

Intro to Quantum Computing

TRIUMF co-op student seminar #2

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<https://github.com/glassnotes/Intro-QC-TRIUMF>

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Recap of Wednesday

We saw:

- The motivation behind quantum computing
- Single qubit systems
- Common unitary operations

Plan for today

We'll go through:

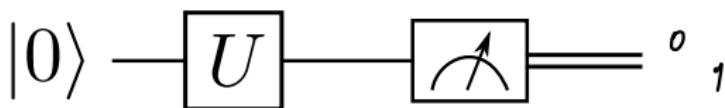
- A brief explanation of measurement
- Multi-qubit systems and entanglement
- The no-cloning theorem
- Quantum teleportation
- Overview of current-generation quantum hardware

Measurement

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad 0: |\alpha|^2 \quad 1: |\beta|^2$$

We know now that *unitary operations* can be used to manipulate our qubits to perform a computation. But after we're done computing, how do we get the answer? We need to measure our system.

A measurement in a circuit is represented by a box with a dial:



The two wires coming out of it indicate a *classical bit* - the outcome of the measurement is not a qubit, it's either 0 or 1!

Measurement

We need a mathematical formalism for measuring qubits, to see what the state system is in after the computation.

If we measure a qubit prepared in state $|\psi\rangle$, the probability of observing it in state $|\varphi\rangle$ is

$$\Pr(\text{observe } \varphi) = |\langle \varphi | \psi \rangle|^2$$

After the measurement the qubit will be left in state $|\varphi\rangle^1$.

¹ Actually things are a bit more subtle than this; for a good overview of projective measurements, see <https://www.people.vcu.edu/~sgharibian/courses/CMSC491/notes/Lecture%203%20-%20Measurement.pdf>

Measurement

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \quad \langle\psi| = (\alpha^* \ \beta^*) \quad \langle\psi|\psi\rangle$$

Measurement is performed with respect to a basis, for example, the computational basis $\{|0\rangle, |1\rangle\}$

Let $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. ↵

Then if we measure $|\psi\rangle$, we will observe the system in state $|0\rangle$ or state $|1\rangle$ with probability

$$\Pr(0) = |\langle 0 |\psi\rangle|^2 = |\alpha|^2$$

$$\Pr(1) = |\langle 1 |\psi\rangle|^2 = |\beta|^2$$

Measurement in other bases

We can measure in any orthonormal basis by applying a suitable unitary transformation to the computational basis vectors.

Example: measuring in the *Hadamard basis*:

$$|+\rangle = H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$\begin{aligned}\langle +|+\rangle &= 1 & \langle +|- \rangle &= \langle -|+ \rangle = 0 \\ \langle -|- \rangle &= 1\end{aligned}$$

Measurement in other bases

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Then, for $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$,

$$\begin{aligned} \Pr(+)&= |\langle + | \psi \rangle|^2 \\ &= \frac{1}{2} \left| \alpha \langle 0 | 0 \rangle + \cancel{\alpha \langle 1 | 0 \rangle} + \beta \cancel{\langle 0 | 1 \rangle} + \beta \langle 1 | 1 \rangle \right|^2 \\ &= \frac{1}{2} |\alpha + \beta|^2 \end{aligned}$$

$$\Pr(-) = \frac{1}{2} |\alpha - \beta|^2$$

$$\Pr(-) = \frac{1}{2} |\alpha - \beta|^2$$

Measurement in other bases

Why would we want to measure in different bases?

Example

Consider $|+\rangle$ and $|-\rangle$.

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

If we measure in the computational basis, for both states we will get 0 with probability 1/2 and also 1 with probability 1/2. It's impossible to know which state we have!

But if we measure in the Hadamard basis, we will get either *only* + or *only* -.

The measurement statistics change depending on which basis we measure in! We will see shortly an example (quantum teleportation) of how this is useful.

Multi-qubit systems

Tensor products

Hilbert spaces compose under the *tensor product*.

Example

Let

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}.$$

The tensor product of A and B , $A \otimes B$ is

$$A \otimes B = \begin{pmatrix} a \begin{pmatrix} e & f \\ g & h \end{pmatrix} & b \begin{pmatrix} e & f \\ g & h \end{pmatrix} \\ c \begin{pmatrix} e & f \\ g & h \end{pmatrix} & d \begin{pmatrix} e & f \\ g & h \end{pmatrix} \end{pmatrix} = \begin{pmatrix} ae & af & be & bf \\ ag & ah & bg & bh \\ ce & cf & de & df \\ cg & ch & dg & dh \end{pmatrix}$$

Multi-qubit systems

Qubit states compose under the tensor product:

$$\underbrace{|01\rangle}_{\sim} = |0\rangle \otimes |1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ 0 & \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad (1)$$

$$|00\rangle, |01\rangle, |10\rangle, |11\rangle$$

Unitary operations also work under the tensor product. For example, apply U_1 to qubit $|\psi\rangle$ and U_2 to qubit $|\varphi\rangle$:

$$(U_1 \otimes U_2)(|\psi\rangle \otimes |\varphi\rangle) = U_1|\psi\rangle \otimes U_2|\varphi\rangle \quad (2)$$

Multi-qubit systems

$$(\alpha|0\rangle + \beta|1\rangle) \otimes |0\rangle = \alpha|00\rangle + \beta|10\rangle$$

Exercise: Consider the 2-qubit state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad (3)$$

Find

$$|\psi\rangle = \underbrace{\alpha|0\rangle + \beta|1\rangle}_{\text{such that}}, \quad |\varphi\rangle = \underbrace{\gamma|0\rangle + \delta|1\rangle}_{\text{such that}} \quad (4)$$

such that

$$|\Psi\rangle = |\psi\rangle \otimes |\varphi\rangle \quad (5)$$

$$|00\rangle = |0\rangle \otimes |0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad |11\rangle = |1\rangle \otimes |1\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Multi-qubit systems

Exercise: Consider the 2-qubit state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle). \quad (3)$$

Find

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad |\varphi\rangle = \gamma|0\rangle + \delta|1\rangle \quad (4)$$

such that

$$|\Psi\rangle = |\psi\rangle \otimes |\varphi\rangle \quad (5)$$

Solution: This is impossible (sorry!)

Entanglement

The state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle). \quad (6)$$

is **entangled**.

We cannot describe the two qubits individually, we can only described their combined state.

Paraphrasing from John Preskill: *it's like you're reading a book, but instead of reading the pages sequentially, you have to read it all at the same time in order to understand it.*

Entanglement

Furthermore, the measurement outcomes of

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle). \quad (7)$$

are *perfectly correlated*.

For example, if I measure the first qubit and get 0, I'll get 0 for the second qubit as well!

Entanglement is not limited to two qubits. In principle we can entangle as many as we like:

$$|\Psi\rangle = \left(\underbrace{|00\cdots 0\rangle}_{\text{ }} + |11\cdots 1\rangle \right) \frac{1}{\sqrt{2}} \quad (8)$$

A measurement outcome of 0 on qubit 1 means we'll get 0 on *all other qubits* too.

Entangling gates

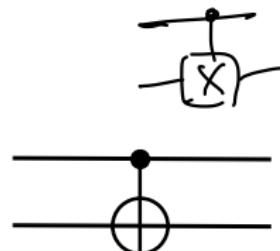
How do we make an entangled state (in theory)? Previous 2-qubit operations we saw were expressed as tensor products of single-qubit ones.

There exist *entangling gates* that will turn a non-entangled, or separable, state into an entangled one. The most commonly used one is the controlled-NOT, or CNOT:

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$\neq U_1 \otimes U_2$

$$\begin{array}{ll} \downarrow & \\ \text{CNOT}|00\rangle & = |00\rangle \\ \text{CNOT}|01\rangle & = |01\rangle \\ \text{CNOT}|10\rangle & = |11\rangle \\ \text{CNOT}|11\rangle & = |10\rangle \end{array}$$



The first qubit is the *control* qubit - it controls whether or not an X (NOT) gate is applied to the second qubit.

Entangling gates: CNOT

Exercise: What happens when we apply a CNOT to qubits in state $|+\rangle \otimes |0\rangle$?

$$\begin{aligned} \text{CNOT} \left[\frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \otimes |0\rangle \right] &= \text{CNOT} \left[\frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{2}} |10\rangle \right] \\ &= \frac{1}{\sqrt{2}} (\text{CNOT} |00\rangle + \frac{1}{\sqrt{2}} (\text{CNOT} |10\rangle)) \\ &= \frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{2}} |11\rangle \end{aligned}$$

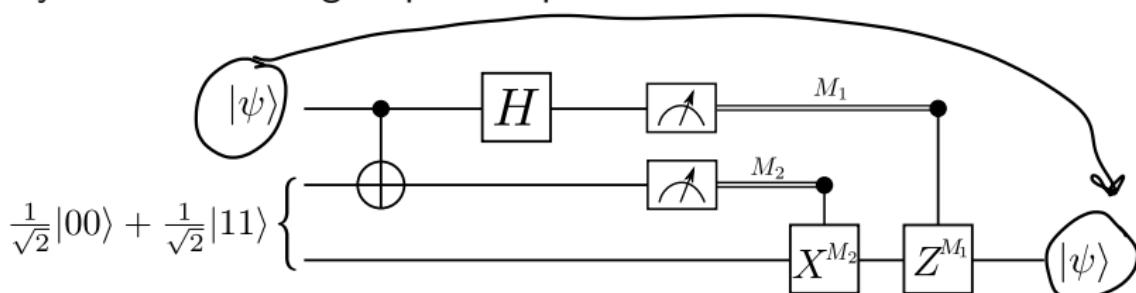
We've gone from a separable state to an entangled one!

Teleporting quantum states

Quantum teleportation

NOT related to Star Trek, despite the photos in media articles.

Alice has a qubit state she wants to send to Bob. She can do so if they share an entangled pair of qubits.



Wait a minute...

Why does she have to do this crazy thing? Why can't she just make a copy of her state and send it to him?

This is forbidden by the *no-cloning theorem*.

The no-cloning theorem

It is impossible to create a copying circuit that works for arbitrary quantum states.

We can prove this!

$$\begin{aligned} \text{CNOT}|00\rangle &\not\rightarrow |00\rangle \\ \text{CNOT}|10\rangle &\not\rightarrow |11\rangle \end{aligned}$$

Proof of the no-cloning theorem

Suppose we want to clone a state $|\psi\rangle$. We want a unitary operation that sends

$$U(|\psi\rangle \otimes |s\rangle) \rightarrow |\psi\rangle \otimes |\psi\rangle \quad (9)$$

where $|s\rangle$ is some arbitrary state.

Let's suppose we find one. If our cloning machine is going to be universal, then we must also be able to clone some other state, $|\varphi\rangle$.

$$U(|\varphi\rangle \otimes |s\rangle) = |\varphi\rangle \otimes |\varphi\rangle \quad (10)$$

Proof of the no-cloning theorem

We purportedly have:

$$U(|\psi\rangle \otimes |s\rangle) = |\psi\rangle \otimes |\psi\rangle \quad (11)$$

$$U(|\psi\rangle \otimes |\psi\rangle) = |\psi\rangle \otimes |\psi\rangle \quad (12)$$

Take the inner product of the LHS of both equations:

$$\langle \psi | \otimes \langle s | U^\dagger U (|\psi\rangle \otimes |s\rangle) = \langle \psi | \psi \rangle \langle s | s \rangle = \langle \psi | \psi \rangle$$

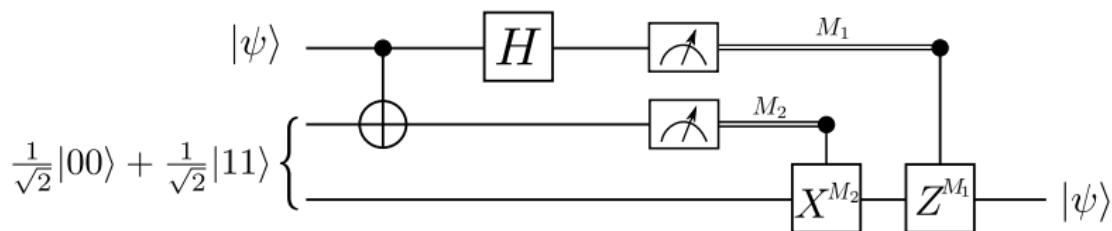
Now take the inner product of the RHS of both equations:

$$(\langle \psi | \otimes \langle \psi |) (|\psi\rangle \otimes |\psi\rangle) : (\langle \psi | \psi \rangle)^2 \quad (14)$$

These two inner products must be equal; but the only numbers that square to themselves are 0 and 1! So either the two states are orthogonal, or are just the same state - they can't be arbitrary!

Back to quantum teleportation

So there is no general protocol for Alice to copy her qubit and send it to Bob; but teleportation allows her to *transfer* arbitrary states from her qubit to the qubit held by Bob.



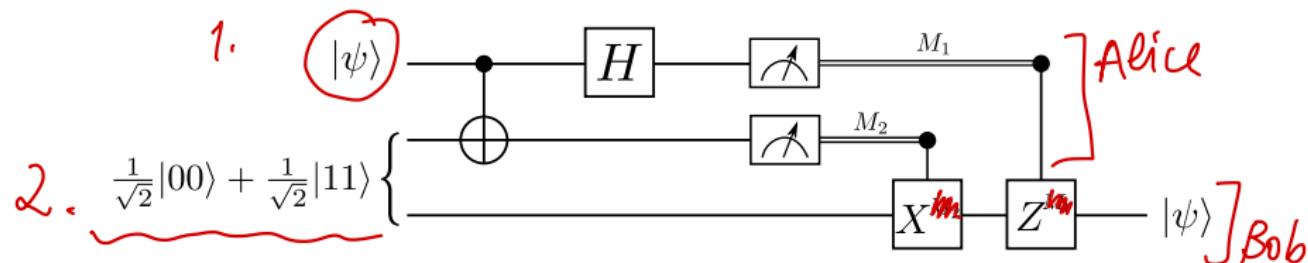
Does this actually work?

Hands-on with Quirk

Go back to

<https://algassert.com/quirk>

and implement the circuit for quantum teleportation:



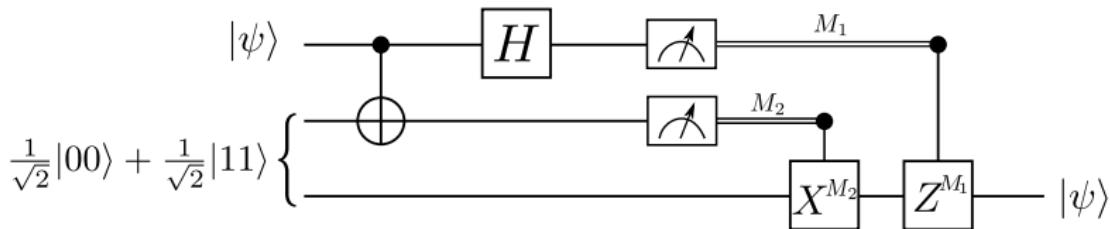
You will need to:

- Prepare a state to teleport (e.g. define some rotation)
- Prepare the shared entangled state

Quantum teleportation

I *really* encourage you to work through this one on your own.

Start by setting $|\psi\rangle_A = \alpha|0\rangle_A + \beta|1\rangle_A$, expand out the linear combination, work through the action of the gates, and then try and factor out Bob's qubit before performing the measurement.



See the appendix slides for the solution!

Quantum advantage

What can we do with a quantum computer?

There are many potential uses for quantum hardware:

- Use the quantum Fourier transform for problems in discrete mathematics (discrete logarithm, order-finding, factoring)
- Random number generation
- Simulate quantum systems
- Quantum key distribution and cryptography for secure quantum communication
- Using Grover's algorithm to search large spaces
- Speeding up linear algebra operations; as 'accelerators' for machine learning algorithms
- *Tons of things we haven't even thought of yet!* Designing quantum algorithms is really hard.

Will quantum computers be better at everything?

Not every quantum algorithm provides the desired exponential speedup over classical computers.

There will still be problems that even quantum computers can't solve efficiently.²

So, more important question: what kinds of problems are complex enough to make it worth pulling out a quantum computer in the first place?

²For those of you who are interested in computational complexity, check out the complete problems in the complexity class QMA
<https://arxiv.org/abs/1212.6312>.

Quantum advantage

Quantum advantage

At what point will quantum computers be able to solve a useful problem that is intractable for a classical computer? How large of a problem, or how many qubits, do we need before we see a *quantum advantage*?³

Need things that are exponentially hard on a classical computer but that quantum computers can solve efficiently.

This is a moving, problem-dependent target! A few years ago, people were estimating we would need around 50 qubits for this, but classical simulators have caught up.

³ See: <https://medium.com/@wjzeng/clarifying-quantum-supremacy-better-terms-for-milestones-in-quantum-computation-d15ccb53954f>

Candidate problem: sampling random circuits

Given a random circuit, sample from the probability distribution given by its output.

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Given a random circuit, sample from the probability distribution given by its output.

Classical:

Do the matrix multiplication to work through the entire circuit, get the final amplitudes, then sample.

Simulating a quantum computer

You might have been thinking: “*if everything is just linear algebra, why bother building a quantum computer at all?*”

For an n -qubit computation, we'd need to:

- Store 2^n complex numbers
- Store $2^n \times 2^n$ unitary matrices
- Multiply the two together repeatedly

This has **insane** time and memory requirements.

The best *full* quantum simulators running on a laptop can manage about 30 qubits with 16 GB RAM. The top supercomputers have managed ~ 50 qubits on circuits of depth ~ 40 .

Candidate problem: sampling random circuits

Given a random circuit, sample from the probability distribution given by its output.

Classical:

Do the matrix multiplication to work through the entire circuit, get the final amplitudes, then sample. Exponentially hard!

Quantum:

Run the circuit many times and keep track of the distribution of measured outputs.

How large a circuit do we need before we can no longer achieve this with classical computers?

Article

Quantum supremacy using a programmable superconducting processor

<https://doi.org/10.1038/s41586-019-1666-5>

Received: 22 July 2019

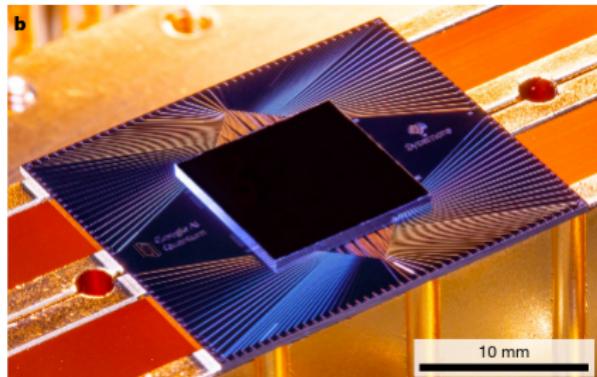
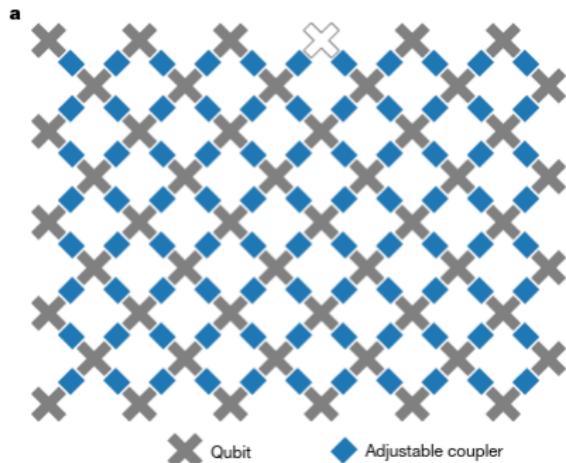
Accepted: 20 September 2019

Published online: 23 October 2019

Frank Arute¹, Kunal Arya¹, Ryan Babbush¹, Dave Bacon¹, Joseph C. Bardin^{1,2}, Rami Barends¹, Rupak Biswas³, Sergio Boixo¹, Fernando G. S. L. Brando^{1,4}, David A. Buell¹, Brian Burkett¹, Yu Chen¹, Zijun Chen¹, Ben Chiaro⁵, Roberto Collins¹, William Courtney¹, Andrew Dunsworth¹, Edward Farhi¹, Brooks Foxen¹⁵, Austin Fowler¹, Craig Gidney¹, Marissa Giustina¹, Rob Graff¹, Keith Guerin¹, Steve Habegger¹, Matthew P. Harrigan¹, Michael J. Hartmann^{1,6}, Alan Ho¹, Markus Hoffmann¹, Trent Huang¹, Travis S. Humble⁷, Sergei V. Isakov¹, Evan Jeffrey¹, Zhang Jiang¹, Dvir Kafri¹, Kostyantyn Kechedzhi¹, Julian Kelly¹, Paul V. Klimov¹, Sergey Knysh¹, Alexander Korotkov^{1,8}, Fedor Kostritsa¹, David Landhuis¹, Mike Lindmark¹, Erik Lucero¹, Dmitry Lyakh⁹, Salvatore Mandrà^{3,10}, Jarrod R. McClean¹, Matthew McEwen⁵, Anthony Megrant¹, Xiao Mi¹, Kristel Michelsen^{11,12}, Masoud Mohseni¹, Josh Mutus¹, Ofer Naaman¹, Matthew Neeley¹, Charles Neill¹, Murphy Yuezhen Niu¹, Eric Ostby¹, Andre Petukhov¹, John C. Platt¹, Chris Quintana¹, Eleanor G. Rieffel¹³, Pedram Roushan¹, Nicholas C. Rubin¹, Daniel Sank¹, Kevin J. Satzinger¹, Vadim Smelyanskiy¹, Kevin J. Sung^{1,13}, Matthew D. Trevithick¹, Amit Vainsencher¹, Benjamin Villalonga^{1,14}, Theodore White¹, Z. Jamie Yao¹, Ping Yeh¹, Adam Zalcman¹, Hartmut Neven¹ & John M. Martinis^{1,5*}

Sycamore

Google ran 53-qubit circuits with depth on their new processor.



They estimated that the same calculation would take 10000 years to run on a 'regular' supercomputer.

Image credit: Arute, F., Arya, K., Babbush, R. et al. Quantum supremacy using a programmable superconducting processor. *Nature* 574, 505–510 (2019) doi:10.1038/s41586-019-1666-5

10000 years!?

IBM doesn't think so:

Recent advances in quantum computing have resulted in two 53-qubit processors: one from our group in IBM and a device described by Google in a paper published in the journal *Nature*. In the paper, it is argued that their device reached “quantum supremacy” and that “a state-of-the-art supercomputer would require approximately 10,000 years to perform the equivalent task.” *We argue that an ideal simulation of the same task can be performed on a classical system in 2.5 days and with far greater fidelity.* This is in fact a conservative, worst-case estimate, and we expect that with additional refinements the classical cost of the simulation can be further reduced.

They argue that Google did not consider the use of disk storage in addition to regular RAM.

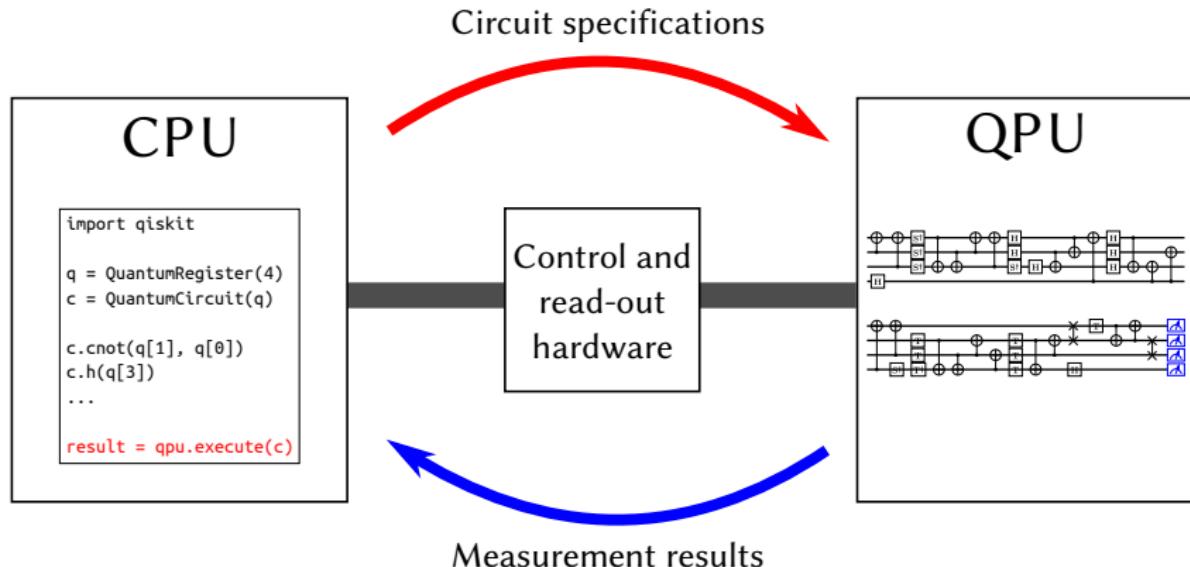
How do we even agree on whether quantum advantage has been achieved??

Source: <https://www.ibm.com/blogs/research/2019/10/on-quantum-supremacy/>

Quantum computing in practice: Hardware and the NISQ era

How do I use a quantum computer?

Now, and likely in the future, quantum computers are being used like special-purpose accelerators (think how GPUs are used today).



How do we build a *quantum processing unit*, or QPU?

What do I need to build a quantum computer?

The DiVincenzo criteria

1. A *scalable* physical system with well-characterized qubits.
2. The ability to *initialize* the state of the qubits to a simple fiducial state.
3. Long relevant decoherence times, much longer than the gate operation times.
4. A *universal* set of quantum gates.
5. A qubit-specific *measurement* capability.

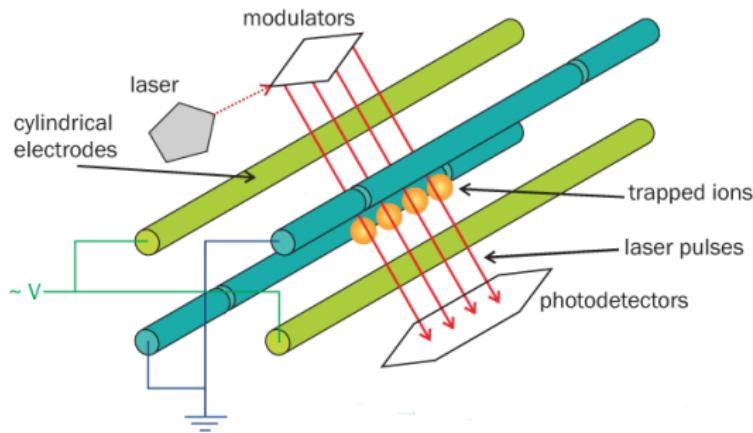
Candidate systems for qubits

- Superconducting qubits
- Trapped ions
- Spin qubits (neutral atoms, diamond NV centres, silicon spins qubits)
- Photons
- Topological qubits
- ... many more.

Superconducting qubits and trapped ions are currently the most well-developed (and most-used commercially), but spin qubits are looking very promising as well!

Candidate systems for qubits: Trapped ions

Qubits are ground state hyperfine levels of ions suspended between electrodes.

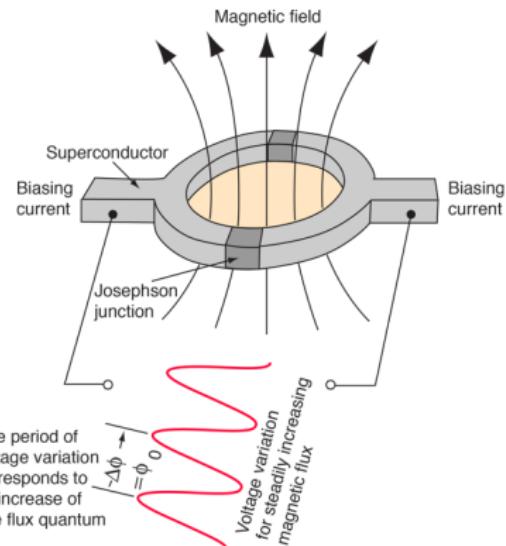


Qubits are individually addressable by applying EM fields of varying frequency and duration. Entangling gates are applied by coupling with a common vibrational mode of the whole chain.

Candidate systems for qubits: superconducting qubits

Multiple ways to make qubits:

- Charge qubits: uses the number of Cooper pairs sitting between a capacitor and a Josephson junction in an electrical circuit
- Flux qubits: uses the direction of magnetic flux induced by current in a superconducting loop



Qubits are individually addressable by applying microwave pulses; qubits are coupled via electric circuit connections.

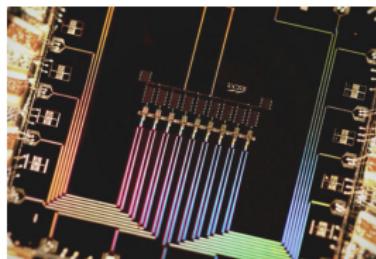
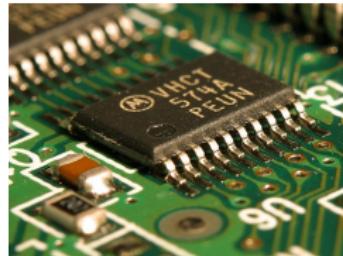
Image credit: <http://hyperphysics.phy-astr.gsu.edu/hbase/Solids/Squid.htm>

Candidate systems for qubits

However, it's important to remember that the “real” qubit of the future might not have even been invented yet!



VS.



VS.



Image credits:

<https://hubpages.com/business/What-Is-a-Transistor-and-Why-is-it-Important>

https://en.wikipedia.org/wiki/Solid-state_electronics

<https://physicsworld.com/a/google-gains-new-ground-on-universal-quantum-computer/>

NISQ-era quantum computing

We are in the era of 'Noisy, intermediate-scale quantum', or **NISQ** devices: medium-sized machines, but they have a number of limitations.

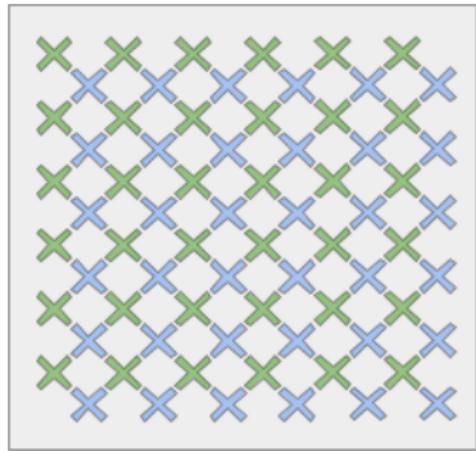
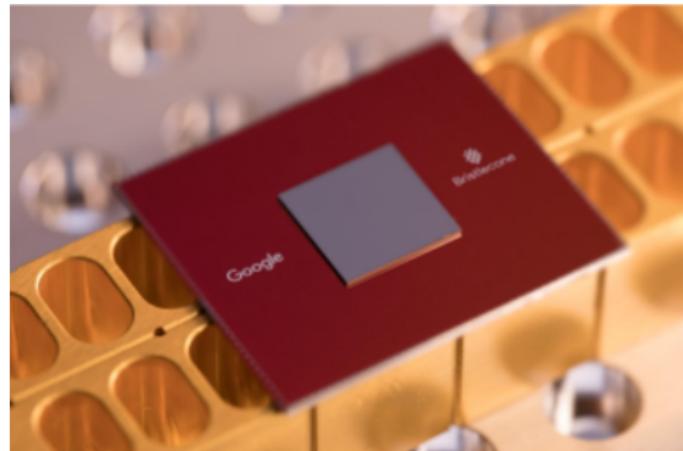
- Qubit count
- Qubit connectivity
- Error rates, coherence times

Useful site:

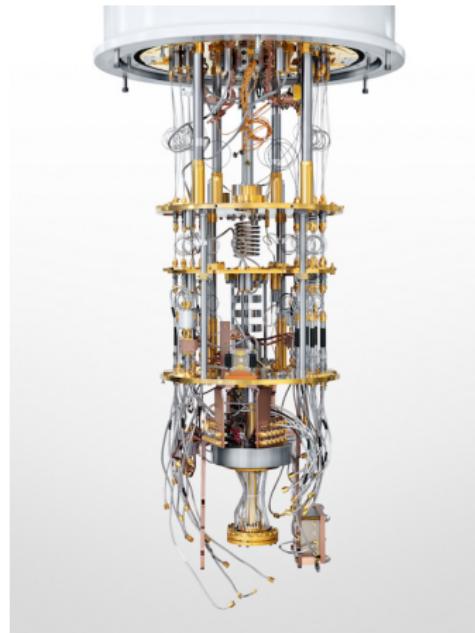
<https://quantumcomputingreport.com/scorecards/qubit-count/>

Important note: things are changing fast; no guarantees any of these numbers will be the same a month from now.

Currently Google's *Bristlecone* is the largest device. It has 72 superconducting qubits arranged in a grid pattern.

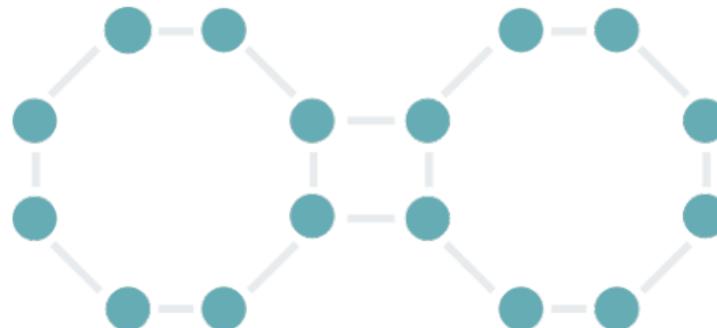


A California-based startup. Full-stack (hardware + software), superconducting qubit chip accessible via the cloud.



Hardware graph - Rigetti

16-qubit unit cell:



32-qubit device available; 128-qubit chip in development; 8 × 16 of the unit cell above.

<https://medium.com/rigetti/the-rigetti-128-qubit-chip-and-what-it-means-for-quantum-df757d1b71ea>

Startup out of U. Maryland. 79 qubits individually addressible; can do pairwise 2-qubit gates on up to 11 qubits. Recently partnered with Microsoft, Amazon, for cloud access.

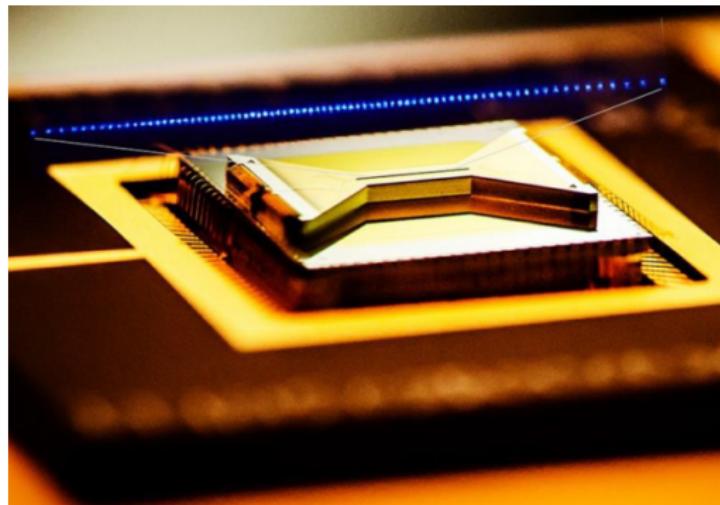
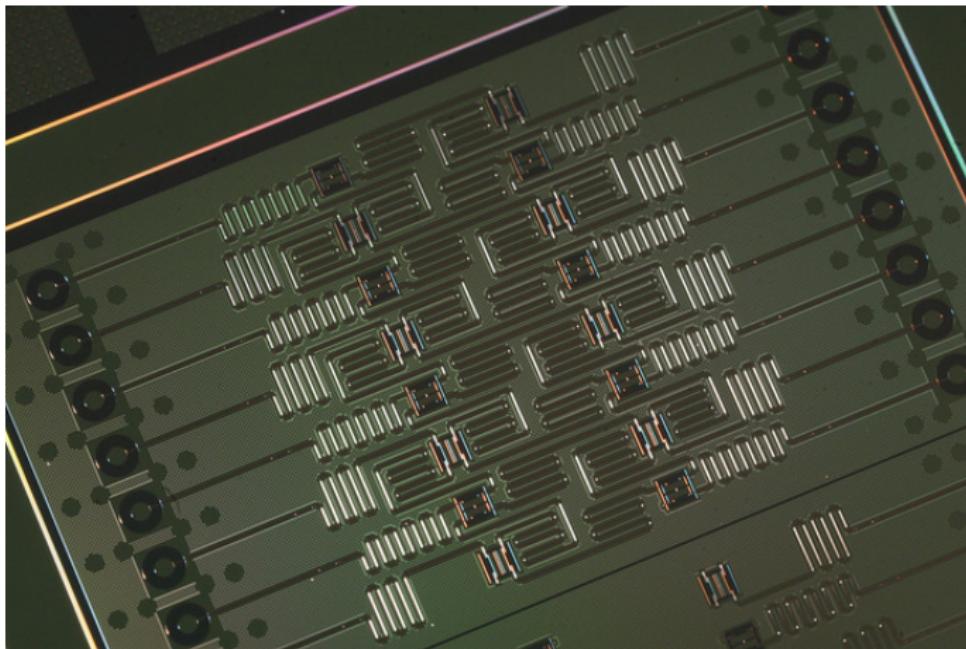


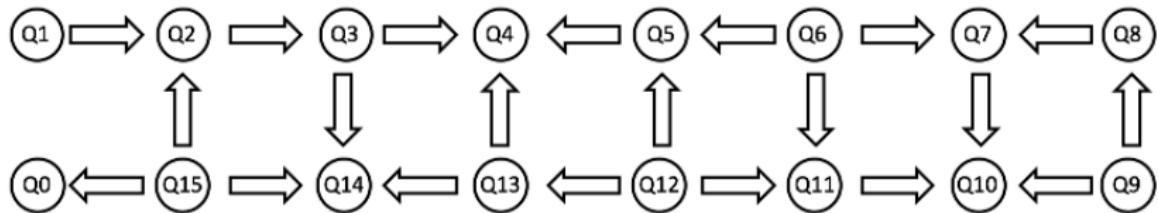
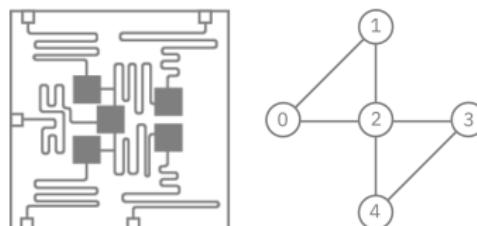
Image credit: <https://physicsworld.com/a/ion-based-commercial-quantum-computer-is-a-first/>

Full-stack, superconducting qubits, available in the cloud.



Hardware graph - IBM

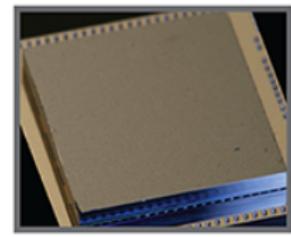
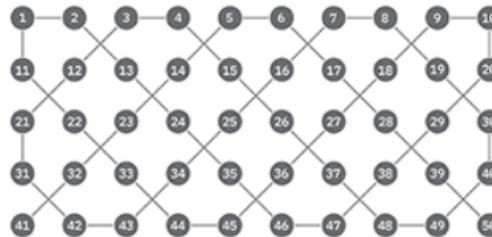
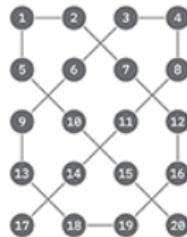
Small machines



<https://www.research.ibm.com/ibm-q/technology/devices/>

Hardware graph - IBM

Larger machines



<https://www.ibm.com/blogs/research/2017/11/the-future-is-quantum/>

Qubit counts

Other players (non-exhaustive):

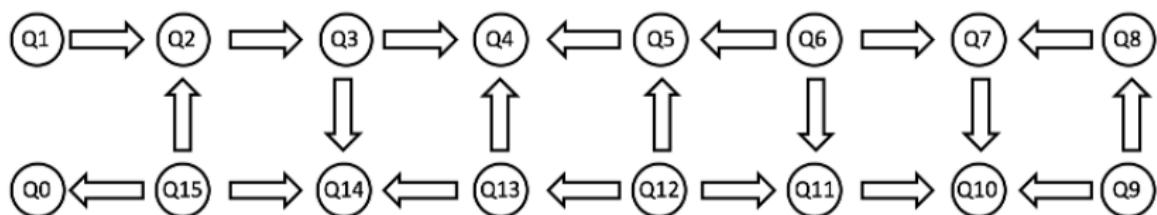
- Superconducting: Intel (49), U. of Science and Technology China (10), Alibaba (11)
- Ion traps: IonQ (11)⁴, Inst. for Quantum Optics and Quantum Information (20)
- Spin qubits⁵: Intel (26)
- Neutral atoms: U. Wisconsin (49)
- Photonic computing: Xanadu, PsiQuantum
- Topological qubits: Microsoft

⁴160 atoms, single-qubit gates on 79, two-qubit gates on all pairs of 11

⁵Being worked on at UBC and SFU!

Qubit connectivity

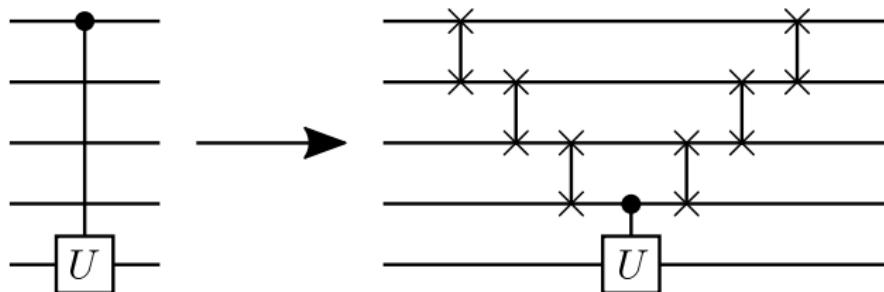
Look closely at the structure of this graph:



The arrows indicate where, and in what direction we can perform CNOT gates. What happens when we need to apply a CNOT on qubits that are far apart?

Qubit connectivity

Naive solution: when you need to operate on two non-adjacent qubits, perform SWAP gates until they are adjacent, perform the operation, then undo all the SWAPS.



Better solution: Heuristic techniques for compiling down to circuits over the universal gate set that fit the topology. For example, 're-allocation' of qubit indexing to minimize the number of SWAPS.

Noise

This is the 20-qubit IBM Q System One. It's in a dilution refrigerator cooled to $\sim 10 - 20\text{mK}$, suspended in a 9-foot cube.



Image credit:

[https://www.hpcwire.com/2019/01/10/
ibm-quantum-update-q-system-one-launch-new-collaborators-and-qc-center-plans/](https://www.hpcwire.com/2019/01/10/ibm-quantum-update-q-system-one-launch-new-collaborators-and-qc-center-plans/)

Quantifying noise in quantum computing

Why so cold? Energy levels are very close together; need thermal excitations to be small enough that they don't cause transition between the qubit states.

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We judge the quality of our qubits using a variety of metrics, for example:

- Gate fidelity (1- and 2-qubit gates)
- Spin relaxation time (time it takes for $|1\rangle$ to 'relax' to $|0\rangle$)
- Decoherence time

NISQ-device qubit quality

All values are averaged over the qubits:

Org.	Machine	1-qubit fid.	2-qubit fid.	Readout fid.
IBM	Q System One	99.96	98.31	TBD
Rigetti	32Q Aspen-7	98.7	95.2	96.4
IonQ	11 Qubit	99.64	97.5	99.3
Google	Sycamore	99.84	99.38	96.2

For relatively up-to-date numbers:

<https://quantumcomputingreport.com/scorecards/qubit-quality/>

NISQ-device coherence times

All times are averages:

Org.	Machine	T_1 (μ s)	T_2 (μ s)
IBM	Q System One	73.9	69.1
Rigetti	32Q Aspen-7	41	35
IonQ	11 Qubit	$> 10^{10}$	$\sim 3 \cdot 10^6$

T_1 is the relaxation time, T_2 is the decoherence time.

Other news:

- 2018: Silicon qubits have seen $T_2 \approx 9.4\text{ms}$
- 2019: IBM reports lifetimes up to $500\mu\text{s}$ lifetime for individual superconducting qubits (<https://www.ibm.com/blogs/research/2019/03/power-quantum-device/>)
- 2019: TU Delft reports lifetimes up to 63s in diamond NV-center qubit <http://arxiv.org/abs/1905.02094>

Applications of NISQ devices

... can we still do interesting things with NISQ devices?

Yes! There is a steady stream of proof-of-concept applications and small/toy problem instances.

A few things that have been done so far:

- Calculations of molecular energies; largest to date has been water on the IonQ platform
<http://arxiv.org/abs/1902.10171>
- Supervised classification (ML) on a toy dataset
<https://www.nature.com/articles/s41586-019-0980-2>
- Simulation of lattice gauge theories
<http://stacks.iop.org/1367-2630/19/i=10/a=103020>

Summary

Today we saw:

- Multi-qubit operations and entanglement
- How quantum state teleportation works
- What quantum advantage is, and a candidate problem
- What ‘real’ quantum computers currently look like, and what their limitations are

Questions? Comments? Interested in research? Feel free to come chat any time (MOB 232).

Quantum teleportation

If you work through the math, you'll find that before the measurements, the combined state of the system looks like this (removing the $\frac{1}{\sqrt{2}}$ for readability):

$$\begin{aligned}(|00\rangle_A + |11\rangle_A) &\otimes (\alpha|0\rangle_B + \beta|1\rangle_B) + \\(|10\rangle_A + |01\rangle_A) &\otimes (\alpha|1\rangle_B + \beta|0\rangle_B) + \\(|00\rangle_A - |11\rangle_A) &\otimes (\alpha|0\rangle_B - \beta|1\rangle_B) + \\(|01\rangle_A - |10\rangle_A) &\otimes (\alpha|1\rangle_B - \beta|0\rangle_B)\end{aligned}$$

This is a superposition of 4 distinct terms.

Quantum teleportation

You can see that Bob's state is always some variation on the original state of Alice:

$$\begin{aligned}(|00\rangle_A + |11\rangle_A) &\otimes (\alpha|0\rangle_B + \beta|1\rangle_B) + \\(|10\rangle_A + |01\rangle_A) &\otimes (\alpha|1\rangle_B + \beta|0\rangle_B) + \\(|00\rangle_A - |11\rangle_A) &\otimes (\alpha|0\rangle_B - \beta|1\rangle_B) + \\(|01\rangle_A - |10\rangle_A) &\otimes (\alpha|1\rangle_B - \beta|0\rangle_B)\end{aligned}$$

Quantum teleportation

The possible states of Alice's qubits are an orthonormal 2-qubit basis of entangled states called the Bell basis.

$$\begin{aligned}(|00\rangle_A + |11\rangle_A) &\otimes (\alpha|0\rangle_B + \beta|1\rangle_B) + \\(|10\rangle_A + |01\rangle_A) &\otimes (\alpha|1\rangle_B + \beta|0\rangle_B) + \\(|00\rangle_A - |11\rangle_A) &\otimes (\alpha|0\rangle_B - \beta|1\rangle_B) + \\(|01\rangle_A - |10\rangle_A) &\otimes (\alpha|1\rangle_B - \beta|0\rangle_B)\end{aligned}$$

Quantum teleportation

So Alice can measure in the Bell basis, and send her results to Bob.

$$\begin{aligned}(|00\rangle_A + |11\rangle_A) &\otimes (\alpha|0\rangle_B + \beta|1\rangle_B) + \\(|10\rangle_A + |01\rangle_A) &\otimes (\alpha|1\rangle_B + \beta|0\rangle_B) + \\(|00\rangle_A - |11\rangle_A) &\otimes (\alpha|0\rangle_B - \beta|1\rangle_B) + \\(|01\rangle_A - |10\rangle_A) &\otimes (\alpha|1\rangle_B - \beta|0\rangle_B)\end{aligned}$$

Once Bob knows the results, he knows exactly what term of the superposition they had, and can adjust his state accordingly.