Practice problems

1. Consider the discrete-time signal $x[t]$ where

$$x[n] = 1 + \cos\,(4\pi n/9)$$

(a) Fin the period N
(b) What is the corresponding fundamental frequency f_0 and ω_0?
(c) What are the coefficients for its Fourier series expansion?

2. Consider the continuous-time signal $x(t)$ where

$$x(t) = 1 + \cos(\pi t) + \cos\,(2\pi t)$$

Suppose that x is the input to an LTI system with frequency response given by

$$H(j\omega) = \begin{cases} e^{j\omega}, & |\omega| < 4\ rad/s \\ 0, & otherwise \end{cases}$$

What will be the output of the system?

3. Suppose that the continuous time signal $x(t)$ is periodic with period T. Let the fundamental frequency be $\omega_0 = 2\pi/T$. Suppose that the Fourier series coefficients for this signal are known constants $C_0, C_1, C_2, \cdots$. Give the Fourier series coefficients $C'_0, C'_1, C'_2, \cdots$ for each of the following signals:
(a) $ax(t)$, where a is a real valued constant
(b) $x(t - t_0)$, where t_0 is a constant
(c) $S(x)$, where S is an LTI system with frequency response $H(j\omega)$ given by

$$H(j\omega) = \begin{cases} 1, & \omega = 0 \\ 0, & otherwise \end{cases}$$

(d) Let $y(t)$ be another periodic signal with period T. Suppose $y(t)$ has Fourier series coefficients $C''_0, C''_1, C''_2, \cdots$. Give Fourier series coefficients of $x(t) + y(t)$.

4. Find the smallest positive integer l such that

$$\sum_{k=0}^{n} e^{j5k\pi/6} = 0$$

Hint: Note that the term being summed is a periodic function of k. What is its period? What is the sum of a complex exponential over one period?

From Lee-Varaiya, 2011.

5. Consider a continuous-time periodic signal $x(t)$ with fundamental frequency $\omega_0 = 1\, rad/s$. Suppose that the Fourier series coefficients are

$$C_k = \begin{cases} 1, & k = 0, 1, 2 \\ 0, & otherwise \end{cases}$$

(a) Given the continuous-time LTI system $Filter$, with a frequency response

$$H(j\omega) = \cos(\pi\omega/2)$$

find $y(t) = Filter(x)$.

(b) What is the fundamental frequency in rad/s for $y(t)$ calculated in (a)?

6. Suppose that the frequency response $H(e^{j\omega})$ of a discrete-time LTI system $Filter$ is given by:

$$H(e^{j\omega}) = |\omega|$$

where ω has units of rad/s. What is the output $y[t]$ of the system $Filter$ for each of the following inputs $x[t]$:

(a) $x[n] = \cos(\pi n/2)$
(b) $x[n] = 5$
(c) $x[n] = \begin{cases} +1, & n\ is\ even \\ -1, & n\ is\ odd \end{cases}$

7. Consider a continuous-time LTI system with impulse response given by

$$h(t) = \delta(t-1) + \delta(t-2)$$

where δ is the Dirac delta function.

(a) Find a simple equation relating the input $x(t)$ and output $y(t)$ of this system
(b) Find the frequency response of this system

8. Suppose the following difference equation relates the input $x[t]$ and output $y[t]$ of a discrete-time, causal LTI system S,

$$y[n] + \alpha y[n-1] = x[n] + x[n-1]$$

for some constant α.

(a) Find the impulse response $h[n]$
(b) Find the frequency response $H(e^{j\omega})$
(c) Find a sinusoidal input with non-zero amplitude such that the output is zero

9. Each of the statements below refers to a discrete-time system S with input $x[n]$ and output $y[n]$. Determine whether the statement is true or false.

(a) Suppose you know that if $x[n]$ is a sinusoid then $y[n]$ is a sinusoid. Then you can conclude that S is LTI.

(b) Suppose you know that S is LT, and that if $x[n] = cos(\pi n/2)$, then $y[n] = 2cos(\pi n/2)$. Then you have enough information to determine the frequency response.

(c) Suppose you know that S is LTI, and that if $x[n] = \delta[n]$, then $y[n] = (0.9)^n u[n]$ then you have enough information to determine the frequency response.

(d) Suppose you know that S is causal, and that input $x[n] = \delta[n]$ produces output $y[n] = \delta[n] + \delta[n-1]$, and input $x'[n] = \delta[n-2]$ produces output $y'[n] = 2\delta[n-2] + \delta[n-3]$. Then you can conclude that S is not LTI.

(e) Suppose you know that S is causal, and that if $x[n] = \delta[n] + \delta[n-2]$ then $y[n] = \delta[n] + \delta[n-1] + 2\delta[n-2] + \delta[n-3]$. Then you can conclude that S is not LTI.

10. Consider an LTI discrete-time system $Filter$ with impulse response

$$h[n] = \delta[n] + \delta[n-2]$$

where δ is the Kronecker delta function.

(a) Sketch $h[n]$

(b) Find the output when the input is the unit step function $u[n]$

(c) Find the output when the input is a ramp

$$r[n] = \begin{cases} n, & for\ n \geq 0 \\ 0, & otherwise \end{cases}$$

(d) Suppose the input signal $x[n]$ is such that

$$x[n] = \cos(\omega n)$$

where $\omega = \pi/2$. Give a simple expression for $y[n] = Filter[x]$.

(e) What is the frequency response $H(e^{j\omega})$ of the system $Filter$?