Filtro de Kalman em um robô 1D

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Modelagem do problema

Consideramos um robô 1D descrito pela seguinte equação diferença:

$$\frac{v_r(k+1) - v_r(k)}{T} = \frac{u(k)}{m}$$

Em que v_r é a velocidade, $u(k)$ é a entrada de força, m é a massa do robô e T é o tempo de amostragem.

considerando x_r a posição do robô, temos o seguinte vetor de estados:

$$x = \begin{bmatrix} x_r & v_r \end{bmatrix}^T$$

A partir da equação de velocidade, calculamos $x_r(k+1)$:

$$v_r(k) = \frac{x_r(k+1) - x_r(k)}{T} \Rightarrow x_r(k+1) = 1 \cdot x_r(k) + T \cdot v_r(k) + 0 \cdot u(k)$$

A partir da equação diferença, calculamos $v_r(k+1)$:

$$v_r(k+1) = 0 \cdot x(k) + 1 \cdot v_r(k) + \left(\frac{T}{m}\right) \cdot u(k)$$

Assim, o processo é descrito pela seguinte equação:

$$x(k+1) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ \frac{T}{m} \end{bmatrix} u(k) + \upsilon(k)$$

De forma que $\upsilon(k)$ é o ruído de processo.

Usando um sensor de velocidade, com ruído caracterizado por $\omega(k)$, temos a seguinte equação de saída:

$$y(k+1) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(k) + \omega(k)$$

Dados do problema

```
m = 1;
T = 0.5;
V = [0.2, 0.05; 0.05, 0.1]; % Process noise covariance matrix
W = 0.5; % Output noise variance
```

Matrizes da modelagem fenomenológica:

```
F = [1, T; 0, 1];
G = [0; T/m];
H = [0, 1];
```

O problema original trabalha com apenas uma iteração.. Como a atividade requer mais iterações. Vamos definir um número de iterações:

```
n_iter = 20;
```

Usaremos as condições iniciais originalmente descritas, mas para que tenhamos mais iterações será necessário especificar mais valores de entrada e saída que o sistema receberia durante essas iterações. Esses valores foram escolhidos arbitrariamente construindo um cenário aonde o robô vai parando.

```
% No more voltage
u = zeros(n_iter, 1);
```

Estimativa inicial e matriz de incerteza (oferecida no problema):

```
x = zeros(2, n_iter);
x(:, 1) = [2; 4];
P = zeros(2, 2, n_iter);
P(:, :, 1) = [1, 0; 0, 2];
```

Sabendo que o estado real na primeira iteração é $x(k+1) = [1.8 \quad 2]^T$. Definimos o vetor de velocidades supostamente retornadas pelo sensor no decorrer das iterações, quando a velocidade do carro vai diminuindo, além de adicionar W*hite Gaussian Noise* com variância W = 0.5

```
% Based on the first real state, we compute the first ideal sensor reading
x_real = [1.8; 2];
y = zeros(n_iter, 1);
y(1) = H * x_real;
y(2) = y(1); % Just for later didatic purposes

% y must be zero at last iteration
dy = y(2)/(n_iter - 2);

for k = 3:n_iter
    y(k) = y(k-1) - dy;
end

% Adding White Gaussian Noise with y as mean
noise = rand(n_iter, 1)*sqrt(W);
y = y + noise;
```

Implementação do filtro de Kalman

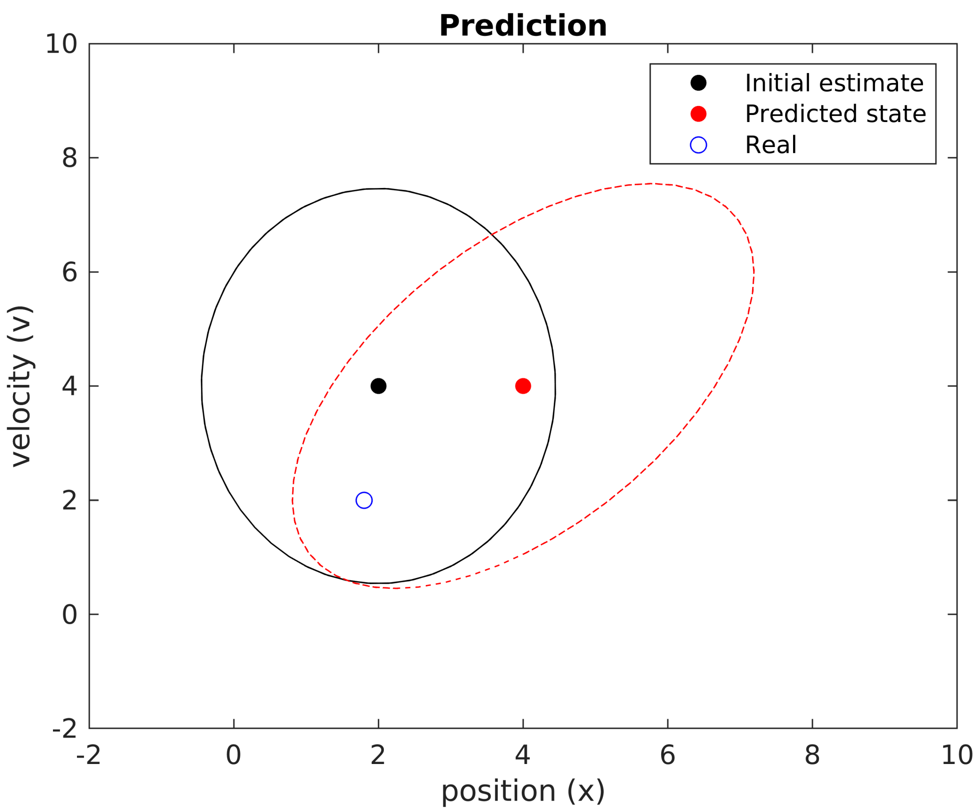
Aplicando as equações demonstradas em sala, temos os passos de predição e correção. Na primeira iteração, temos:

Predição

```
x(:, 2) = F*x(:, 1) + G*u(1);
P(:, :, 2) = F*P(:, :, 1)*F' + V;
```

Para visualizar a predição usamos o seguinte trecho de código:

```
% Choosing confidence level to plot
p = 0.95;
% Computing scale factor
s = - 2 * log(1 - p);

% Computing eigenvalues of eigenvectors uncertainity matrix P
[V1, D1] = eig(P(:,:,1) * s);
[V2, D2] = eig(P(:,:,2) * s);

% Ellipse equation
vel = linspace(-2,10);
a = (V1 * sqrt(D1)) * [cos(vel(:))'; sin(vel(:))'];
b = (V2 * sqrt(D2)) * [cos(vel(:))'; sin(vel(:))'];

% Ploting ellipses with x as mean
plot(a(1, :) + x(1,1), a(2, :) + x(2,1), 'k-');
hold on;
plot(b(1, :) + x(1,2), b(2, :) + x(2,2), 'r--');

% Adding states
s1 = scatter(x(1,1), x(2,1), 'k', 'filled');
s2 = scatter(x(1,2), x(2,2), 'r', 'filled');
s3 = scatter(x_real(1), x_real(2), 'b');

legend([s1, s2, s3], {'Initial estimate', 'Predicted state', 'Real'});
title('Prediction');
xlabel('position (x)');
ylabel('velocity (v)');
axis([-2 10 -2 10]);
hold off;
```



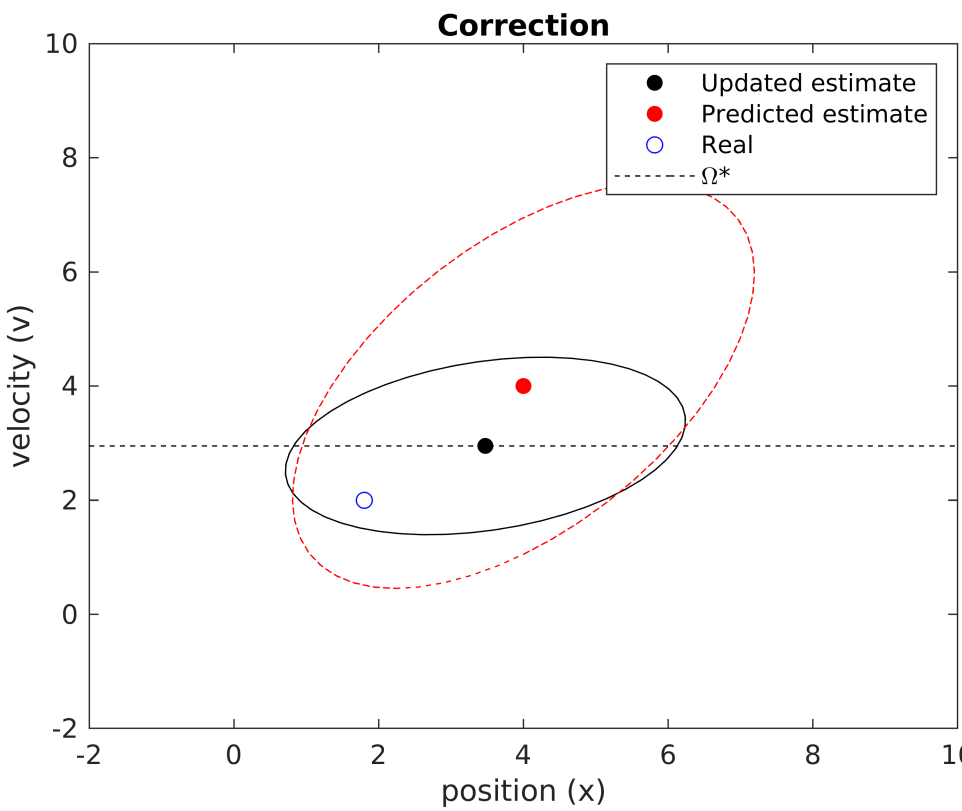
Correção

```
R = P(:,:,2)*H'/(H*P(:,:,2)*H' + W);
nu = y(2) - H*x(:, 1);

% Saving predicted values (for plot)
x_predicted = x(:, 2);
P_predicted = P(:,:,2);

x(:, 2) = x(:, 2) + R*nu;
P(:,:,2) = P(:,:,2) - R*H*P(:,:,2);
```

Podemos plotar a iteração da seguinte maneira:

```
% Choosing confidence level to plot
p = 0.95;
% Computing scale factor
s = - 2 * log(1 - p);

% Computing eigenvalues of eigenvectors uncertainity matrix P
[V1, D1] = eig(P(:,:,2) * s);
[V2, D2] = eig(P_predicted * s);

% Ellipse equation
vel = linspace(-2,10);
a = (V1 * sqrt(D1)) * [cos(vel(:))'; sin(vel(:))'];
b = (V2 * sqrt(D2)) * [cos(vel(:))'; sin(vel(:))'];
```

```
% Ploting ellipses with x as mean
plot(a(1, :) + x(1,2), a(2, :) + x(2,2), 'k-');
hold on;
plot(b(1, :) + x_predicted(1), b(2, :) + x_predicted(2), 'r--');

% Adding states
s1 = scatter(x(1,2), x(2,2), 'k', 'filled');
s2 = scatter(x_predicted(1), x_predicted(2), 'r', 'filled');
s3 = scatter(x_real(1), x_real(2), 'b');

omega = ones(length(vel), 1) * x(2,2);
o1 = plot(vel, omega, 'k--');


legend([s1, s2, s3, o1], {'Updated estimate', 'Predicted estimate', 'Real', '\Omega*'}
title('Correction');
xlabel('position (x)');
ylabel('velocity (v)');
axis([-2 10 -2 10]);
hold off;
```

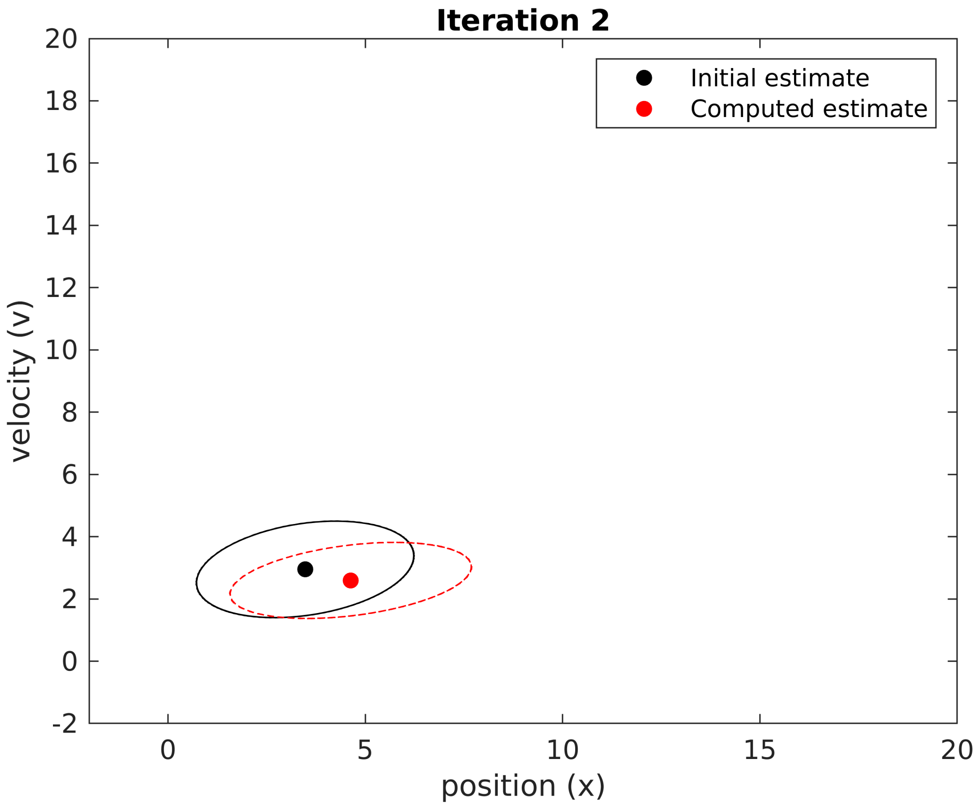


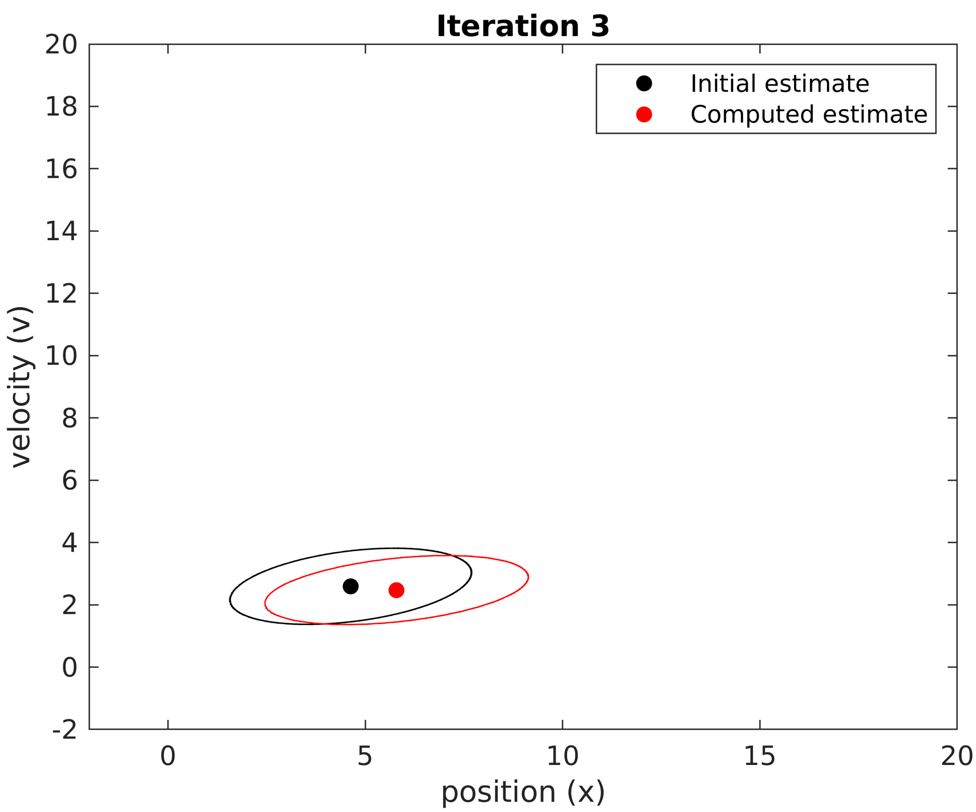
Agora aplicamos as iterações seguintes

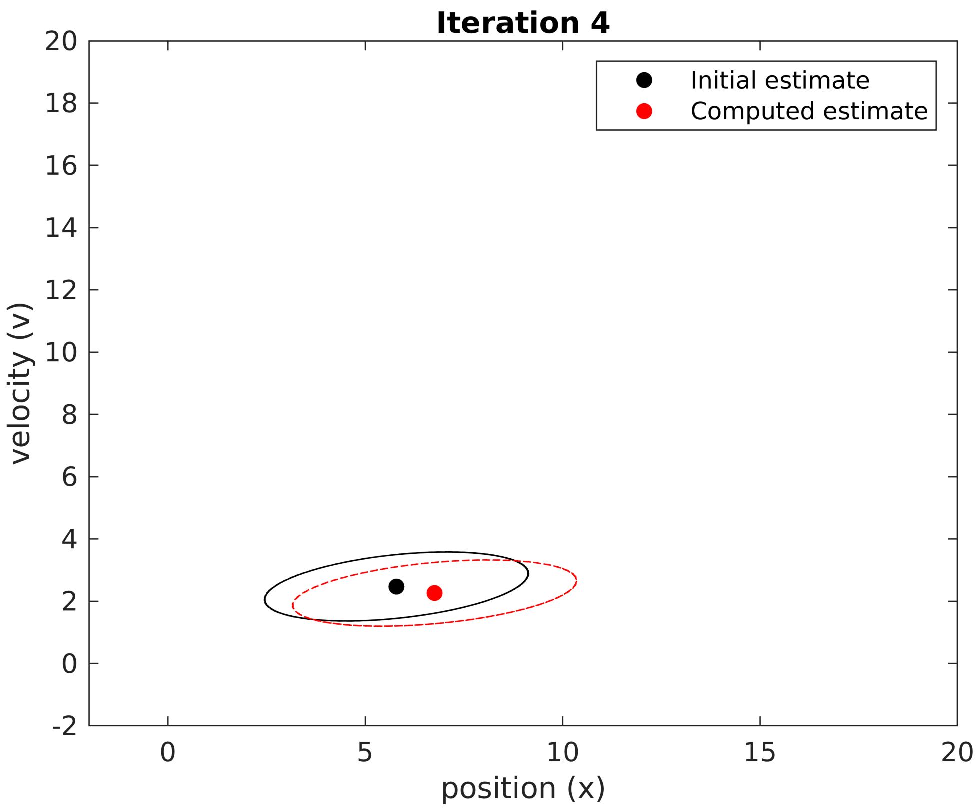
```
for k = 3:n_iter
    % Prediction
    x(:, k) = F*x(:, k-1) + G*u(k-1);
```

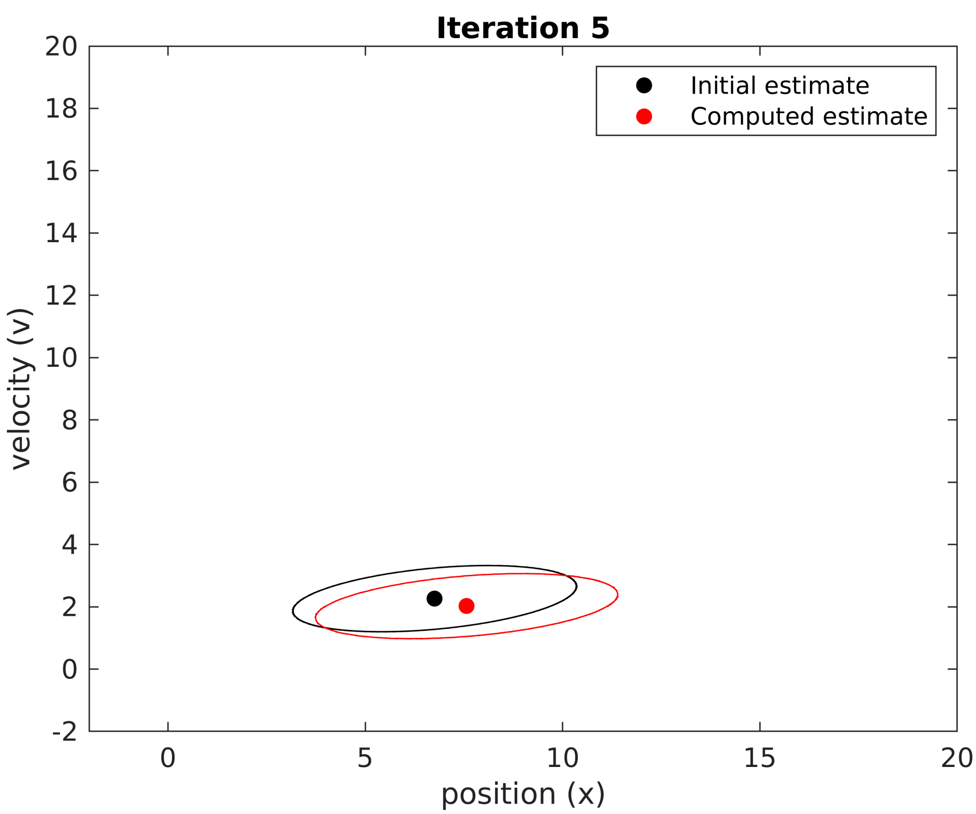
```
    P(:, :, k) = F*P(:, :, k-1)*F' + V;

    % Correction
    R = P(:,:,k)*H'/(H*P(:,:,k)*H' + W);
    nu = y(k) - H*x(:, k-1);

    x(:, k) = x(:, k) + R*nu;
    P(:,:,k) = P(:,:,k) - R*H*P(:,:,k);
end
```

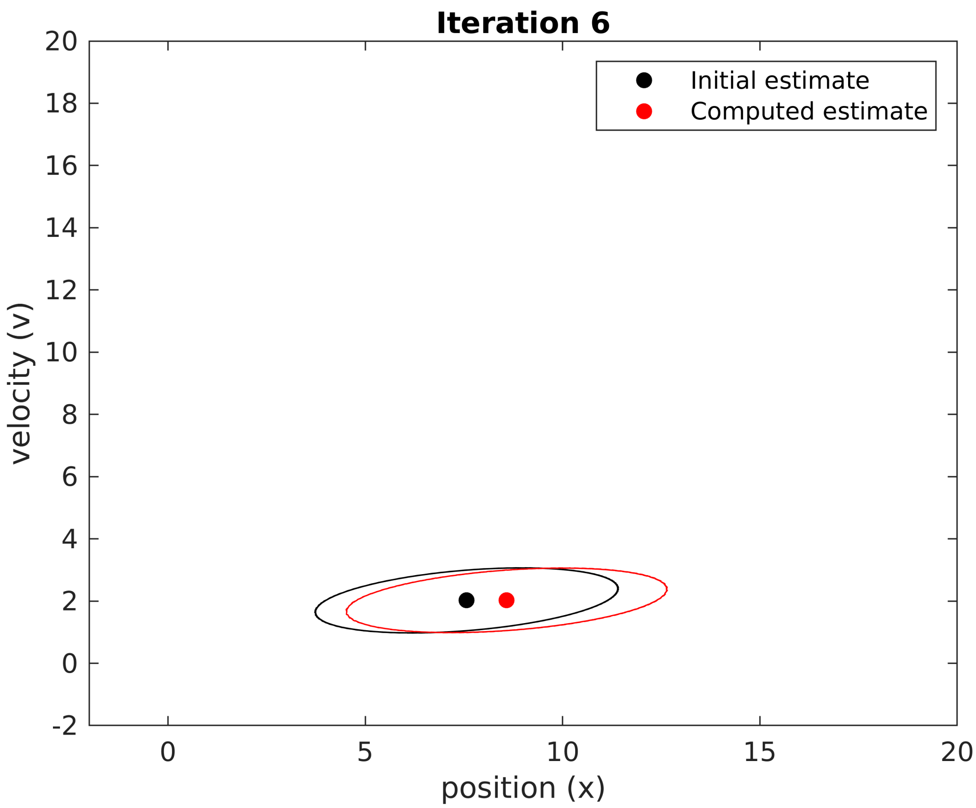
```
% Choosing confidence level to plot
p = 0.95;
% Computing scale factor
s = - 2 * log(1 - p);

for k = 3:n_iter
    % Computing eigenvalues of eigenvectors uncertainity matrix P
    [V1, D1] = eig(P(:,:,k-1) * s);
    [V2, D2] = eig(P(:,:,k) * s);

    % Ellipse equation
    vel = linspace(-2,20);
    a = (V1 * sqrt(D1)) * [cos(vel(:))'; sin(vel(:))'];
    b = (V2 * sqrt(D2)) * [cos(vel(:))'; sin(vel(:))'];

    % Ploting ellipses with x as mean
    figure
    plot(a(1, :) + x(1,k-1), a(2, :) + x(2,k-1), 'k-');
    hold on;
    plot(b(1, :) + x(1,k), b(2, :) + x(2,k), 'r--');

    % Adding states
    s1 = scatter(x(1,k-1), x(2,k-1), 'k', 'filled');
    s2 = scatter(x(1,k), x(2,k), 'r', 'filled');

    legend([s1, s2], {'Initial estimate', 'Computed estimate'});
    title(['Iteration ', num2str(k-1)]);
    xlabel('position (x)');
    ylabel('velocity (v)');
    axis([-2 20 -2 20]);
    hold off;
end
```

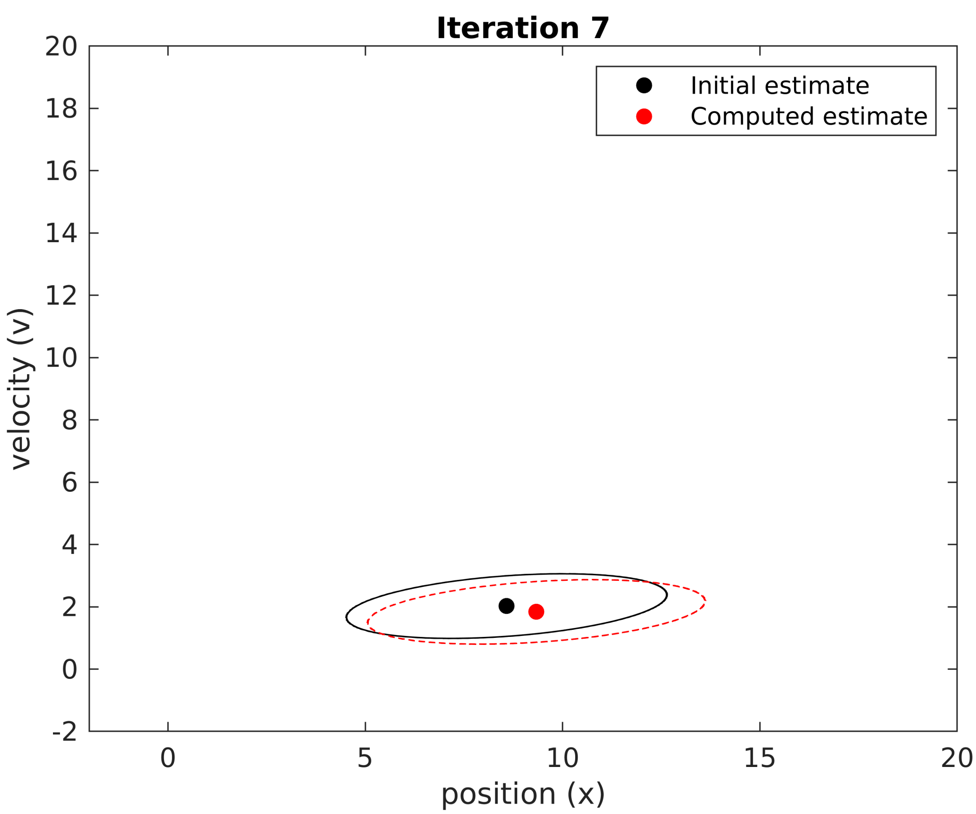

Iteration 2
Initial estimate
Computed estimate
velocity (v)
position (x)
20
18
16
14
12
10
8
6
4
2
0
-2
0
5
10
15
20

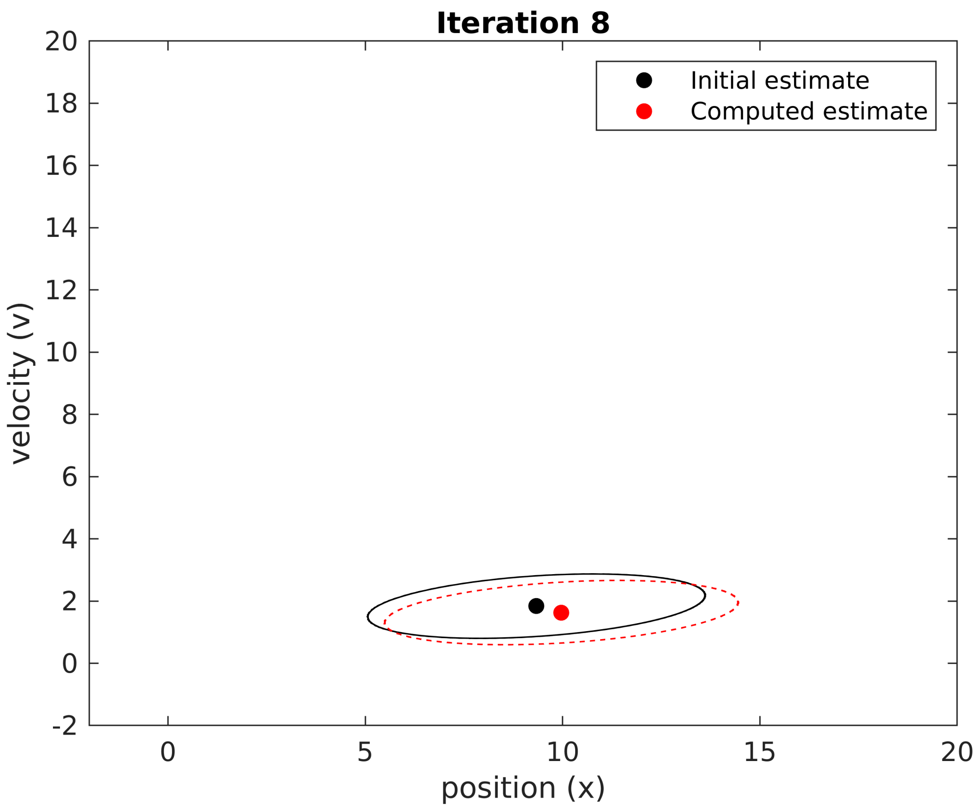

Iteration 3
Initial estimate
Computed estimate
velocity (v)
position (x)
20
18
16
14
12
10
8
6
4
2
0
-2
0
5
10
15
20

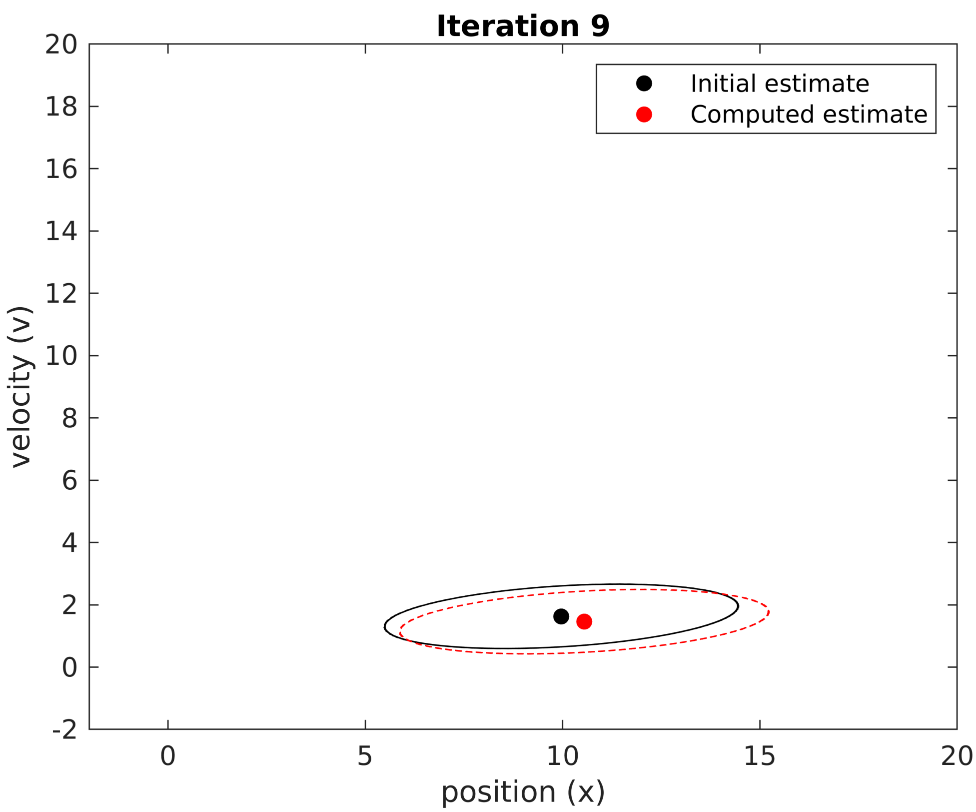

Iteration 4
Initial estimate
Computed estimate
velocity (v)
position (x)
20
18
16
14
12
10
8
6
4
2
0
-2
0
5
10
15
20

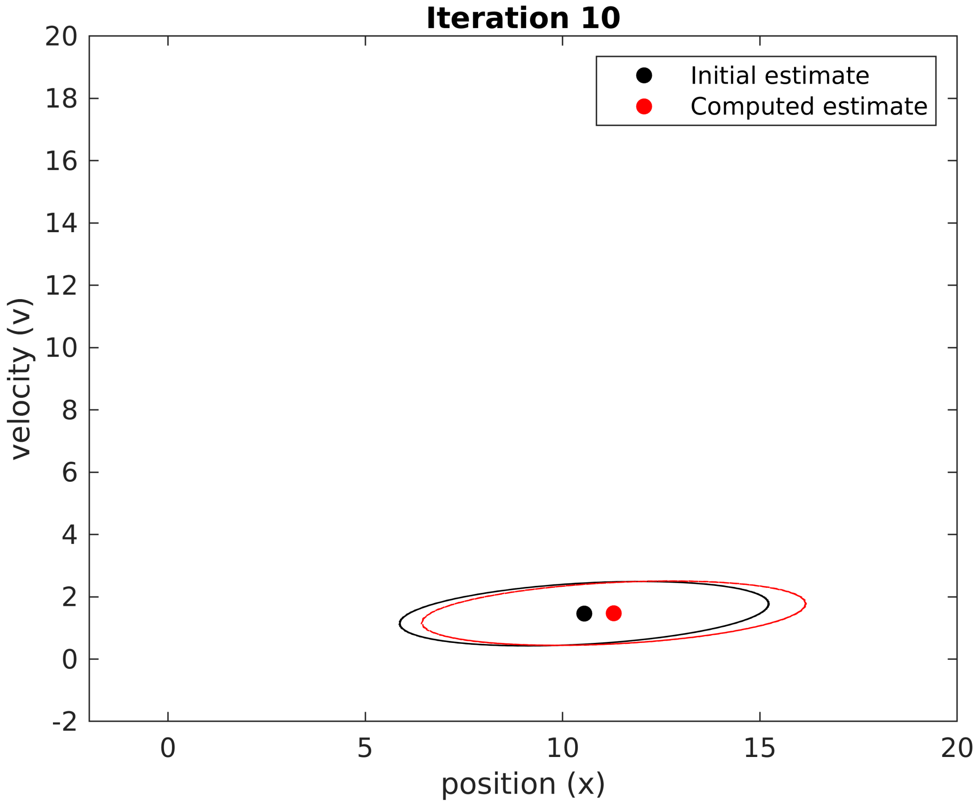

Iteration 5
Initial estimate
Computed estimate
velocity (v)
position (x)
20
18
16
14
12
10
8
6
4
2
0
-2
0
5
10
15
20

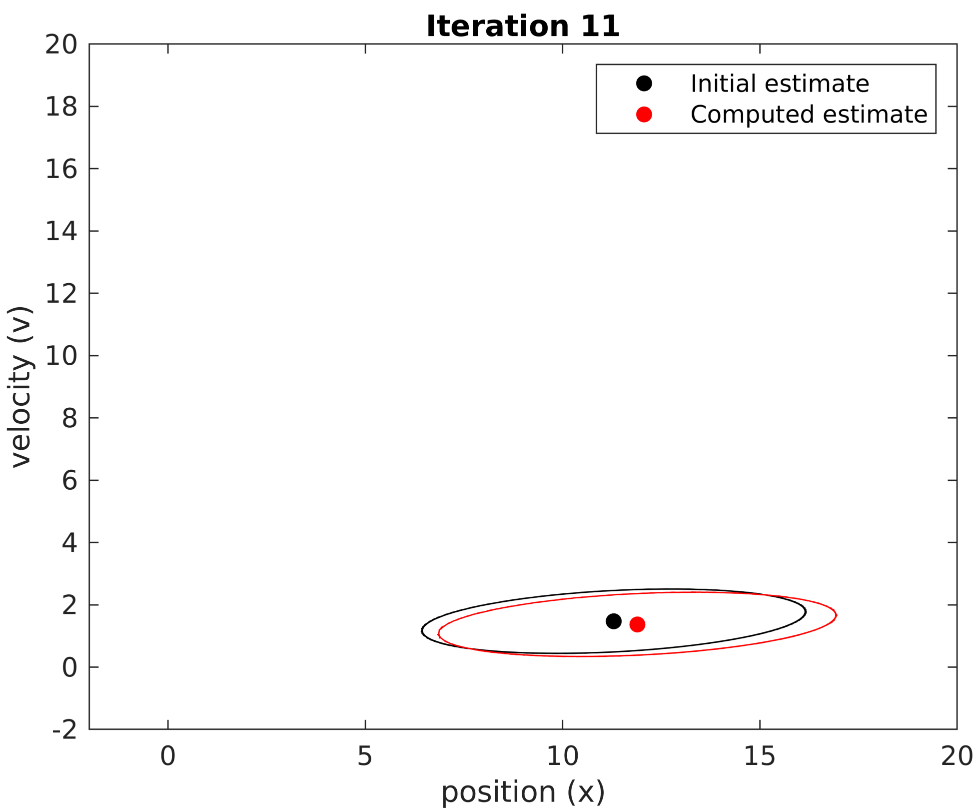

Iteration 6
Initial estimate
Computed estimate
velocity (v)
position (x)
20
18
16
14
12
10
8
6
4
2
0
-2
0
5
10
15
20

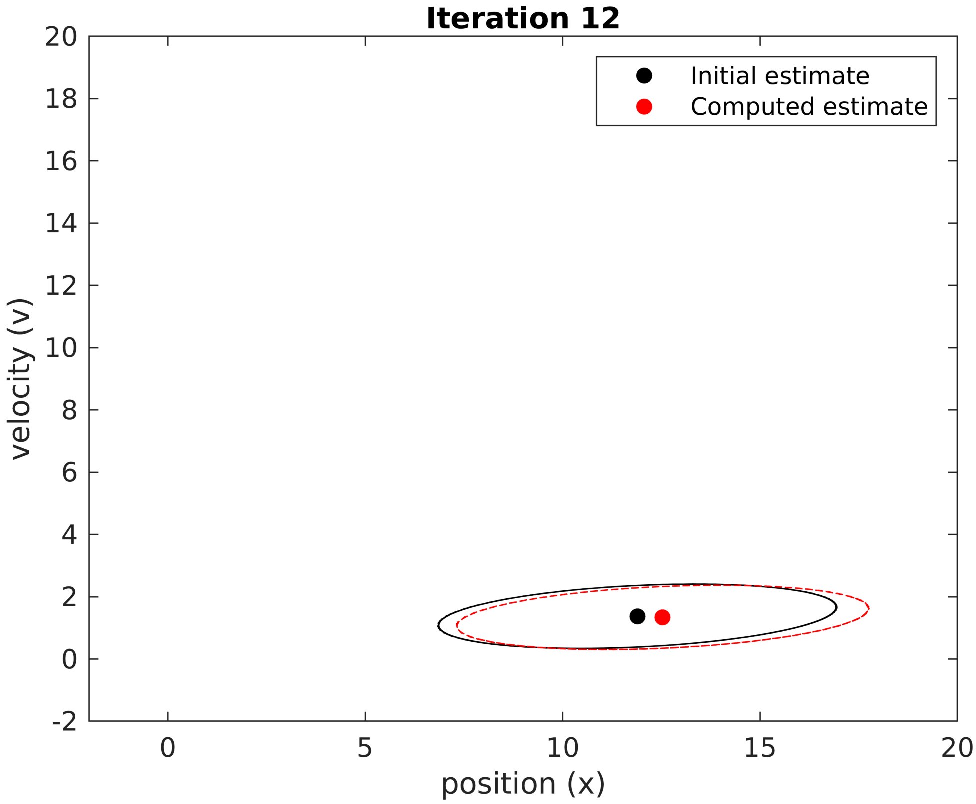

Iteration 7
Initial estimate
Computed estimate
velocity (v)
position (x)
20
18
16
14
12
10
8
6
4
2
0
-2
0
5
10
15
20

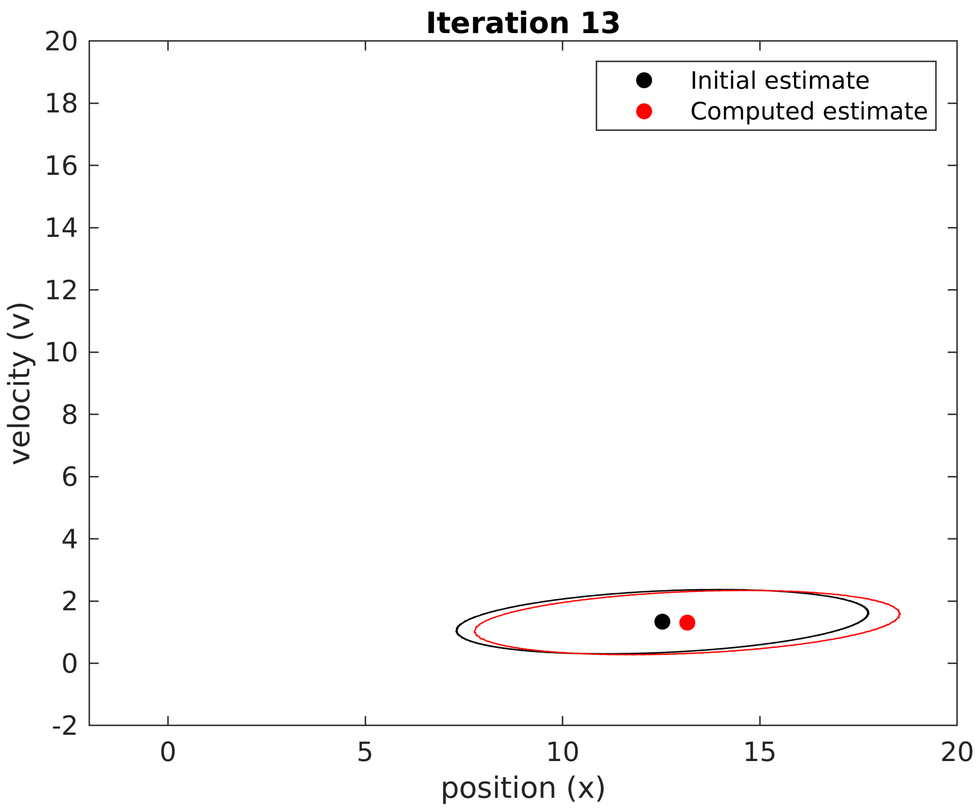

Iteration 8
Initial estimate
Computed estimate
velocity (v)
position (x)
20
18
16
14
12
10
8
6
4
2
0
-2
0
5
10
15
20

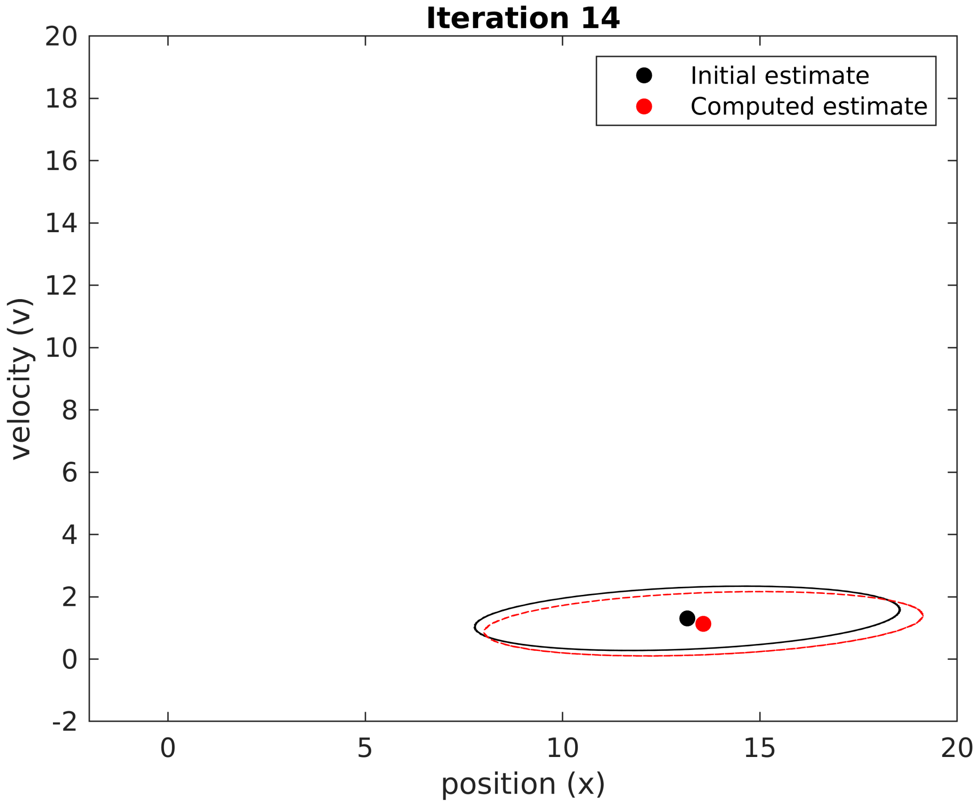

Iteration 9
Initial estimate
Computed estimate
velocity (v)
position (x)
20
18
16
14
12
10
8
6
4
2
0
-2
0
5
10
15
20


Iteration 10
Initial estimate
Computed estimate
velocity (v)
position (x)
20
18
16
14
12
10
8
6
4
2
0
-2
0
5
10
15
20

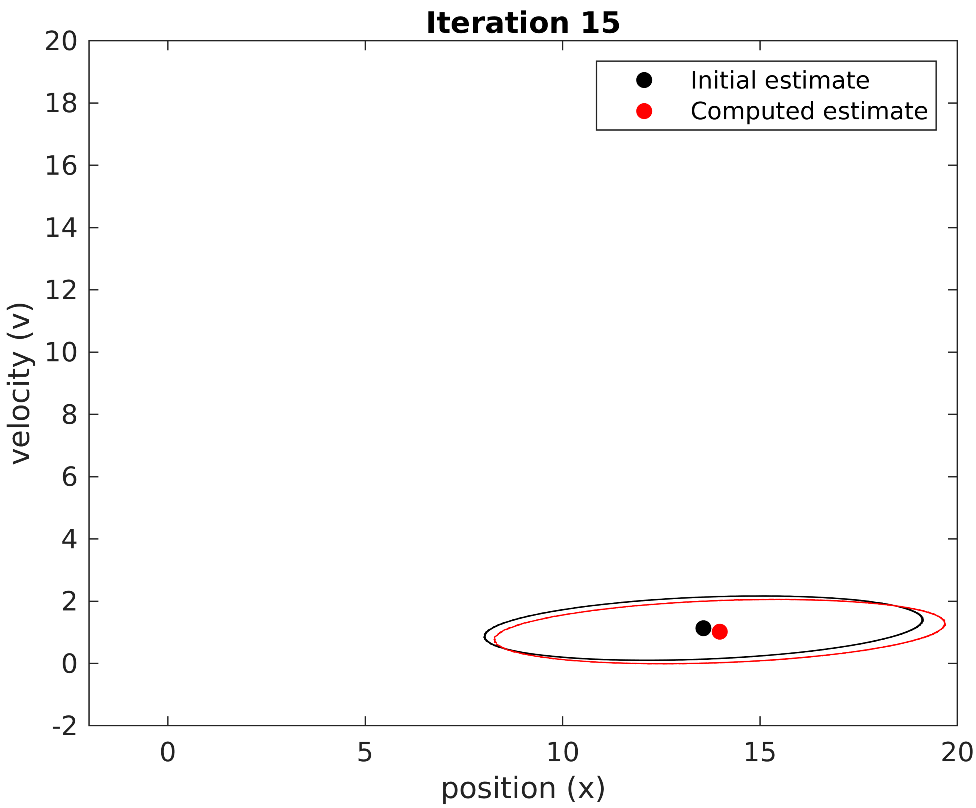

Iteration 11
Initial estimate
Computed estimate
velocity (v)
position (x)
20
18
16
14
12
10
8
6
4
2
0
-2
0
5
10
15
20


Iteration 12
Initial estimate
Computed estimate
velocity (v)
position (x)
20
18
16
14
12
10
8
6
4
2
0
-2
0
5
10
15
20

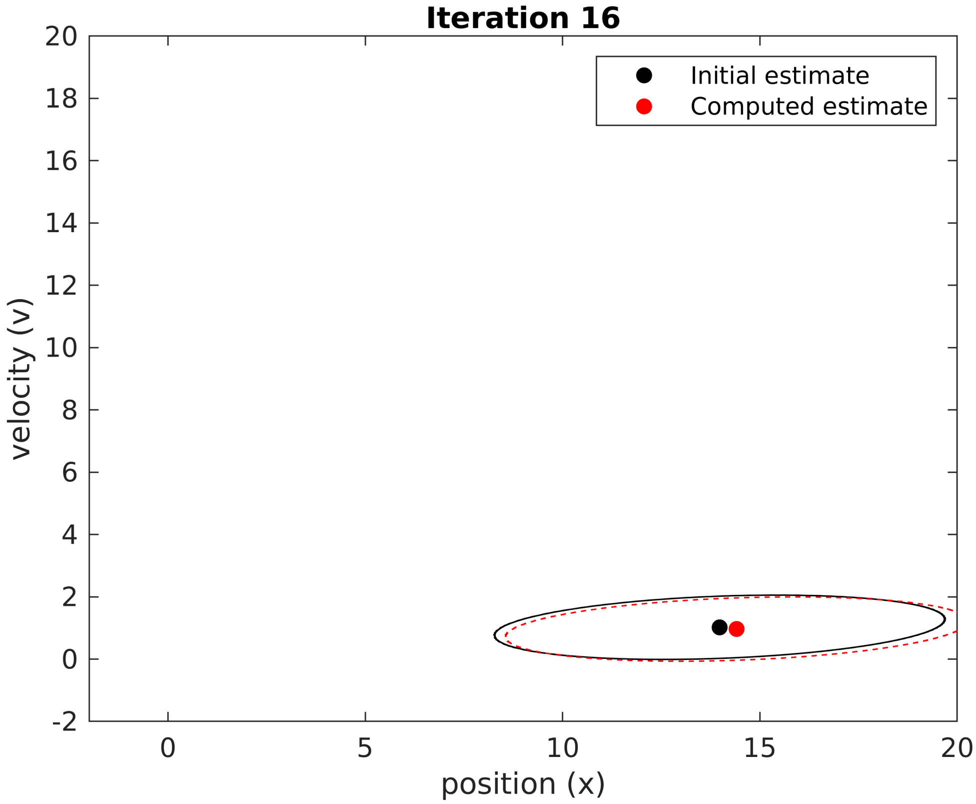

Iteration 13
Initial estimate
Computed estimate
velocity (v)
position (x)
20
18
16
14
12
10
8
6
4
2
0
-2
0
5
10
15
20



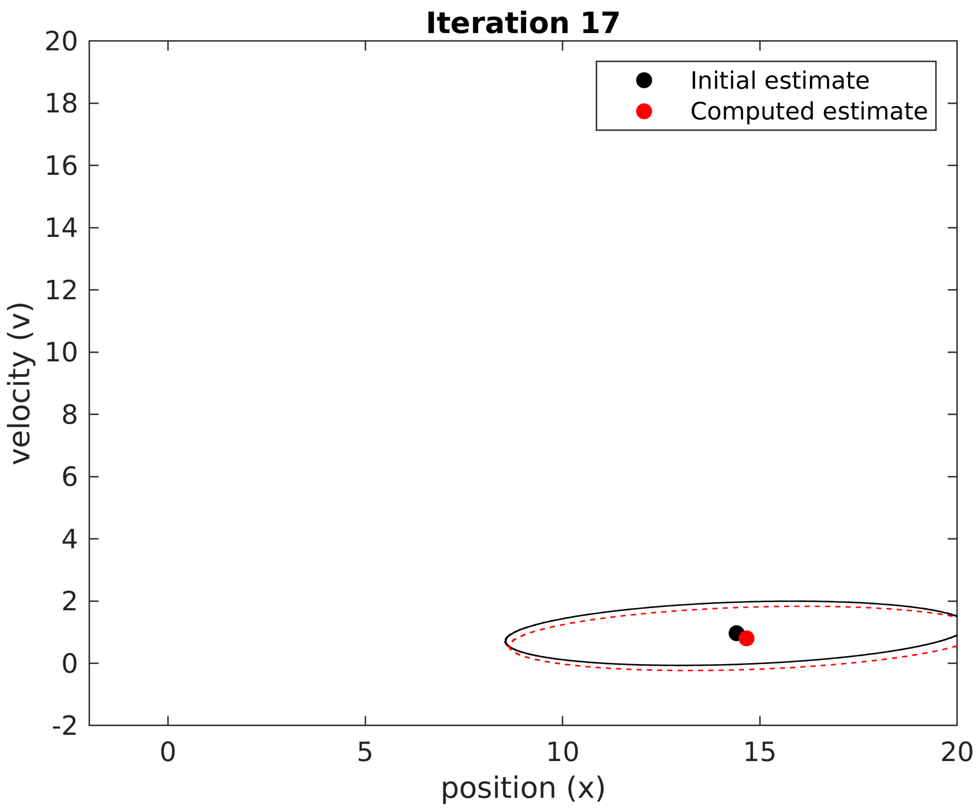
Iteration 14
Initial estimate
Computed estimate
velocity (v)
position (x)



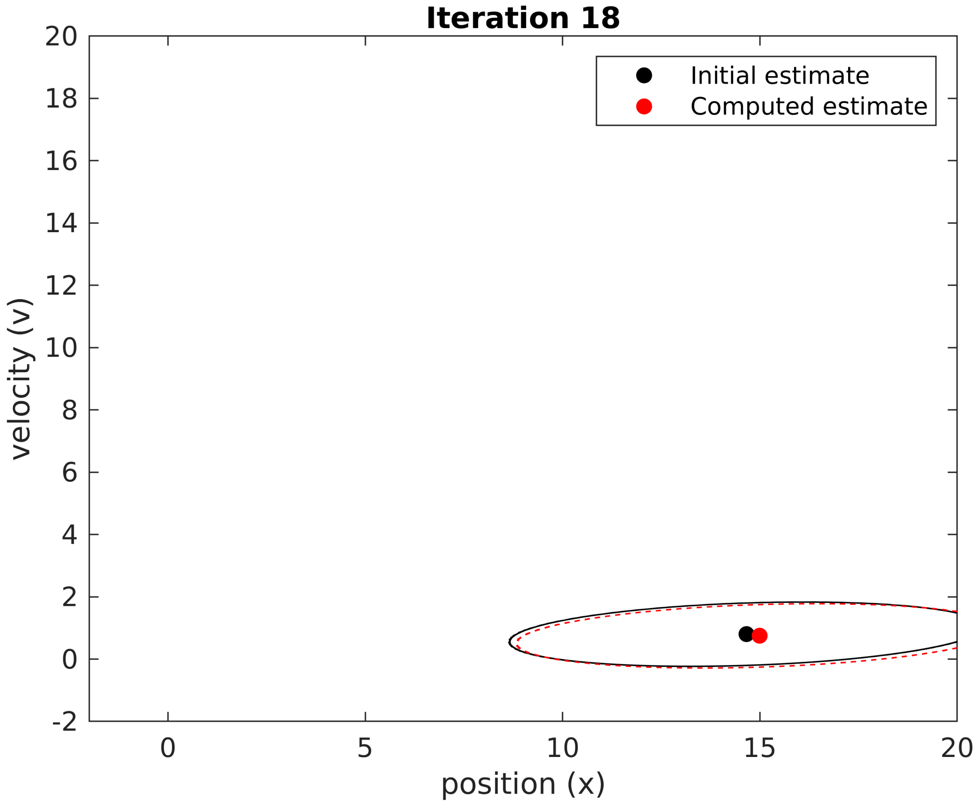
Iteration 15
Initial estimate
Computed estimate
velocity (v)
position (x)

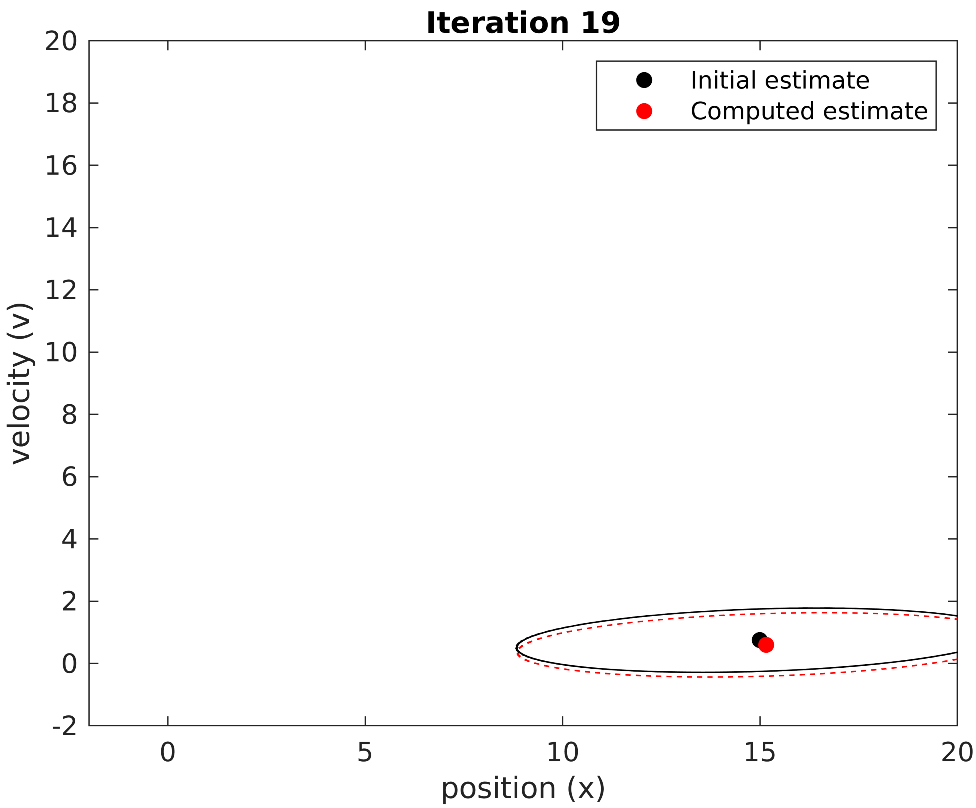


Iteration 16
Initial estimate
Computed estimate
velocity (v)
position (x)
20
18
16
14
12
10
8
6
4
2
0
-2
0
5
10
15
20



Iteration 17
Initial estimate
Computed estimate
velocity (v)
position (x)
20
18
16
14
12
10
8
6
4
2
0
-2
0
5
10
15
20


Iteration 18
Initial estimate
Computed estimate
velocity (v)
position (x)
20
18
16
14
12
10
8
6
4
2
0
-2
0
5
10
15
20


Iteration 19
Initial estimate
Computed estimate
velocity (v)
position (x)
20
18
16
14
12
10
8
6
4
2
0
-2
0
5
10
15
20