MA4011/7091 Computer Class 5 – 01.03.2018

Remember to save all of your programs for future use!

We shall be concerned with the implementation of the upwind and Lax-Wendroff methods, described in Sections 6.2 and 6.3 of the notes for the numerical approximation of the solution of the advection initial/boundary value problem

$$\begin{aligned} u_t - 2u_x &= 0, \quad \text{for} \quad 0 \le t \le 1,\ 0 \le x \le 1 \\ u(0,x) &= u_0(x) := \begin{cases} 10^5(0.8-x)^2(0.9-x)^2, & \text{if } \ 0.8 < x < 0.9; \\ 0, & \text{otherwise.} \end{cases} \\ u(t,1) &= 0, \quad \text{for} \quad 0 \le t \le 1. \end{aligned}$$

The exact solution can be found by the method of characteristics to be $u(t,x) = u_0(x+2t)$.

Task 1.
Write a MATLAB program implementing the upwind method for the above problem. Use $N_x = 100$ and $N_t = 250$ (i.e., $\nu = 0.4$) and plot the approximate solutions for $t = 0.1,\ 0.2,\ 0.3$, respectively. Now, use $N_x = 100$ and $N_t = 200$ (i.e., $\nu = 0.5$) and plot the approximate solutions for $t = 0.1,\ 0.2,\ 0.3$, respectively.

Task 2.
Write a MATLAB program implementing the Lax-Wendroff method for the above problem. Use $N_x = 100$ and $N_t = 250$ (i.e., $\nu = 0.4$) and plot the approximate solutions for $t = 0.1,\ 0.2,\ 0.3$, respectively.

Task 3.
Calculate the maximum error of the approximation for the upwind and the Lax-Wendroff methods with $N_x = 100$ and $N_t = 250$ at times $t = 0.1,\ 0.2,\ 0.3$, respectively. Compare the two results appropriately. What do you observe? Explain.