I. Introduction

To send our astronauts to the Moon, we need to make many calculations surrounding both the gravity of the Earth and Moon, as well as calculations predicting the performance of Saturn V. Launching a spacecraft is essentially a battle against the Earth's gravity; thus we need to comprehensively understand the effects of gravitational fields around the Earth and Moon. However, these calculations can be hard, impossible, or simply inefficient to do analytically or numerically by hand. We use the power of computers and programming language python to create programs which can calculate important factors such as the gravitational potential field and the gravitational force field over large areas.

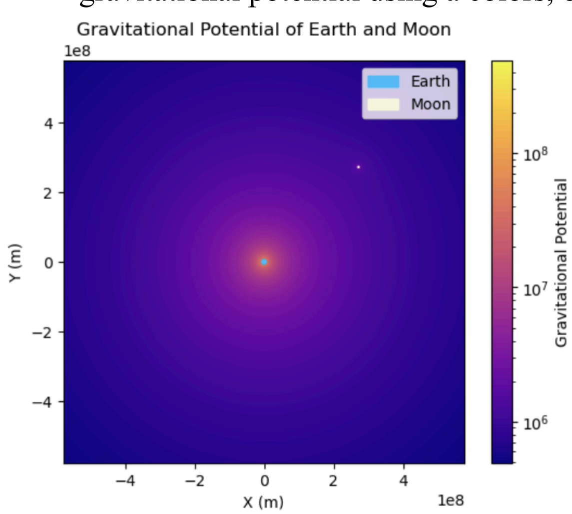
II. The Gravitational Potential of the Earth-Moon system

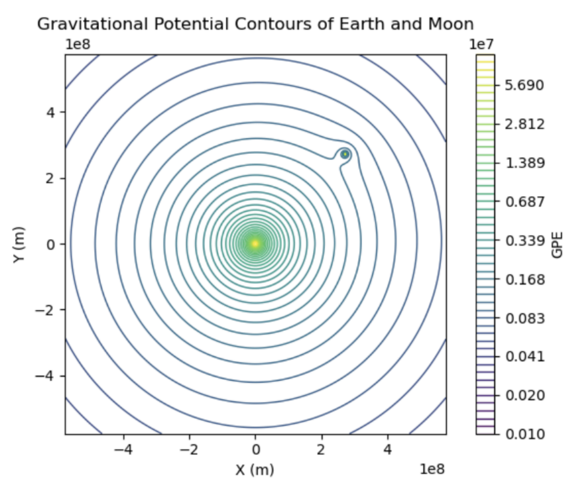
Gravitational potential is the gravitational energy felt by a test particle from a body massive enough to exert non-negligible gravitational pull. Gravitational potential is inversely proportional to the distance of the test particle from the massive body.

$$\Phi(r) = -\frac{GM}{r}$$

Gravitational potential depends on mass and is inversely proportional to distance

To graph the gravitational potential field in python, we create a function which allows python to find the gravitational potential at a certain point by taking the coordinates of that point as well as the coordinates and mass in kg of the object creating the potential. We can then find the potential along a wide array of coordinate points (in an area around 1.5x the distance between the Earth and Moon), and graph our results along a coordinate plane. Below are two graphs showcasing the gravitational potential energy the Apollo 11 rocket would feel at different locations. We can represent the strength of the gravitational potential using a colors, or by taking a contour plot:





Overall, by calculating the gravitational potential field, we can see how Apollo would be affected by the Earth and Moon's gravity.

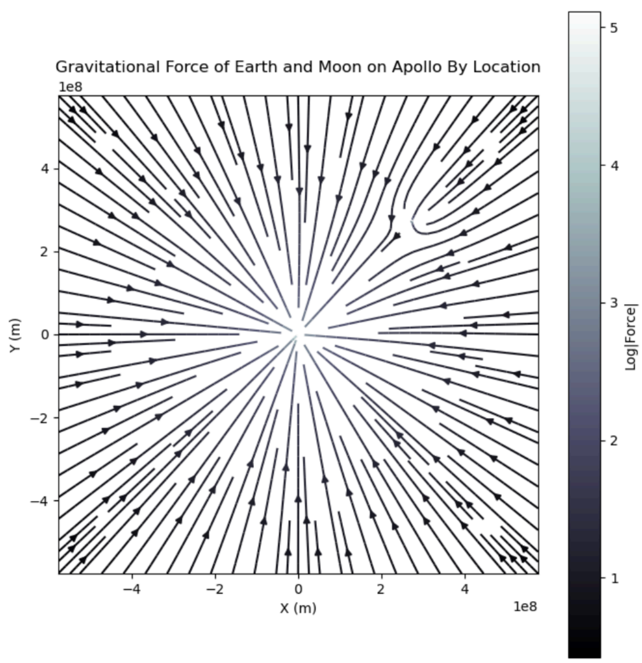
III. The Gravitational Force of the Earth-Moon system

Gravitational force is the actual force acting between two objects due to gravity. Whereas gravitational potential is characteristic of only a single mass, gravitational force depends on the mass of both objects being considered. Gravitational force helps us understand the acceleration felt by the spacecraft.

To illustrate the gravitational force the rocket would feel due to the Earth and Moon at different locations using python, we first create a function that describes the gravitational force by taking the coordinate locations and masses of the two bodies and calculates the force they exert on and experience from each other according to the equation:

$$\vec{F}_{21} = -G\frac{M_1 m_2}{|\vec{r}_{21}|^2}\hat{r}_{21}$$

We can then plot this function over a series of coordinate points in the x and y directions. We illustrate the gravitational force using arrows, with the length and direction of arrows symbolizing the magnitude and direction of the force. Finally, inputting the masses and locations of the Earth, Moon, and the mass of Apollo 11, we can graph the gravitational force felt by Apollo at any given location about 1.5x the distance between the Earth and Moon:



IV. Projected performance of the Saturn V Stage 1

Aside from the effects of gravity, we also need to calculate the efficiency of the Saturn V rocket; namely how high it can launch the Apollo 11 spacecraft given a certain amount of fuel. We must consider the mass with and without the fuel, the burn rate, exhaust speed, as well as the force of gravity near the surface of Earth.

First we can calculate the total time it takes for the rocket to burn its fuel:

$$T = \frac{m_0 - m_f}{\dot{m}}$$

Where T is the burn time, m0 is the mass of the rocket with fuel, mf is the mass of the rocket without fuel, and m* is how many kilograms of fuel the rocket burns each second.

Finally, we can calculate the final altitude of the rocket by first finding the change in velocity in terms of the initial masses, and then integrating the change in velocity over the total burn rate time to get overall displacement:

$$h = \int_0^T \Delta v(t) dt$$

$$\Delta v(t) = v_e \ln\left(\frac{m_0}{m(t)}\right) - gt$$

Calculating these values in python, we estimate an burn time of **157.7 seconds** and a final altitude of **74.2 km**.

V. Discussion and Future Work

Approximations: it is important to note some of the approximations taken in making these calculations and creating graphs. Formulas for gravitational potential and gravitational force must be approximated at very low distances from the Earth and Moon's coordinate locations, as the classical formulas blow up to high numbers near the location of the body, which the computer cannot handle.

Future work: to make the calculations more realistic, we can use more exact numbers for the masses of the Earth, Moon, Apollo, and the wet and dry masses as well as the burn rate.

Comparing with test results: The calculated burn time using python was around 157.7 seconds. In practice, the rocket burned for about 160 seconds, making our calculation an underestimate. Potential reasons for the longer burn time in experiment include impurities in the fuel causing it to take longer to burn, and perhaps the wet and dry mass used in the calculations was not perfectly accurate.

The calculated altitude was also around 74.2 km with a very small calculated error (however, this error assumes all the measured values are exact). In experiment the system was lifted to about 70 km, making our calculation an overestimate. This discrepancy can be attributed to several factors, with the most obvious being air resistance and drag which would counteract the rocket's ascent. Additionally, the fuel efficiency in practice might be lower than ideal due to impurities or other mechanical imperfections.