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AD1 - Matemática Comp.

1

A) $f(x) = 2 + (x-1)^3$

R: Como não foi dado nenhuma restrição no domínio e imagem, os mesmos pertecem aos reais.

$D = \mathbb{R} \rightarrow D =]-\infty, \infty[\rightarrow (-\infty, \infty)$

$I = \mathbb{R} \quad\;\; I =]-\infty, \infty[\rightarrow (-\infty, \infty)$

B) $f(x) = 2x^4 - 4$

R: Como não temos nenhuma restrição o domínio será pertencente aos reais, porém a imagem não.

$f(0) = 2 \cdot 0^4 - 4 = -4$

$f(1) = 2 \cdot 1^4 - 4 = -2$

$f(-1) = 2 \cdot (-1)^4 - 4 = -2$

$f(2) = 2 \cdot (2)^4 - 4 = 4$

$f(-2) = 2 \cdot (-2)^4 - 4 = 4$

Então:

$D = \mathbb{R} \rightarrow]-\infty, \infty[\rightarrow (-\infty, \infty)$

$I =]-\infty, -4] \rightarrow (-\infty, -4]$

(1)

$\frac{N}{D} \rightarrow D \neq 0$

(c) $f(x) = \frac{-2}{(x-1)^2}$

$(A-B)^2 = A^2 - 2AB + B^2$

$(x-1)^2 \neq 0$

$x^2 - 2 \cdot x \cdot 1 + 1^2$

$x^2 - 2x + 1 \neq 0$

$\Delta = 4 - 4$

$\Delta = 0$

$x \neq \frac{2 \pm 0}{2}$

$\boxed{x \neq 1}$

$\Delta = B^2 - 4AC$

$x = \frac{-B \pm \sqrt{\Delta}}{2A}$

$f(0) = \frac{-2}{(x-1)^2} = \frac{-2}{(0-1)^2} = -2$

$f(-1) = \frac{-2}{(-1-1)^2} = \frac{-2}{4} = \frac{-1}{2} = -0,5$

$f(-2) = \frac{-2}{(-2-1)^2} = \frac{-2}{9} = -0,222\ldots$

$f(2) = \frac{-2}{(2-1)^2} = -2$

$f(-3) = \frac{-2}{(-3-1)^2} = \frac{-2}{(-4)^2} = \frac{-2}{16} = \frac{-1}{8} = -0,125$

Logo, teremos que

$D = \mathbb{R} - 1 \rightarrow]-\infty, 1[\cup]1, \infty[\rightarrow (-\infty, 1) \cup (1, \infty)$

$I = \mathbb{R} < 0 \rightarrow]-\infty, 0[\rightarrow (-\infty, 0)$

1

D) $f(x) = \dfrac{x-1}{x+4}$ $\quad \dfrac{N}{D} \to D \neq 0$

$$\boxed{\begin{aligned} x+4 &\neq 0 \\ x &\neq -4 \end{aligned}}$$

$f(0) = \dfrac{0-1}{0+4} = \dfrac{-1}{4} = -0{,}25$

$f(1) = \dfrac{1-1}{1+4} = \dfrac{0}{5} = 0$

$f(-1) = \dfrac{-1-1}{-1+4} = \dfrac{-2}{3} = -0{,}666\ldots$

$f(2) = \dfrac{2-1}{2+4} = \dfrac{1}{6} = 0{,}1666\ldots$

$f(-2) = \dfrac{-2-1}{-2+4} = \dfrac{-3}{2} = -1{,}5$

$f(3) = \dfrac{3-1}{3+4} = \dfrac{2}{7} = 0{,}28571$

$f(-3) = \dfrac{-3-1}{-3+4} = \dfrac{-4}{1} = -4$

$f(4) = \dfrac{4-1}{4+4} = \dfrac{3}{8} = 0{,}375$

$f(-5) = \dfrac{-5-1}{-5+4} = \dfrac{-6}{-1} = 6$

$f(5) = \dfrac{5-1}{5+4} = \dfrac{4}{9} = 0{,}444\ldots$

$D = \mathbb{R} - 4 \to\ ]-\infty, 4[\ \cup\ ]4, \infty[\ \to (-\infty, 4) \cup (4, \infty)$

$I = \mathbb{R} - 1 \to\ ]-\infty, 1[\ \cup\ ]1, \infty[\ \to (-\infty, 1) \cup (1, \infty)$

2

A $f(x) = 3x+4$

$f(x) = y$

$y = 3x+4$

$y-4 = 3x$

$\frac{y-4}{3} = x$

$f^{-1}(x) = \frac{x-4}{3}$

B $f(x) = \frac{1}{x+A}$

$f(x) = y$

$y = \frac{1}{x+A}$

$x - A = \frac{1}{y}$

$x = \frac{1}{y} + A$

$f^{-1}(x) = \frac{1}{x} + A$

C $f(x) = \frac{x+A}{x-A}$

$f(x) = y$

$y = \frac{x+A}{x-A}$

$y(x-A) = x+A$

$yx - yA = x + A$

$yx - x - yA = A$

$x(y-1) = A + yA$

$x = \frac{A+yA}{y-1}$

$f^{-1}(x) = \frac{A+xA}{x-1}$

(2)

(D) $f(x) = \frac{1}{x}, \; x > 0$

$f(x) = y$

$y = \frac{1}{x}$

$x = \frac{1}{y}$

$$\boxed{f^{-1}_{(x)} = \frac{1}{x}}$$

$x > 0$

$\frac{1}{y} > 0$

$y > 0$

(E) $f(x) = x^2 - 4, \; x \leq 0$

$f(x) = y$

$y = x^2 - 4$

$\sqrt{y+4} = x$

$$\boxed{f^{-1}_{(x)} = \sqrt{x+4}}$$

$x \leq 0$

$\sqrt{y+4} \leq 0$

$y + 4 \leq 0$

$y \leq -4$

+ OBS.: [redacted] As raízes negativas fazem parte do conjunto dos números imaginários.

(f) $f(x) = x^2 - 4, \; x \geq 0$

$f(x) = y$

$y = x^2 - 4$

$\sqrt{y+4} = x$

$$\boxed{f^{-1}_{(x)} = \sqrt{x+4}}$$

$x \geq 0$

$\sqrt{y+4} \geq 0$

$y + 4 \geq 0$

$y \geq -4$

③

Ⓐ $\lim_{s \to 1/2} \frac{s+4}{2s}$

$\frac{1/2+4}{2 \cdot 1/2} = \frac{1}{2} + 4 = \frac{9}{2}$

Ⓑ $\lim_{x \to 4} (e^{x} + 4x)$

$(e^{4} + 16) \cong 70,598150$

Ⓒ $\lim_{H \to 0} \frac{(x+H)^{4} - x^{4}}{H}$

$\frac{\cancel{x^{4}} + 4x^{3}H^{1} + 6x^{2}H^{2} + 4xH^{3} + H^{4} - \cancel{x^{4}}}{H}$

		1			$\to n0$
		1	1		$\to n1$
	1	2	1		$\to n2$
1	3	3	1		$\to n3$
1	4	6	4	1	$\to n4$

Triângulo de Pascal

$\frac{\cancel{H}(4x^{3} + 6x^{2}H + 4xH^{2} + H^{3})}{\cancel{H}}$

$4x^{3} + \underbrace{6x^{2} \cdot 0} + \underbrace{4x \cdot 0^{2}} + \underbrace{0^{3}}$

$4x^{3} + 0 + 0 + 0 = 4x^{3}$

4

A $\lim_{x \to 2^+} f(x)$, $\lim_{x \to 2^-} f(x)$ e $\lim_{x \to 2} f(x)$

$$f(x) \begin{cases} x^2+1 & \text{se } x < 2 \\ 2 & \text{se } x = 2 \\ 9-x^2 & \text{se } x > 2 \end{cases}$$

$$\lim_{x \to 2^+} f(x) = 9-2^2 = 5$$

$$\lim_{x \to 2^-} f(x) = 2^2+1 = 5$$

$$\lim_{x \to 2} f(x) = 2$$

4

B) $\lim_{x \to 1/5^+} f(x)$, $\lim_{x \to 1/5^-} f(x)$ e $\lim_{x \to 1/5} f(x)$

$$f(x) = 2 + |5x - 1|$$

$$\lim_{x \to 1/5^+} f(x) = 2 + \left| 5 \cdot \frac{1}{5} - 1 \right| = 2$$

$$\lim_{x \to 1/5^-} f(x) = 2 + \left| 5 \cdot \frac{1}{5} - 1 \right| = 2$$

$$\lim_{x \to 1/5} f(x) = 2 + \left| 5 \cdot \frac{1}{5} - 1 \right| = 2$$

(4)

(c) $\lim_{x \to 0} f(x)$, $\lim_{x \to 5^+} f(x)$, $\lim_{x \to 5^-} f(x)$, $\lim_{x \to 5} f(x)$ e $\lim_{x \to -5} f(x)$

$$f(x) = \frac{x^5 - 25}{x - 5}$$

$$\lim_{x \to 0} f(x) = \frac{0^5 - 25}{0 - 5} = \frac{-25}{-5} = 5$$

$$\lim_{x \to 5^+} f(x) = \frac{5^5 - 25}{5 - 5} = \frac{3100}{0} = +\infty$$

$$\lim_{x \to 5^-} f(x) = \frac{5^5 - 25}{5 - 5} = \frac{3100}{0} = -\infty$$

$$\lim_{x \to 5} f(x) = \frac{5^5 - 25}{5 - 5} = \frac{3100}{0} = +\infty$$

$$\lim_{x \to -5} f(x) = \frac{-5^5 - 25}{-5 - 5} = \frac{-3150}{-10} = \frac{315\cancel{0}}{1\cancel{0}} = 315$$

(5)

(A) $\lim_{x \to +\infty} = \frac{x^2+3x+1}{x^3-2}$

NOTE QUE:
$+ \lim_{x \to \infty} = \frac{1}{x^N} = 0$

$$\lim_{x \to +\infty} = \frac{\frac{1}{x^3} + \frac{3}{x^2} + \frac{1}{x^3}}{1 - \frac{2}{x^3}}$$

$$\lim_{x \to +\infty} = \frac{1}{x^3} + \frac{3}{x^2} + \frac{1}{x^3} = 0+0+0 = 0$$

$$\lim_{x \to +\infty} = 1 - \frac{2}{x^3} = 1$$

$$\lim_{x \to +\infty} = \frac{x^2+3x+1}{x^3-2} = \frac{0}{1} = 0$$

(B) $\lim_{x \to +\infty} = \frac{x\sqrt{x}+3x-10}{x^3}$

NOTE QUE:
$+ \sqrt{x} = x^{1/2}$
$+ \lim_{x \to \infty} = \frac{1}{x^N} = 0$

$$\lim_{x \to +\infty} = \frac{1 \cdot 1^{1/2}}{x^2} + \frac{3}{x^2} - \frac{10}{x^3} = 0+0-0$$

$$\lim_{x \to +\infty} = \frac{x\sqrt{x}+3x-10}{x^3} = 0$$

⑥

(A) ANULADO

(B) $f(x) = x - |x|$ EM $x = 0$

$+ f(x) = x - x = 0 \geq 0$

$+ f(x) = x + x = 2x < 0$

$\lim_{x \to 0^+} 0 = 0$

$\lim_{x \to 0^-} 2x = 0$

$\lim_{x \to 0} 0 - 0 = 0$

$f(A) = 0 - |0| = 0$

R: Logo f é CONTINUA EM $x = 0$

6

c) $f(x) \begin{cases} \dfrac{x^3-8}{x^2-4} & \text{se } x \neq 2 \\ 3 & \text{se } x = 2 \end{cases}$ em $x = 2$

$$\lim_{x \to +2} \frac{x^3-8}{x^2-4} = \frac{1-\frac{8}{x^3}}{\frac{1}{x}-\frac{4}{x^3}} = \frac{1-0}{0-0} = \frac{1}{0} = +\infty$$

$$\lim_{x \to -2} \frac{x^3-8}{x^2-4} = \frac{1-\frac{8}{x^3}}{\frac{1}{x}-\frac{4}{x^3}} = \frac{1-0}{0-0} = \frac{1}{0} = -\infty$$

$$\lim_{x \to 2} \frac{\cancel{(x^2-4)}.(x+2)}{\cancel{(x^2-4)}} = 2+2 = 4$$

$f(2) = 3$

$\boxed{\text{R.: } \lim_{x \to 2} f(x) \neq f(2), \text{ Logo temos descontinuidade de } x = 2}$

7. Plano yx

$x = f(y) = y^4 - 6y^2$

$f'(y) = 4y^3 - 12y$

$\rightarrow f'(y)$ nos da Inclinação:

$x = f(y) = y^4 - 6y^2 \rightarrow y^4 - 6y^2 = 0$

$y^2(y^2 - 6) = 0 \rightarrow y = 0 \rightarrow y = \sqrt{6}, \rightarrow y = -\sqrt{6}$

Logo os pontos são: $(0,0), (\sqrt{6}, 0) (-\sqrt{6}, 0)$

$$\boxed{\begin{array}{l} f'(y) = y^4 - 6y^2 \\ f'(y) = 4y^3 - 12y = 0 \longrightarrow \text{Inclinação nula.} \end{array}}$$

$$\boxed{\begin{array}{l} f''(y) = 4y^3 - 12y \\ f''(y) = 12y^2 - 12 = -12 < 0 \longrightarrow \text{maximo Local} \end{array}}$$

$y = \sqrt{6} \rightarrow f'(\sqrt{6}) = 4y^3 - 12y = 12\sqrt{6} \rightarrow$ Inclinação é de $12\sqrt{6}$

$y = -\sqrt{6} \rightarrow f'(-\sqrt{6}) = 4y^3 - 12y = -12\sqrt{6} \rightarrow$ Inclinação

(7) Plano X Y

$(x)' = (y^4 - 6y^2)' \rightarrow 1 = 4y^3 \cdot y' - 12y' \rightarrow y' = \dfrac{1}{4y^3 - 12y}$

$\boxed{f'(x) \rightarrow \text{Inclinação}}$

$(0,0) \rightarrow f'(0) = \dfrac{1}{4y^3 - 12y} = \dfrac{1}{0} = \infty \rightarrow$ $\boxed{\text{Ponto } (0,0) \text{ é vertical}}$

$(0, \sqrt{6}) \rightarrow f'(0) = \dfrac{1}{4y^3 - 12y} = \dfrac{1}{12\sqrt{6}} \Rightarrow$ $\boxed{\text{Inclinação é } \dfrac{\sqrt{6}}{72}}$

$(0, -\sqrt{6}) \rightarrow f'(0) = \dfrac{1}{4y^3 - 12y} = \dfrac{-1}{12\sqrt{6}} \Rightarrow$ $\boxed{\text{Inclinação é } \dfrac{-\sqrt{6}}{72}}$

(8)

$$f'(x) = \cos x^{-5/2}$$

$$f'(x) = -\operatorname{sen} x^{-5/2} \cdot -5/2\, x^{-7/2} \cdot (-1)$$

$$f'(x) = \left(\operatorname{sen} x^{-5/2}\right) \cdot \frac{x^{-7/2} \cdot 5}{2}$$

$$f'(x) = \frac{5 \operatorname{sen}\left(x^{-5/2}\right) \cdot x^{-7/2}}{2}$$

$$f'(x) = \frac{5 \operatorname{sen}\left(x^{-5/2}\right)}{2x^{7/2}}$$

$$f'(x) = \frac{5 \operatorname{sen}\left(\frac{1}{x^{5/2}}\right)}{2x^{7/2}}$$

Digitalizado com CamScanner

9

A) $f(x) = (10 - 5x^2)^4$

$f'(x) = 4(10 - 5x^2)^{4-1} \cdot (0 - 10x)^3$

$f'(x) = -40\left[(10 - 5x^2)^3\right]$

$f''(x) = -40.(10 - 5x^2)^3 + (-40x)\, 3(10 - 5x^3)^2 \cdot (-10x)$

$f''(x) = -40(10 - 5x^2)^3 + 1200x^2(10 - 5x^2)^2$

B) $f(x) = \dfrac{x^2 + 4}{2 - x^4}$

$f'(x) = \dfrac{(x^2 + K).(2 - x^4) - (x^2 + 4)(2 - x^4)}{(2 - x^4)^2}$

$f'(x) = \dfrac{(2x).(2 - x^4) - (x^2 + 4).(-4x^3)}{(2 - x^4)^2}$

$f'(x) = \dfrac{4x - 2x^5 - (-4x^5 - 16x^3)}{(2 - x^4)^2}$

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$$f'(x) = \frac{4x + 2x^5 + 16x^3}{(2-x^4)^2}$$

$$f''(x) = \frac{4x + 2x^5 + 16x^3}{(2-x^4)^2}$$

$$f''(x) = \frac{(4+10x^4+48x^2)(2-x^4)^2 - (-8x^3)(2-x^4)(4x+2x^5+16x^3)}{((2-x^4)^2)^2}$$

$$f''(x) = \frac{(2-x^4)(4+10x^4+48x^2) + 8x^3(2-x^4)(4x+2x^5+16x^3)}{(2-x^4)^4}$$

$$f''(x) = \frac{8 + 20x^4 + 96x^2 - 4x^4 - 10x^8 + 96x^2 - 48x^6 + (4x+2x^5+16x^3)8x^3}{(2-x^4)^3}$$

$$f''(x) = 8 + 20x^4 + 96x^2 - 4x^4 - 10x^8 + 96x^2 - 48x^6 + \underbrace{(4x+2x^5+16x^3)\,8x^3}_{-48x^6 + 32x^4 + 16x^8 + 128x^6}$$

$$f'''(x) = \frac{8 + 48x^4 + 6x^8 + 96x^2 + 80x^6}{(2-x^4)}$$

$$f''(x) = \frac{6x^8 + 80x^6 + 48x^4 + 96x^2 + 8}{(2-x^4)^3}$$

ⓒ $f_{(x)} = sen\,(cos\,x^3)$

$f'_{(x)} = cos\,(cos\,x^3).\,(-sen\,(x^3).\,3x^2)$

$f'(x) = -3x^2.\,cos\,(cos\,(x^3))\,sen\,x^3$

$f''(x) = -3x^2.\,cos(cos(x^3))\,sen\,x^3$

$f''(x) = -6x.\,cos(cos(x^3)).\,sen\,x^3.\,(-sen(cos(x^3)).\,sen\,x^3.\,(-sen(x^3)).\,3x^2.\,sen\,x^3.$

$\hookrightarrow cos\,x^3.\,3x^2$

$f''(x) = -6x.\,cos(cos(x^3)).\,sen\,x^3 - 9x^4.\,sen\,(cos(x^3)).\,sen\,x^3.\,cos\,x^3$