ELEC 221 Lecture 24
Feedback systems

Thursday 5 December 2024

Announcements

- Last class!
- Please come pick up your midterms
- Will post final exam info (incl. practice final) on PrairieLearn
- Assignment 5 due Sunday at 23:59

Last time

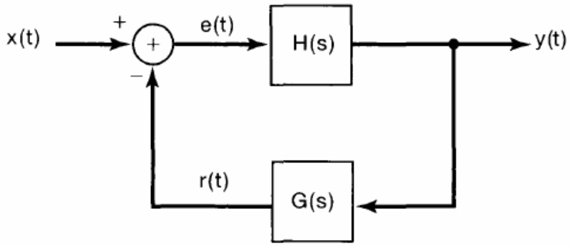
The Laplace transform, with info about input/output relationships, can help characterize systems described by differential equations.

Today

Learning outcomes:

- compute system functions for feedback systems
- use the Laplace transform and feedback systems to design inverse systems and stabilize unstable systems
- identify the z-transform

An important application of Laplace transforms is the analysis of **feedback systems**.

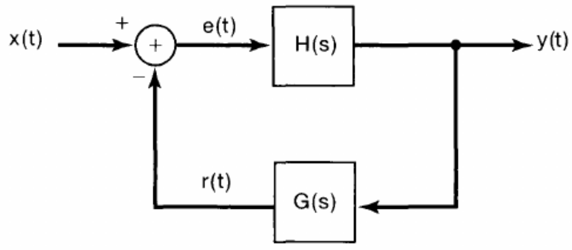


Some important applications of feedback include:

- reducing sensitivity to disturbance and errors
- stabilizing unstable systems
- designing tracking systems

Image credit: Oppenheim 11.1

An important application of Laplace transforms is the analysis of **feedback systems**.



- $H(s)$ is the system function of the forward path
- $G(s)$ is the system function of the feedback path
- the combined function $Q(s)$ is the closed-loop system function

Let's compute $Q(s)$ in terms of $H(s)$ and $G(s)$.

Image credit: Oppenheim 11.1

Feedback systems

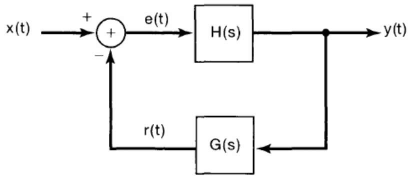
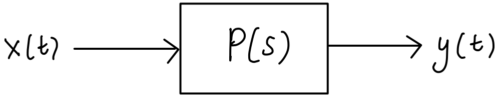
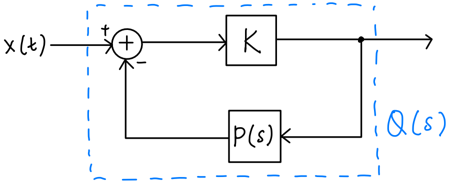


Image credit: Oppenheim 11.1

Suppose we have some LTI system



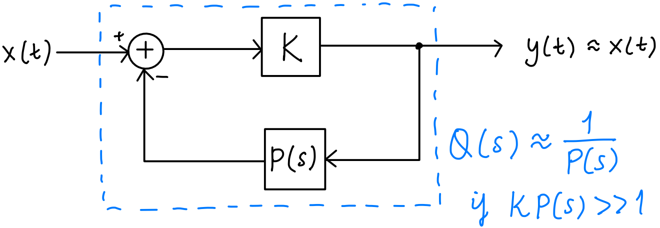
Let's use it as part of a larger system:



where the transfer function K is simply gain of strength K.

Exercise: What is $Q(s)$, and under what conditions can it act as the inverse of $P(s)$?

Solution: we can directly apply the expression for the closed-loop system function here

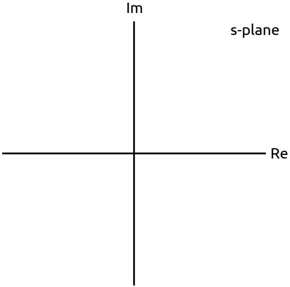


Application of feedback: stabilizing an unstable system

Consider a system described by the first order DE

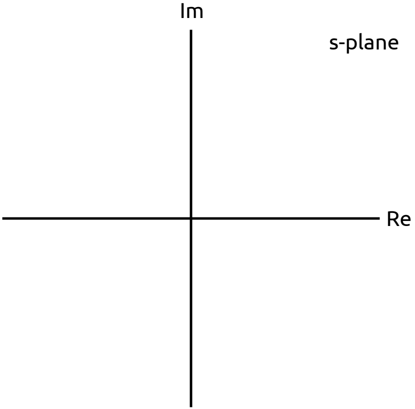
$$\frac{dy(t)}{dt} - ay(t) = bx(t)$$

Exercise: compute the system function and draw the ROC. Under what conditions is it stable?



Application of feedback: stabilizing an unstable system

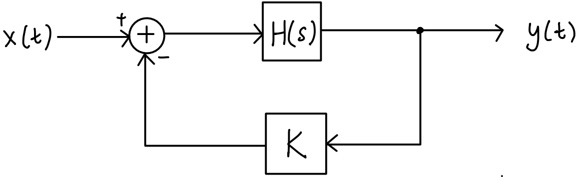
Suppose we have this setup ($a > 0$):

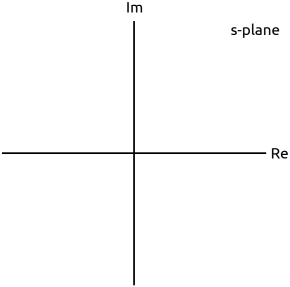


How can we make it stable?

Application of feedback: stabilizing an unstable system

Show that the following system will move the pole (under certain conditions on K):





Called a *proportional feedback system* since feeding back in a rescaled version of the output.

Real-world example: audio feedback

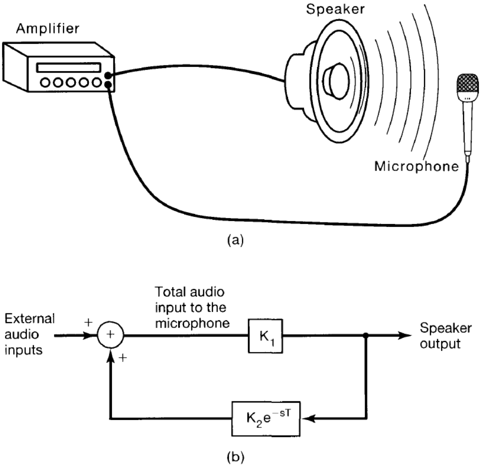


Image credit: Oppenheim, Fig. 11.8

The z-transform

CT

Fourier series coefficients

$$c_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}\,dt$$

Fourier transform (spectrum)

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

Laplace transform

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$$

DT

Fourier series coefficients

$$c_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk\frac{2\pi n}{N}}$$

Fourier transform

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

z-transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

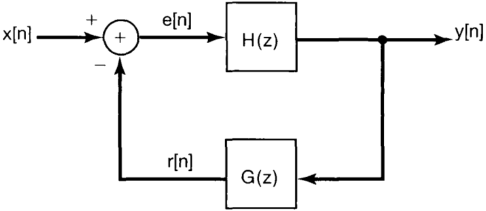
Consider a DT complex exponential signal

If we put this in a system with impulse response $h[n]$, obtain

where

- $z = e^{j\omega}$: discrete-time Fourier transform
- $z = re^{j\omega}$: z-transform

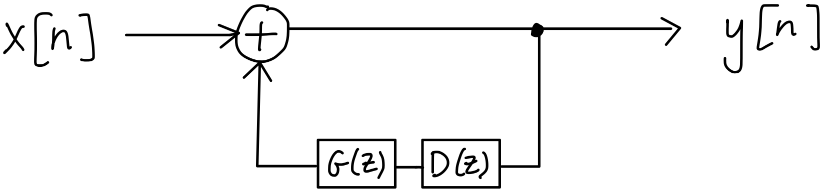
The z-transform can help us analyze feedback systems (using them for stabilization, etc.), just like Laplace transform in CT.



The closed-loop system function has the same form:

Image credit: Oppenheim 11.1

One type of system with this structure is called the **comb filter**

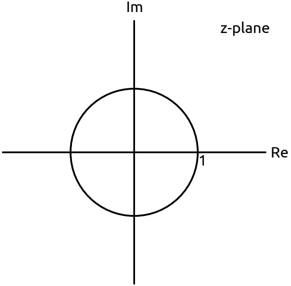


- $D(z)$ is a system that causes a delay of K steps
- $G(z)$ is a system with gain g

Difference equation:
System function:

Example: comb filters

What are the poles and zeros?



Why is it called the comb filter? Let's look at its frequency response (take $z = e^{j\omega}$).

Another example of this is the Karplus-Strong algorithm!

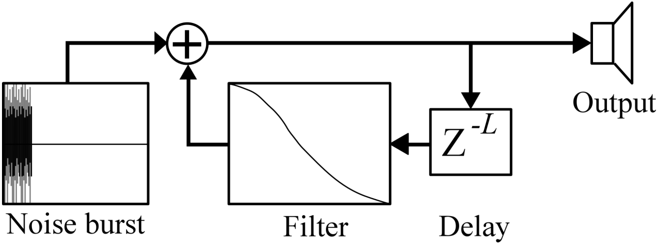
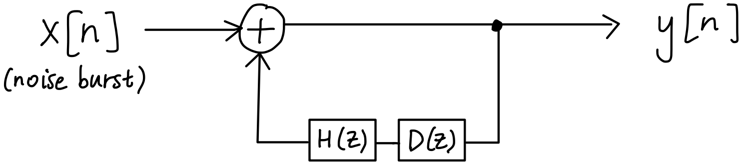


Image credit: https://commons.wikimedia.org/wiki/File:Karplus-strong-schematic.svg Author: PoroCYon CC BY-SA 3.0



- $D(z)$ is a system that causes a delay of K steps
- $H(z)$ is a lowpass filter described by DE $y[n] = \frac{1}{2}(x[n] + x[n-1])$

Difference equation:
System function:

For next time

Action items:

1. Assignment 5 due Sunday at 23:59

Recommended reading:

- From this class: Oppenheim 11.1-11.2
- Suggested problems: 11.2-11.4