ELEC 221 Lecture 12
The CT Fourier transform properties: convolution and multiplication

Thursday 17 October 2024

Announcements

- Quiz 6 Tuesday
- Please prepare a 4-5 second excerpt of your favourite song (as a `.wav` file) for Monday's tutorial assignment

Last time

We saw the Dirichlet conditions for the Fourier transform.

If the signal

1. is single-valued
2. is absolutely integrable ($\int_{-\infty}^{\infty} |x(t)|dt < \infty$)
3. has a finite number of maxima and minima within any finite interval
4. has a finite number of finite discontinuities within any finite interval

then the Fourier transform converges to

- $x(t)$ where it is continuous
- the average of the values on either side at a discontinuity

Last time

We computed Fourier transforms of periodic signals.

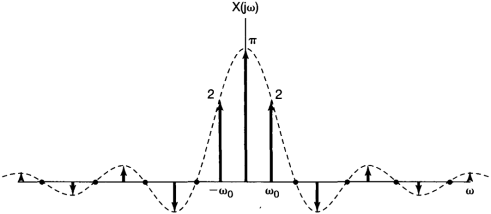


Image credit: Oppenheim chapter 4.2

Last time

Duality: for any transform pair $(x(t) \leftrightarrow X(j\omega))$, there is a *dual pair* with the time and frequency variables interchanged.

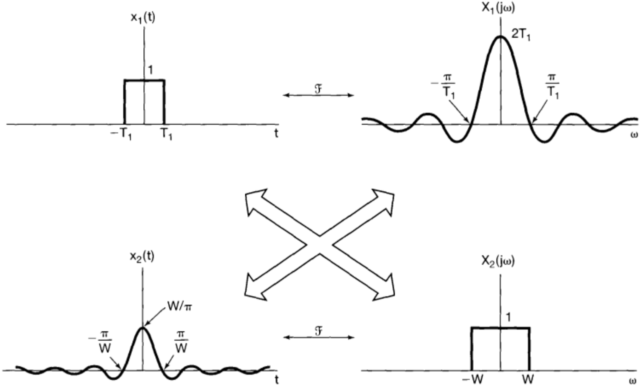


Image credit: Oppenheim chapter 4.3

Today

We will make a big step towards answering the question
"Why are we even doing this?"

Learning outcomes:

- Leverage key properties of Fourier transform to simplify its computation
- Apply the *convolution property* of the Fourier transform to characterize LTI system behaviour
- Describe the *multiplication property* of the Fourier transform and provide applications of its use

Clarification

Last class, I wrote

and you asked about $u(0)$.

There are different conventions:

- we are treating it as undefined
- sometimes it's defined as 1
- sometimes it's defined as 0
- sometimes it's defined as $1/2$ ("half-maximum convention")

For how we're using it (in integrals), it doesn't matter.

What is the Fourier transform of $x(t) = e^{-2|t-1|}$?

Linearity.

Our example:

Time shifting. If

then

Notice: $|X(j\omega)|$ does not change; we just add a linear phase shift.

Our example:

Time scaling. If

then

Time reversal follows from this:

Our example: we have

Important properties of the Fourier transform

Conjugation. If

then

If $x(t)$ is purely real,

Implications for even/odd parts of a (real) signal:

Convolution and the Fourier transform

Recall complex exponentials are eigenfunctions of LTI systems. If we input signal $x(t)$ into LTI system with impulse response $h(t)$

where

This came from the convolution integral:

Convolution and the Fourier transform

Let's express $x(t)$ using the inverse Fourier transform:

and put this into the convolution integral...

We have **two** ways to write $y(t)$:

This has an important implication:

Example: convolution

This can be helpful for evaluating the output of systems given $h(t)$ and $x(t)$ (or $h(t)$ given $y(t)$ and $x(t)$, etc.)

Example: suppose a signal $x(t) = \frac{\sin(\omega_0 t)}{\pi t}$ is input into a lowpass filter with frequency response

Method 1: inverse FT $H(j\omega)$ to get $h(t)$, then convolve.

Example: convolution

Method 2: compute $X(j\omega)$ then use convolution property.

We just computed $h(t)$ and found

Exercise: convolution

Consider an LTI system that sends

What is its impulse response?

Exercise: convolution

The multiplication property

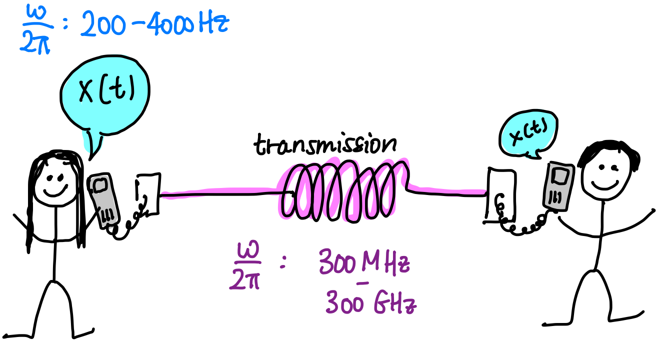
We know that:

Something similar holds when we interchange time and frequency:

This is the **multiplication property**.

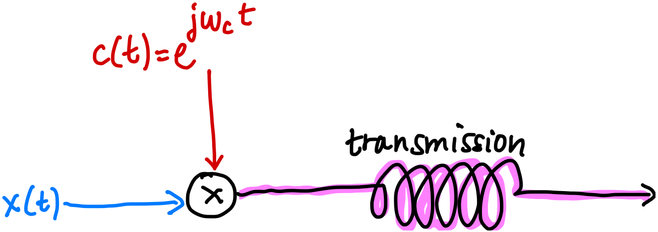
We are going to take a much closer look at this when we discuss communication systems and signal **modulation**.

For now, here is a taste:



Example: the multiplication property

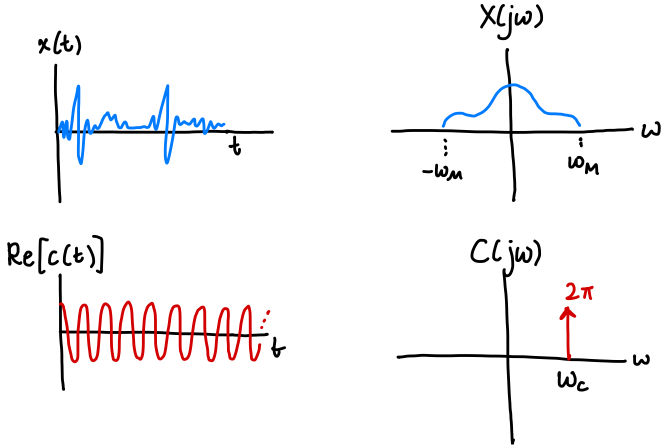
To shift our signal into the frequency range of transmission, we can multiply it by a **carrier signal** (amplitude modulation):



Is this doing what we think it is?

Example: the multiplication property

Consider the Fourier spectrum of both signals:



Example: the multiplication property

The multiplication property tells us

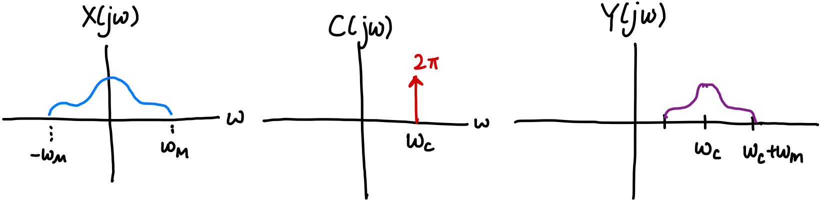
We have

Example: the multiplication property

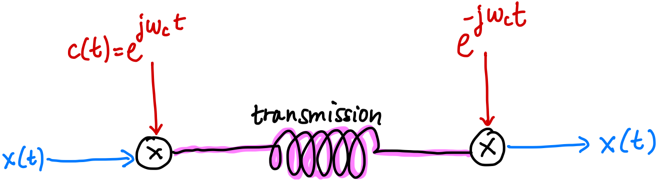
Let's convolve them:

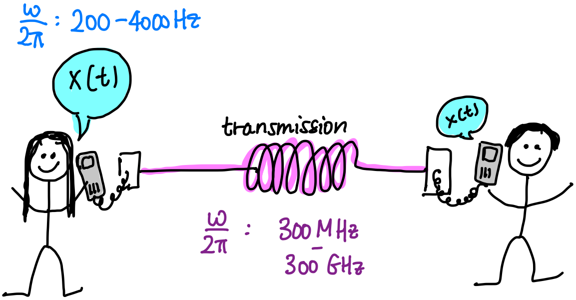
Example: the multiplication property

Multiplication with complex exponential carrier signal shifts the spectrum. We can move it into the desired frequency range.



Example: the multiplication property





Example: frequency-selective filtering with variable centre frequency

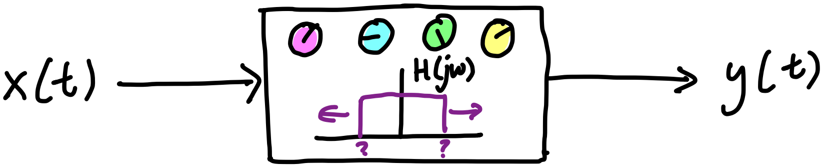


Image: `https://www.euronics.ee/UserFiles/Products/Images/162391-panasonic-rf-2400d-radio.png`

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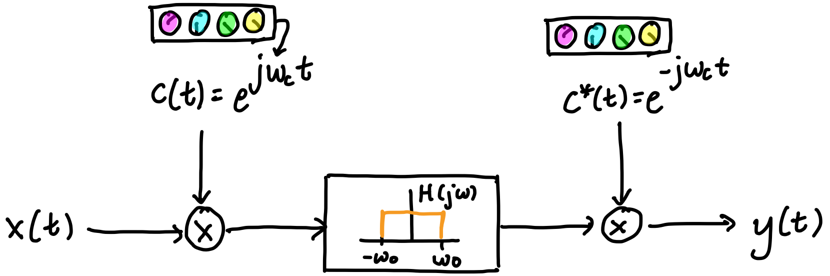
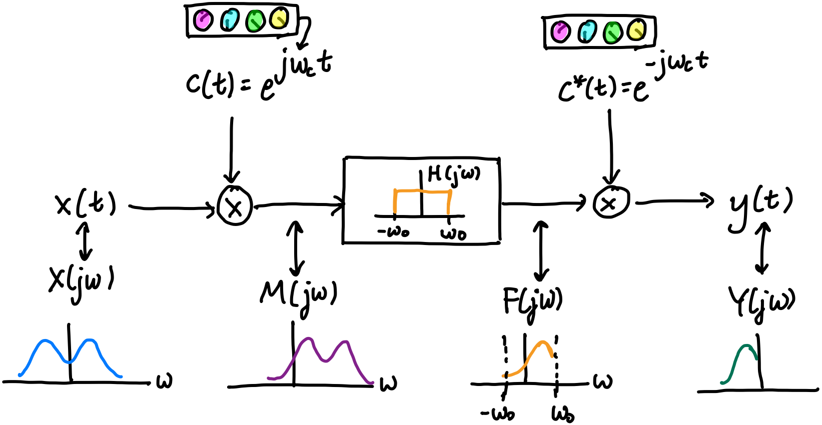
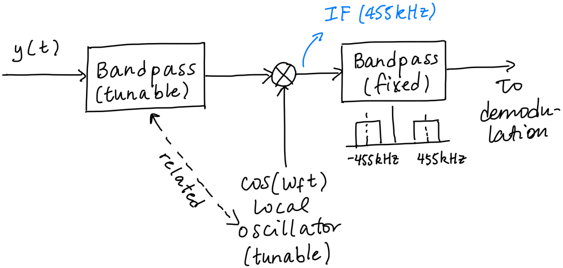


Example: frequency-selective filtering with variable centre frequency



Real-world example: radio

“Superheterodyne receiver”



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Recap

Today's learning outcomes were:

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For next time

Content:

- Behaviour of the Fourier transform under differentiation and integration
- LTI systems based on differential equations

Action items:

1. Tutorial Assignment 3 on Monday - bring music!

Recommended reading:

- From today's class: Oppenheim 4.4-4.6
- Suggested problems: 4.4, 4.6, 4.9, 4.12, 4.15, 4.17, 4.19, 4.26, 4.32
- For Tuesday's class: Oppenheim chapter 4.7, 6.1-6.2