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AD1 - Matemática Conf.

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(A) $f(x) = 2 + (x-1)^3$

R: Como NÃO foi dada nenhuma restrição no domínio e imagem, os mesmos pertencem aos Reais.

$$D = \mathbb{R} \rightarrow D =]-\infty, \infty[\rightarrow (-\infty, \infty)$$

$$I = \mathbb{R} \quad I =]-\infty, \infty[\rightarrow (-\infty, \infty)$$

(B) $f(x) = 2x^4 - 4$

R: Como NÃO temos nenhuma restrição o domínio será pertencente aos Reais, porém a imagem NÃO.

Então:

$$D = \mathbb{R} \rightarrow]-\infty, \infty[\rightarrow (-\infty, \infty)$$

$$I =]-\infty, -4] \rightarrow (-\infty, -4]$$

$$f(0) = 2 \cdot 0^4 - 4 = -4$$

$$f(1) = 2 \cdot 1^4 - 4 = -2$$

$$f(-1) = 2 \cdot (-1)^4 - 4 = -2$$

$$f(2) = 2 \cdot (2)^4 - 4 = 4$$

$$f(-2) = 2 \cdot (-2)^4 - 4 = 4$$

$$\frac{N}{D} \rightarrow D \neq 0$$

$$c) f(x) = \frac{-2}{(x-1)^2}$$

$$(x-1)^2 \neq 0$$

$$x^2 - 2 \cdot x \cdot 1 + 1^2$$

$$x^2 - 2x + 1 \neq 0$$

$$\Delta = 4 - 4$$

$$\Delta = 0$$

$$x \neq \frac{2 \pm 0}{2}$$

$$x \neq 1$$

$$(A-B)^2 = A^2 - 2AB + B^2$$

$$\Delta = B^2 - 4AC$$

$$x = \frac{-B \pm \sqrt{\Delta}}{2A}$$

$$f(0) = \frac{-2}{(0-1)^2} = \frac{-2}{(-1)^2} = -2$$

$$f(-1) = \frac{-2}{(-1-1)^2} = \frac{-2}{4} = -\frac{1}{2} = -0,5$$

$$f(-2) = \frac{-2}{(-2-1)^2} = \frac{-2}{9} = -0,222...$$

$$f(2) = \frac{-2}{(2-1)^2} = -2$$

$$f(-3) = \frac{-2}{(-3-1)^2} = \frac{-2}{(-4)^2} = \frac{-2}{16} = -\frac{1}{8} = -0,125$$

Logo, teremos que

$$D = \mathbb{R} - 1 \rightarrow]-\infty, 1[\cup]1, \infty[\rightarrow (-\infty, 1) \cup (1, \infty)$$

$$I = \mathbb{R} \setminus \{0\} \rightarrow]-\infty, 0[\cup]0, \infty[\rightarrow (-\infty, 0) \cup (0, \infty)$$

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$$\textcircled{D} f(x) = \frac{x-1}{x+4}$$

$$\frac{N}{D} \rightarrow D \neq 0$$

$$\begin{array}{l} x+4 \neq 0 \\ x \neq -4 \end{array}$$

$$f(0) = \frac{0-1}{0+4} = \frac{-1}{4} = -0,25$$

$$f(1) = \frac{1-1}{1+4} = \frac{0}{5} = 0$$

$$f(-1) = \frac{-1-1}{-1+4} = \frac{-2}{3} = -0,666\dots$$

$$f(2) = \frac{2-1}{2+4} = \frac{1}{6} = 0,1666\dots$$

$$f(-2) = \frac{-2-1}{-2+4} = \frac{-3}{2} = -1,5$$

$$f(3) = \frac{3-1}{3+4} = \frac{2}{7} = 0,28571$$

$$f(-3) = \frac{-3-1}{-3+4} = \frac{-4}{1} = -4$$

$$f(4) = \frac{4-1}{4+4} = \frac{3}{8} = 0,375$$

$$f(-5) = \frac{-5-1}{-5+4} = \frac{-6}{-1} = 6$$

$$f(5) = \frac{5-1}{5+4} = \frac{4}{9} = 0,444\dots$$

$$D = \mathbb{R} - 4 \rightarrow]-\infty, 4[\cup]4, \infty[\rightarrow (-\infty, 4) \cup (4, \infty)$$

$$I = \mathbb{R} - 1 \rightarrow]-\infty, 1[\cup]1, \infty[\rightarrow (-\infty, 1) \cup (1, \infty)$$

②

① $f(x) = 3x + 4$

$$f(x) = y$$

$$y = 3x + 4$$

$$y - 4 = 3x$$

$$\frac{y - 4}{3} = x$$

$$f^{-1}(x) = \frac{x - 4}{3}$$

② $f(x) = \frac{1}{x - A}$

$$f(x) = y$$

$$y = \frac{1}{x - A}$$

$$x - A = \frac{1}{y}$$

$$x = \frac{1}{y} + A$$

$$f^{-1}(x) = \frac{1}{x} + A$$

③ $f(x) = \frac{x + A}{x - A}$

$$f(x) = y$$

$$y = \frac{x + A}{x - A}$$

$$y(x - A) = x + A$$

$$yx - yA = x + A$$

$$yx - x - yA = A$$

$$x(y - 1) = A + yA$$

$$x = \frac{A + yA}{y - 1}$$

$$f^{-1}(x) = \frac{A + xA}{x - 1}$$

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(D) $f(x) = \frac{1}{x}, x > 0$

$x > 0$

$f(x) = y$

$\frac{1}{y} > 0$

$y = \frac{1}{x}$

$x = \frac{1}{y}$

$$f^{-1}(x) = \frac{1}{x}$$

$y > 0$

(E) $f(x) = x^2 - 4, x \leq 0$

$f(x) = y$

$y = x^2 - 4$

$\sqrt{y+4} = x$

$$f^{-1}(x) = \sqrt{x+4}$$

$x \leq 0$
 $\sqrt{y+4} \leq 0$

$y+4 \leq 0$

$y \leq -4$

+ OBS.: ~~As~~ As raízes negativas fazem parte do conjunto dos números imaginários.

(F) $f(x) = x^2 - 4, x \geq 0$

$f(x) = y$

$y = x^2 - 4$

$\sqrt{y+4} = x$

$$f^{-1}(x) = \sqrt{x+4}$$

$x \geq 0$
 $\sqrt{y+4} \geq 0$

$y+4 \geq 0$

$y \geq -4$

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(A) $\lim_{x \rightarrow 2^+} f(x)$, $\lim_{x \rightarrow 2^-} f(x)$ e $\lim_{x \rightarrow 2} f(x)$

$$f(x) = \begin{cases} x^2 + 1 & \text{se } x < 2 \\ 2 & \text{se } x = 2 \\ 9 - x^2 & \text{se } x > 2 \end{cases}$$

$$\lim_{x \rightarrow 2^+} f(x) = 9 - 2^2 = 5$$

$$\lim_{x \rightarrow 2^-} f(x) = 2^2 + 1 = 5$$

$$\lim_{x \rightarrow 2} f(x) = 2$$

④

⑧ $\lim_{x \rightarrow 1/5^+} f(x), \lim_{x \rightarrow 1/5^-} f(x) \in \lim_{x \rightarrow 1/5} f(x)$

$$f(x) = 2 + |5x - 1|$$

$$\lim_{x \rightarrow 1/5^+} f(x) = 2 + \left| 5 \cdot \frac{1}{5} - 1 \right| = 2$$

$$\lim_{x \rightarrow 1/5^-} f(x) = 2 + \left| 5 \cdot \frac{1}{5} - 1 \right| = 2$$

$$\lim_{x \rightarrow 1/5} f(x) = 2 + \left| 5 \cdot \frac{1}{5} - 1 \right| = 2$$

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$$(C) \lim_{x \rightarrow 0} f(x), \lim_{x \rightarrow 5^+} f(x), \lim_{x \rightarrow 5^-} f(x), \lim_{x \rightarrow 5} f(x) \in \lim_{x \rightarrow -5} f(x)$$

$$f(x) = \frac{x^5 - 25}{x - 5}$$

$$\lim_{x \rightarrow 0} f(x) = \frac{0^5 - 25}{0 - 5} = \frac{-25}{-5} = 5$$

$$\lim_{x \rightarrow 5^+} f(x) = \frac{5^5 - 25}{5 - 5} = \frac{3100}{0} = +\infty$$

$$\lim_{x \rightarrow 5^-} f(x) = \frac{5^5 - 25}{5 - 5} = \frac{3100}{0} = -\infty$$

$$\lim_{x \rightarrow 5} f(x) = \frac{5^5 - 25}{5 - 5} = \frac{3100}{0} = +\infty$$

$$\lim_{x \rightarrow -5} f(x) = \frac{-5^5 - 25}{-5 - 5} = \frac{-3150}{-10} = \frac{3150}{10} = 315$$

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$$(A) \lim_{x \rightarrow +\infty} \frac{x^2 + 3x + 1}{x^3 - 2}$$

Note Que 1
 $\lim_{x \rightarrow \infty} \frac{1}{x^n} = 0$

$$\lim_{x \rightarrow +\infty} = \frac{\frac{1}{x^3} + \frac{3}{x^2} + \frac{1}{x^3}}{1 - \frac{2}{x^3}}$$

$$\lim_{x \rightarrow +\infty} = \frac{1}{x^3} + \frac{3}{x^2} + \frac{1}{x^3} = 0 + 0 + 0 = 0$$

$$\lim_{x \rightarrow +\infty} = 1 - \frac{2}{x^3} = 1$$

$$\lim_{x \rightarrow +\infty} \frac{x^2 + 3x + 1}{x^3 - 2} = \frac{0}{1} = 0$$

$$(B) \lim_{x \rightarrow +\infty} \frac{x\sqrt{x} + 3x - 10}{x^3}$$

Note Que 1
 $\sqrt{x} = x^{1/2}$
 $\lim_{x \rightarrow \infty} \frac{1}{x^n} = 0$

$$\lim_{x \rightarrow +\infty} = \frac{1 \cdot 1^{1/2}}{x^2} + \frac{3}{x^2} - \frac{10}{x^3} = 0 + 0 - 0$$

$$\lim_{x \rightarrow +\infty} \frac{x\sqrt{x} + 3x - 10}{x^3} = 0$$

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(A) ANULADO

(B) $f(x) = x - |x|$ em $x = 0$

$$+ f(x) = x - x = 0 \geq 0$$

$$+ f(x) = x + x = 2x < 0$$

$$\lim_{x \rightarrow 0^+} 0 = 0$$

$$\lim_{x \rightarrow 0^-} 2x = 0$$

$$\lim_{x \rightarrow 0} 0 = 0 = 0$$

$$f(0) = 0 - |0| = 0$$

R.: Logo f é contínua em $x = 0$

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(c) $f(x) = \begin{cases} \frac{x^3 - 8}{x^2 - 4} & \text{se } x \neq 2 \\ 3 & \text{se } x = 2 \end{cases}$ em $x = 2$

$$\lim_{x \rightarrow 2^+} \frac{x^3 - 8}{x^2 - 4} = \frac{1 - \frac{8}{x^3}}{\frac{1}{x} - \frac{4}{x^3}} = \frac{1 - 0}{0 - 0} = \frac{1}{0} = +\infty$$

$$\lim_{x \rightarrow 2^-} \frac{x^3 - 8}{x^2 - 4} = \frac{1 - \frac{8}{x^3}}{\frac{1}{x} - \frac{4}{x^3}} = \frac{1 - 0}{0 - 0} = \frac{1}{0} = -\infty$$

$$\lim_{x \rightarrow 2} \frac{(x^2 - 4) \cdot (x + 2)}{(x^2 - 4)} = 2 + 2 = 4$$

$$\lim_{x \rightarrow 2} f(x) = 4$$

R.: $\lim_{x \rightarrow 2} f(x) \neq f(2)$, Logo Temos descontinuidade de $x = 2$

7. Plano y x

$$X = f(y) = y^4 - 6y^2$$

$$f'(y) = 4y^3 - 12y$$

→ $f'(y)$ nos da Inclinação:

$$X = f(y) = y^4 - 6y^2 \rightarrow y^4 - 6y^2 = 0$$

$$y^2(y^2 - 6) = 0 \rightarrow y = 0 \rightarrow y = \sqrt{6}, \rightarrow y = -\sqrt{6}$$

Logo os pontos são $(0,0)$, $(\sqrt{6},0)$ $(-\sqrt{6},0)$

$$f'(y) = y^4 - 6y^2$$

$$f'(y) = 4y^3 - 12y = 0 \rightarrow \text{Inclinação nula.}$$

$$f''(y) = 4y^3 - 12y$$

$$f''(y) = 12y^2 - 12 = -12 < 0 \rightarrow \text{máximo local}$$

$$y = \sqrt{6} \rightarrow f'(\sqrt{6}) = 4y^3 - 12y = 12\sqrt{6} \rightarrow \text{Inclinação é de } 12\sqrt{6}$$

$$y = -\sqrt{6} \rightarrow f'(-\sqrt{6}) = 4y^3 - 12y = -12\sqrt{6} \rightarrow \text{Inclinação}$$

⑦ Plano $X \ Y$

$$(x)' = (y^4 - 6y^2)' \rightarrow 1 = 4y^3 \cdot y' - 12y \rightarrow y' = \frac{1}{4y^3 - 12y}$$

$$f'(x) \rightarrow \text{Inclinação}$$

Ponto $(0,0)$ é vertical

$$(0,0) \rightarrow f'(0) = \frac{1}{4y^3 - 12y} = \frac{1}{0} = \infty \rightarrow$$

$$(0, \sqrt{6}) \rightarrow f'(0) = \frac{1}{4y^3 - 12y} = \frac{1}{12\sqrt{6}} \Rightarrow \text{Inclinação é } \frac{\sqrt{6}}{72}$$

$$(0, -\sqrt{6}) \rightarrow f'(0) = \frac{1}{4y^3 - 12y} = \frac{-1}{12\sqrt{6}} \Rightarrow \text{Inclinação é } -\frac{\sqrt{6}}{72}$$

(8)

$$f'(x) = \cos x^{-5/2}$$

$$f'(x) = -\sin x^{-5/2} \cdot -\frac{5}{2} x^{-7/2} \cdot (-1)$$

$$f'(x) = \left(\sin x^{-5/2} \right) \cdot \frac{x^{-7/2} \cdot 5}{2}$$

$$f'(x) = \frac{5 \sin (x^{-5/2}) \cdot x^{-7/2}}{2}$$

$$f'(x) = \frac{5 \sin (x^{-5/2})}{2 x^{7/2}}$$

$$f'(x) = \frac{5 \sin \left(\frac{1}{x^{5/2}} \right)}{2 x^{7/2}}$$

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$$\textcircled{A} f(x) = (10 - 5x^2)^4$$

$$f'(x) = 4(10 - 5x^2)^{4-1} \cdot (0 - 10x)^3$$

$$f'(x) = -40 \left[(10 - 5x^2)^3 \right]$$

$$f''(x) = -40 \cdot (10 - 5x^2)^3 + (-40x) 3(10 - 5x^2)^2 \cdot (-10x)$$

$$f''(x) = -40(10 - 5x^2)^3 + 1200x^2(10 - 5x^2)^2$$

$$\textcircled{B} f(x) = \frac{x^2 + 4}{2 - x^4}$$

$$f'(x) = \frac{(x^2 + 4) \cdot (2 - x^4)' - (x^2 + 4)(2 - x^4)'}{(2 - x^4)^2}$$

$$f'(x) = \frac{(2x) \cdot (2 - x^4) - (x^2 + 4) \cdot (-4x^3)}{(2 - x^4)^2}$$

$$f'(x) = \frac{4x - 2x^5 - (-4x^5 - 16x^3)}{(2 - x^4)^2}$$

$$f'(x) = \frac{4x + 2x^5 + 16x^3}{(2-x^4)^2}$$

$$f''(x) = \frac{4x + 2x^5 + 16x^3}{(2-x^4)^2}$$

$$f''(x) = \frac{(4+10x^4+48x^2)(2-x^4)^2 - (-8x^3)(2-x^4)(4x+2x^5+16x^3)}{((2-x^4)^2)^2}$$

$$f''(x) = \frac{(2-x^4)(4+10x^4+48x^2) + 8x^3(2-x^4)(4x+2x^5+16x^3)}{(2-x^4)^4}$$

$$f''(x) = \frac{8 + 20x^4 + 96x^2 - 4x^4 - 10x^8 + 96x^2 - 48x^6 + (4x + 2x^5 + 16x^3)8x^3}{(2-x^4)^3}$$

$$f''(x) = \frac{8 + 20x^4 + 96x^2 - 4x^4 - 10x^8 + 96x^2 - 48x^6 + (4x + 2x^5 + 16x^3)8x^3}{(2-x^4)^3}$$

$$- 48x^6 + 32x^4 + 16x^8 + 128x^6$$

$$f''(x) = \frac{8 + 48x^4 + 6x^8 + 96x^2 + 80x^6}{(2-x^4)^3}$$

$$f''(x) = \frac{6x^8 + 80x^6 + 48x^4 + 96x^2 + 8}{(2 - x^4)^3}$$

$$c) f(x) = \sin(\cos x^3)$$

$$f'(x) = \cos(\cos x^3) \cdot (-\sin(x^3)) \cdot 3x^2$$

$$f'(x) = -3x^2 \cdot \cos(\cos(x^3)) \sin x^3$$

$$f''(x) = -3x^2 \cdot \cos(\cos(x^3)) \sin x^3$$

$$f''(x) = -6x \cdot \cos(\cos(x^3)) \cdot \sin x^3 \cdot (-\sin(\cos(x^3))) \cdot \sin x^3 \cdot (-\sin(x^3)) \cdot 3x^2 \cdot \sin x^3$$

$$f''(x) = -6x \cdot \cos(\cos(x^3)) \cdot \sin x^3 - 9x^4 \cdot \sin(\cos(x^3)) \cdot \sin x^3 \cdot \cos x^3$$