Tecnologia em Sistemas de Computação

AP1X - Matemática Computacional

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A) $f(x) = \sqrt{10 - 3x}$

$y = \sqrt{10 - 3x}$

$y^2 = 10 - 3x$

$y^2 - 10 = -3x$

$\frac{y^2 - 10}{-3} = x$

$\frac{-y^2 - 10}{3} = x$

$f^{-1}(x) = \frac{-x^2 - 10}{3}$

(1)

(B) $f(x) = \dfrac{4x-1}{2x+3}$

$$y = \frac{4x-1}{2x+3}$$

$$y \cdot (2x+3) = 4x-1$$

$$2xy + 3y = 4x-1$$

$$2xy = 4x - 1 - 3y$$

$$2xy - 4x = -1 - 3y$$

$$2x(y-2) = -1 - 3y$$

$$\frac{2x(y-2)}{2(y-2)} = \frac{-1-3y}{2(y-2)}$$

$$x = \frac{-1-3y}{2(y-2)}$$

$$f^{-1}(x) = \frac{-1-3x}{2(x-2)}$$

(1)

(C) $f(x) = e^{x^3}$

$$\boxed{\ln(e) = \log_e e = 1}$$

$$y = e^{x^3}$$

$$\ln(y) = \ln(e^{x^3})$$

$$\ln(y) = x^3 \ln(e)$$

$$\ln(y) = x^3 \log_e e$$

$$\ln(y) = x^3$$

$$x^3 = \ln(y)$$

$$x = \sqrt[3]{\ln(y)}$$

$$f^{-1}(x) = \sqrt[3]{\ln(x)}$$

(1) (D) $f(x) = \frac{1 + e^x}{1 - e^x}$

$$y = \frac{1 + e^x}{1 - e^x}$$

$$y \cdot (1 - e^x) = 1 + e^x$$

$$y - ye^x = 1 + e^x$$

$$-ye^x = 1 + e^x - y$$

$$-ye^x - e^x = 1 - y$$

$$-e^x(y + 1) = 1 - y$$

$$e^x = \frac{1 - y}{-(y + 1)}$$

$$\ln(e^x) = \ln\left(\frac{1 - y}{-(y + 1)}\right)$$

$$x \cancel{\ln} = \ln\left(\frac{1 - y}{-(y + 1)}\right)$$

$$x = \ln\left(\frac{1 - y}{-(y + 1)}\right)$$

$$f^{-1}(x) = \ln\left(\frac{1 - x}{-(x + 1)}\right)$$

② (A) $f(x) = \dfrac{1}{x-a}$

$$\lim_{x \to a^-} \frac{1}{x-a} \Rightarrow \lim_{x \to a^-} \frac{1}{-a-a} \Rightarrow \lim_{x \to a^-} \frac{1}{-2a}$$

$$\lim_{x \to a^+} \frac{1}{x-a} \Rightarrow \lim_{x \to a^+} \frac{1}{a-a} = +\infty$$

(B) $f(x) = \dfrac{1}{(x-a)^2} \Rightarrow f(x) \dfrac{1}{(x-a).(x-a)}$

$$\lim_{x \to a^-} \frac{1}{(-a-a).(-a-a)} \Rightarrow \lim_{x \to a^-} \frac{1}{-4a} \Rightarrow \lim_{x \to a^-} \frac{-1}{4a}$$

$$\lim_{x \to a^+} \frac{1}{(a-a).(a-a)} \Rightarrow \lim_{x \to a^+} +\infty$$

$$\lim_{x \to a} \frac{1}{(a-a).(a-a)} \Rightarrow \lim_{x \to a} +\infty$$

② ⓒ $f(x)\ \frac{-1}{x-a}$

$$\lim_{x \to a^-} \frac{-1}{x-a} \Rightarrow \lim_{x \to a^-} \frac{-1}{-a-a} \Rightarrow \lim_{x \to a^-} \frac{1}{2a}$$

$$\lim_{x \to a^+} \frac{-1}{x-a} \Rightarrow \lim_{x \to a^+} \frac{-1}{a-a} \Rightarrow \lim_{x \to a^+} -\infty$$

ⓓ $f(x) = \frac{-1}{(x-a)^2} \Rightarrow f(x) = \frac{-1}{(x-a)\cdot(x-a)}$

$$\lim_{x \to a^-} \frac{-1}{(x-a)\cdot(x-a)} \Rightarrow \lim_{x \to a^-} \frac{-1}{(-a-a)\cdot(-a-a)} \Rightarrow \lim_{x \to a^-} \frac{1}{4a}$$

$$\lim_{x \to a^+} \frac{-1}{(x-a)\cdot(x-a)} \Rightarrow \lim_{x \to a^+} \frac{-1}{(a-a)\cdot(a-a)} \Rightarrow \lim_{x \to a^+} -\infty$$

$$\lim_{x \to a} \frac{-1}{(x-a)\cdot(x-a)} \Rightarrow \lim_{x \to a} \frac{-1}{(a-a)\cdot(a-a)} \Rightarrow \lim_{x \to a} -\infty$$

(3) $f(x) = \begin{cases} x^2 - 3 & x \leq 3 \\ \sqrt{x+13} & x > 3 \end{cases}$

$$\lim_{x \to 3} f(x)$$

$$\lim_{x \to 3} x^2 - 3 \Rightarrow \lim_{x \to 3} 3^2 - 3 \Rightarrow 6$$

$$\lim_{x \to 3^+} \sqrt{x+13} \Rightarrow \lim_{x \to 3^+} \sqrt{3+13} \Rightarrow \lim_{x \to 3^+} 4$$

$$\lim_{x \to 3^-} x^2 - 3 \Rightarrow \lim_{x \to 3^-} -3^2 - 3 \Rightarrow \lim_{x \to 3^-} 6$$

(4) $f(x) = \sqrt{9 - x^2}$

$9 - x^2 \geq 0$ $\boxed{x^2 = -x^2}$

$9 \geq x^2$

$\sqrt{9} \geq x$

$\boxed{3 \geq x}$

PORTANTO TEREMOS QUE $3 \geq x \geq -3$;

$\lim\limits_{x \to 3} \sqrt{9 - x^2} \Rightarrow \lim\limits_{x \to 3} \sqrt{9 - 3^2} \Rightarrow \lim\limits_{x \to 3} 0$

$\lim\limits_{x \to 3^-} \sqrt{9 - x^2} \Rightarrow \lim\limits_{x \to 3^-} \sqrt{9 - (-3)^2} \Rightarrow \lim\limits_{x \to 3^-} 0$

$\lim\limits_{x \to 3^+} \sqrt{9 - x^2} \Rightarrow \lim\limits_{x \to 3^+} \sqrt{9 - 3^2} \Rightarrow \lim\limits_{x \to 3^+} 0$

$f(3) = \sqrt{9 - 3^2} \Rightarrow f(3) = 0$

$f(-3) = \sqrt{9 - (-3)^2} \Rightarrow f(-3) = 0$

Logo NÃO HÁ descontinuidade NA função,

Logo A MESMA É UMA FUNÇÃO CONTINUA.

$\lim f(x) = f(3)$

(5) (A) $f(x) = \frac{3}{\sqrt{x}+3}$

$f'(x) = -3\left(\frac{1}{2}x\right)^{-1/2}$

$f'(x) = \frac{-3 \cdot \frac{1}{2\sqrt{x}}}{(\sqrt{x}+2)^2}$

$f'(x) = \frac{-3}{2\sqrt{x}(\sqrt{x}+2)^2}$

$f'(1) = \frac{-3}{2 \cdot 1(1+2)^2}$

$f'(1) = \frac{-3}{2 \cdot 9}$

$f'(1) = \frac{-1}{2 \cdot 3}$

$f'(1) = \frac{-1}{6}$

$$f'(x) = \frac{-3}{2\sqrt{x}\,(\sqrt{x}+2)^2}$$

$$f''(x) = 3\left[\frac{x^{-1/2}(\sqrt{x}+2)^2 + 2\sqrt{x}\cdot 2(\sqrt{x}+8)\cdot \frac{1}{2}x^{-1/2}}{\left[2\sqrt{x}\,(\sqrt{x}+2)^2\right]^2}\right]$$

$$f''(x) = \frac{3\left[x^{-1/2}(\sqrt{x}+2)\left[(\sqrt{x}+2)+2\sqrt{x}\right]\right]}{4x(\sqrt{x}+2)^4}$$

$$f''(1) = \frac{3\left[1(1+2)\left[(1+2)+2\cdot 1\right]\right]}{4\cdot(3)^4}$$

$$f''(1) = \frac{\cancel{45}^{\,5}}{4\cdot\cancel{81}^{\,9}} = \frac{5}{36}$$

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B $f(x) = (2x^7 - x^2)\left(\frac{x-1}{x+1}\right)$

$$f'(x) = (14x^6 - 2x)\left(\frac{x-1}{x+2}\right) + (2x^7 - x)\left(\frac{(x+1)-(x-1)}{(x+1)^2}\right)$$

$$f'(1) = \cancel{(14-2)\cdot 0} + (2\cdot 1 - 1)\left(\frac{2-0}{4}\right).$$

$$f'(1) = \frac{2}{4}$$

$$f'(1) = \frac{1}{2}$$

Obs.: Só consegui desenvolver até aqui

Favor considerar.

⑥ $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$

$f'(x) = 12x^3 + 12x^2 - 24x$

$12x(x^2 + x - 2)$

$12x(x+2)(x-1)$

$$\boxed{\begin{array}{l} x = 0 \\ x + 2 = 0 \Rightarrow x = -2 \\ x - 1 = 0 \Rightarrow x = 1 \end{array}}$$

Intervalo	$(12x)$	$(x+2)$	$(x-1)$	$f'(x)$	Conclusão
$x < -2$	$(-)$	$(-)$	$(-)$	$(-)$	f é decrescente em $(-\infty, -2]$
$-2 < x < 0$	$(-)$	$(+)$	$(-)$	$(+)$	f é crescente em $[-2, 0]$
$0 < x < 1$	$(+)$	$(+)$	$(-)$	$(-)$	f é decrescente em $[0, 1]$
$1 < x$	$(+)$	$(+)$	$(+)$	$(+)$	f é crescente em $[1, +\infty)$