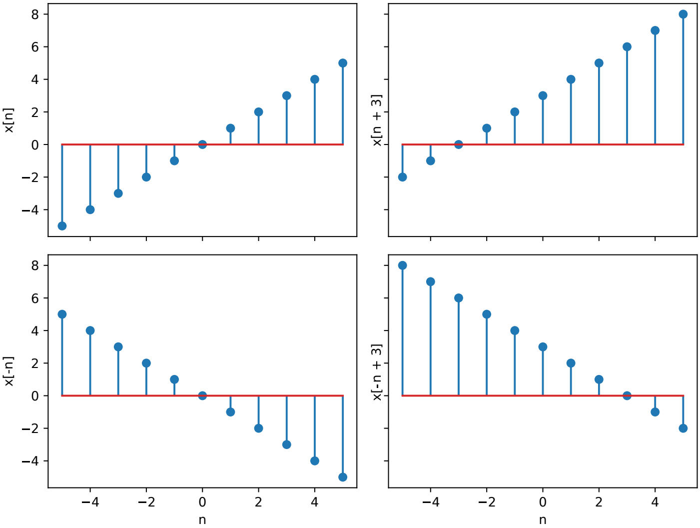
ELEC 221 Lecture 02
LTI systems, DT impulse response and the convolution sum

Tuesday 10 September 2024

- Quiz 1 today
- Tutorial assignment 1 Monday 16 Sept 23:59
- Assignment 1 due Thursday 19 Sept 23:59

Last time

We saw continuous-time and discrete-time signals, and applied some simple transformations to them.



We introduced systems, which respond to signals, transform them, and output new signals.

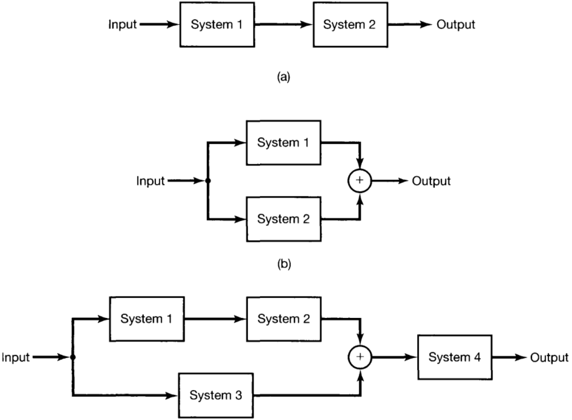


Image credits: Signals and Systems 2nd ed., Oppenheim

Last time

We explored some properties of systems:

1. Memory
2. Invertibility
3. Causality
4. Stability
5. Linearity
6. Time invariance

Quiz time

Available at `us.prairielearn.com`

- Work individually
- You may use lecture slides, notes, or the textbook.
- Please do not search online for answers

Once you start the quiz you will have 10 minutes to complete it.

You have one attempt per question.

Questions will re-enabled later for practice with random variants.

Learning outcomes:

- Define what it means for a system to be LTI (linear, time-invariant)
- Define the DT unit impulse and unit step functions
- Define the convolution sum and use it to compute the output of a system

Consider a function f such that $y = f(x)$.

If f is linear, what key properties does it have?

Properties of systems: linearity

A **linear** system $x(t) \to y(t)$ sends

Thus, a linear system sends

for arbitrary a, b (which may be complex).

Example: linearity

Is the following system linear?

$$x(t) \to y(t) = x(n+1) - x(n-1)$$

Example: linearity

Is the following system linear?

$$x(t) \rightarrow y(t) = x(n+1) - x(n-1)$$

Exercise: linearity

Is the following system linear?

$$x[n] \to y[n] = x[n] + 1$$

Properties of systems: time invariance

A system is **time invariant** if a time-shifted input leads to an output time-shifted by the same amount.

Intuition: behaviour of the system is fixed over time.

Example: time invariance

Is this system time-invariant?

$$y(t) = \cos(3t)x(t)$$

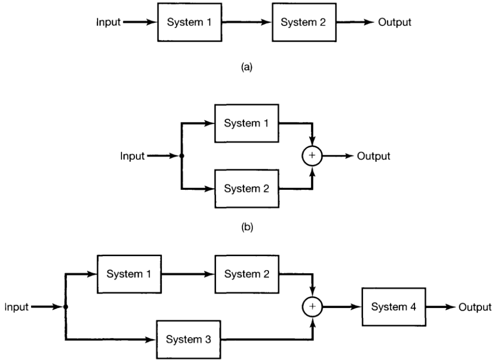
Exercise: time invariance

Is this system time-invariant?

$$y(t) = x(t+1) - x(t-1)$$

LTI systems

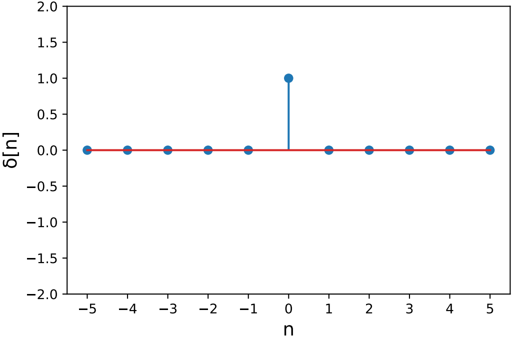
We are most interested in systems that are both **linear** and **time-invariant**, i.e., LTI systems.



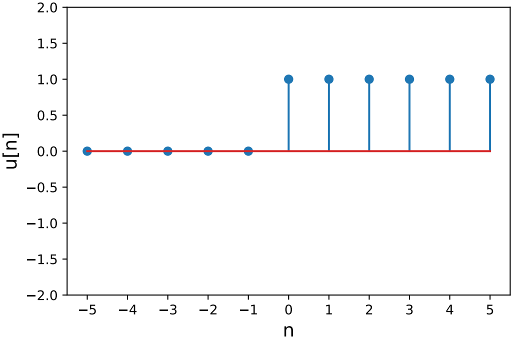
How can we characterize the behaviour of LTI systems?

Image credits: Signals and Systems 2nd ed., Oppenheim

The DT unit impulse



The DT unit step

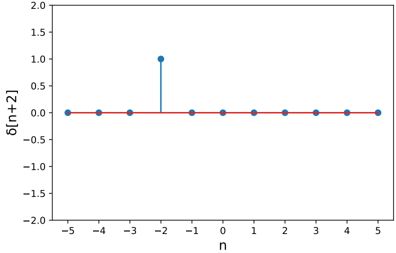


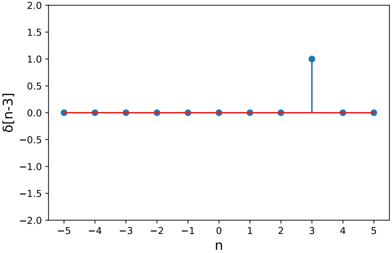
Relationships between basic signals

We can express these in terms of each other:

The unit impulse is an important tool for characterizing the behaviour of systems.

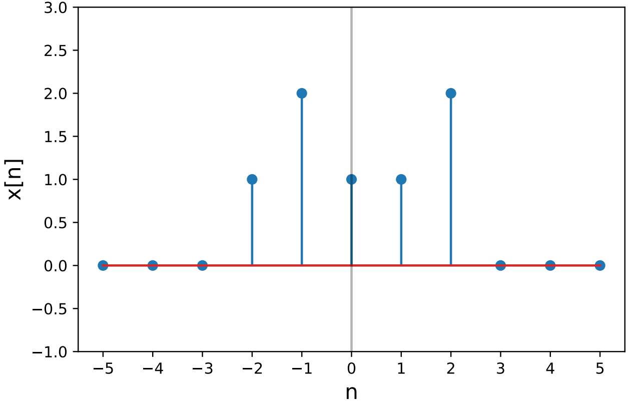
By considering unit impulses time-shifted as various points, we can pick out, or *sift* out specific parts of the signal.





The sifting property

The value of a DT at every point is a *weighted, shifted impulse*.



The unit impulse as a sampler

Multiplying by a shifted impulse "samples" the signal at that point:

Any signal can be written as a **superposition of weighted impulses.**

Given a signal

how does an LTI system respond to it?

For a **linear** system $z[n] \to w[n]$,

More generally,

The impulse response

Suppose the system sends $\delta[n-k] \to h_k[n]$. Then

$h_k[n]$ is called the **impulse response**.

Real-world example: nerve conduction study

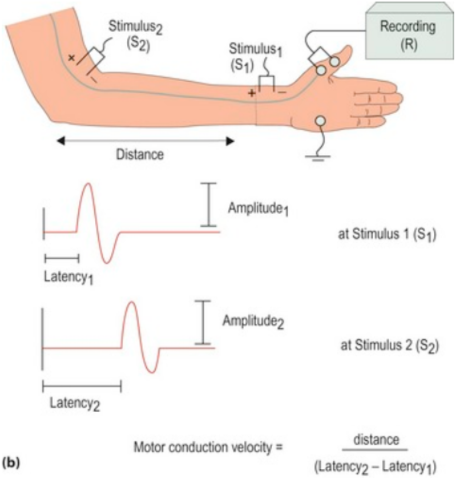


Image source: `https://neupsykey.com/nerve-conduction-studies-and-electromyography/`

The impulse response and time-invariance

What if the system is also time invariant?

Then

The convolution sum

If we know how a **linear** system responds to the unit impulse, we can learn how it responds to **any other signal**!

This is the **convolution sum**. We are "convolving" the sequences $x[n]$ and $h[n]$.

Exercise: impulse response

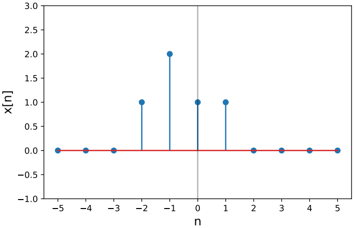
Consider an LTI system with input/output relationship

$$y[n] = 2x[n] + x[n-1]$$

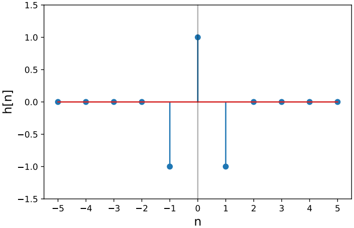
What is the impulse response of the system?

Example: convolution sum

Consider the signal

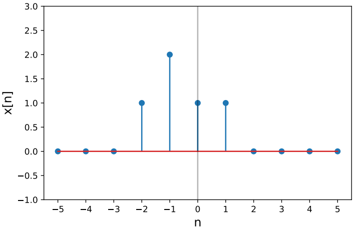


input to a system with impulse response



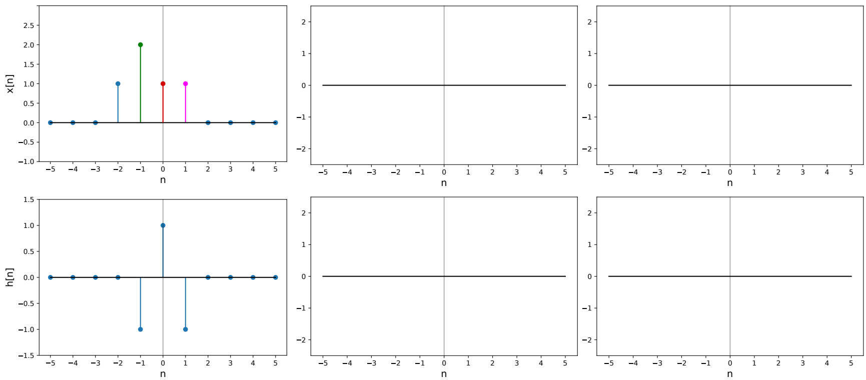
Example: convolution sum

To learn the system output, we must consider the contribution of each weighted impulse response:

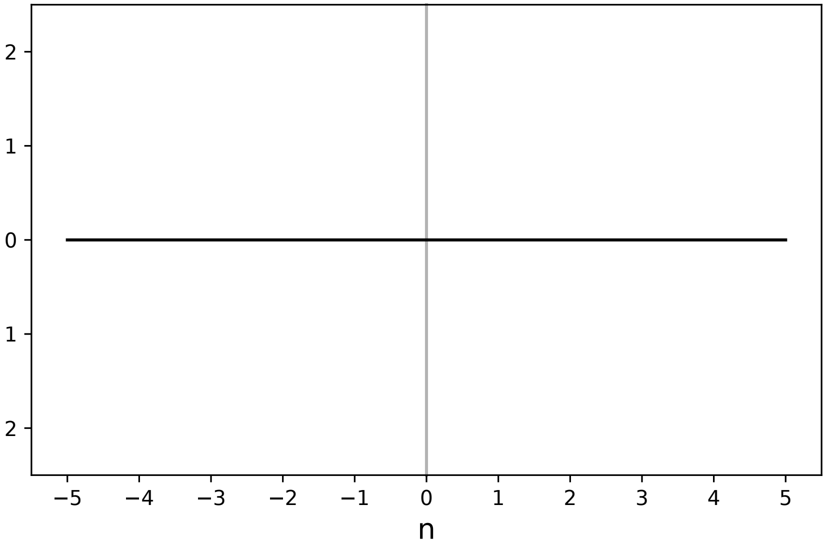


Only $x[k] \neq 0$ only for $k \in \{-2, -1, 0, 1\}$. So need to determine $x[k]h[n-k]$ for these cases, and sum them.

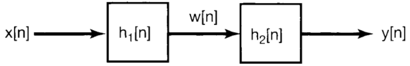
Example: convolution sum



Example: convolution sum



Properties of convolutions



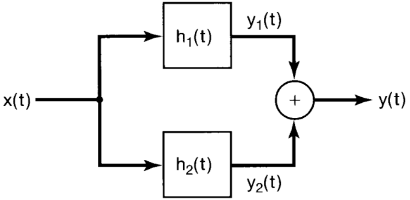


Image credits: Signals and Systems 2nd ed., Oppenheim

Convolution is:

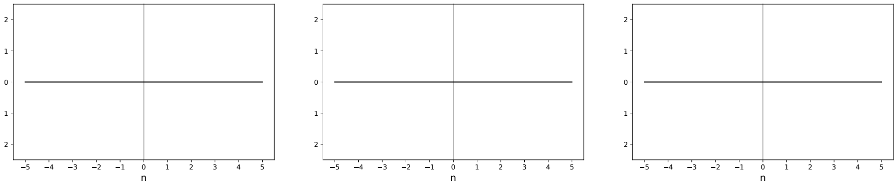
- Associative:
  $x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$
- Commutative:
  $x[n] * h[n] = h[n] * x[n]$
- Distributive:
  $x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$

Consider an LTI system with impulse response

$$h[n] = 3\delta[n] + 2\delta[n+1]$$

What is output of the system if

$$x[n] = \left(\frac{2}{3}\right)^n u[n]$$



Example/exercise: convolution sum

What is output of the system

$$x[n] = \left(\frac{2}{3}\right)^n u[n], \quad h[n] = 3\delta[n] + 2\delta[n+1]$$

Example/exercise: convolution sum

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$$x[n] = \left(\frac{2}{3}\right)^n u[n], \quad h[n] = 3\delta[n] + 2\delta[n+1]$$

Today's learning outcomes were:

- Define what it means for a system to be LTI (linear, time-invariant)
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For next time

Content:

- Continuous-time unit impulse and step
- Convolution integral
- Characterizing systems with the impulse response

Action items:

1. Work on Tutorial Assignment 1
2. Work on Assignment 1

Recommended reading:

- from today's class: Oppenheim 1.6.5-6, 1.4, 2.1
- practice problems: 1.16-1.20, 2.1-2.7, 2.21
- for next class: Oppenheim 1.4, 2.2-2.3