Tec. em Sistemas de Comp.
Glauber de Souza Faria
17213050160
Angra dos Reis - RJ

APX 2 - Mat. Comp.

① $y = F(x)$

$\Rightarrow F(x) = x^4 + 3x^3 - 5x + 1$

$\therefore F(0) = 0^4 + 3.(0)^3 - 5.(0) + 1 = 1$

$\boxed{(x,y) = (0,1)}$

$\Rightarrow F(1) = 1^4 + 3.(1)^3 - 5.(1) + 1 = 0$

$\boxed{(x,y) = (1,0)}$

Como Assíntotas horizontais, tem-se:

$\lim\limits_{x \to +\infty} x^4 + 3x^3 - 5x + 1 = +\infty$

$\lim\limits_{x \to -\infty} x^4 + 3x^3 - 5x + 1 = +\infty$ $\boxed{\infty^4 > \infty^3}$

$\nexists$ Assíntotas verticais, pois se trata de um polinômio.

Como pontos críticos, teremos:

$$f'(x) = 0$$

$$f'(x^*) = (x^4 + 3x^3 - 5x + 1)' = 0$$

$$f'(x^*) = 4x^3 + 9x^2 - 5 = 0$$

Intuitivamente vemos que uma das 3 raízes é $x = -1$

$$f'(-1) = 4\cdot(-1)^3 + 9\cdot(-1)^2 - 5 = -4 + 9 - 5 = 0$$

Utilizando o método de Briot-Ruffini, temos:

$$\cancel{\begin{array}{l|l} 4x^3 + 9x^2 + 5 & x - 1 \\ \hline 4x^3 \quad 4x^2 & 4x^2 + 5x \\ 0 + 5x^2 - 5 & \end{array}}$$

$$\begin{array}{r|rrrr} -1 & 4 & 9 & 0 & -5 \\ \hline & 4 & 5 & -5 & 0 \end{array}$$

$\to$ Coeficientes

$\to$ Novos coeficientes

$$\boxed{4x^2 + 5x - 5 = 0}$$

Aplicando Bhaskara, teremos

$$\frac{-5 \pm \sqrt{5^2 + 80}}{8} = x_2 \text{ e } x_3$$

$$\frac{-5 + \sqrt{105}}{8} = x_2$$

$$X_3 = \frac{-5 - \sqrt{105}}{8}$$

$$X_1 = -1$$

$$f'(-1) = f'\left(\frac{-5+\sqrt{105}}{8}\right) = f'\left(\frac{-5-\sqrt{105}}{8}\right) = 0$$

$$f''(x) = (4x^3 + 9x^2 - 5)'$$

$$f''(x) = 12x^2 + 18x$$

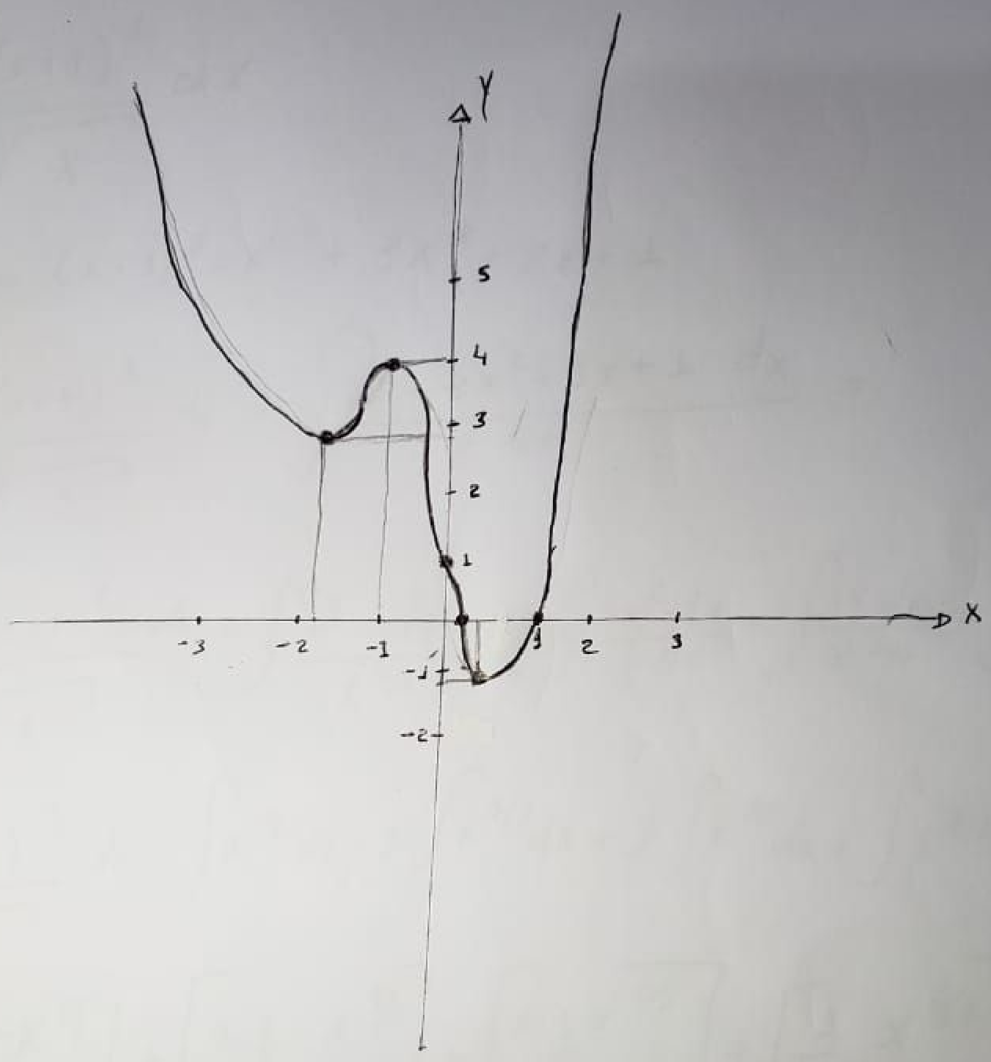
$$f''(-1) = 12.(-1)^2 + 18.(-1) = -6 < 0 \Rightarrow \text{máximo Local}$$

$$f''\left(\frac{-5+\sqrt{105}}{8}\right) = 12.\left(\frac{-5+\sqrt{105}}{8}\right)^2 + 18.\left(\frac{-5+\sqrt{105}}{8}\right).$$

$$f''\left(\frac{-5+\sqrt{105}}{8}\right) = \frac{105 + 3\sqrt{105}}{8} > 0 \Rightarrow \text{mínimo Local}$$

$$f''\left(\frac{-5-\sqrt{105}}{8}\right) = 12.\left(\frac{-5-\sqrt{105}}{8}\right)^2 + 18.\left(\frac{-5-\sqrt{105}}{8}\right)$$

$$f''\left(\frac{-5-\sqrt{105}}{8}\right) = \frac{105 - 3\sqrt{105}}{8}. > 0 \Rightarrow \text{mínimo LOCAL}$$


Y
X
5
4
3
2
1
-1
-2
-3
-2
-1
1
2
3

(2)

(A) $$\int \frac{(x+1)^3}{\sqrt[3]{x}}\,dx$$

Como $(x+1)^3 = x^3 + 3x^2 + 3x + 1$

$$\int \frac{(x+1)^3}{\sqrt[3]{x}}\,dx = \int \frac{x^3 + 3x^2 + 3x + 1\,dx}{x^{1/3}} =$$

$$\int \frac{(x+1)^3}{\sqrt[3]{x}}\,dx = \int \frac{x^3\,dx}{x^{1/3}} + \int \frac{3x^2\,dx}{x^{1/3}} + \int \frac{3x\,dx}{x^{1/3}} + \int \frac{1\,dx}{x^{1/3}} =$$

$$\int \frac{(x+1)^3}{\sqrt[3]{x}}\,dx = \int x^{8/3}dx + 3\int x^{5/3}dx + 3\int x^{2/3}dx + \int x^{-1/3}dx =$$

$$= \left[\frac{1}{\frac{11}{3}} \cdot x^{\frac{11}{3}}\right] + \left[3 \cdot \frac{1}{\frac{8}{3}} \cdot x^{8/3}\right] + \left[3 \cdot \frac{1}{5/3}\, x^{5/3}\right] + \left[\frac{1}{2/3}\, x^{2/3}\right] + C$$

$$= \left[\frac{3}{11}\, x^{\frac{11}{3}}\right] + \left[3 \cdot \frac{3}{8}\, x^{8/3}\right] + \left[3 \cdot \frac{3}{5} \cdot x^{5/3}\right] + \left[\frac{3}{2}\, x^{2/3}\right] + C$$

$$= \frac{3}{11} x^{11/3} + \frac{9}{8} x^{8/3} + \frac{9}{5} x^{5/3} + \frac{3}{2} x^{2/3} + C$$

$$= \frac{3}{11}\sqrt[3]{x^{11}} + \frac{9}{8}\sqrt[3]{x^{8}} + \frac{9}{5}\sqrt[3]{x^{5}} + \frac{3}{2}\sqrt[3]{x^{2}} + C$$

$$= \frac{120\sqrt[3]{x^{11}} + 495\sqrt[3]{x^{8}} + 792\sqrt[3]{x^{5}} + 660\sqrt[3]{x^{2}}}{440} + C$$

(B) $\int 3x^2 \sqrt[3]{1-2x^3}\,dx$

Se $U = 1 - 2x^3 \therefore du = -6x^2 dx \Rightarrow dx = \frac{dU}{-6x^2}$

Logo:

$$\int 3x^2 \sqrt[3]{U}\,\frac{dU}{-6x^2} = \int \frac{-3}{6}\sqrt[3]{U}\,dU$$

$$= \frac{-1}{2}\int U^{1/3}\,dU = \frac{-1}{2}\left(\frac{1}{\frac{4}{3}}\cdot U^{4/3}\right) + C$$

$$= \frac{-1}{2}\left(\frac{1}{\frac{4}{3}}\, U^{4/3}\right) + C = \frac{-1}{2}\left(\frac{3}{4}\, U^{4/3}\right) + C$$

$$= \frac{-3}{8}\, U^{4/3} + C -$$

$$\Rightarrow \frac{-3}{8}\left(1 - 2x^3\right)\frac{4}{3} + C$$

$$\therefore \int 3x^2 \cdot \sqrt[3]{1-2x^3}\,dx = -\frac{3}{8}\sqrt[3]{(1-2x^3)^4} + C$$

(c) $\int sen^2 \frac{x}{2} \, dx$

Se $u = \frac{x}{2} \therefore du = \frac{1}{2} dx \Rightarrow dx = 2du$

Logo:

$$\int sen^2 u \cdot 2du = 2\int sen^2 u \, du$$

Como $sen^2(x) = \frac{1 - cos(2x)}{2}$

Logo:

$$2\int \frac{1 - cos(2u)}{2} du = \frac{2}{2}\int 1 - cos(2u) \, du$$

$$= \int 1 \, du - \int cos(2u) \, du$$

$$= u - 1/2 \, sen(2u) + C$$

Portanto:

$$\frac{x}{2} - \frac{1}{2} sen\left(2 \cdot \frac{x}{2}\right) + C = \int sen^2 \frac{x}{2} dx$$

$$\int sen^2 \frac{x}{2} dx = \frac{1}{2}\left(x - sen(x)\right) + C$$

③

Ⓐ $\lim_{x \to \pi/2} \frac{1 - \operatorname{sen} x}{\cos x}$

Como $\lim_{x \to \pi/2} \frac{1 - \operatorname{sen} x}{\cos x} = \frac{0}{0}$,

Logo: por L'Hospital:

$$\lim_{x \to \pi/2} \frac{(1 - \operatorname{sen} x)'}{(\cos x)'} = \lim_{x \to \pi/2} \frac{(-\cos x)}{(-\operatorname{sen} x)} = \lim_{x \to \pi/2} \frac{\cos x}{\operatorname{sen} x}$$

$$= \frac{\cos\left(\frac{\pi}{2}\right)}{\operatorname{sen}\left(\frac{\pi}{2}\right)} = \frac{0}{1} = 0$$

$$\therefore \lim_{x \to \pi/2} \frac{1 - \operatorname{sen} x}{\cos x} = 0$$

③ Ⓑ $\lim_{x \to 0} \frac{e^x - 1}{x^4}$

Como $\lim_{x \to 0} \frac{e^x - 1}{x^4} = \frac{1-1}{0} = \frac{0}{0}$

Por L'Hospital

$$\lim_{x \to 0} \frac{(e^x - 1)'}{(x^4)'} = \lim_{x \to 0} \frac{(e^x)' - (1)'}{(x^4)'} =$$

$$\lim_{x \to 0} \frac{e^x - 0}{4x^3} = \lim_{x \to 0} \frac{e^x}{4x^3} = L$$

$$\lim_{x \to 0^+} \frac{e^x}{4x^3} = \frac{1}{0} = +\infty$$

$$\lim_{x \to 0^-} \frac{e^x}{4x^3} = \frac{-1}{0} = -\infty$$

$\therefore \lim_{x \to 0^+} \frac{e^x}{4x^3} \neq \lim_{x \to 0^-} \frac{e^x}{4x^3} \Rightarrow L$ diverge

Assim $\lim_{x \to 0} \frac{e^x - 1}{x^4}$ diverge

(3) (c) $\lim\limits_{x \to 0^+} \dfrac{\ln x}{\operatorname{cosec} x}$

Como cosec 0, é não definida, Aplicaremos a Regra de L'Hospital.

$$\lim_{x \to 0^+} \frac{(\ln x)'}{(\operatorname{cossec} x)'} = \lim_{x \to 0} \frac{1/x}{-\cot(x)\operatorname{cossec}(x)}$$

$$\lim_{x \to 0^+} \frac{-1}{x \cdot \cot(x) \cdot \operatorname{cossec}(x)} = \frac{-1}{\infty} = 0$$

(4) (A)

$$f(x) = y = -x^2 - 3x + 6$$

$$g(x) = y = 3 - x$$

Os pontos em comum de $g(x)$ e $f(x)$ são dados por:

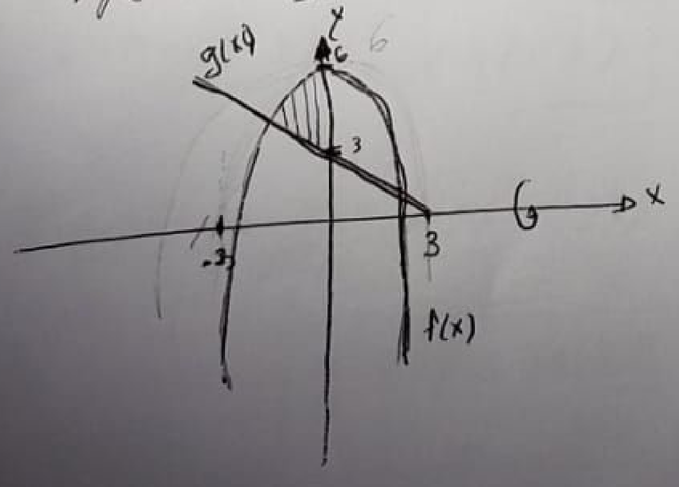
$$f(x) = y = g(x) \Rightarrow -x^2 - 3x + 6 = 3 - x$$

$$= -x^2 - 2x + 3 = 0$$

Por soma e produto temos as seguintes raízes:

$$\left.\begin{array}{l} -3 + 1 = -2 \\ -3 \cdot 1 = -3 \end{array}\right\} \begin{array}{l} x_1 = -3 \\ x_2 = 1 \end{array}$$

Como é delimitado pelo 2º Quadrante, usar-se-á apenas x_1.



$$\int_{-3}^{0} \pi . [f(x)^2 - g(x)^2]\, dx$$

$$\int_{-3}^{0} \pi [(-x^2 - 3x + 6)^2 - (3 - x)^2]\, dx$$

$$\pi \int_{-3}^{0} (-x^2 - 3x + 6)^2 - (3 - x)^2\, dx$$

$$= \pi \int_{-3}^{0} x^4 + 6x^3 - 4x^2 - 30x + 27\, dx$$

Aplicando Regra da Soma, Temos:

$$\pi \left[\int_{-3}^{0} x^4\, dx + \int_{-3}^{0} 6x^3\, dx - \int_{-3}^{0} 4x^2\, dx - \int_{-3}^{0} 30x\, dx + \int_{-3}^{0} 27\, dx \right]$$

$$\pi . \left[\frac{x^5}{5} + \frac{6x^4}{4} - \frac{4x^3}{3} - \frac{30x^2}{2} + 27x \right] \Bigg|_{-3}^{0}$$

$$\pi . \left[\frac{0^5}{5} + \frac{6.(0)^4}{4} - \frac{4.(0)^3}{3} - \frac{30(0)^2}{2} + 27.0 \right] - \pi \left[\frac{-3^5}{5} + \frac{6.(-3)^4}{4} \right.$$

$$\left. - \frac{4.(-3)^3}{3} - \frac{30.(-3)^2}{2} + 27(-3) \right]$$

$$= 0 - \pi \left(-\frac{243}{5} + \frac{243}{2} + 36 + 135 - 81 \right)$$

$$= \pi . \frac{1071}{10}$$

$$\boxed{V = \pi . \frac{1071}{10}}$$

4

B