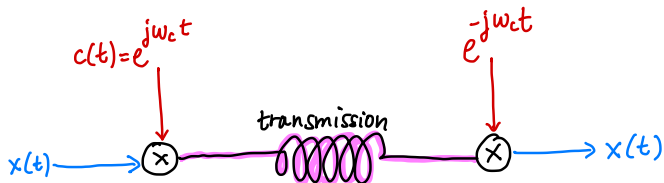


## Complex exponential amplitude modulation

Convolution of the spectra leads to the original spectrum being moved into a different frequency regime.

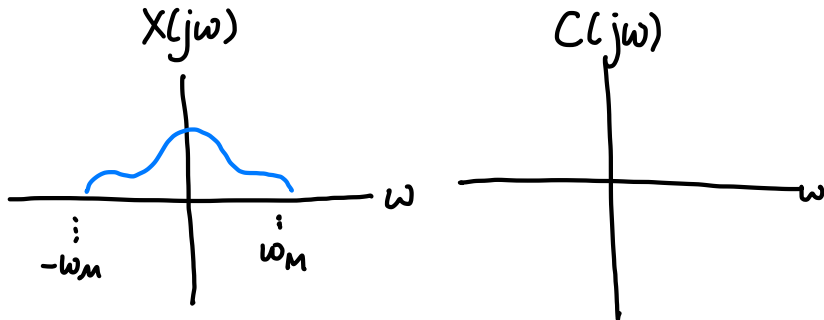


Demodulation is straightforward.

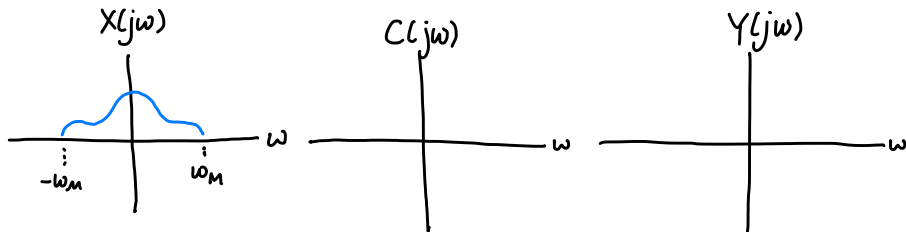


## Sinusoidal amplitude modulation

What if we modulate instead by a sinusoidal signal?

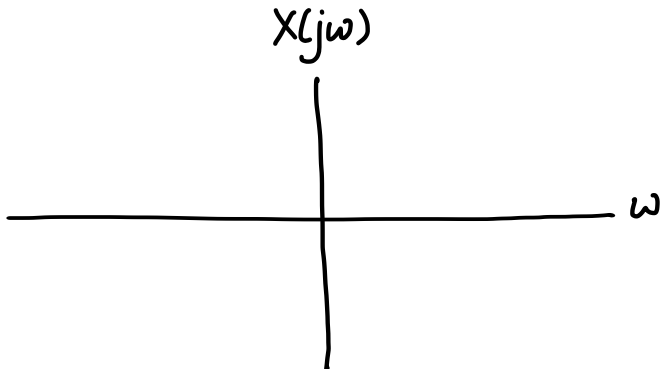
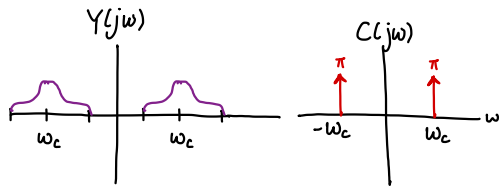


## Sinusoidal amplitude modulation



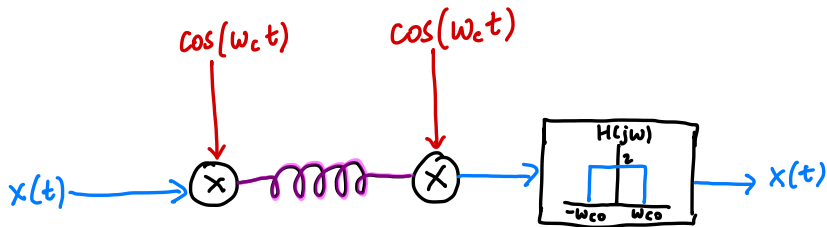
- Any foreseeable problems with this?
- How do we recover the signal?

## Sinusoidal amplitude modulation





## Sinusoidal amplitude modulation



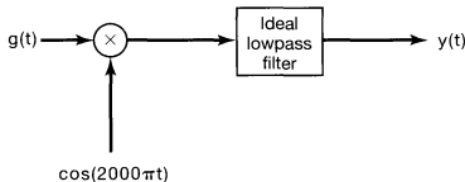
Let's check the math:

## Exercise: sinusoidal amplitude modulation

- 8.3.** Let  $x(t)$  be a real-valued signal for which  $X(j\omega) = 0$  when  $|\omega| > 2,000\pi$ . Amplitude modulation is performed to produce the signal

$$g(t) = x(t) \sin(2,000\pi t).$$

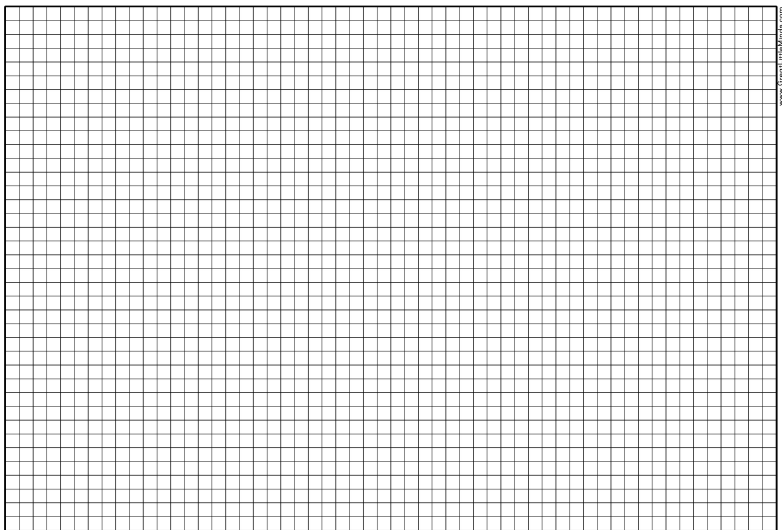
A proposed demodulation technique is illustrated in Figure P8.3 where  $g(t)$  is the input,  $y(t)$  is the output, and the ideal lowpass filter has cutoff frequency  $2,000\pi$  and passband gain of 2. Determine  $y(t)$ .



**Figure P8.3**

## Exercise: sinusoidal amplitude modulation

## Exercise: sinusoidal amplitude modulation



## Synchronous demodulation

More generally, we need to consider the phases in both the modulating and demodulating signals:

Output after the lowpass filter is

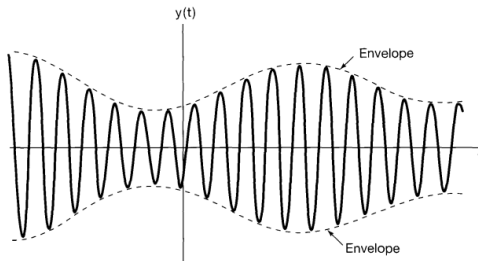
If  $\phi_c = \theta_c$  we call this synchronous demodulation. What could go wrong?

# Asynchronous demodulation

Suppose the following is true:

- $x(t)$  is positive
- $\omega_c$  is much larger than  $\omega_m$

The transmitted signal will look something like this:



# Asynchronous demodulation

Design a system to track the envelope (we won't go into details).

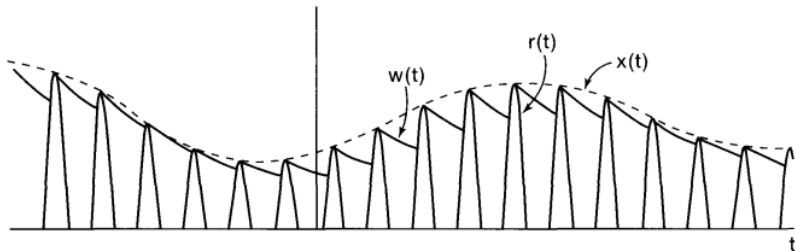
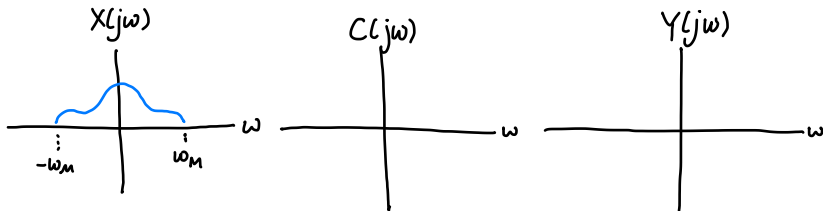
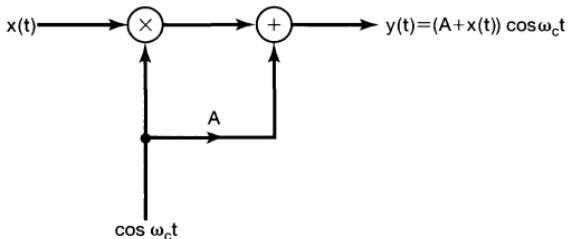


Image credit: Oppenheim 8.2

# Asynchronous demodulation

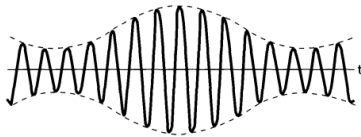
If  $x(t)$  is not positive, choose  $A$  sufficiently large and add to signal:



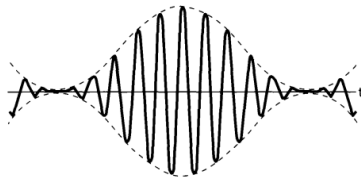


# Asynchronous demodulation

Suppose  $|x(t)| \leq K$ . Must have  $A > K$ . Then  $m = K/A$  is known as the *modulation index*.



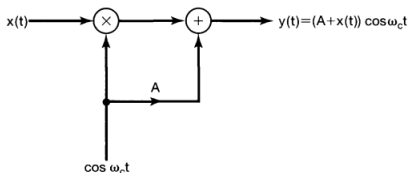
(a)



(b)

## Exercise: modulation index

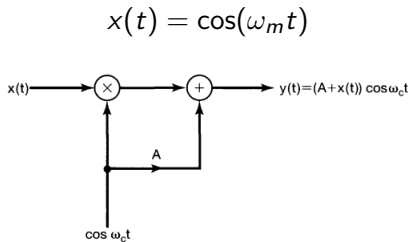
(Oppenheim 8.27)



There is inefficiency because we are sending the carrier signal too. Suppose  $x(t) = \cos(\omega_m t)$ ,  $\omega_m < \omega_c$  and  $A + x(t) > 0$ .

1. What is the modulation index  $m$ ?
2. What is the average power as a function of  $m$ ?
3. What is the *efficiency* (ratio of power in sideband to total power)?

## Exercise: modulation index



What is the modulation index  $m$ ?

## Exercise: modulation index

$$y(t) = (A + \cos(\omega_m t)) \cos(\omega_c t)$$

What is the average power as a function of  $m$ ? For periodic signal,

$$P = \frac{1}{T} \int_T |y(t)|^2 dt$$

*Hint: use Parseval's theorem.*

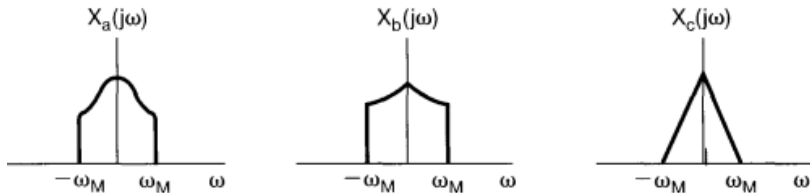
## Exercise: modulation index

$$y(t) = (A + \cos(\omega_m t)) \cos(\omega_c t)$$

What is the *efficiency* (ratio of power in sideband to total power)?

# Frequency-division multiplexing (FDM)

Suppose we have more than one signal we wish to transmit:



Modulation can help us send them at the same time!

Image credit: Oppenheim 8.3

# Frequency-division multiplexing (FDM)

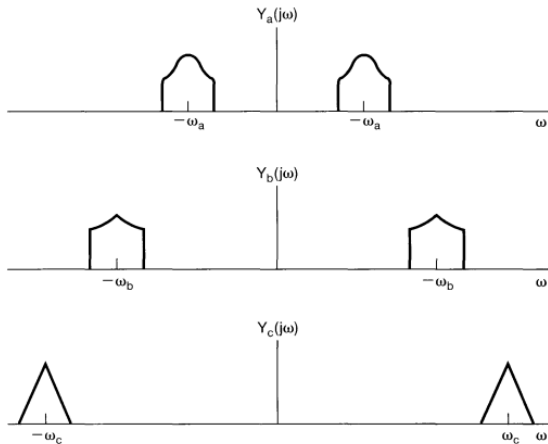
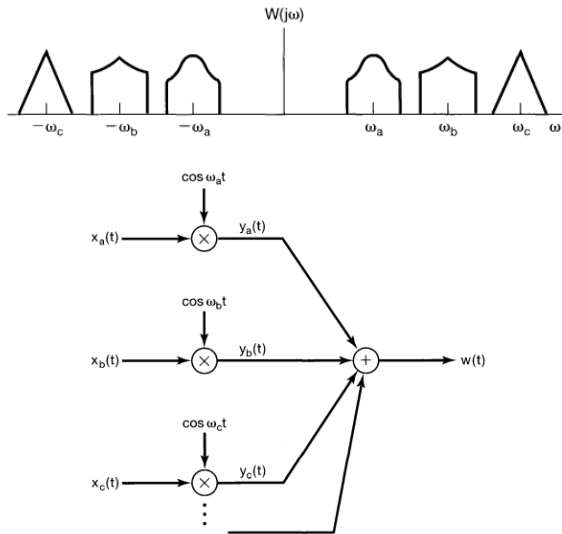


Image credit: Oppenheim 8.3

# Frequency-division multiplexing (FDM)

This is called *frequency-division multiplexing*.





# Frequency-division multiplexing (FDM)

How to separate out the channels?

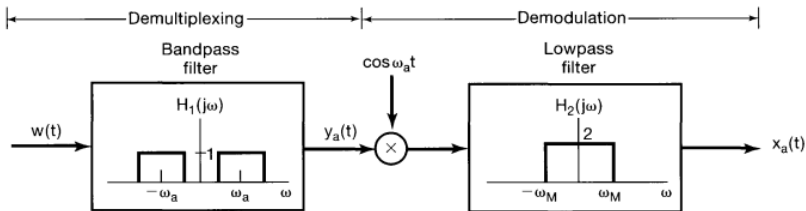


Image credit: Oppenheim 8.3

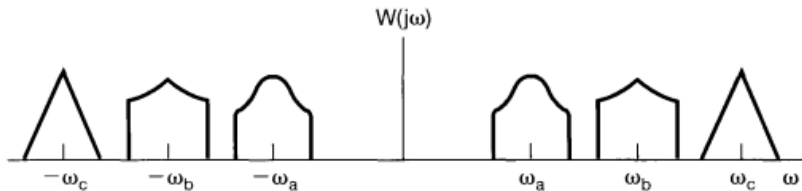
## AM radios

See Oppenheim problem 8.36.



# Single-sideband modulation (SSB)

This is inefficient...

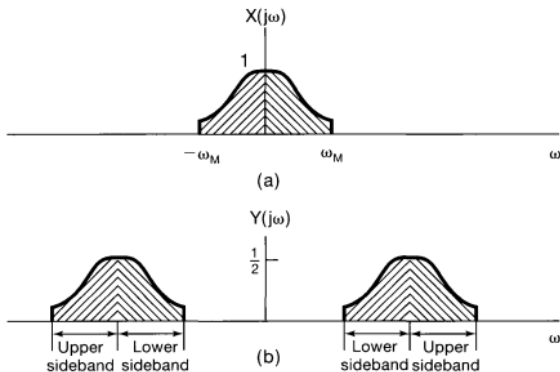


We are using twice as much bandwidth as we need to!

Image credit: Oppenheim 8.3

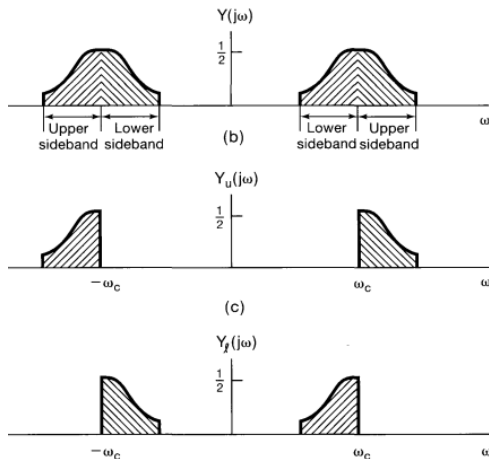
# Single-sideband modulation (SSB)

A modulated signal's spectrum can be divided into upper/lower *sidebands*:



# Single-sideband modulation (SSB)

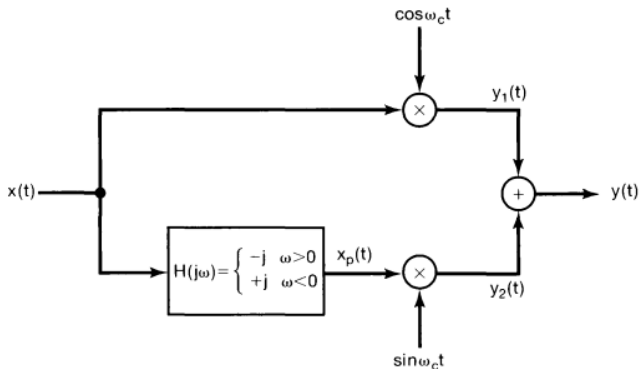
In single-sideband modulation, keep and transmit only one band:



How do you think this is done?

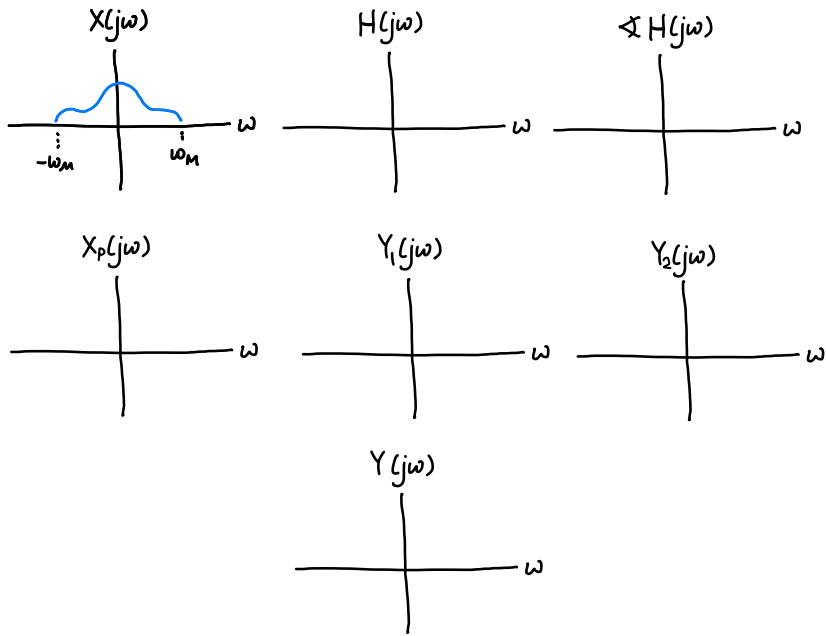
# Single-sideband modulation (SSB)

Alternative: “90° phase-shift network”

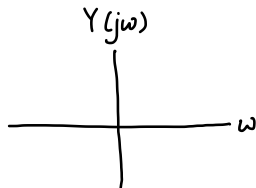
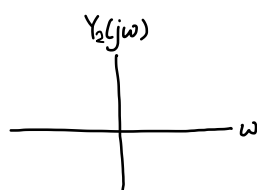
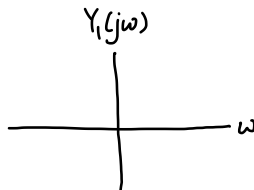
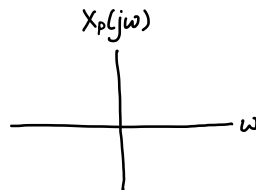
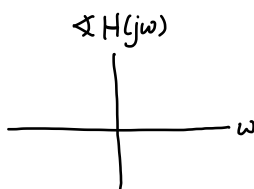
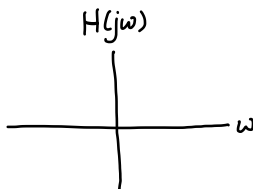
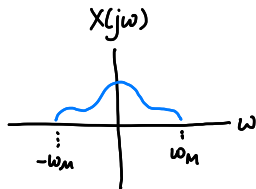


Exercise: see how this works by analyzing what happens to the spectra of  $x_p(t)$ ,  $y_1(t)$ ,  $y_2(t)$ , and  $y(t)$  in the frequency domain.

# Single-sideband modulation (SSB)



# Single-sideband modulation (SSB)





Learning outcomes:

- perform sinusoidal amplitude modulation (AM) and demodulation
- describe the process of frequency-domain multiplexing
- differentiate between double- and single-sideband modulation

Oppenheim practice problems: 8.1, 8.2, 8.4, 8.7-8.9, 8.21, 8.22, 8.26, 8.28

# For next time

## Content:

- pulse-amplitude modulation, frequency modulation
- Quiz 9 on Tuesday

## Action items:

1. Assignment 6 (computational) available soon

## Recommended reading:

- From this class: Oppenheim 8.0-8.4
- For next class: Oppenheim 8.5-8.9