Implementation Notes of the Stationary and Spatially Varying Velocity Models for SFBA

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1 Introduction

This note outlines the steps for implementing the stationary and spatially varying models presented in Lavrentiadis et al., 2025: Lavrentiadis et al., 2025 "Data-driven Characterization of Near-Surface Velocity in the San Francisco Bay Area: A Stationary and Spatially Varying Approach"

The required files and a Python implementation of the velocity models are available at: `https://github.com/glavrentiadis/svm_sfba`

2 Stationary model

1. Read V_{S30} at the given location
2. Compute V_{S30scl}, n and k

$$V_{S30scl} = (\ln(V_{S30}) - V_{S30ref})/V_{S30w} \tag{1}$$

$$n = 1 + s_2\ S(V_{S30scl}) \tag{2}$$

$$k = \exp\left[r_1 + r_2 S(V_{S30scl}) + r_3 V_{S30w} H(V_{S30scl})\right]; \tag{3}$$

where the two functions $S(x)$ and $H(x)$ for a given variable x are defined as follows:

$$S(x) = 1/(1 + \exp(-x)) \tag{4}$$

$$H(x) = x \ln(1 + \exp(x)) \tag{5}$$

NOTE: To compute the coefficients, use the Median values of Table 1.

Table 1: Model coefficients for the stationary model

Coefficient	V_{S30ref}	V_{S30w}	r_1	r_2	r_3	s_2	σ
Mean	6.5045	0.4368	-2.2960	5.4669	0.4236	7.1685	0.3759
Median	6.4990	0.4354	-2.2986	5.3966	0.3886	7.0741	0.3759
5^{th} Percentile	6.3505	0.3866	-2.4135	3.9247	0.0335	5.7509	0.3686
95^{th} Percentile	6.6780	0.4916	-2.1700	7.2551	0.9193	8.9065	0.3834

3. Compute V_{S0}

$$V_{S0} = V_{S30} \begin{cases} \frac{(1+(30-z^*)k)^{1/n} + 2.5k\left(1-\frac{1}{n}\right) - 1}{z^* k\left(1-\frac{1}{n}\right)} & \text{for } n \neq 1 \\ \frac{z^* + \frac{1}{k}\ln(1+k(30-z^*))}{30} & \text{for } n = 1 \end{cases} \tag{6}$$

4. Compute V_s for a given depth z

$$V_S(z) = \begin{cases} V_{S0} & \text{for } z \leq z^* \\ V_{S0}(1 + k(z - z^*))^{1/n} & \text{for } z > z^* \end{cases} \tag{7}$$

NOTE: $z^* = 2.5$ m.

5. Truncation criteria: $z_{\max} = \min\{z_1, z_2\}$ where
   (a) z_1: Depth at which $V_S(z) \geq 1000$ m/s
   (b) z_2: Depth at which $V_{\mathrm{USGS}}(z) \geq 1000$ m/s

   NOTE: If $z_{\max} = z_1$, for $z_{\max} \leq z \leq z_2$, $V_S(z) = 1000$ m/s.

3 Spatially varying model

1. Load the regression output file for the spatially varying model:
   (`Reporting/EQ_Spectra/data/svarying_model_stan_parameters.csv`)

   NOTES:

   (a) Regression file can be found on GitHub

   (b) Columns `X`, `Y` represent the profile location in UTM zone 10S

   (c) `param_dBr_med` and `param_dBr_std` contain the median and standard deviation of the profile-specific slope adjustment term.

2. Read V_{S30} and location, $\vec{x}_s$, for the profile of interest. Convert the profile location to UTM coordinates.

3. Compute the profile-specific slope adjustments (δB_r).
   Using the covariance function:

$$\kappa_{\delta B_r}(\vec{x}_s, \vec{x}_{s'}) = \omega_{\delta Br}^2 \exp\left(\frac{|\vec{x}_s - \vec{x}_{s'}|}{\ell_{\delta Br}}\right) \tag{8}$$

   compute the covariance matrices between the regression data:

$$\mathbf{K}_{ij} = \kappa_{\delta B_r}(\vec{x}_{s\ i}, \vec{x}_{s\ j}) \tag{9}$$

   where $\vec{x}_{s\ i}$ and $\vec{x}_{s\ i}$ are the coorindates of the i^{th} and j^{th} site in the regression dataset. Compute the covariance matrix between regression data and the the new location:

$$\mathbf{k}_i = \kappa_{\delta B_r}(\vec{x}_{s\ i}, \vec{x}_s) \tag{10}$$

   The mean of the δB_r can be computed as follows:

$$\mu_{\delta B_r} = \mathbf{k}^T\ \mathbf{K}^{-1}\ \delta \mathbf{B}_{r\ med} \tag{11}$$

   where $\delta \mathbf{B}_{r\ med}$ are the median slope adjustments of the regression dataset. The variance of δB_r is given by:

$$\sigma_{\delta B_r}^2 = \omega_{\delta B_r}^2 - \mathbf{k}^T\ \mathbf{K}^{-1}\ \mathbf{k} + \mathbf{k}^T\ \mathbf{K}^{-1}\ \operatorname{diag}(\delta \mathbf{B}_{r\ std})\ \mathbf{K}^{-T}\mathbf{k} \tag{12}$$

   where $\operatorname{diag}(\delta \mathbf{B}_{r\ std})$ is a diagonal matrix with the standard deviation of the slope adjustment of the regression dataset.

   NOTES:

   (a) For a single profile realization using the mean value of δB_r (Equation 11)

   (b) For multiple realizations sample δB_r from a normal distribution $\mathcal{N}(\mu_{\delta B_r}, \sigma_{\delta B_r})$

   (c) For a cross-section compute all slope adjustments together

4. Compute V_{S30scl}, n and k

$$V_{S30scl} = (\ln(V_{S30}) - V_{S30ref})/V_{S30w} \tag{13}$$

$$n = 1 + s_2\ S(V_{S30scl}) \tag{14}$$

$$k = \exp\left[r_1 + r_2 S(V_{S30scl}) + r_3 V_{S30\ w} H(V_{S30scl}) + \delta B_r\right] \tag{15}$$

   where the two functions $S(x)$ and $H(x)$ for a given variable x are defined as follows:

$$S(x) = 1/(1 + \exp(-x)) \tag{16}$$

$$H(x) = x \ln(1 + \exp(x)) \tag{17}$$

   NOTE: Use the Median values of Table 2 to compute k and Fixed Values to compute n and V_{S30scl}.

Table 2: Posterior distributions of spatially varying model coefficients

Coefficient	V_{S30ref}	V_{S30w}	r_1	r_2	r_3	s_2	σ	$\ell_{\delta Br}$ (km)	$\omega_{\delta Br}$
Fixed Value	6.4990	0.4355	-	-	0.3897	7.0713	-	-	-
Mean	-	-	-2.6097	5.9316	-	-	0.2807	1.9471	0.3159
Median	-	-	-2.6102	5.9329	-	-	0.2807	1.9104	0.3156
5th Percentile	-	-	-2.7168	5.6428	-	-	0.2746	1.4690	0.2860
95th Percentile	-	-	-2.5012	6.2179	-	-	0.2869	2.5526	0.3458

5. Compute V_{S0}

$$V_{S0} = V_{S30} \begin{cases} \frac{(1+(30-z^*)k)^{1/n}+2.5k\left(1-\frac{1}{n}\right)-1}{z^*k\left(1-\frac{1}{n}\right)} & \text{for } n \neq 1 \\ \frac{z^*+\frac{1}{k}\ln(1+k(30-z^*))}{30} & \text{for } n = 1 \end{cases} \tag{18}$$

6. Compute V_s for a given depth z

$$V_S(z) = \begin{cases} V_{S0} & \text{for } z \leq z^* \\ V_{S0}(1+k(z-z^*))^{1/n} & \text{for } z > z^* \end{cases} \tag{19}$$

NOTE: $z^* = 2.5$ m.

7. Truncation criteria: $z_{\max} = \min\{z_1, z_2\}$ where
   (a) z_1: Depth at which $V_S(z) \geq 1000$ m/s
   (b) z_2: Depth at which $V_{\text{USGS}}(z) \geq 1000$ m/s

   NOTE: If $z_{\max} = z_1$, for $z_{\max} \leq z \leq z_2$, $V_S(z) = 1000$ m/s.