ELEC 221 Lecture 16
DT systems based on difference equations

Thursday 31 October 2024

Announcements

- Assignment 3 due Saturday 23:59; solutions posted shortly after deadline
- Midterm 2 Monday 4 Nov during tutorial time
- No class on Tuesday 5 Nov

Last time

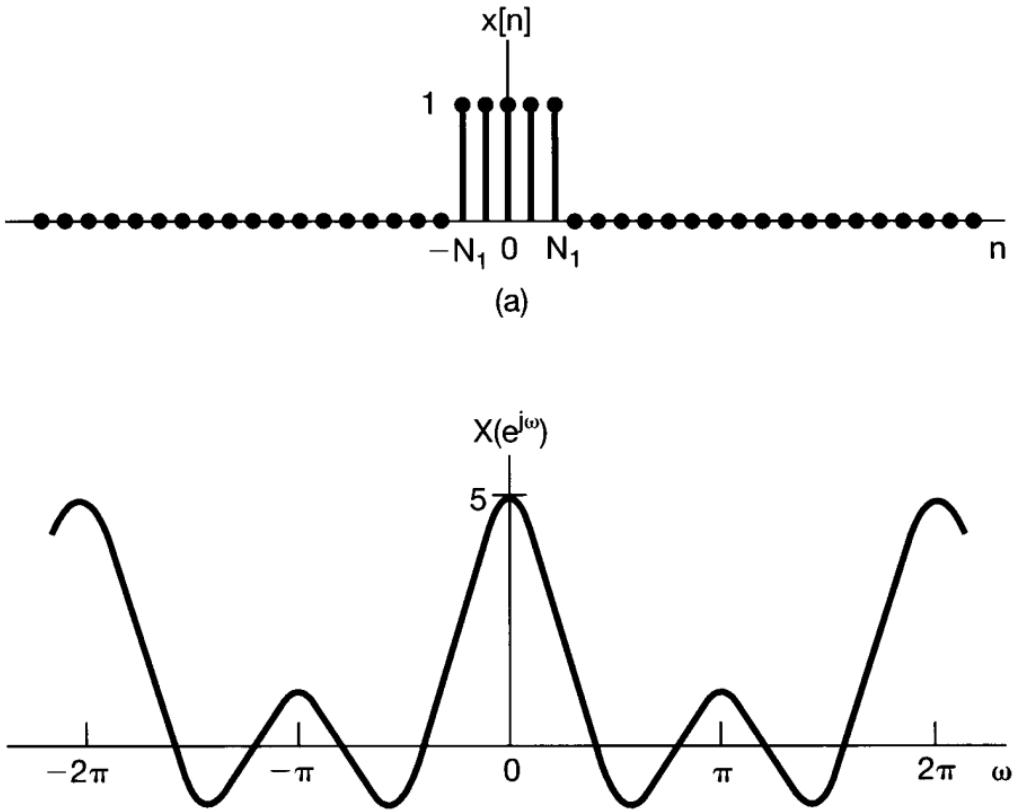
We derived the **discrete-time Fourier transform** (DTFT)

Inverse DTFT (synthesis equation)

DTFT (analysis equation)

Last time

We computed the DTFT for a square pulse



The DTFT is **continuous**, and 2π-**periodic**!

Image credit: Oppenheim chapter 5.1

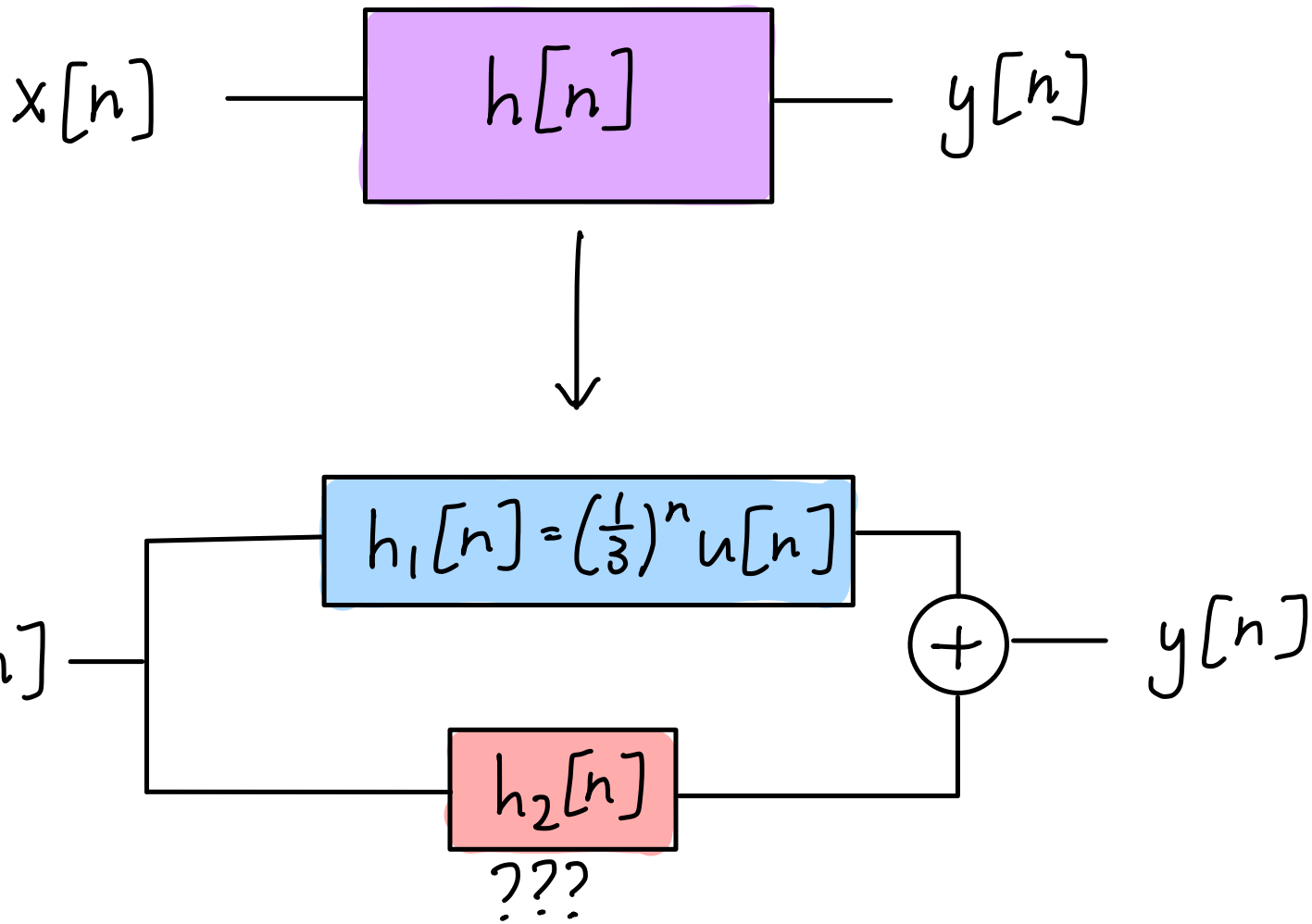
Today

Learning outcomes:

- Leverage key properties of the DTFT to simplify its computation
- Use the convolution property of the DTFT to analyze the behaviour of LTI systems
- Construct and analyze DT systems based on difference equations

Example

$$H(e^{j\omega}) = \frac{-12 + 5e^{-j\omega}}{12 - 7e^{-j\omega} + e^{-2j\omega}}$$



Example

What is the DTFT of

$$x[n] = a^n u[n], \quad |a| < 1$$

Properties of the DT Fourier transform

Linearity: If

then

Example

Properties of the DT Fourier transform

Time shift: If

then

Frequency shift:

Periodicity:

Example: time and frequency shift

What is the DTFT of

$$x[n] = \delta[n] + 2\delta[n-1] + e^{3jn}\delta[n-2]$$

Properties of the DT Fourier transform

Conjugation: If

then

If $x[n]$ is real,

Consequences for odd/even functions:

Properties of the DT Fourier transform

Time expansion:

Properties of the DT Fourier transform

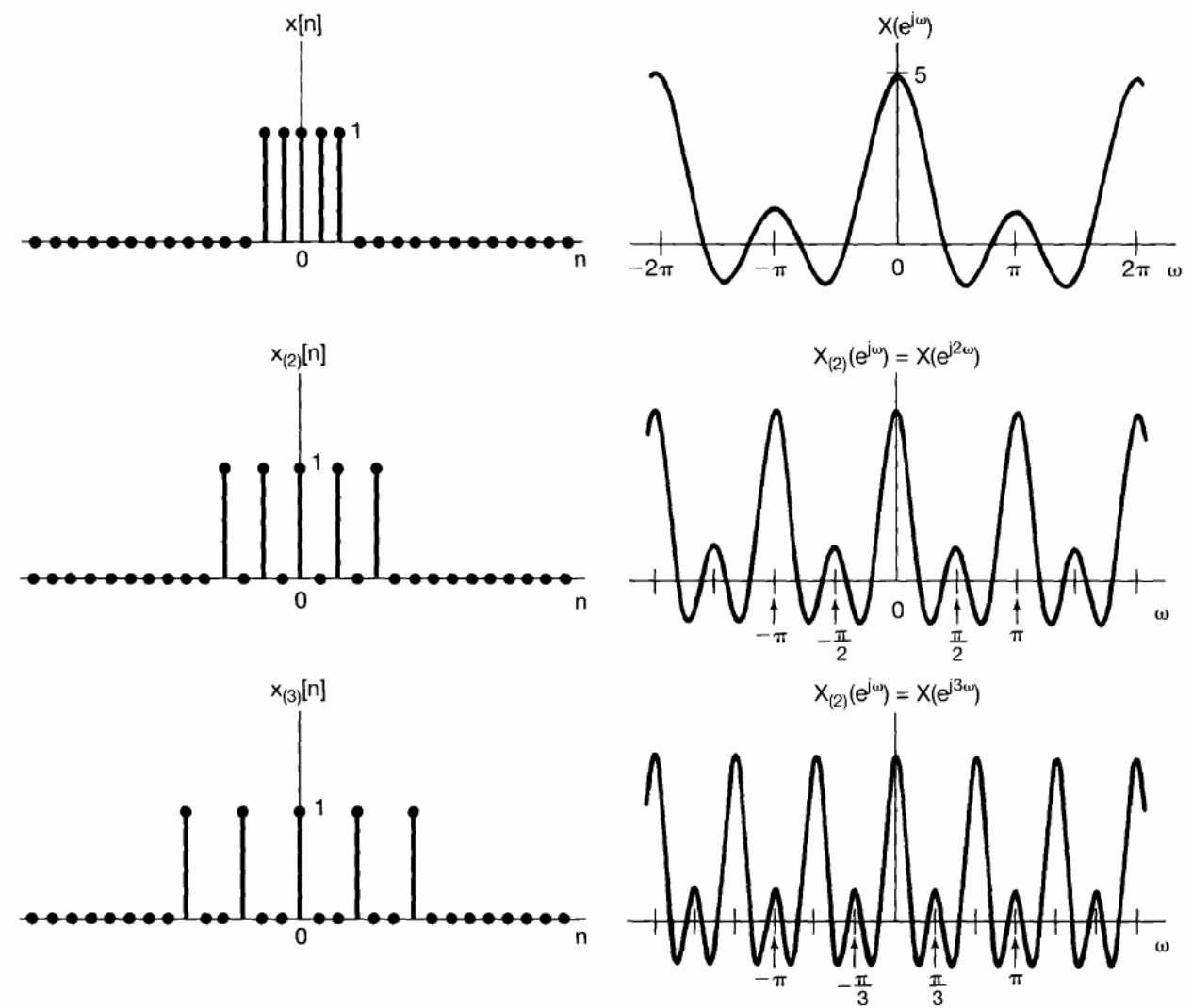


Image credit: Oppenheim Figure 5.14

Properties of the DT Fourier transform

Differentiation in frequency:

Differencing:

Example: differentiation in frequency

What is the DTFT of

$$x[n] = n\delta[n+3]$$

Example: difference equations

What is the frequency response of a system defined by a linear, constant-coefficient difference equation

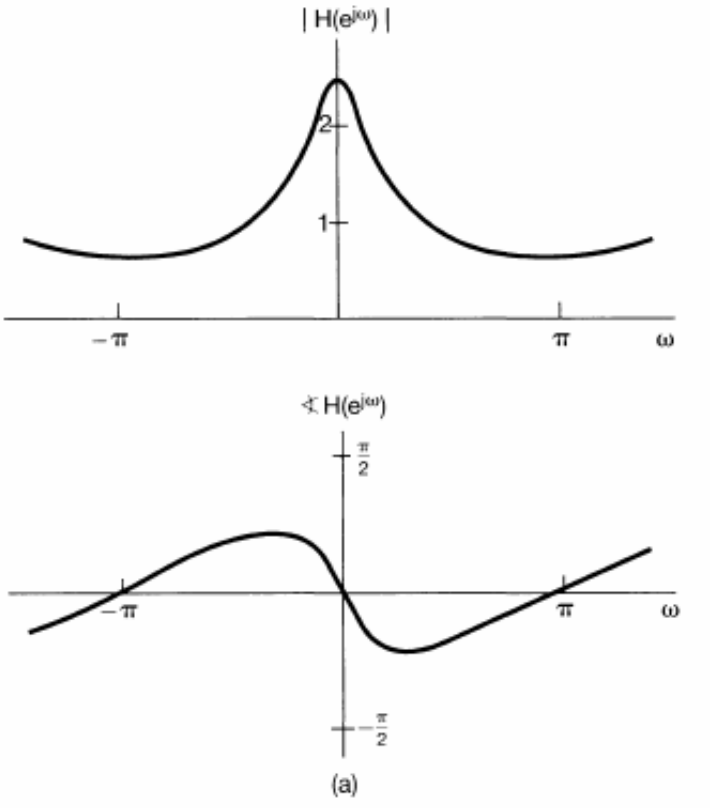
DT systems based on difference equations

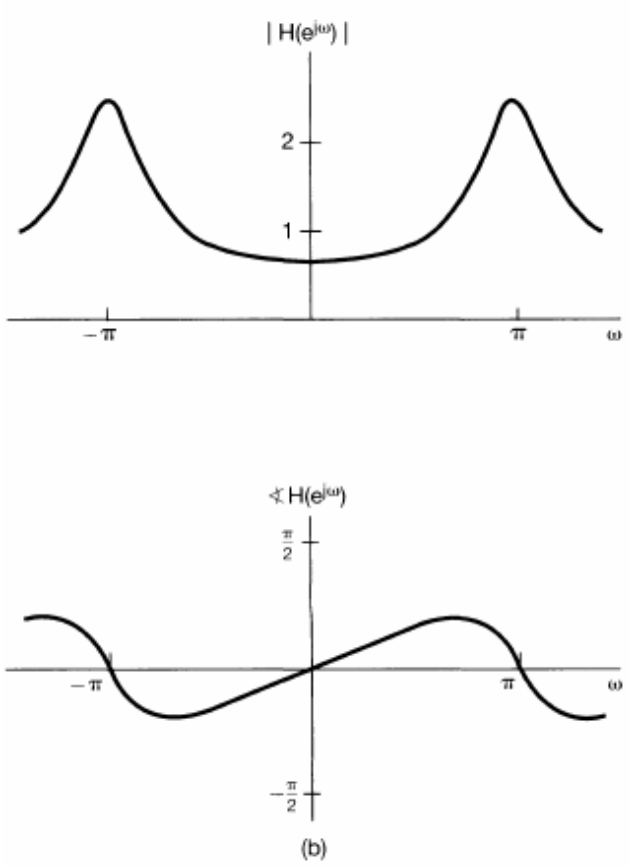
Exercise: compute frequency response of a system characterized by

$$y[n] - ay[n-1] = x[n]$$

Example

"First-order recursive DT filters"





$0 < a < 1 \ \ (a = 0.6)$

$-1 < a < 0 \ \ (a = -0.6)$

Image credit: Oppenheim Figure 3.34

DT systems based on difference equations

Exercise: compute frequency response of a system characterized by

$$y[n] = \frac{1}{3}(x[n+1] + x[n] + x[n-1])$$

DT systems based on difference equations

Exercise: compute frequency response of a system characterized by

$$y[n] = \frac{1}{5}(x[n+2] + x[n+1] + x[n] + x[n-1] + x[n-2])$$

DT systems based on difference equations

Exercise: compute frequency response of a system characterized by

$$y[n] = \frac{1}{N+M+1} \sum_{k=-N}^{M} x[n-k]$$

DT systems based on difference equations

"Non-recursive DT filters"

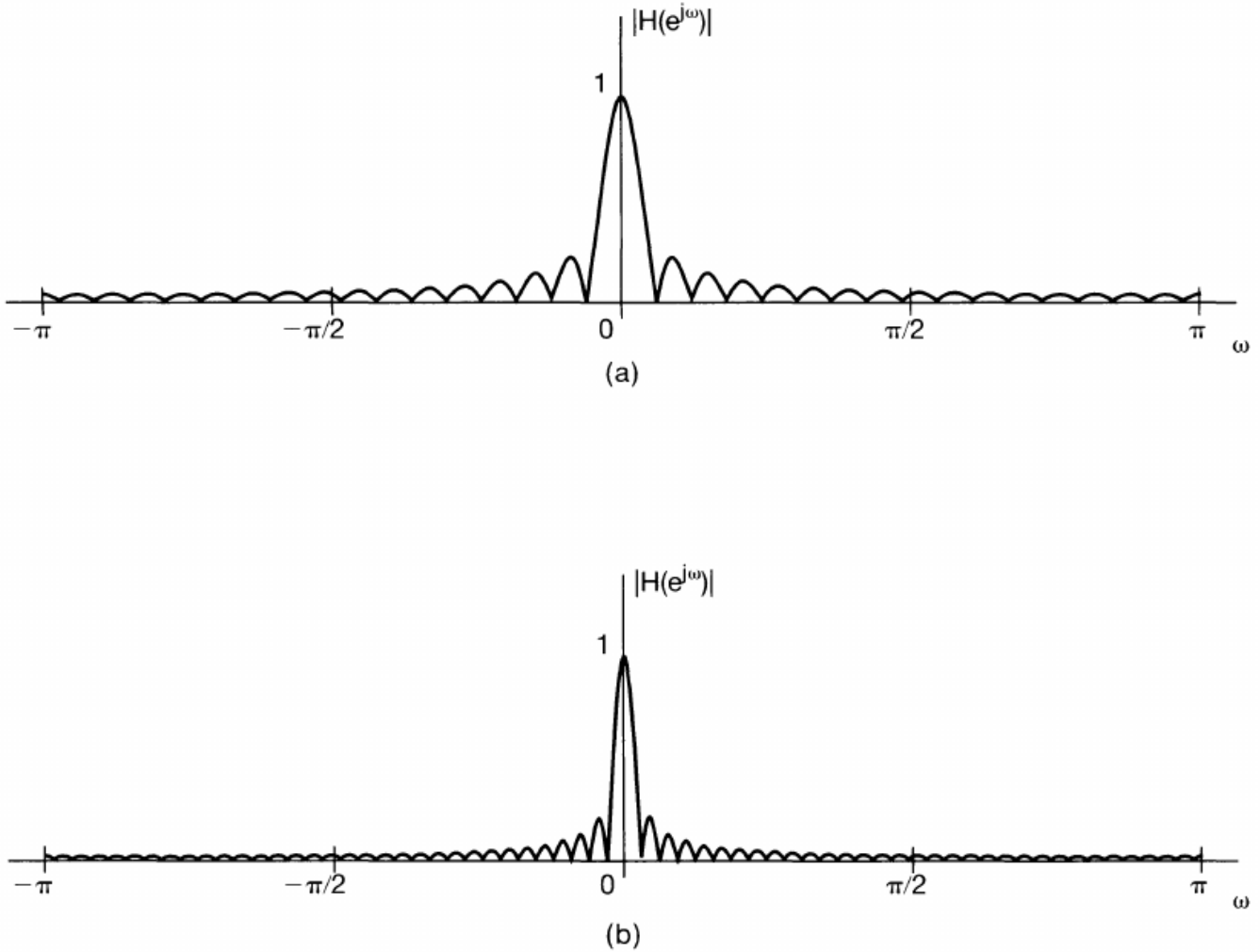


Image credit: Oppenheim Figure 3.36

For next time

Content:

- The sampling theorem
- Basics of interpolation
- The Nyquist rate and aliasing

Action items:

1. Assignment 3
2. Midterm 2

Recommended reading:

- From this class: Oppenheim 5.2-5.9
- Suggested problems: 5.4b, 5.6, 5.8, 5.19, 5.22bcdfgh, 5.25, 5.29, 5.31, 5.33-5.36
- For next class (7 Nov): Oppenheim 7.1-7.3