MA4011/7091 – Computer Class 2 – 01.02.2018

Remember to save all of your programs for future use!

Here we shall test the divided difference formulas for the approximation of the second derivative of a function seen in Section 3.1.2 of the Lecture notes. In particular, let us look back to the second central divided difference formula (3.14), named δ_h^2 and the second right divided difference formula (3.17), named $\delta_{h,right}^2$. These can be implemented as a scalar product between the vector of coefficients of the divided difference formula and the vector collecting the relevant function values, and then dividing through by h^2. So, for instance, the second central divided difference of a function f at x is given by

$$\delta_h^2 f(x) = [1 \quad -2 \quad 1] \cdot [f(x+h) \quad f(x) \quad f(x-h)]/h^2.$$

Task 1

Create an M-file in MATLAB implementing the second central and second right divided difference formula as described above, for the function $f(x) = \cos(\pi x)$ at a given point x with spacing h. Now set $x = 0.2$ and create a loop to calculate the divided difference formulas for $h = 1/n$ with $n = 4, 8, 16, 32, 64, 128, 256, 512$, and 1024 and record the error of approximation

$$E_h^c = |f''(x) - \delta_h^2 f(x)|,$$

and

$$E_h^r = |f''(x) - \delta_{h,right}^2 f(x)|.$$

Plot the errors E_h^c and E_h^r against h, using the `loglog` plotting command. What do you observe? Explain.

For the remainder of the class, we shall be concerned with implementing a difference method for the numerical approximation of the solution of the following boundary value problem:

$$\text{Find } u : [0,1] \to \mathbb{R} \text{ function, such that } -u''(x) = -\pi^2 \cos(\pi x) \text{ and } u(0)' = 0,\ u(1) = 0. \tag{1}$$

(The difference method for a similar (but not identical) problem is described in the notes, Section 3.2.

Task 2.

Write a MATLAB program implementing the difference method for the above problem using an equispaced subdivision of the solution domain $[0,1]$ made of N subintervals.

Task 3.

Calculate the error of approximation

$$E_N := \max_{1 \le i \le N+1} |u(x_i) - u_i|,$$

when $N = 4, 8, 16, 32, 64, 128, 256, 512$, and 1024 points; the notation above is explained in the notes. (To do so, you need to calculate the exact solution of the problem (1).) Plot the error E_N against N, using the `loglog` plotting command. What do you observe? Explain.