Countable Infinities

$\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ (rational numbers), $\mathbb{Z}\times\mathbb{Z}$, $\mathbb{N}\times\mathbb{N}$

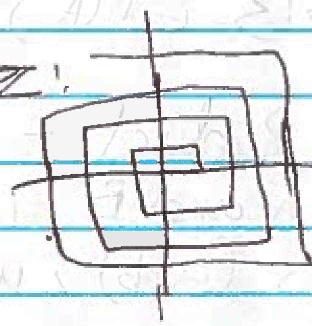
natural — integers

$\bigcup_{n=0}^{\infty}\{0,1\}$

- Uncountable infinite

  $\mathbb{R}$ – real numbers b/c Cantor's Diagonalization

To count $\mathbb{Z}\times\mathbb{Z}$:



LHRR ex:

$d_0=0 \quad d_1=1$

$d_{n+1}=3d_n-\frac{9}{4}d_{n-1}$

① $x^{n+1}=3x^n-\frac{9}{4}x^{n-1}$

② $4x^2-12x+9=0$

$(2x-3)(2x-3)=0$

$\boxed{x=\frac{3}{2}}$ $m=2$

③ $d_n=C_1\left(\frac{3}{2}\right)^n+C_2 n\left(\frac{3}{2}\right)^n$

④ $d_0=C_1+C_2\cdot 0\cdot 1=\boxed{C_1=0}$

$d_1=C_1\left(\frac{3}{2}\right)^1+C_2\cdot 1\left(\frac{3}{2}\right)^1=1 \rightarrow \boxed{C_2=\frac{2}{3}}$

$d_n=\frac{2}{3}\cdot n\left(\frac{3}{2}\right)^n$