MA4011/7091 Computer Class 4 – 15.02.2018

Remember to save all of your programs for future use!

Computer Class 4

We shall be concerned with the implementation of the implicit Euler and Crank-Nicolson methods, described in Section 4.2 of the notes for the the numerical approximation of the solution of the initial/boundary value problem

$$\begin{aligned} u_t(t,x) &= u_{xx}(t,x) + f(t,x) \quad \text{for all } t \in [0,1] \text{ and } x \in [-1,1], \\ u(0,x) &= \sin(\pi x), \qquad \text{for all } x \in [-1,1], \\ u(t,-1) = u(t,1) &= 0, \qquad \text{for all } t \in [0,1]. \end{aligned}$$

In the following, we shall use the notation from the lecture notes. Also, let

$$f(t,x) = (\cos t + \pi^2 \sin t)\sin(\pi x),$$

in which case the exact solution to the above problem is given by

$$u(t,x) = (\mathrm{e}^{-\pi^2 t} + \sin t)\sin(\pi x).$$

Task 1.
Write a MATLAB program implementing the implicit Euler method for the above problem. Use $N_x = 39$ and $N_t = 400$ (i.e., $\mu = 1$) and plot the approximate solutions at time steps $n = 100, 200, 400$.

Task 2.
Write a MATLAB program implementing the Crank-Nicolson method for the above problem. Read carefully in the notes how the Crank-Nicolson method was derived before implementing your method. In particular, you need to decide at which points your program should evaluate the function $f(t,x)$ in view of maintaining the order of convergence of the method established in the notes for the simpler problem $u_t = u_{xx}$. (Your answers should include the program.) Use $N_x = 39$ and $N_t = 20$ (i.e., $\mu = 1/h$) and plot the approximate solutions at time steps $n = 5, 10, 20$.

Task 3.
Calculate the maximum error of the approximation for the implicit Euler method for $n = 100, 200, 400$ and for the Crank-Nicolson method for $n = 5, 10, 20$; that is: calculate

$$E_{IE}^n := \max_{1 \le i \le N_x} |u_i^n - u(t_n, x_i)|$$

when u_i^n is the implicit Euler approximation and the grid point (t_n, x_i), for $n = 100, 200, 400$, and calculate calculate

$$E_{CN}^n := \max_{1 \le i \le N_x} |u_i^n - u(t_n, x_i)|$$

when u_i^n is the Crank-Nicolson approximation and the grid point (t_n, x_i), for $n = 5, 10, 20$. Compare the two results appropriately. What do you observe? Explain.