

# Finding Provably Optimal Markov Chains

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Dear reviewers,

Please find in *main.pdf* the updated version of our paper. We are in the progress of preparing a camera-ready version. In particular, we want to highlight that we have updated parts of our implementation to address some of its weaknesses.

The changes are as follows: = We compute CurMax using a slightly better sampling strategy (which improves the CurMax estimate in early iterations). = We added a default option that adds an initial bound-computation in our loop. More precisely, we use parameter lifting on the complete region to improve the lower and upper bounds from the standard  $[0,1]$ . This allows the monotonicity checker to create the initial ordering more quickly. You may toggle this option, see the README for more details.

The overall message of the paper remains as is. The experiments have been slightly improved by these measures. You can see the old and new table side-by-side in this document.

Best regards,

The authors; Jip Spel, Sebastian Junges, Joost-Pieter Katoen

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\* Supported by the DFG RTG 2236 “UnRAVeL”.

**Table 1.** New implementation

					$\epsilon: 0.1$						$\epsilon: 0.05$					
name	instance	#states	#trans	V	integrated			vanilla			integrated			vanilla		
					#	i	#	i <sub>b</sub>	t	#i	t	#	i	#	i <sub>b</sub>	t
NRP	(5,1)	56	75	5	469	2	<1			2575	<1	5143	2	<1	48701	3
	(6,1)	76	102	6	1539	2	<1			10085	<1	27037	2	<b>2</b>	351709	26
	(7,1)	99	133	7	4139	2	<1			29991	2	116253	2	<b>12</b>	1873247	196
	(8,1)	125	168	8	10187	2	<b>1</b>			76495	8	454435	2	<b>62</b>	9554453	1388
	(9,1)	154	207	9	25407	2	<b>3</b>			194751	23	1806581	2	<b>281</b>		MO
	(10,1)	186	250	10	66219	2	<b>10</b>			512909	72	7168029	2	<b>1406</b>		TO
	(11,1)	221	297	11	165593	2	<b>28</b>			1266655	229					TO
	(12,1)	259	348	12	425643	2	<b>88</b>			3304325	661					TO
	(13,1)	300	403	13	1103811	2	<b>274</b>				TO					TO
	(14,1)	344	462	14	2608869	2	<b>718</b>				MO					MO
	(15,1)	391	525	15			TO				MO					MO
EVADE	(1,2,0,1)	129	249	40	0	2	<1			2410	2	1	2	<1	4619	4
	(1,2,1,1)	257	497	80	0	2	<1			MO		1	2	<1		MO
	(1,2,3,1)	513	993	160	0	2	<b>3</b>			MO		1	2	<b>3</b>		MO
	(1,2,0,2)	425	842	141	0	2	<b>2</b>			MO		0	2	<b>2</b>		MO
	(1,2,1,2)	849	1682	281	0	2	<b>6</b>			MO		0	2	<b>6</b>		MO
	(1,2,3,2)	1697	3362	561	0	2	<b>19</b>			MO		0	2	<b>19</b>		MO
Herman	(11,10)	21500	242926	1	3	3	3			3	<b>2</b>	9	6	5	9	<b>3</b>
	(11,15)	31740	369706	1	5	4	6			5	<b>3</b>	11	7	9	11	<b>5</b>
	(13,15)	126888	1713246	1	7	5	41			7	<b>18</b>	11	7	48	11	<b>22</b>
	(13,25)	208808	2889206	1	7	5	86			7	<b>31</b>	11	7	97	11	<b>39</b>
	(13,35)	290728	4065166	1	5	4	162			5	<b>38</b>			TO	11	<b>53</b>
Maze	(25)	360	660	24	0	2	<1			1	<1	0	2	<1	40	<1
	(1000)	14985	26985	999	0	2	1			1	<1	0	2	<b>1</b>		MO
	(10000)	149985	269985	9999	0	2	301			1	<1	0	2	<b>380</b>		TO

**Table 2.** Old implementation

					$\varepsilon: 0.1$					$\varepsilon: 0.05$					
					integrated			vanilla		integrated			vanilla		
name	instance	#states	#trans	V	# i	# i <sub>b</sub>	t	#i	t	# i	# i <sub>b</sub>	t	#i	t	
NRP	(5,1)	56	75	5	683	0	<1	2575	<1	7911	0	<1	48701	0	2
	(6,1)	76	102	6	2627	0	<1	10085	<1	53819	0	<b>3</b>	351709	21	
	(7,1)	99	133	7	7435	0	<1	29991	2	271625	0	<b>22</b>	1873247	151	
	(8,1)	125	168	8	18955	0	<b>2</b>	76495	6	1319643	0	<b>136</b>	9554453	1010	
	(9,1)	154	207	9	48573	0	<b>5</b>	194751	19	6301539	0	<b>812</b>	TO	TO	
	(10,1)	186	250	10	128791	0	<b>15</b>	512909	55			TO	TO	MO	MO
Evade	(1,2,-,1)	65	125	24	381	381	<1	65	<1	416	413	<1	5779	0	2
	(1,2,0,1)	129	249	48	3219	3217	3	144	<1	36562	36539	<b>42</b>	531078	372	
	(1,2,1,1)	257	497	96	5426	5424	11	5089	<b>6</b>			TO	MO	MO	MO
	(1,2,0,2)	593	1186	249	15902	15892	<b>74</b>		MO	30457	30215	<b>146</b>	MO	MO	MO
	(1,2,1,2)	1185	2370	497			TO		MO			TO		MO	MO
Herman	(11,10)	21500	242926	1	3	1	<1	3	<1	9	4	2	9	1	
	(11,15)	31740	369706	1	5	2	1	5	1	11	5	3	11	<b>2</b>	
	(13,15)	126888	1713246	1	7	3	7	7	7	11	5	15	11	<b>9</b>	
	(13,25)	208808	2889206	1	5	2	16	7	<b>13</b>	11	5	28	11	<b>17</b>	
	(13,35)	290728	4065166	1			TO	5	<b>11</b>	11	5	39	11	<b>22</b>	
Maze	(25)	360	660	25	0	0	<1	1	<1	0	0	<1	40	0	<1
	(1000)	14985	4001	1000	0	0	1	1	<1	0	0	<1	TO	TO	TO
	(10000)	149985	40001	10000	0	0	187	1	<1	0	0	<b>187</b>	TO	TO	TO