Update_accept

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```
#Inputs
  library(Deriv)
g <- function(x) {
  (3/sqrt(2*pi))*exp(-(x^2)/2)
}
X <- c(-1.7,1.5)
X <- sort(X)
#x_star <- runif(1,min = X[1], max = X[2])

  g_prime <- Deriv(g)
  update_H <- function(x) {log(g(x))}
  update_H_prime <- function(x) {g_prime(x)/g(x)}

  H <- sapply(X, update_H)
  H_prime <- sapply(X, update_H_prime)


  z <- function(i) {
  (H[i+1]-H[i]-X[i+1]*H_prime[i+1]+X[i]*H_prime[i])/(H_prime[i]-H_prime[i+1])
  }
  Z <- c(-Inf,Inf)
  for (z_order in 1:length(X)-1) {
      Z <- append(Z,z(z_order),z_order)
    }
x_accept <- c()
length_accept <- 1
```

```
#Note: added H, H_prime, Z as parameters to function
update_accept <- function(x_star, g, X, H, H_prime, Z) {
  library(Deriv)
  #g_prime <- Deriv(g)

  #update_H <- function(x) {log(g(x))}
  #update_H_prime <- function(x) {g_prime(x)/g(x)}

  l <- function(x,i) {
   #leval <- max(((X[i+1]-x)*H[i] + (x-X[i])*H[i+1])/(X[i+1]-X[i]),-Inf)
   #if (!(is.numeric(leval))){
  #   leval <- -Inf
  # }
   if (x < X[length(X)] & x > X[1]){ #x in [x_i,x_{i+1}]
      leval <- ((X[i+1]-x)*H[i] + (x-X[i])*H[i+1])/(X[i+1]-X[i])
    } else { #x < x_1 or x > x_k
      leval <- -Inf
    }
   return(leval)
  }
```

```r
u <- function(x,i) {H[i] + (x-X[i])*H_prime[i]}

#H <- sapply(X, update_H)
#H_prime <- sapply(X, update_H_prime)
z <- function(i) {
(H[i+1]-H[i]-X[i+1]*H_prime[i+1]+X[i]*H_prime[i])/(H_prime[i]-H_prime[i+1])
}

#Z <- c()
#for (z_order in 1:length(X)-1) {
#    Z[z_order] <- z(z_order)
#  }


w <- runif(1)

i_star_x <- max(which.max(X[X<x_star]),0)
  #i_star_z <- which.min(Z[x_star<Z])
  i_star_z <- length(Z[x_star>=Z])
print(paste0("x_star is: ",x_star))
print(paste0("X :",list(X)))
print(paste0("Z is: ",list(Z)))
print(paste0("i_star_x is: ", i_star_x))
print(paste0("i_star_z is: ", i_star_z))
print(paste0("L is: ",l(x_star,i_star_x)))
print(paste0("U is: ",u(x_star,i_star_z)))

if(w <= exp(l(x_star,i_star_x) - u(x_star,i_star_z))){
  x_accept[length_accept] <<- x_star
  length_accept <<- length_accept+1
  print(paste0("No update. New x_accept"))

} else {
  #Update step
  if (x_star %in% X){#x* in X
    print("x* already in X, no modifications to abscissae, H, or H_prime")
  }
  else {#x* NOT in X
    X <- append(X, x_star,i_star_x)
    #X <- sort(X)
    H <- append(H,update_H(x_star),i_star_x)
    H_prime <- append(H_prime,update_H_prime(x_star),i_star_x)

    if (x_star != X[1] & x_star != X[length(X)]){ #x_star outside other X
      Z[i_star_x+1] <- z(i_star_x) #Overwrite a z
      Z <- append(Z,z(i_star_x+1),i_star_x+1) #Append a new z
    } else {
      Z <- append(Z,z(i_star_z),i_star_z)
    }

    #Z <- append(Z,Z_2,i_star_x+1)
    #for (z_order in 1:length(X)-1) {
    #  Z[z_order] <- z(z_order)}
```

```
      #X <<- X
      #H <<- H
      #H_prime <<- H_prime
      #Z <<- Z
      print(paste0("Abscissae is updated"))
      print(paste0("New H is updated"))
      print(paste0("New H_prime is updated"))
    }

    if (w <= exp(log(g(x_star)) - u(x_star,i_star_z))) {
      x_accept[length_accept] <<- x_star
      length_accept <<- length_accept+1
      print(paste0("Update step is done. New x_accept"))
    }
  }
  #return(list(X, H, H_prime, Z))
  X <<- X
  H <<- H
  H_prime <<- H_prime
  Z <<- Z
}

#update_accept(0.2,g,X)
```

Test

```
for (i in (1:25)) {
  x_star <- runif(1,min=-2,max=2)
  update_accept(x_star,g,X,H,H_prime,Z)
}
```

```
## [1] "x_star is: -0.795404932461679"
## [1] "X :c(-1.7, 1.5)"
## [1] "Z is: c(-Inf, -0.1, Inf)"
## [1] "i_star_x is: 1"
## [1] "i_star_z is: 1"
## [1] "L is: -1.17486673778273"
## [1] "U is: 0.272485370278583"
## [1] "Abscissae is updated"
## [1] "New H is updated"
## [1] "New H_prime is updated"
## [1] "Update step is done. New x_accept"
## [1] "x_star is: -0.179629785940051"
## [1] "X :c(-1.7, -0.795404932461679, 1.5)"
## [1] "Z is: c(-Inf, -1.24770246623084, 0.352297533769161, Inf)"
## [1] "i_star_x is: 2"
## [1] "i_star_z is: 2"
## [1] "L is: -0.35359681330466"
## [1] "U is: 0.353129841001869"
## [1] "Abscissae is updated"
## [1] "New H is updated"
## [1] "New H_prime is updated"
```

```
## [1] "x_star is: 0.35220683645457"
## [1] "X :c(-1.7, -0.795404932461679, -0.179629785940051, 1.5)"
## [1] "Z is: c(-Inf, -1.24770246623084, -0.487517359200865, 0.660185107029974, Inf)"
## [1] "i_star_x is: 3"
## [1] "i_star_z is: 3"
## [1] "L is: -0.18757029201305"
## [1] "U is: 0.259074024100828"
## [1] "No update. New x_accept"
## [1] "x_star is: -1.95779237803072"
## [1] "X :c(-1.7, -0.795404932461679, -0.179629785940051, 1.5)"
## [1] "Z is: c(-Inf, -1.24770246623084, -0.487517359200865, 0.660185107029974, Inf)"
## [1] "i_star_x is: 0"
## [1] "i_star_z is: 1"
## [1] "L is: -Inf"
## [1] "U is: -1.70357328718878"
## [1] "Abscissae is updated"
## [1] "New H is updated"
## [1] "New H_prime is updated"
## [1] "Update step is done. New x_accept"
## [1] "x_star is: -1.9483199538663"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 1.5)"
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, 0.660185107029974, Inf)"
## [1] "i_star_x is: 1"
## [1] "i_star_z is: 1"
## [1] "L is: -1.71947766181911"
## [1] "U is: -1.71825670244358"
## [1] "No update. New x_accept"
## [1] "x_star is: 1.9621376413852"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 1.5)"
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, 0.660185107029974, Inf)"
## [1] "i_star_x is: 5"
## [1] "i_star_z is: 5"
## [1] "L is: -Inf"
## [1] "U is: -1.63853270661436"
## [1] "Abscissae is updated"
## [1] "New H is updated"
## [1] "New H_prime is updated"
## [1] "Update step is done. New x_accept"
## [1] "x_star is: -0.911278015933931"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 1.5, 1.9621376413852)"
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, 0.660185107029974, 1.73"
## [1] "i_star_x is: 2"
## [1] "i_star_z is: 3"
## [1] "L is: -0.281235879846848"
## [1] "U is: -0.22882676996212"
## [1] "No update. New x_accept"
## [1] "x_star is: 1.74813713878393"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 1.5, 1.9621376413852)"
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, 0.660185107029974, 1.73"
## [1] "i_star_x is: 5"
## [1] "i_star_z is: 6"
## [1] "L is: -1.3748687087413"
## [1] "U is: -1.3254198649776"
## [1] "No update. New x_accept"
```

```
## [1] "x_star is: -0.308292386122048"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 1.5, 1.9621376413852)"
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, 0.660185107029974, 1.73
## [1] "i_star_x is: 3"
## [1] "i_star_z is: 4"
## [1] "L is: 0.100815074396359"
## [1] "U is: 0.14042869013582"
## [1] "No update. New x_accept"
## [1] "x_star is: -1.89199499879032"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 1.5, 1.9621376413852)"
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, 0.660185107029974, 1.73
## [1] "i_star_x is: 1"
## [1] "i_star_z is: 1"
## [1] "L is: -1.61646516613419"
## [1] "U is: -1.60798413470291"
## [1] "No update. New x_accept"
## [1] "x_star is: -0.799991752021015"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 1.5, 1.9621376413852)"
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, 0.660185107029974, 1.73
## [1] "i_star_x is: 2"
## [1] "i_star_z is: 3"
## [1] "L is: -0.142383733905086"
## [1] "U is: -0.140309126730554"
## [1] "No update. New x_accept"
## [1] "x_star is: 0.972575300373137"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 1.5, 1.9621376413852)"
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, 0.660185107029974, 1.73
## [1] "i_star_x is: 4"
## [1] "i_star_z is: 5"
## [1] "L is: -0.59712831276315"
## [1] "U is: -0.154189195096268"
## [1] "No update. New x_accept"
## [1] "x_star is: 1.49838128779083"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 1.5, 1.9621376413852)"
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, 0.660185107029974, 1.73
## [1] "i_star_x is: 4"
## [1] "i_star_z is: 5"
## [1] "L is: -0.944257594843504"
## [1] "U is: -0.942898176222815"
## [1] "No update. New x_accept"
## [1] "x_star is: 0.127375240437686"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 1.5, 1.9621376413852)"
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, 0.660185107029974, 1.73
## [1] "i_star_x is: 4"
## [1] "i_star_z is: 4"
## [1] "L is: -0.0391398207329239"
## [1] "U is: 0.218687572635756"
## [1] "Abscissae is updated"
## [1] "New H is updated"
## [1] "New H_prime is updated"
## [1] "Update step is done. New x_accept"
## [1] "x_star is: -0.208914778195322"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 0.127375240437686, 1.5, 1
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, -0.0261272727511826, 0.8
```

```
## [1] "i_star_x is: 3"
## [1] "i_star_z is: 4"
## [1] "L is: 0.149263383376496"
## [1] "U is: 0.158279868574933"
## [1] "No update. New x_accept"
## [1] "x_star is: -1.15769576188177"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 0.127375240437686, 1.5, 1
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, -0.0261272727511826, 0.8
## [1] "i_star_x is: 2"
## [1] "i_star_z is: 3"
## [1] "L is: -0.588691909189009"
## [1] "U is: -0.424828660535119"
## [1] "No update. New x_accept"
## [1] "x_star is: 1.36177327763289"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 0.127375240437686, 1.5, 1
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, -0.0261272727511826, 0.8
## [1] "i_star_x is: 5"
## [1] "i_star_z is: 6"
## [1] "L is: -0.83285287176302"
## [1] "U is: -0.737986160985901"
## [1] "No update. New x_accept"
## [1] "x_star is: 0.0886168368160725"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 0.127375240437686, 1.5, 1
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, -0.0261272727511826, 0.8
## [1] "i_star_x is: 4"
## [1] "i_star_z is: 5"
## [1] "L is: 0.170548878142335"
## [1] "U is: 0.176498390505442"
## [1] "No update. New x_accept"
## [1] "x_star is: -0.969729073345661"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 0.127375240437686, 1.5, 1
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, -0.0261272727511826, 0.8
## [1] "i_star_x is: 2"
## [1] "i_star_z is: 3"
## [1] "L is: -0.354165408333264"
## [1] "U is: -0.275319029335011"
## [1] "No update. New x_accept"
## [1] "x_star is: -0.792436891235411"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 0.127375240437686, 1.5, 1
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, -0.0261272727511826, 0.8
## [1] "i_star_x is: 3"
## [1] "i_star_z is: 3"
## [1] "L is: -0.135213776208118"
## [1] "U is: -0.134299953197624"
## [1] "No update. New x_accept"
## [1] "x_star is: 0.61544291395694"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 0.127375240437686, 1.5, 1
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, -0.0261272727511826, 0.8
## [1] "i_star_x is: 5"
## [1] "i_star_z is: 5"
## [1] "L is: -0.225573094246471"
## [1] "U is: 0.109393792260781"
## [1] "No update. New x_accept"
## [1] "x_star is: -1.30892798118293"
```

```
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 0.127375240437686, 1.5, 1
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, -0.0261272727511826, 0.8
## [1] "i_star_x is: 2"
## [1] "i_star_z is: 2"
## [1] "L is: -0.777384722184635"
## [1] "U is: -0.600503812547549"
## [1] "No update. New x_accept"
## [1] "x_star is: 0.932591898366809"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 0.127375240437686, 1.5, 1
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, -0.0261272727511826, 0.8
## [1] "i_star_x is: 5"
## [1] "i_star_z is: 6"
## [1] "L is: -0.48363329662576"
## [1] "U is: -0.0942140920867762"
## [1] "Abscissae is updated"
## [1] "New H is updated"
## [1] "New H_prime is updated"
## [1] "Update step is done. New x_accept"
## [1] "x_star is: -0.350446664728224"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 0.127375240437686, 0.9325
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, -0.0261272727511826, 0.5
## [1] "i_star_x is: 3"
## [1] "i_star_z is: 4"
## [1] "L is: 0.0802641318112584"
## [1] "U is: 0.132856526093336"
## [1] "No update. New x_accept"
## [1] "x_star is: 0.844155802391469"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 0.127375240437686, 0.9325
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, -0.0261272727511826, 0.5
## [1] "i_star_x is: 5"
## [1] "i_star_z is: 6"
## [1] "L is: -0.208320391177257"
## [1] "U is: -0.172715282356476"
## [1] "No update. New x_accept"
```

Test

```
x_star <- -5
update_accept(x_star,g,X,H,H_prime,Z)
```

```
## [1] "x_star is: -5"
## [1] "X :c(-1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 0.127375240437686, 0.9325
## [1] "Z is: c(-Inf, -1.82889618901536, -1.24770246623084, -0.487517359200865, -0.0261272727511826, 0.5
## [1] "i_star_x is: 0"
## [1] "i_star_z is: 1"
## [1] "L is: -Inf"
## [1] "U is: -7.69281263695256"
## [1] "Abscissae is updated"
## [1] "New H is updated"
## [1] "New H_prime is updated"
## [1] "Update step is done. New x_accept"
```

```
  x_star <- 6
  update_accept(x_star,g,X,H,H_prime,Z)
```

```
## [1] "x_star is: 6"
## [1] "X :c(-5, -1.95779237803072, -1.7, -0.795404932461679, -0.179629785940051, 0.127375240437686, 0.9
## [1] "Z is: c(-Inf, -3.47889618901536, -1.82889618901536, -1.24770246623084, -0.487517359200865, -0.0
## [1] "i_star_x is: 9"
## [1] "i_star_z is: 9"
## [1] "L is: -Inf"
## [1] "U is: -9.66816003097741"
## [1] "Abscissae is updated"
## [1] "New H is updated"
## [1] "New H_prime is updated"
```

```
print(x_accept)
```

```
##  [1] -0.79540493  0.35220684 -1.95779238 -1.94831995  1.96213764
##  [6] -0.91127802  1.74813714 -0.30829239 -1.89199500 -0.79999175
## [11]  0.97257530  1.49838129  0.12737524 -0.20891478 -1.15769576
## [16]  1.36177328  0.08861684 -0.96972907 -0.79243689  0.61544291
## [21] -1.30892798  0.93259190 -0.35044666  0.84415580 -5.00000000
```

```
print(X)
```

```
##  [1] -5.0000000 -1.9577924 -1.7000000 -0.7954049 -0.1796298  0.1273752
##  [7]  0.9325919  1.5000000  1.9621376  6.0000000
```

```
print(Z)
```

```
##  [1]        -Inf -3.47889619 -1.82889619 -1.24770247 -0.48751736
##  [6] -0.02612727  0.52998357  1.21629595  1.73106882  3.98106882
## [11]         Inf
```