ELEC 221
Tutorial 2

Monday 3 October 2022

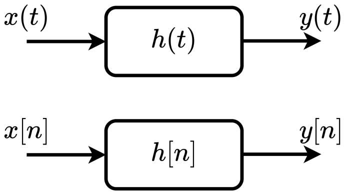
LTI System

- CT:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

- DT:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



Fourier Series for Periodic Signals

- CT synthesis equation:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega t}$$

- CT analysis equation:

$$c_k = \frac{1}{T} \int_T e^{-jk\omega t} x(t) dt$$

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- DT synthesis equation:

$$x[n] = \sum_{k=0}^{N-1} c_k e^{jk\frac{2\pi n}{N}}$$

- DT analysis equation:

$$c_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\frac{2\pi n}{N}}$$

$$\begin{aligned} x(t) \to y(t) &= \sum_{k=-\infty}^{\infty} c_k H(jk\omega) e^{jk\omega t} \\ x[n] \to y[n] &= \sum_{k=0}^{N-1} c_k H(e^{jk\omega}) e^{jk\omega n} \end{aligned}$$

$H(j\omega)$ in CT, and $H(e^{j\omega})$ in DT, are called the **frequency response** of the system, where

$$\begin{aligned} H(jw) &= \int_{-\infty}^{\infty} e^{-jw\tau} h(\tau) d\tau, \\ H(e^{jw}) &= \sum_{m=-\infty}^{\infty} e^{-j\omega m} h[m]. \end{aligned}$$

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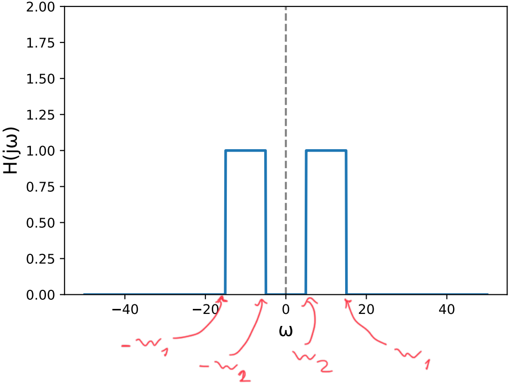

$$x(t) = 2 + cos(\omega_0 t) \rightarrow y(t) = ?$$

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- Approach 1:
  1. compute $h(t)$:

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega)e^{j\omega t}d\omega$$

  2. compute the convolution:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

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- Approach 2:
  - Identify the frequency components of $x(t)$:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega t}$$

  - The output is:

$$y(t) = \sum_{k=-\infty}^{\infty} c_k H(jk\omega) e^{jk\omega t}$$

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$$x(t) = 2 + cos(\omega_0 t) = 2e^{j \times 0 \times t} + \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

- $\omega_1 \leq \omega_0 \leq \omega_2$:

$$y(t) = 0 \times 2e^{j \times 0 \times t} + 1 \times \frac{e^{j\omega_0 t}}{2} + 1 \times \frac{e^{-j\omega_0 t}}{2} = cos(\omega_0 t)$$

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- otherwise:

$$y(t) = 0 \times 2e^{j\times 0\times t} + 0 \times \frac{e^{j\omega_0 t}}{2} + 0 \times \frac{e^{-j\omega_0 t}}{2} = 0$$