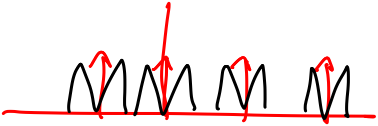
ELEC 221 Lecture 19
Amplitude modulation

Tuesday 19 November 2024

Announcements



- Quiz 8 today
- Assignment 4 due Saturday 23:59
- Tutorial assignment 4 due Monday 23:59
- Friday office hour cancelled
- Class next Tuesday (26 Nov) on Zoom

We performed CT signal processing by sampling, converting to DT, processing, then converting back.

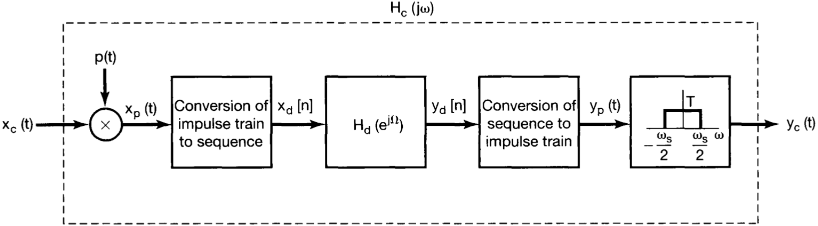


Image credit: Oppenheim 7.4

We sampled DT signals (decimation). If original signal was CT, this is called *downsampling*.

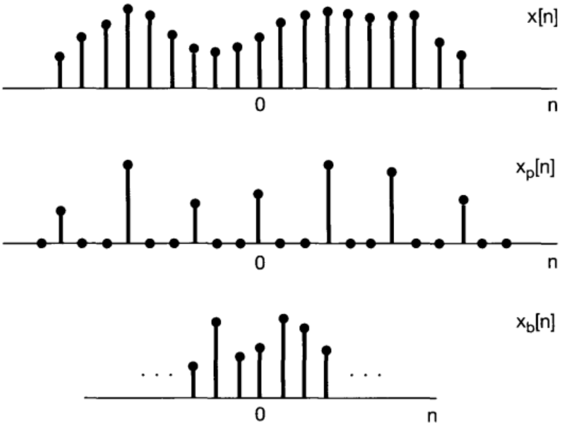


Image credit: Oppenheim 7.34

Decimation spreads out the spectrum.

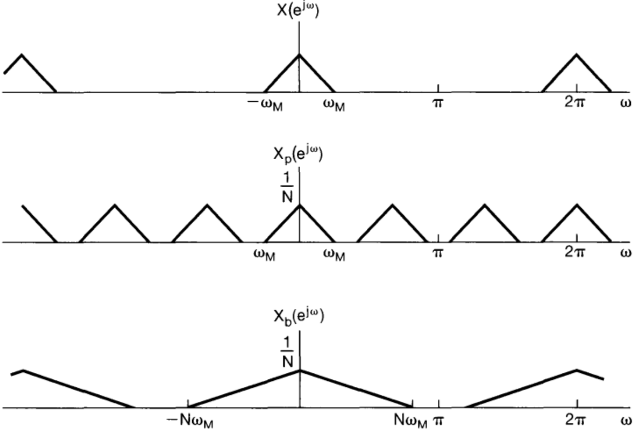


Image credit: Oppenheim 7.35

The opposite of decimation is interpolation (upsampling).

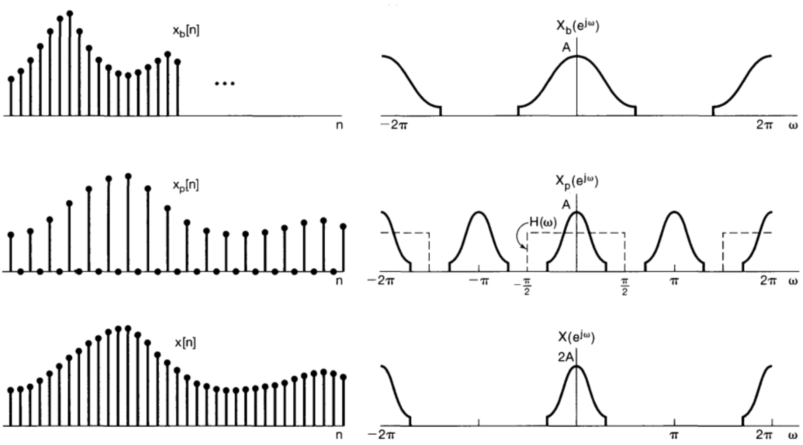


Image credit: Oppenheim 7.5

Today

First: finish example from last time.

Learning outcomes:

- perform sinusoidal amplitude modulation (AM) and demodulation in CT and DT
- differentiate between synchronous and asynchronous modulation techniques, and identify pros/cons of each method
- carry out frequency-division multiplexing

Example: down/upsampling

Oppenheim problem 7.19,

7.19. Consider the system shown in Figure P7.19, with input $x[n]$ and the corresponding output $y[n]$. The zero-insertion system inserts two points with zero amplitude between each of the sequence values in $x[n]$. The decimation is defined by

$$y[n] = w[5n],$$

where $w[n]$ is the input sequence for the decimation system. If the input is of the form

$$x[n] = \frac{\sin \omega_1 n}{\pi n},$$

determine the output $y[n]$ for the following values of ω_1:

(a) $\omega_1 \le \frac{3\pi}{5}$

(b) $\omega_1 > \frac{3\pi}{5}$

(a) $\dfrac{\sin\left(\frac{\omega_1 5n}{3}\right)}{5\pi n}$

(b) $\frac{1}{5}\delta[n]$

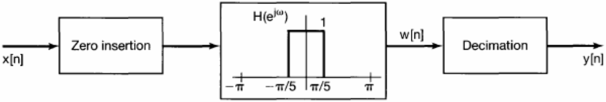
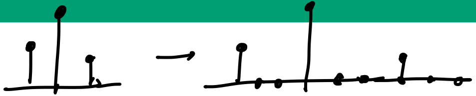
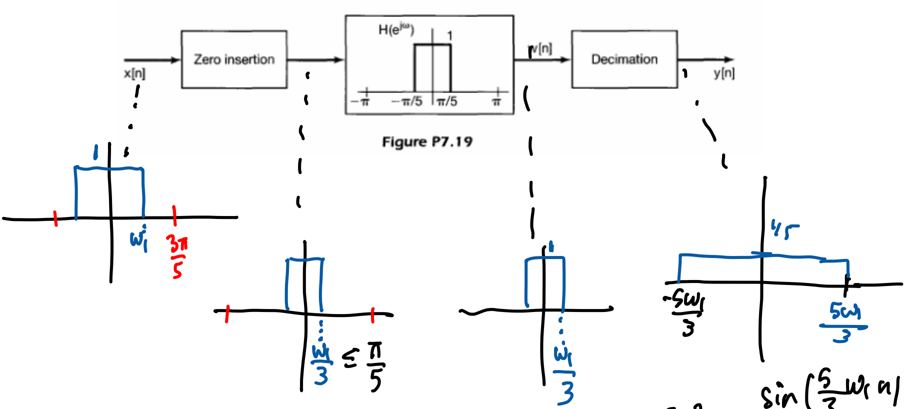


Figure P7.19

Example: down/upsampling

First case: $\omega_1 \leq \frac{3\pi}{5}$





$$y[n] = \frac{\sin\left(\frac{5}{3}\omega_1 n\right)}{5\pi n}$$

Example: down/upsampling

$\delta[n] \xleftrightarrow{\mathcal{F}} 1$

Second case: $\omega_1 > \frac{3\pi}{5}$

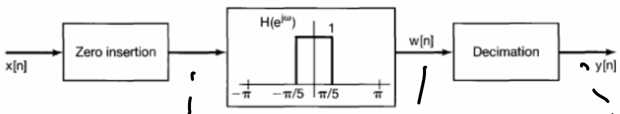
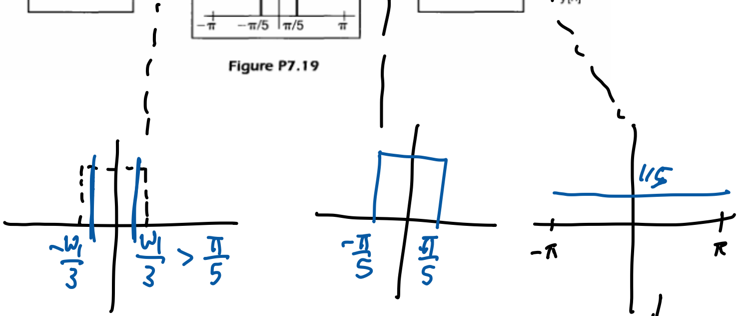


Figure P7.19



$y[n] = \frac{1}{5}\delta[n]$

Impulse train sampling

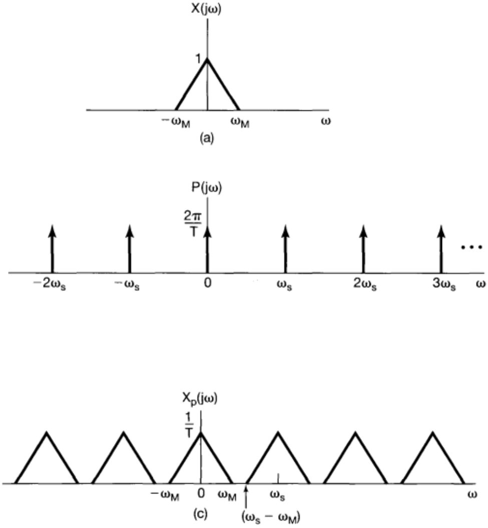
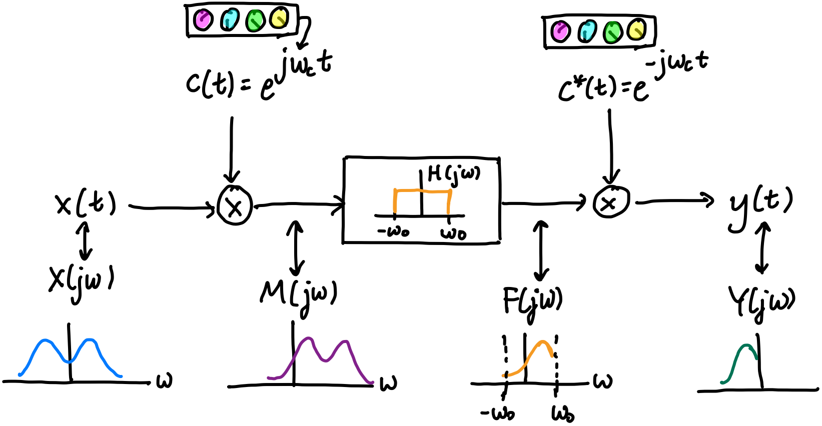


Image credit: Oppenheim 7.1

Frequency-selective filtering with variable centre frequency

Recall from Assignment 3:



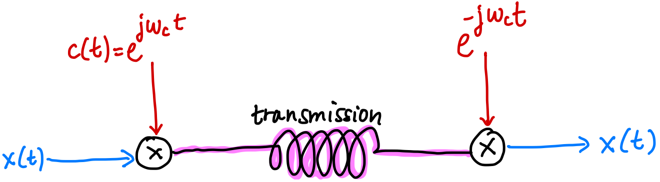
Multiplication property

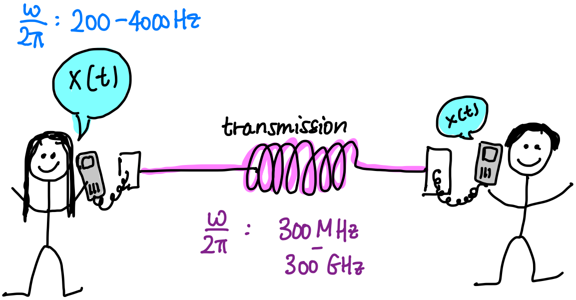
Both results come from the multiplication property:

$$y(t) = x(t)\,p(t) \quad \Rightarrow \quad Y(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)\,P(j(\omega-\theta))\,d\theta$$

This is also known as the *modulation property*.

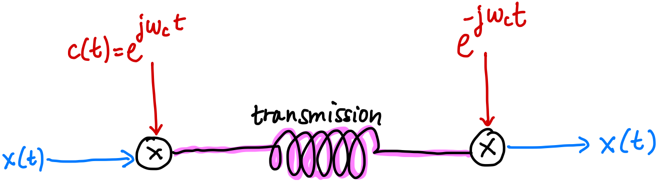
Motivation: communication systems





Modulation

The process of embedding an information-bearing signal into a second signal. (Extracting the signal: demodulation)



Two main types (we will only discuss AM):

- amplitude modulation (AM)
- frequency modulation (FM)

We focus on two types of **carrier signal**, $c(t)$:

- complex exponential signal

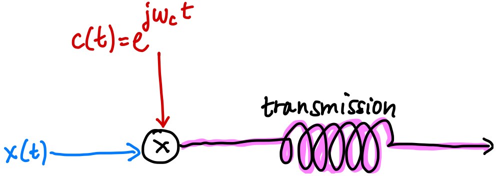
$$c(t) = e^{j(\omega_c t + \theta_c)}$$

- sinusoidal signal

$$c(t) = \cos(\omega_c t + \theta_c)$$

Complex exponential amplitude modulation

We've already seen what happens with the first one.



In practice:

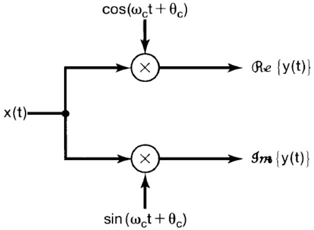
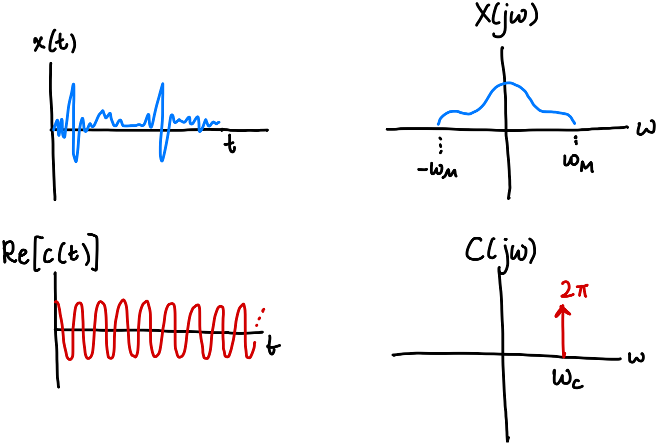


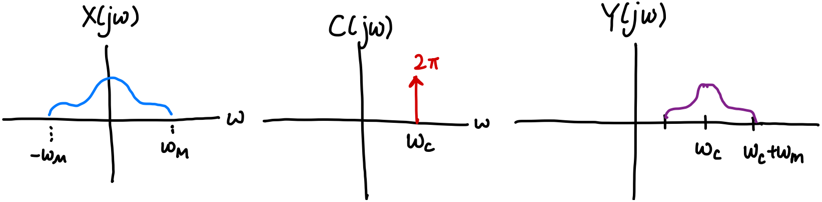
Figure 8.2 Implementation of amplitude modulation with a complex exponential carrier $c(t) = e^{j(\omega_c t + \theta_c)}$.

Image credit: Oppenheim Fig 8.2

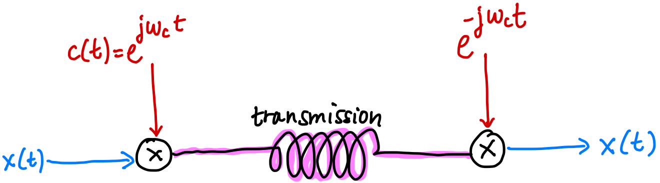
Consider the Fourier spectrum of both signals:



Multiplication by $c(t)$ shifts spectrum to different frequency range.



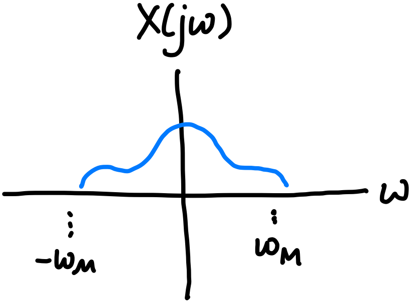
Demodulation is straightforward.

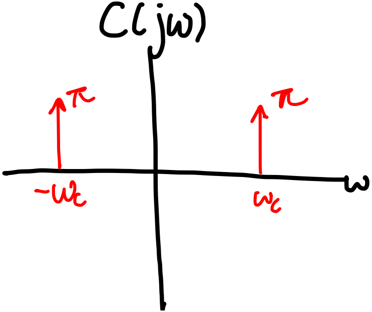


Sinusoidal amplitude modulation

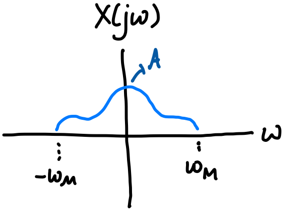
What if we use a sinusoidal signal instead?

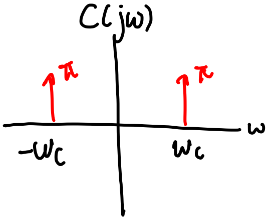
$$c(t) = \cos(\omega_c t)$$

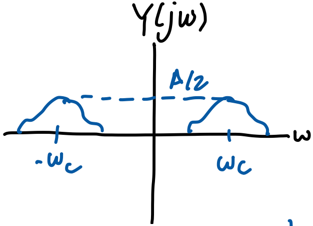


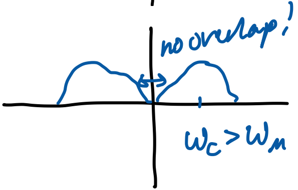


$y(t) = x(t)\,c(t)$



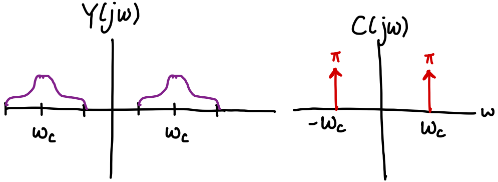


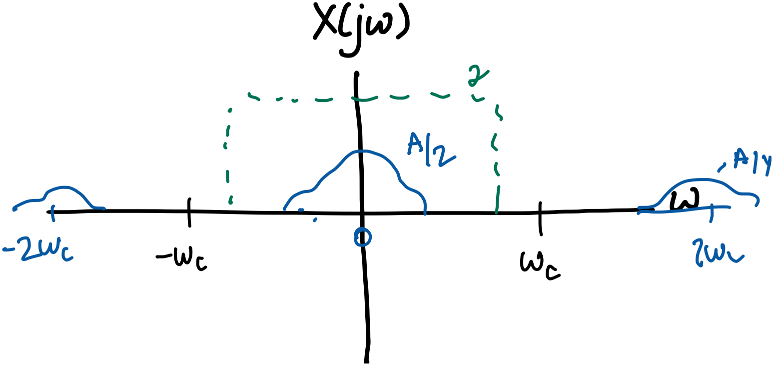


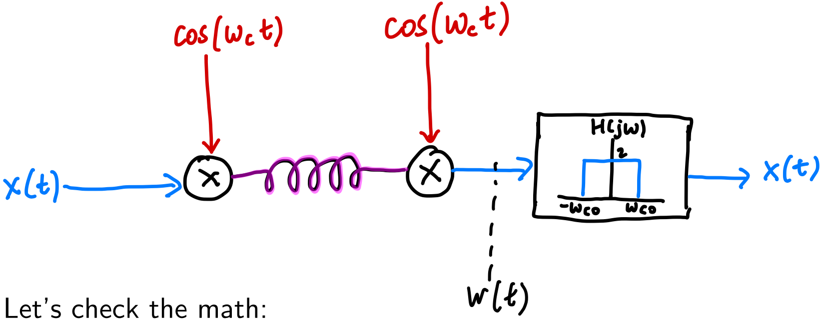


- Any foreseeable problems with this?
- How do we recover the signal?

Sinusoidal amplitude modulation



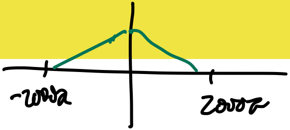




Let's check the math:

$$
\begin{aligned}
w(t) &= \cos(w_c t)\left[\cos(w_c t)\, x(t)\right] \\
&= \left[\frac{1}{2} + \frac{1}{2}\cos(2w_c t)\right] x(t) \\
&= \frac{1}{2} x(t) + \frac{1}{2}\cos(2w_c t)\, x(t)
\end{aligned}
$$

Exercise: sinusoidal amplitude modulation



8.3. Let $x(t)$ be a real-valued signal for which $X(j\omega) = 0$ when $|\omega| > 2{,}000\pi$. Amplitude modulation is performed to produce the signal

$$g(t) = x(t)\sin(2{,}000\pi t).$$

A proposed demodulation technique is illustrated in Figure P8.3 where $g(t)$ is the input, $y(t)$ is the output, and the ideal lowpass filter has cutoff frequency $2{,}000\pi$ and passband gain of 2. Determine $y(t)$.

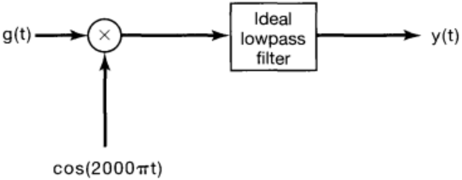


Figure P8.3

Exercise: sinusoidal amplitude modulation

$\sin(2\theta) = 2\sin\theta\cos\theta$

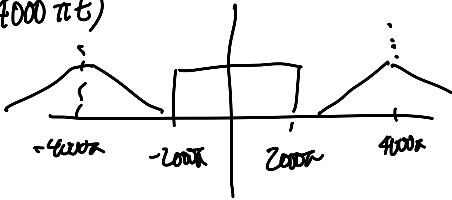
Solution option 1: mathematical

Evaluate $w(t) = g(t)\cos(2000\pi t)$ (input to filter):

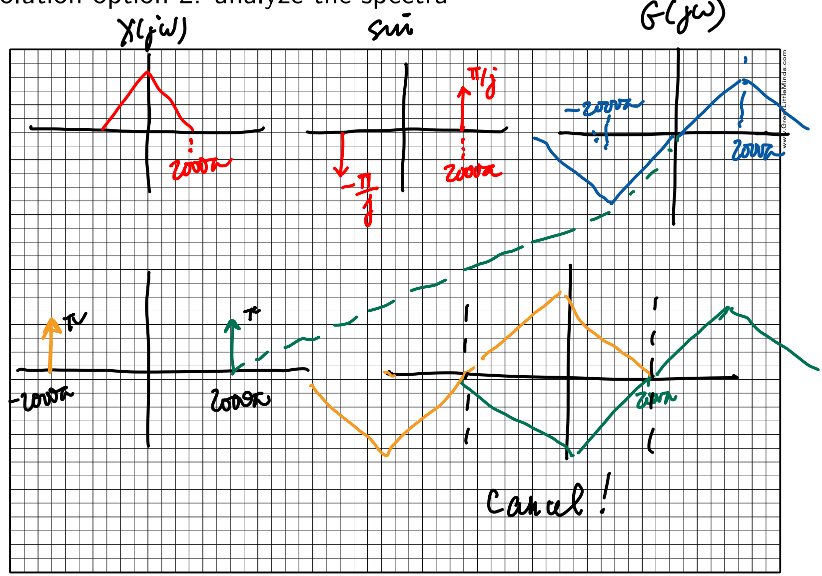
$$= x(t)\sin(2000\pi t)\cos(2000\pi t)$$

$$= x(t)\cdot\frac{1}{2}\sin(4000\pi t)$$

$y(t) = 0$



Solution option 2: analyze the spectra



More generally, must consider phases in both modulating and demodulating signals:

$$w(t) = \cos(\omega_c t + \theta_c)\cos(\omega_c t + \phi_c)\, x(t)$$

$$= \left[\frac{1}{2}\cos(\theta_c - \phi_c) + \frac{1}{2}\cos(2\omega_c t + \theta_c + \phi_c)\right] x(t)$$

Output after the lowpass filter is

$$x'(t) = \frac{1}{2}\cos(\theta_c - \phi_c)\, x(t)$$

Synchronous demodulation: $\phi_c = \theta_c$. What could go wrong?

Asynchronous demodulation

Suppose the following is true:

- $x(t)$ is positive
- ω_c is much larger than ω_m

The transmitted signal will look something like this:

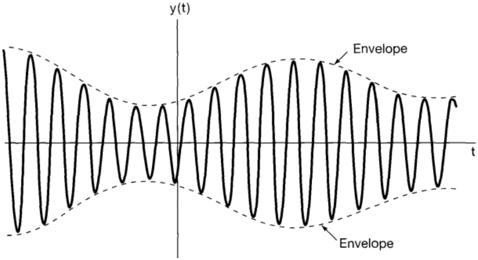


Image credit: Oppenheim 8.2

Asynchronous demodulation

Design a system to track the envelope (we won't go into details).

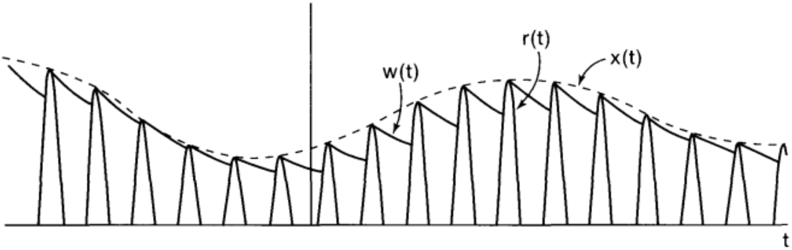


Image credit: Oppenheim 8.2

Asynchronous demodulation

If $x(t)$ is not positive, choose A sufficiently large and add to signal:

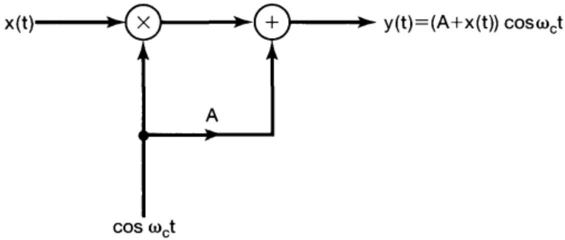
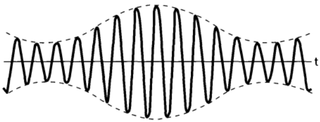
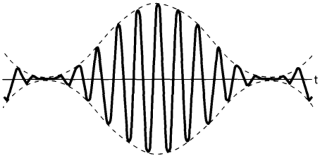


Image credit: Oppenheim 8.2

Suppose $|x(t)| \leq K$. Must have $A > K$. Then $m = K/A$ is known as the *modulation index*.



(a)



(b)

You will explore this in Assignment 4.

Image credit: Oppenheim 8.2

For next time

Content:

- single-sideband modulation
- pulse amplitude modulation and time-division multiplexing
- cellphone communication systems

Action items:

1. Assignment 4 due Saturday 23:59
2. Tutorial assignment 4 due Monday 23:59

Recommended reading:

- From this class: Oppenheim 8.0-8.3
- Suggested problems: 8.1, 8.2, 8.4-8.6, 8.8, 8.21-8.23, 8.40
- For next class: Oppenheim 8.4-8.6